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CBSE Examination 2026

Class 10th

Mathematics - Standard

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion - Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study based questions carrying 4 marks each; Internal choice is provided in 2 marks questions in each case study.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.
10. Use of calculators is not allowed.

SECTION-A

This section has 20 Multiple Choice Questions (MCQs) carrying 1 mark each. **$20 \times 1 = 20$**

1. The LCM of 960 and 240 is ;

(A) 960 (B) 240

(C) 60 (D) 15

Sol :

We know that,

$$\text{LCM}\ (a, b) = \frac{a \times b}{\text{HCF}\,(a, b)}$$

Since, $960 = 240 \times 4$

$$\text{HCF}\ (960, 240) = 240$$

Therefore,

$$\text{LCM}\ (960, 240) = 960$$

Thus (A) is correct option.

2. The natural number 1 is :

(A) a prime number.

(B) a composite number.

(C) prime as well as composite.

(D) neither prime nor composite.

Sol :

A prime number has exactly two distinct positive factors: 1 and itself. But 1 has only one factor, that is 1.

Therefore, 1 is neither a prime number nor a composite number.

Thus (D) is correct option.

3. For any natural number n, 5^n ends with the digit :

(A) 0 (B) 5

(C) 3 (D) 2

Sol :

Let us observe the pattern:

$$5^1 = 5$$

$$5^2 = 25$$

$$5^3 = 125$$

$$5^4 = 625$$

We see that every power of 5 ends with the digit 5.

Therefore, for any natural number n, 5^n ends with 5.

Thus (B) is correct option.

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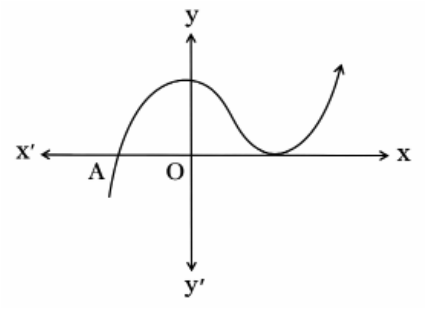
4. The graph of $y = f(x)$ is given. The number of distinct zeroes of $y = f(x)$ is :



(A) 0 (B) 1

(C) 2 (D) 3

Sol :

The zeroes of a function are the points where its graph intersects or touches the x-axis.

From the given graph:

1. The curve cuts the x-axis once on the left side of the origin.
2. It touches the x-axis once on the right side (at the minimum point).

Thus, the graph meets the x-axis at two distinct points.

Therefore, the number of distinct zeroes is 2.

Thus (C) is correct option.

5. If α and β are two zeroes of a polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then value of p is :

(A) $-\frac{2}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

Sol :

For a quadratic polynomial $ax^2 + bx + c$:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Here, $a = p, b = -2, c = 3p$

So, $$\alpha + \beta = -\frac{-2}{p} = \frac{2}{p}$$

$$\alpha\beta = \frac{3p}{p} = 3$$

Given that, $\alpha + \beta = \alpha\beta$

$$\frac{2}{p} = 3$$

$$p = \frac{2}{3}$$

Thus (B) is correct option.

6. If the pair of linear equations : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is consistent and dependent, then

(A) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (B) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (D) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Sol :

For two linear equations to be consistent and dependent, they must represent the same line.

Therefore, the condition is:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Thus (D) is correct option.

7. Which of the following sequence is not an A.P. ?

(A) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$, ...

(B) –1.2, –3.2, –5.2, –7.2, ...

(C) $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, ...

(D) $1^2, 3^2, 5^2, 7^2$,

Sol :

In an Arithmetic Progression (A.P.), the difference between consecutive terms is constant.

(A) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$, ...

Common difference:

$$\frac{5}{2} - 2 = \frac{1}{2}$$

$$3 - \frac{5}{2} = \frac{1}{2}$$

So, it is an A.P.

(B) –1.2, –3.2, –5.2, –7.2, ...

Common difference:

$$-3.2 - (-1.2) = -2$$

Constant difference, so it is an A.P.

(C) $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, ...

$$\sqrt{8} = 2\sqrt{2}$$

$$\sqrt{18} = 3\sqrt{2}$$

Sequence becomes:

$$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, \ldots$$

Common difference $= \sqrt{2}$

So, it is an A.P.

(D) $1^2, 3^2, 5^2, 7^2$,

That is : 1, 9, 25, 49,…

Differences:

$$9 - 1 = 8$$

$$25 - 9 = 16$$

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Difference is not constant.
Therefore, it is not an A.P.
Thus (D) is correct option.

8. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$, then $AB : AC$ is equal to:
(A) $PQ : PR$
(B) $PQ : QR$
(C) $QR : QP$
(D) $PR : QR$

Sol :

Given,

$$\angle A = \angle Q$$
$$\angle B = \angle R$$

Therefore, triangles ABC and PQR are similar (by AA similarity).
Corresponding vertices:

$$A \leftrightarrow Q,\ B \leftrightarrow R,\ C \leftrightarrow P$$

Hence, corresponding sides are:

$$AB \leftrightarrow QR,\ AC \leftrightarrow QP$$

Therefore, $AB : AC = QR : QP$
Thus (C) is correct option.

9. The distance of the point $A(4a, 3a)$ from x-axis is :
(A) $3a$ (B) $-3a$
(0) $4a$ (D) $-4a$

Sol :

The distance of a point (x, y) from the x-axis is equal to the absolute value of its y-coordinate.
Here, the given point is $A(4a, 3a)$.
Therefore, distance from x-axis $= |3a|$
Hence, the distance is $3a$.
Thus (A) is correct option.

10. If $\cos A = \frac{4}{5}$, then the value of $\tan A$ is :
(A) $\frac{3}{5}$ (B) $\frac{3}{4}$
(C) $\frac{4}{3}$ (D) $\frac{5}{3}$

Sol :

We have $\cos A = \frac{4}{5}$

We know that, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$

$$\text{Perpendicular} = \sqrt{5^2 - 4^2}$$
$$= \sqrt{25 - 16} = 3$$

Now, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

Thus (B) is correct option.

11. If $2 \sin A = 1$, then the value of $\tan A + \cot A$ is :
(A) $\sqrt{3}$ (B) $\frac{4}{\sqrt{3}}$
(C) $\frac{\sqrt{3}}{2}$ (D) 1

Sol :

Given,

$$2 \sin A = 1$$
$$\sin A = \frac{1}{2}$$

Since, $\sin A = \frac{1}{2}$
$$A = 30^\circ$$

Now, $\tan 30^\circ = \frac{1}{\sqrt{3}}$
$$\cot 30^\circ = \sqrt{3}$$

Therefore,

$$\tan A + \cot A = \frac{1}{\sqrt{3}} + \sqrt{3}$$
$$= \frac{1+3}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

Thus (B) is correct option.

12. From a point on the ground, which is 60 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is found to be 45°. The height (in metres) of the tower is :
(A) $10\sqrt{3}$ (B) $30\sqrt{3}$
(C) 60 (D) 30

Sol :

Let the height of the tower be h metres.
Using trigonometric ratio:

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

Here, $\tan 45^\circ = \frac{h}{60}$
Since, $\tan 45^\circ = 1$

$$1 = \frac{h}{60}$$
$$h = 60$$

Thus (C) is correct option.

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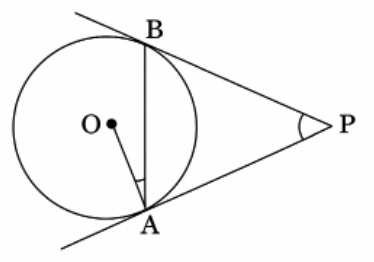
13. In, the, given figure, PA and PB are tangents to a circle centred at O. If $\Delta OAB = 15^\circ$, then ΔAPB equals :



(A) 80° (B) 15°

(C) 45° (D) 10°

Sol :

Given,

PA and PB are tangents to the circle at A and B respectively, and O is the centre of the circle. Join OA and OB.

Since the radius is perpendicular to the tangent at the point of contact

$$OA \perp PA \text{ and } OB \perp PB$$

Therefore, $\angle OAP = 90^\circ$

$\angle OBP = 90^\circ$

In quadrilateral $OAPB$, the sum of all interior angles is 360°.

$$\angle OAP + \angle OBP + \angle AOB + \angle APB = 360^\circ$$

$$90^\circ + 90^\circ + \angle AOB + \angle APB = 360^\circ$$

$$180^\circ + \angle AOB + \angle APB = 360^\circ$$

$$\angle APB = 180^\circ - \angle AOB$$

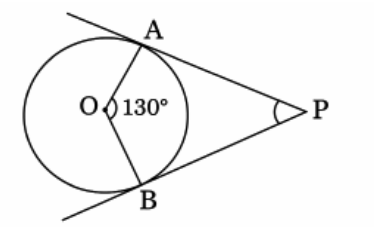
If $\angle AOB = 135^\circ$

$\angle APB = 180^\circ - 135^\circ$

$= 45^\circ$

Thus (C) is correct option.

14. In the given figure, PA and PB are tangents to a circle centred at O. If $\angle AOB = 130^\circ$, then $\angle APB$ is equal to :



(A) 130° (B) 50°

(C) 120° (D) 90°

Sol :

Since PA and PB are tangents, we know that:
Radius is perpendicular to tangent.

$\angle OAP = 90^\circ$

$\angle OBP = 90^\circ$

We use the property:
Angle between two tangents = 180° – Central angle

Given, $\angle AOB = 130^\circ$

Therefore, $\angle APB = 180^\circ - 130^\circ = 50^\circ$

Thus (B) is correct option.

15. Area of a segment of a circle of radius 'r' and central angle 60° is :

(A) $\frac{\pi r^2}{2} - \frac{1}{2}r^2$ (B) $\frac{2\pi r}{4} - \frac{\sqrt{3}}{4}r^2$

(C) $\frac{\pi r^2}{6} - \frac{\sqrt{3}}{4}r^2$ (D) $\frac{2\pi r}{4} - r^2 \sin 60^\circ$

Sol :

Area of segment = Area of sector – Area of triangle

$$\text{Area of sector} = \frac{60^\circ}{360^\circ}\pi r^2 = \frac{1}{6}\pi r^2$$

$$\text{Area of triangle} = \frac{1}{2}r^2 \sin 60^\circ$$

$$= \frac{1}{2}r^2 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}r^2$$

Therefore,

$$\text{Area of segment} = \frac{\pi r^2}{6} - \frac{\sqrt{3}}{4}r^2$$

Thus (C) is correct option.

16. A hemispherical bowl is made of steel of thickness 1 cm. The outer radius of the bowl is 6 cm. The volume of steel used (in cm^3) is :

(A) 182π (B) $\frac{182}{3}\pi$

(C) $\frac{682}{3}\pi$ (D) $\frac{364}{3}\pi$

Sol :

Outer radius $R = 6$ cm

Thickness $= 1$ cm

Inner radius: $r = 6 - 1 = 5$ cm

Volume of steel used = Volume of outer hemisphere – Volume of inner hemisphere

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Volume of hemisphere $= \frac{2}{3}\pi r^3$

Therefore,

$$\text{Volume} = \frac{2}{3}\pi(6^3 - 5^3)$$

$$= \frac{2}{3}\pi(216 - 125)$$

$$= \frac{2}{3}\pi(91) = \frac{182}{3}\pi$$

Thus (B) is correct option.

17. The mean and median of a frequency distribution are 43 and 43.4 respectively. The mode of the distribution is :

(A) 43.4 (B) 42.4

(C) 44.2 (D) 49.3

Sol :

We use the empirical relation between mean, median and mode :

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Given, Mean = 43,

Median = 43.4

$$\text{Mode} = 3(43.4) - 2(43)$$

$$= 130.2 - 86 = 44.2$$

Thus (C) is correct option.

18. The probability for a randomly selected number out of 1, 2, 3, 4, ..., 25 to be a composite number is :

(A) $\frac{15}{25}$ (B) $\frac{10}{25}$

(C) $\frac{11}{25}$ (D) $\frac{9}{25}$

Sol :

Total numbers from 1 to 25 = 25

Prime numbers between 1 and 25:

2, 3, 5, 7, 11, 13, 17, 19, 23

Total primes = 9

We know that, 1 is neither prime nor composite.

Therefore, composite numbers:

$$= 25 - (9 + 1)$$

$$= 25 - 10 = 15$$

$$\text{Probability} = \frac{\text{Number of composite numbers}}{\text{Total numbers}}$$

$$= \frac{15}{25}$$

Thus (A) is correct option.

19. **Assertion (A) :** The surface area of the cuboid formed by joining two cubes of sides 4 cm each, end-to-end, is 160 cm^2.

Reason (R) : The surface area of a cuboid of dimensions $l \times b \times h$ is $(lb + bh + hl)$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Sol :

Two cubes of side 4 cm are joined end-to-end.

So, the new cuboid has dimensions:

$$l = 8 \text{ cm}, b = 4 \text{ cm}, h = 4 \text{ cm}$$

$$\text{Surface area} = 2(lb + bh + hl)$$

$$= 2(8 \times 4 + 4 \times 4 + 4 \times 8)$$

$$= 2(32 + 16 + 32)$$

$$= 2(80) = 160 \text{ cm}^2$$

Thus (A) is correct option.

20. **Assertion (A) :** The mean of first 'n' natural numbers is $\frac{n-1}{2}$.

Reason (R) : The sum of first 'n' natural numbers is $\frac{n(n+1)}{2}$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Sol :

Sum of first n natural numbers:

$$= \frac{n(n+1)}{2}$$

$$\text{Mean} = \frac{\text{sum}}{\text{n}} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

Thus, the mean is $\frac{n+1}{2}$, not $\frac{n-1}{2}$.

Thus (D) is correct option.

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SECTION-B

This section has 5 Very Short Answer (VSA) type questions carrying 2 marks each. $5 \times 2 = 10$

21. If α, β are the zeroes of the quadratic polynomial $px^2 + qx + r$, then find the value of $\alpha^3\beta + \beta^3\alpha$.

Sol :

For quadratic polynomial $ax^2 + bx + c$:

$$\alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a}$$

Here, $$\alpha + \beta = -\frac{q}{p},\ \alpha\beta = \frac{r}{p}$$

Now, $\alpha^3\beta + \beta^3\alpha$

Take $\alpha\beta$ common,

$$= \alpha\beta(\alpha^2 + \beta^2)$$

We know, $\alpha^2 + \beta^2 = (\alpha + \beta) - 2\alpha\beta$

Substituting values,

$$= \alpha\beta\left[\left(-\frac{q}{p}\right)^2 - 2\left(\frac{r}{p}\right)\right]$$

$$= \frac{r}{p}\left[\frac{q^2}{p^2} - \frac{2r}{p}\right]$$

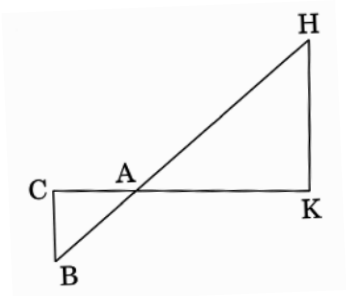
Taking LCM inside the bracket,

$$= \frac{r}{p}\left[\frac{q^2 - 2pr}{p^2}\right]$$

$$= \frac{r(q^2 - 2pr)}{p^3}$$

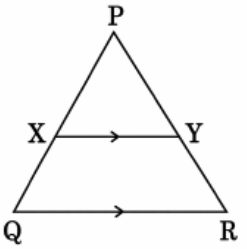
$$\alpha^3\beta + \beta^3\alpha = \frac{r(q^2 - 2pr)}{p^3}$$

22. (a) In the given figure, $\Delta AHK \sim \Delta ABC$. If $AK = 10$ cm, $BC = 3.5$ cm and $HK = 7$ cm, find the length of AC.



or

(b) In the given figure, $XY \| QR$, $\frac{PQ}{XQ} = \frac{7}{3}$ and $PR = 6.3$ cm. Find the length of YR.



Sol :

(a) Given,

$$\Delta AHK \sim \Delta ABC$$

Therefore, corresponding sides are proportional:

$$\frac{HK}{BC} = \frac{AK}{AC}$$

Substituting the given values,

$$\frac{7}{3.5} = \frac{10}{AC}$$

$$2 = \frac{10}{AC}$$

$$AC = \frac{10}{2} = 5 \text{ cm}$$

or

(b) Since $XY \| QR$, triangles ΔPXY and ΔPQR are similar.

Therefore,

$$\frac{PX}{PQ} = \frac{PY}{PR}$$

Given, $$\frac{PQ}{XQ} = \frac{7}{3}$$

Let $PQ = 7k$ and $XQ = 3k$

Then, $$PX = PQ - XQ$$

$$= 7k - 3k = 4k$$

Thus, $$\frac{PX}{PQ} = \frac{4k}{7k} = \frac{4}{7}$$

So, $$\frac{PY}{PR} = \frac{4}{7}$$

$$PY = \frac{4}{7} \times 6.3$$

$$= 3.6 \text{ cm}$$

Now, $$YR = PR - PY$$

$$= 6.3 - 3.6$$

$$= 2.7 \text{ cm}$$

23. If the points $A(4, 5)$, $B(m, 6)$, $C(4, 3)$ and D$(1, n)$ taken in this order are the vertices of a parallelogram $ABCD$, then find the values of m and n.

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Sol :

In a parallelogram, diagonals bisect each other.

$$\text{Midpoint of } AC = \text{Midpoint of } BD$$

$$\text{Midpoint of } AC = \left(\frac{4+4}{2}, \frac{5+3}{2}\right) = (4, 4)$$

$$\text{Midpoint of } BD = \left(\frac{m+1}{2}, \frac{6+n}{2}\right)$$

Equating coordinates:

$$\frac{m+1}{2} = 4$$

$$m+1 = 8$$

$$m = 7$$

$$\frac{6+n}{2} = 4$$

$$6+n = 8$$

$$n = 2$$

24. (a) If $\tan\theta + \frac{1}{\tan\theta} = 2$, find the value of $\tan^2\theta + \frac{1}{\tan^2\theta}$

or

(b)Prove that : $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

Sol :

(a) Let $x = \tan\theta$

Given,

$$x + \frac{1}{x} = 2$$

Squaring both sides,

$$\left(x + \frac{1}{x}\right)^2 = 4x^2 + \frac{1}{x^2} + 2$$

$$= 4x^2 + \frac{1}{x^2} = 2$$

or

(b) Proof :

Start with RHS :

$$\sec\theta - \tan\theta = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

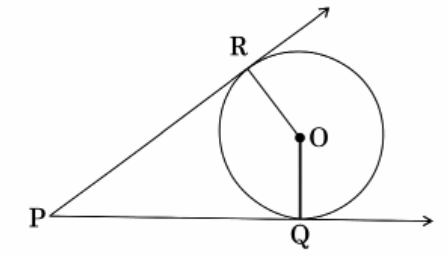
Multiply numerator and denominator by $\sqrt{\frac{1+\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta(1+\sin\theta)}}$$

Since $\cos^2\theta = 1 - \sin^2\theta$

$$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$ Hence proved

25. In the given figure, O is the centre of the circle. PQ and PR are tangents. Show that the quadrilateral $PQOR$ is cyclic.



Sol :

Since PQ and PR are tangents:

$$\angle OQP = 90^\circ$$

$$\angle ORP = 90^\circ$$

Thus, $\angle OQP + \angle ORP = 180^\circ$

Opposite angles of quadrilateral $PQOR$ are supplementary.

Therefore, $PQOR$ is a cyclic quadrilateral.

Hence proved

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SECTION-C

This section has 6 Short Answer (SA) type questions carrying 3 marks each. $6 \times 3 = 18$

26. Prove that $\sqrt{5}$ is an irrational number.

Sol :

Assume that $\sqrt{5}$ is rational.

Then it can be written as:

$$\sqrt{5} = \frac{a}{b}$$

where a and b are integers having no common factor and $b \neq 0$.

Squaring both sides:

$$5 = \frac{a^2}{b^2}$$

$$a^2 = 5b^2$$

Thus, a^2 is divisible by $5 \Rightarrow a$ is divisible by 5.

Let $a = 5k$

Substituting:

$$(5k)^2 = 5b^2$$

$$25k^2 = 5b^2$$

$$5k^2 = b^2$$

Thus, b^2 is divisible by 5 $\Rightarrow$ b is divisible by 5.

Therefore, both a and b are divisible by 5, which contradicts the assumption that they have no common factor.

Hence, our assumption is wrong.

$\sqrt{5}$ is irrational.

27. Find the coordinates of the points of trisection of the line segment joining the points $A(-1, 4)$ and $B(-3, -2)$.

Sol :

Points of trisection divide the line internally in the ratios 1 : 2 and 2 : 1.

Using section formula:

First point (ratio 1 : 2),

$$\left(\frac{1(-3)+2(-1)}{3}, \frac{1(-2)+2(4)}{3}\right)$$

$$= \left(\frac{-3-2}{3}, \frac{-2+8}{3}\right) = \left(-\frac{5}{3}, 2\right)$$

Second point (ratio 2 : 1),

$$\left(\frac{2(-3)+1(-1)}{3}, \frac{2(-2)+1(4)}{3}\right)$$

$$= \left(\frac{-6-1}{3}, \frac{-4+4}{3}\right) = \left(-\frac{7}{3}, 0\right)$$

28. (a) Prove that :

$$\frac{\sec^3\theta}{\sec^2\theta - 1} + \frac{\text{cosec}^3\theta}{\text{cosec}^2\theta - 1} = \sec\theta.\,\text{cosec}\,\theta(\sec\theta + \text{cosec}\,\theta)$$

or

(b) If $\frac{\sec\alpha}{\text{cosec}\,\beta} = p$ and $\frac{\tan\alpha}{\text{cosec}\,\beta} = q$, then prove that $(p^2 - q^2)\sec^2\alpha = p^2$.

Sol :

(a) $$\text{LHS} = \frac{\sec^3\theta}{\sec^2\theta - 1} + \frac{\text{cosec}^3\theta}{\text{cosec}^2\theta - 1}$$

Using identities,

$$\sec^2\theta - 1 = \tan^2\theta$$

$$\text{cosec}^2\theta - 1 = \cot^2\theta$$

Substituting, $$= \frac{\sec^3\theta}{\tan^2\theta} + \frac{\text{cosec}^3\theta}{\cot^2\theta}$$

Now, $$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$$

Also, $$\sec\theta = \frac{1}{\cos\theta}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta}$$

Therefore,

$$\frac{\sec^3\theta}{\tan^2\theta} = \frac{\frac{1}{\cos^3\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} = \frac{1}{\cos\theta\sin^2\theta}$$

$$= \text{cosec}\,\theta\sec^2\theta$$

Similarly,

$$\frac{\text{cosec}^3\theta}{\cot^2\theta} = \frac{\frac{1}{\sin^3\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{1}{\sin\theta\cos^2\theta}$$

$$= \text{cosec}\,\theta\sec^2\theta$$

Thus, $$\text{LHS} = \sec\theta\,\text{cosec}^2\theta + \text{cosec}\,\theta\sec^2\theta$$

Taking common factor $\sec\theta\ \text{cosec}\,\theta$,

$$= \sec\theta\,\text{cosec}\,\theta(\text{cosec}\,\theta + \sec\theta)$$

$$= \sec\theta\,\text{cosec}\,\theta(\sec\theta + \text{cosec}\,\theta)$$

$$\text{LHS} = \text{RHS}$$

or

(b) Given $$\frac{\sec\alpha}{\text{cosec}\,\beta} = p$$

$$\frac{\tan\alpha}{\text{cosec}\,\beta} = q$$

Squaring both equations,

$$\frac{\sec^2\alpha}{\text{cosec}^2\beta} = p^2 \quad \ldots(1)$$

$$\frac{\tan^2\alpha}{\text{cosec}^2\beta} = q^2 \quad \ldots(2)$$

Subtracting (2) from (1),

$$\frac{\sec^2\alpha - \tan^2\alpha}{\text{cosec}^2\beta} = p^2 - q^2$$

Using identity,

$$\sec^2\alpha - \tan^2\alpha = 1$$

So, $$\frac{1}{\text{cosec}^2\beta} = p^2 - q^2$$

$$p^2 - q^2 = \sin^2\beta$$

From (1), $$p^2 = \frac{\sec^2\alpha}{\text{cosec}^2\beta}$$

$$= \sec^2\alpha\sin^2\beta$$

Substitute,

$$\sin^2\beta = p^2 - q^2$$

$$p^2 = \sec^2\alpha(p^2 - q^2)$$

$$(p^2 - q^2)\sec^2\alpha = p^2$$ Hence Proved

29. (a) Prove that the lengths of tangents drawn from an external point to a circle are equal.

or

(b) Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\ \angle OPQ$.

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Sol :

(a) Let P be an external point.
From P, two tangents PA and PB are drawn to a circle with centre O.
Join OA, OB and OP.

We know, $OA = OB$ (radii)

$$\angle OAP = \angle OBP = 90^\circ$$

$OP = OP$ (common)

Thus, triangles ΔOAP and ΔOBP are congruent (RHS).

Therefore, $PA = PB$ Hence proved.

or

(b) Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Since a radius is perpendicular to the tangent at the point of contact,

$$OP \perp TP$$

and $OQ \perp TQ$

$$\angle OPT = 90^\circ$$

$$\angle OQT = 90^\circ$$

In quadrilateral $OPTQ$,

$$\angle OPT + \angle OQT + \angle PTQ + \angle POQ = 360^\circ$$

$$90^\circ + 90^\circ + \angle PTQ + POQ = 360^\circ$$

$$180^\circ + \angle PTQ + \angle POQ = 360^\circ$$

$$\angle PTQ = 180^\circ - \angle POQ \ldots(1)$$

In ΔOPQ, $OP = OQ$

Therefore, $\angle OPQ = \angle OQP$

$$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$$

$$2\angle OPQ + \angle POQ = 180^\circ$$

$$2\angle OPQ = 180^\circ - \angle POQ \ldots(2)$$

From (1) and (2),

$$\angle PTQ = 2\angle OPQ$$

Hence proved

30. Find the area of the sector of a circle of radius 42 cm and of central angle 30°. Also, find the area of the corresponding major sector. [Use $\pi = \frac{22}{7}$]

Sol :

$$\text{Area of sector} = \frac{\theta}{360^\circ}\pi r^2$$

$$= \frac{30}{360} \times \frac{22}{7} \times 42^2$$

$$= \frac{1}{12} \times \frac{22}{7} \times 1764$$

$$= 462 \text{ cm}^2$$

$$\text{Area of circle} = \frac{22}{7} \times 1764 = 5544 \text{ cm}^2$$

Area of major sector $= 5544 - 462 = 5082 \text{ cm}^2$

31. Two dice are thrown at the same time. Determine the probability that the
(i) sum of the numbers on the two dice is 5 and
(ii) difference of the numbers on the two dice is 3.

Sol :

(i) Sum of the numbers is 5
Possible outcomes giving sum 5:
(1, 4), (2, 3), (3, 2), (4, 1)
Number of favourable outcomes = 4

$$\text{Probability} = \frac{4}{36} = \frac{1}{9}$$

(ii) Difference of the numbers is 3
Possible outcomes giving difference 3:
(1, 4), (4, 1), (2, 5), (5, 2), (3, 6), (6, 3)
Number of favourable outcomes = 6

$$\text{Probability} = \frac{6}{36} = \frac{1}{6}$$

SECTION-D

This section has 4 Long Answer (LA) type questions carrying 5 marks each. **$4 \times 5 = 20$**

32. Aarush bought 2 pencils and 3 chocolates for ₹11 and Tanish bought 1 pencil and 2 chocolates for ₹7 from the same shop. Represent this situation in the form of a pair of linear equations. Find the price of 1 pencil and 1 chocolate, graphically.

Sol :

Let the price of one pencil be ₹x and the price of one chocolate be ₹y.
According to the given information,
Aarush bought 2 pencils and 3 chocolates for ₹11,

$$2x + 3y = 11 \quad \ldots(1)$$

Tanish bought 1 pencil and 2 chocolates for ₹7,

$$x + 2y = 7 \quad \ldots(2)$$

Thus, the required pair of linear equations is:

$$2x + 3y = 11$$

$$x + 2y = 7$$

To solve graphically, we find two points for each equation.

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For equation (1): $2x+3y=11$
If $x=1$,

$$2(1)+3y=11$$
$$3y=9$$
$$y=3$$

Point obtained: (1, 3)
If $x=4$,

$$8+3y=11$$
$$3y=3$$
$$y=1$$

Point obtained: (4, 1)
For equation (2): $x+2y=7$
If $x=1$,

$$1+2y=7$$
$$2y=6$$
$$y=3$$

Point obtained: (1, 3)
If $x=3$,

$$3+2y=7$$
$$2y=4$$
$$y=2$$

Point obtained: (3, 2)
Plotting these points on the graph paper, the two lines intersect at the point: (1, 3)
Hence,
Price of one pencil = ₹1
Price of one chocolate = ₹3

33. (a) A person on a tour has 4,200 for expenses. If he extends his tour for 3 days, he has to cut down his daily expenses by ₹70. Find the original duration of the tour.

or

(b) The area of a right-angled triangle is 600 cm^2. If the base of the triangle exceeds the altitude by 10 cm, find all the three dimensions of the triangle.

Sol :

(a) Let the original duration be x days.
Total money = ₹4200
Let original daily expense be : $\frac{4200}{x}$

If duration is extended by 3 days:
New duration = $x+3$
New daily expense: $\frac{4200}{x+3}$
Given reduction in daily expense = ₹70

$$\frac{4200}{x}-\frac{4200}{x+3}=70$$

Taking LCM,

$$\frac{4200(x+3)-4200x}{x(x+3)}=70$$
$$\frac{4200x+12600-4200x}{x(x+3)}=70$$
$$\frac{12600}{x(x+3)}=70$$
$$12600=70x(x+3)$$
$$12600=70x^2+210x$$

Divide by 70, $180=x^2+3x$

$$x^2+3x-180=0$$

Factorising,

$$x^2+15x-12x-180=0$$
$$x(x+15)-12(x+15)=0$$
$$(x+15)(x-12)=0$$
$$x=-15 \text{ or } x=12$$

Since duration cannot be negative,

$$x=12$$

or

(b) Area of right triangle = $\frac{1}{2}\times$ base $\times$ height
base $\times$ height = 1200
Let height = x
Then base = $x+10$

$$x(x+10)=1200$$
$$x^2+10x-1200=0$$
$$x^2+40x-30x-1200=0$$
$$x(x+40)-30(x+40)=0$$
$$(x-30)(x+40)=0$$
$$x=30$$

Height = 30 cm
Base = 40 cm

$$\text{Hypotenuse}=\sqrt{30^2+40^2}$$
$$=\sqrt{900+1600}$$
$$=\sqrt{2500}=50 \text{ cm}$$

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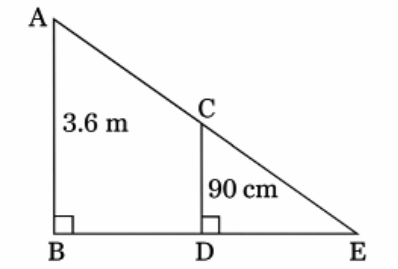
34. (a) Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

or

(b) As shown,in the given figure, a girl of height 90 cm is walking away from the base of a lamp post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.



Sol :

(a) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Let in ΔABC, $DE \| BC$.

Then, $\frac{AD}{DB} = \frac{AE}{EC}$

Thus proved using Basic Proportionality Theorem.

(b) Height of lamp post $= 3.6$ m

Height of girl $= 90$ cm $= 0.9$ m

Speed $= 1.2$ m/s

Distance walked in 4 seconds

$= 1.2 \times 4 = 4.8$ m

Let length of shadow $= x$

Using similar triangles,

$$\frac{3.6}{4.8+x} = \frac{0.9}{x}$$

$$3.6x = 0.9(4.8+x)$$

$$3.6x = 4.32 + 0.9x$$

$$2.7x = 4.32$$

$$x = 1.6 \text{ m}$$

35. An SBI health insurance agent found the following data for distribution of age of 100 policy holders. The health insurance policies are given to persons of age 15 years and onwards, but less than 60 years.

Age (in yrs)	Number of policy holders
15-20	2
20-25	4
25-30	18
30-35	21
35-40	33
40-45	11
45-50	3
50-55	6
55-60	2

Find the modal age and median age of the policy holders.

Sol :

(i) Median Age

First, find cumulative frequency,

Class	f	C.F.
15-20	2	2
20-25	4	6
25-30	18	24
30-35	21	45
35-40	33	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100

$$\frac{N}{2} = \frac{100}{2} = 50$$

The class whose cumulative frequency just exceeds 50 is 35–40.

So, median class $=$ 35–40

Here, $l = 35$, $h = 5$, $f = 33$, $cf = 45$

Median formula:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right)$$

$$h = 35 + \left(\frac{50-4}{33}\right) \times 5$$

$$= 35 + \frac{25}{33} = 35.76$$

(ii) Modal Age

Modal class $=$ Class with highest frequency

Highest frequency $= 33 \rightarrow$ class 35–40

Here

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$l = 35,\ h = 5$

$f_1 = 33,\ f_0 = 21,\ f_2 = 11$

Mode formula:

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 35 + \frac{33 - 21}{2(33) - 21 - 11} \times 5$$

$$= 35 + \frac{12}{66 - 32} \times 5$$

$$= 35 + \frac{12}{34} \times 5$$

$$= 35 + 1.76$$

$$\approx 36.76 \text{ years}$$

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SECTION-E

This section has 3 case study based questions carrying 4 marks each. $3 \times 4 = 12$

36. Your elder brother wants to buy a car and plans to take a loan from a bank for his car. He repays his total loan of 1,18,000 by paying every month, starting with the first instalment of ₹1,000 and he increases the instalment by ₹100 every month.
Based on the information, given above, answer the following questions :

(i) Find the amount paid by him in the 30th Instalment.

(ii) If the total number of instalments is 40, what is the amount paid in the last instalment

(iii) (a) What amount does he still have to pay after the 30th installment?

or

(b) Find the ratio of the tenth installment to the last installment.

Sol :

(i) 30th instalment

$$a_{30} = a + (n-1)\,d$$

$$= 1000 + 29 \times 100$$

$$= 1000 + 2900$$

$$= 3900$$

(ii) Last installment (40th)

$$a_{40} = 1000 + 39 \times 100$$

$$= 1000 + 3900$$

$$= 4900$$

(iii) (a) Amount paid after 30th installment

Sum of first 30 terms,

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$S_{30} = \frac{30}{2}[2(1000) + 29(100)]$$

$$= 15\ [2000 + 2900]$$

$$= 15 \times 4900$$

$$= 73{,}500$$

Total Loan = ₹1,18,000

Amount still to pay:

$$= 1{,}18{,}000 - 73{,}500$$

$$= 44{,}500$$

or

(b) Ratio of 10th installment to last installment

10th term,

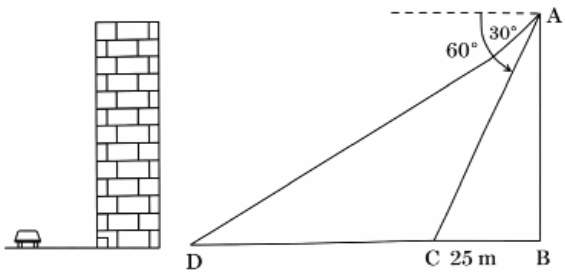
$$a_{10} = 1000 + 9(100)$$

$$= 1900$$

Last installment = 4900

Ratio, $1900 : 4900 = 19 : 49$

37. Tajas is standing at the top of a building and observes a car at an angle of depression of 30° as it approaches the base of the building at a uniform speed. 6 seconds later, the angle of depression increases to 60° and at that moment, the car is 25 m away from the building.



Based on till information given above, answer the following questions :

(i) What is the height of the building ?

(ii) What is the distance between the two positions of the car ?

(iii) (a) What would be the total time taken by the car to reach the foot of the building from the starting point?

or

(b) What is the distance of the observer from the car when it makes an angle of 60°?

Sol :

(i) Height of the building
When angle of depression = 60°
Horizontal distance = 25 m

$$\tan 60^\circ = \frac{h}{25}$$

$$\sqrt{3} = \frac{h}{25}$$

$$h = 25\sqrt{3} \text{ m}$$

(ii) Distance between the two positions of the car
When angle of depression = 30°,
Let distance from building = x

$$\tan 30^\circ = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{25\sqrt{3}}{x}$$

$$x = 75$$

So initial distance = 75 m
Second distance = 25 m
Distance between positions:

$$75 - 25 = 50 \text{ m}$$

(iii) (a) Total time taken to reach the foot of the building
Distance covered in 6 seconds = 50 m

$$\text{Speed} = \frac{50}{6}$$

Total distance from starting point to building = 75 m

$$\text{Total time} = \frac{75}{\frac{50}{6}}$$

$$= \frac{75 \times 6}{50} = 9 \text{ second}$$

or

(b) Distance of observer from the car when angle is 60°
This is the slant height.
Using Pythagoras:

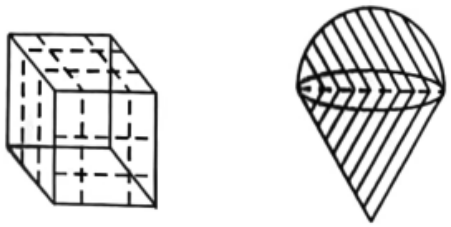
$$= \sqrt{(25)^2 + (25\sqrt{3})^2}$$

$$= \sqrt{625 + 1875}$$

$$= \sqrt{2500}$$

$$= 50 \text{ m}$$

38. On a Sunday your parents took you to a fair. You could see lot of toys displayed and you wanted them to buy a Rubik's cube and a strawberry ice-cream for you.



Based on the information given above, answer the following questions :

(i) Find the length of the diagonal of Rubik's cube if each edge measures 6 cm.

(ii) Find the volume of Rubik's cube if the length of the edge is 7 cm.

(iii) (a) What is the curved surface area of hemisphere (ice-cream) if the base radius is 7 cm?

or

(b) If,two cubes of edges 4 cm are joined end-to-end, then find the surface area of the resulting cuboid.

Sol :

(i) Find the length of the diagonal of Rubik's cube if each edge measures 6 cm.
For a cube of side a, diagonal:

$$d = a\sqrt{3}$$

Here, $a = 6$

$$d = 6\sqrt{3} \text{ cm}$$

(ii) Find the volume of Rubik's cube if length of edge is 7 cm.
Volume of cube :

$$V = a^3$$

$$7^3 = 343 \text{ cm}^3$$

(iii) (a) What is the curved surface area of hemisphere (ice-cream) if base radius is 7 cm?
Curved surface area of hemisphere:

$$= 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times 7^2$$

$$= 2 \times \frac{22}{7} \times 49$$

$$= 2 \times 22 \times 7$$

$$= 308 \text{ cm}^2$$

or

(b) If two cubes of edge 4 cm are joined end-to-end, find the surface area of the resulting cuboid.

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New dimensions, $8 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm}$

$$\begin{aligned}\text{Surface area} &= 2(lb + bh + hl)\\ &= 2(8 \times 4 + 4 \times 4 + 8 \times 4)\\ &= 2(32 + 16 + 32)\\ &= 2(80)\\ &= 160\end{aligned}$$

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CBSE Examination 2025

Class 10th

Mathematics - Standard

Time : 3 Hours **Max. Marks : 80**

General Instructions :

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion - Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = \frac{22}{7}$ wherever required if not stated.
11. Use of calculators is not allowed.

SECTION-A

This section consists of 20 questions of 1 mark each.

1. If $x = ab^3$ and $y = a^3b$, where a and b are prime numbers, then [HCF (x, y) – LCM (x, y)] is equal to

(a) $1 - a^3b^3$

(b) $ab(1 - ab)$

(c) $ab - a^4b^4$

(d) $ab(1 - ab)(1 + ab)$

Sol :

We have $x = ab^3$

$y = a^3b$

$\text{HCF}\ (x, y) = ab$

$\text{LCM}\ (x, y) = a^3b^3$

$\text{HCF}(x, y) - \text{LCM}(x, y) = ab - a^3b^3$

$= ab(1 - a^2b^2)$

$= ab(1 - ab)(1 + ab)$

Thus (d) is correct option.

2. $\left(1 + \sqrt{3}\right)^2 - \left(1 - \sqrt{3}\right)^2$ is

(a) a positive rational number

(b) a negative integer

(c) a positive irrational number

(d) a negative irrational number

Sol :

$\left(1 + \sqrt{3}\right)^2 - \left(1 - \sqrt{3}\right)^2 = (1 + 2\sqrt{3} + 3) - (1 - 2\sqrt{3} + 3)$

$= 4\sqrt{3}$

which is a positive irrational number

Thus (c) is correct option

3. The value of a for which $ax^2 + x + a = 0$ has equal and positive roots is

(a) 2

(b) –2

(c) $\frac{1}{2}$

(d) $-\frac{1}{2}$

Sol :

We have $D = B^2 - 4AC$

$= 1 - 4a \times a$

$= 1 - 4a^2$

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For equal and positive roots

$$\begin{aligned} D &= 0 \\ 1-4a^2 &= 0 \\ 1 &= 4a \\ a^2 &= \frac{1}{4} \\ a &= \pm\frac{1}{2} \end{aligned}$$

For $a=\frac{1}{2}$ we have

$$\begin{aligned} \frac{a^2}{2}+x+\frac{a}{2} &= 0 \\ x^2+2x+1 &= 0 \end{aligned}$$

Here x have negative roots.

For $a=-\frac{1}{2}$ we have

$$\begin{aligned} -\frac{x^2}{2}+x-\frac{1}{2} &= 0 \\ x^2-2x+1 &= 0 \\ (x-1)^2 &= 0 \\ x &= 1 \end{aligned}$$

Here x have positive roots.

Thus (d) is correct option

4. The distance of a point A from x-axis is 3 units. Which of the following cannot be coordinates of the point A?

(a) (1, 3)

(b) (–3, –3)

(c) (–3, 3)

(d) (3, 1)

Sol :

The distance of a point from the x-axis is the absolute value of its y-coordinate. The distance of point A from the x-axis is 3 units. The y-coordinate of point A must be either 3 or -3.

Thus (d) is correct option

5. The number of red balls in a bag is 10 more than the number of black balls. If the probability of drawing a red ball at random from this bag is $\frac{3}{5}$, then the total number of balls in the bag is

(a) 50

(b) 80

(c) 60

(d) 40

Sol :

Let b be the number of black ball, then $b+10$ will be red ball in bag and total $b+b+10=2b+10$ ball bag in bag.

Now as per question we have

$$\begin{aligned} P(\text{red}) &= \frac{b+10}{2b+10} \\ \frac{3}{5} &= \frac{b+10}{2b+10} \\ 3(2b+10) &= 5(b+10) \\ 6b+30 &= 5b+50 \\ b &= 20 \end{aligned}$$

Thus total $2\times 20+10=50$ ball are there in bag.

Thus (a) is correct option

6. The value of p for which the equations $px+3y=p-3,\ 12x+py=p$ has infinitely many solutions is

(a) 6 only

(b) 0 only

(c) ± 6 only

(d) Any real number except $\pm$ 6

Sol :

For infinitely many solutions, the following condition must be satisfied,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Substituting the values, we get,

$$\frac{p}{12}=\frac{3}{p}=\frac{p-3}{p}$$

Let's consider the first two ratios,

$$\begin{aligned} \frac{p}{12} &= \frac{3}{p} \\ p^2 &= 36 \Rightarrow p=\pm 6 \end{aligned}$$

Now let's consider the last two ratios,

$$\begin{aligned} \frac{3}{p} &= \frac{p-3}{p} \\ 3 &= p-3 \Rightarrow p=6 \end{aligned}$$

For infinitely many solutions, all three ratios must be equal. For all three ratios to be equal, the common value of p must be 6.

Thus (a) is correct option

7. ΔABC and ΔPQR are shown in the adjoining figures. The measure of $\angle C$ is

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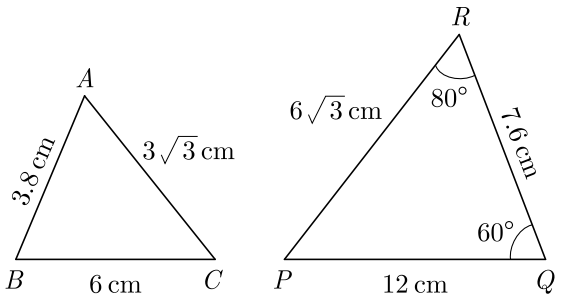


(a) 140°

(b) 80°

(c) 60°

(d) 40°

Sol :

Here $\frac{RQ}{AB} = \frac{RP}{AC} = \frac{PQ}{BC}$

Due to SSS symmetry we have

$$\triangle RQP \sim \triangle ABC$$

Thus
$$\angle C = \angle P$$
$$= 180^\circ - 80^\circ - 40^\circ$$
$$= 40^\circ$$

Thus (d) is correct option

8. $\tan 2A = 3\tan A$ is true, when the measure of $\angle A$ is

(a) 90°

(b) 60°

(c) 45°

(d) 30°

Sol :

Substituting $A = 90^\circ$ we will get

$$\tan 180^\circ = 3\tan 90^\circ$$
$$0 = \infty \qquad \text{Not possible}$$

Substituting $A = 60^\circ$ we will get

$$\tan 120^\circ = 3\tan 60^\circ$$
$$-\sqrt{3} = 3 \times \sqrt{3} \qquad \text{Not possible}$$

Substituting $A = 45^\circ$ we will get

$$\tan 90^\circ = 3\tan 45^\circ$$
$$\infty = 3 \qquad \text{Not possible}$$

Substituting $A = 30^\circ$ we will get

$$\tan 60^\circ = 3\tan 30^\circ$$
$$\sqrt{3} = 3 \times \frac{1}{\sqrt{3}} \qquad \text{Not possible}$$
$$\sqrt{3} = \sqrt{3}$$

Thus (d) is correct option

9. Which of the following statements is true?

(a) $\sin 20^\circ > \sin 70^\circ$

(b) $\sin 20^\circ > \cos 20^\circ$

(c) $\cos 20^\circ > \cos 70^\circ$

(d) $\tan 20^\circ > \tan 70^\circ$

Sol :

Since sine is an increasing function in the first quadrant and $20^\circ < 70^\circ$, we know that $\sin 20^\circ < \sin 70^\circ$. Thus (a) is false.

$$\cos 20^\circ = \sin(90^\circ - 20^\circ)$$
$$= \sin 70^\circ$$

Thus (b) is false due to the reason given for (a).

Since cosine is an decreasing function in the first quadrant and $20^\circ < 70^\circ$, we know that $\cos 20^\circ > \cos 70^\circ$. Thus (c) is true.

Since tangent is an decreasing function in the first quadrant and $20^\circ < 70^\circ$, we know that $\tan 20^\circ \leq \tan 70^\circ$. Thus (c) is false.

Thus (c) is correct option

10. A 30 m long rope is tightly stretched and tied from the top of pole to the ground. If the rope makes an angle of 60° with the ground, the height of the pole is

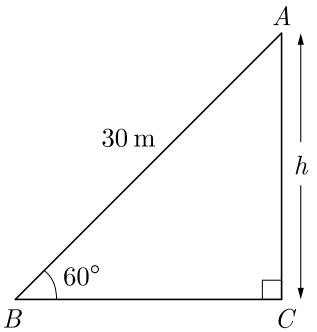
(a) $10\sqrt{3}$ m

(b) $\frac{30}{\sqrt{3}}$ m

(c) 15 m

(d) $15\sqrt{3}$ m

Sol :



$$\sin 60^\circ = \frac{h}{30}$$
$$h = 30\sin 30^\circ$$
$$= 30 \times \frac{\sqrt{3}}{2}$$

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$$= 15\sqrt{3}$$

Thus (d) is correct option

11. On the top face of the wooden cube of side 7 cm, hemispherical depressions of radius 0.35 cm are to be formed by taking out the wood. The maximum number of depressions that can be formed is

(a) 400

(b) 100

(c) 20

(d) 10

Sol :

Diameter of hemispherical depression is $0.35 \times 2 = 0.7$ cm. Each hemispherical depression occupies an area equal to that of a square with side 0.7 cm. Therefore, 10 depressions can be arranged in one row, and similarly, 10 such rows can be formed. Hence, a total of $10 \times 10 = 100$ depressions can be made

Thus (b) is correct option

12. The cumulative frequency for calculating median is obtained by adding the frequencies of all the

(a) classes up to the median class

(b) classes following the median class

(c) classes preceding the median class

(d) all classes

Sol :

To calculate the median in a grouped frequency distribution, we use cumulative frequency to identify the median class. The cumulative frequency of the class just before the median class is used in the median formula.

Thus (a) is correct option

13. If mean and median of given set of observations are 10 and 11 respectively, then the value of mode is

(a) 10.5

(b) 8

(c) 13

(d) 21

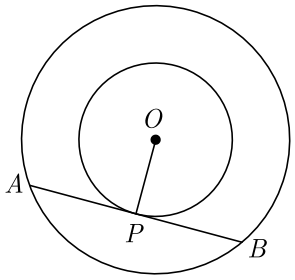
Sol :

We use the empirical relationship between mean, median, and mode:

$$\begin{aligned}\text{Mode} &= 3 \times \text{Median} - 2 \times \text{Mean} \\ &= 3 \times 11 - 2 \times 10 \\ &= 33 - 20 = 13\end{aligned}$$

Thus (c) is correct option

14. In the adjoining figure, AB is the chord of the larger circle touching the smaller circle. The centre of both the circles is O. If $AB = 2r$ and $OP = r$, then the radius of larger circle is



(a) $2r$

(b) $3r$

(c) $2\sqrt{2}\,r$

(d) $\sqrt{2}\,r$

Sol :

Let R be the radius of the larger (outer) circle. Because the circles are concentric, the perpendicular from the common centre O to the chord AB meets AB at its midpoint M and which is P.

Since the chord is tangent to the inner circle

$$OP = r$$

The half-length of the chord is

$$AP = \tfrac{1}{2} = AB = r$$

In right-triangle $\triangle OPA$ we have

$$\begin{aligned}OA^2 &= OP^2 + AP^2 \\ &= r^2 + r^2 = 2r^2\end{aligned}$$

$$OA = \sqrt{2}\,r$$

Hence the radius of the larger circle is $\sqrt{2}\,r$.

Thus (d) is correct option

15. A parallelogram having one of its sides 5 cm circumscribes a circle. The perimeter of parallelogram is

(a) 20 cm

(b) less than 20 cm

(c) more than 20 cm but less than 40 cm

(d) 40 cm

Sol :

If a parallelogram circumscribes a circle, it must be a rhombus. This is because only in a rhombus (a parallelogram with all sides equal), the sum

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of lengths of opposite sides equals the sum of the lengths of the other pair of opposite sides — a necessary condition for a circle to be inscribed.

In such a case, opposite sides are equal, and all four sides are the same length.

One side = 5 cm

All sides = 5 cm (since it's a rhombus)

Perimeter = $4\times5 = 20$ cm

Thus (a) is correct option

16. E and F are points on the sides AB and AC respectively of a triangle ΔABC such that $\frac{AE}{EB} = \frac{AF}{FC} = \frac{1}{2}$. Which of the following relation is true?

(a) $EF = 2BC$

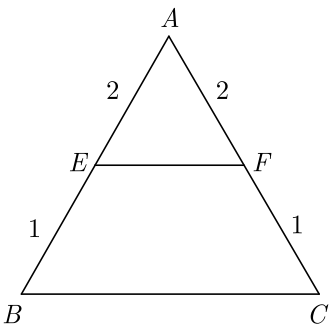
(b) $BC = 2EF$

(c) $EF = 3BC$

(d) $BC = 3EF$

Sol :

As per question statement we have shown the figure below,



Point E divides AB in the ratio 1:2 and point F divides AC in the same ratio. Hence

$$\frac{AE}{AE+EB} = \frac{1}{1+2}$$

$$\frac{AE}{AB} = \frac{1}{3}$$

Similarly $$\frac{AF}{AC} = \frac{1}{3}$$

Since the two sides of $\triangle ABC$ are cut in the same proportion, the line EF is parallel to BC (Basic Proportionality/Thales' theorem).

The similarity of triangles $\triangle AEF \sim \triangle ABC$ then gives

$$\frac{EF}{BC} = \frac{AE}{AB} = \frac{1}{3}$$

$$BC = 3EF$$

Thus (d) is correct option.

17. Which of the following statements is true for a polynomial $p(x)$ of degree 3 ?

(a) $p(x)$ has at most two distinct zeroes.

(b) $p(x)$ has at least two distinct zeroes.

(c) $p(x)$ has exactly three distinct zeroes.

(d) $p(x)$ has at most three distinct zeroes.

Sol :

A polynomial of degree 3 (also called a cubic polynomial) can have at most 3 real zeroes, because the number of zeroes (real or complex) cannot exceed the degree of the polynomial.

Thus (d) is correct option.

18. A pair of dice is thrown. The probability that sum of numbers appearing on top faces is at most 10 is:

(a) $\frac{1}{11}$

(b) $\frac{10}{11}$

(c) $\frac{5}{6}$

(d) $\frac{11}{12}$

Sol :

Total number of outcomes is 36.

Here, all non-favorable possible outcome is (6, 6), (5, 6), (6, 5) and all other are favourable outcome.

$$n(S) = 36$$

No. of favourable outcome,

$$n(E) = 36 - 3 = 33$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{33}{36} = \frac{11}{12}$$

Thus (d) is correct option.

19. **Assertion (A):** 4^n ends with digit 0 for some natural number n.

Reason (R): For a number x having 2 and 5 as its prime factors, x^n always ends with digit 0 for every natural number n.

(a) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A)

(b) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A)

(c) Assertion (A) is true but Reason (R) is false

(d) Assertion (A) is false but Reason (R) is true

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Sol :

A number that has 2 and 5 as its prime factors is divisible by 10. So if a number x has prime factors 2 and 5, then x^n will also be divisible by 10 for all n, and hence will always end with 0. So Reason is true.

Assertion is false because 4 does not have prime factor 5.

Thus (d) is correct option.

20. **Assertion (A):** Tangents drawn at the end points of a diameter of a circle are always parallel to each other.

Reason (R): The lengths of tangents drawn to a circle from a point outside the circle are always equal.

(a) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A)

(b) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A)

(c) Assertion (A) is true but Reason (R) is false

(d) Assertion (A) is false but Reason (R) is true

Sol :

Tangents drawn at the endpoints of a diameter of a circle are perpendicular to the radius at those points. Since the diameter is a straight line and both tangents are perpendicular to it, they are parallel to each other (as perpendiculars to the same line are parallel). Assertion is true.

From any point outside a circle, the lengths of the two tangents drawn to the circle are equal — this is a known geometrical theorem. Reason is true.

The reason talks about the equality of tangent lengths from an external point, while the assertion concerns the direction (parallel nature) of tangents at the ends of a diameter — which is due to their perpendicularity to the same line (the diameter), not due to their lengths.

Thus (b) is correct option.

Section - B

This section consists of 5 questions of 2 marks each.

21. Solve the following system of equations algebraically:
$30x+44y=10;\ 40x+55y=13$

Sol :

We have $30x+44y = 10$ ...(1)

$$40x+55y = 13 \quad ...(2)$$

Multiplying equation (1) by 4, we get

$$120x+176y = 40 \quad ...(3)$$

Multiplying equation (2) by 3, we get

$$120x+165y = 39 \quad ...(4)$$

Now subtracting (4) from (3), we get

$$11y = 1$$

$$y = \tfrac{1}{11}$$

Substituting $y=\frac{1}{11}$ in eq (1) we have

$$30x+44\times\tfrac{1}{11} = 10$$

$$30x+4 = 10$$

$$30x = 6$$

$$x = \tfrac{6}{30} = \tfrac{1}{5}$$

22. (A) A 1.5 m tall boy is walking away from the base of a lamp post which is 12 m high, at the speed of 2.5 m/sec. Find the length of his shadow after 3 seconds.

OR

(B) In parallelogram $ABCD$, side AD is produced to a point E and BE intersects CD at F. Prove that

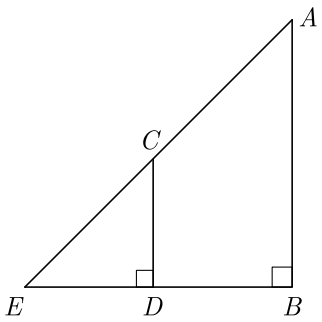
$\Delta ABE \sim \Delta CFB$.

Sol :

(A) After 3 seconds boy will be $2.5\times 3 = 7.5$ m away from post.

Let AB be the lamp post, CD be the boy, ED be the length of the shadow.

As per question we have shown the figure below. After 3 seconds boy is at point D.



Here ED is length of shadow of boy.

Here $AB = 12$ m

$CD = 1.5$ m

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$$DB = 2.5 \times 3 = 7.5 \text{ m}$$

In ΔABE and ΔCDE:

$$\angle ABE = \angle CDE = 90^\circ$$

$$\angle AEB = \angle CED \text{ (Common)}$$

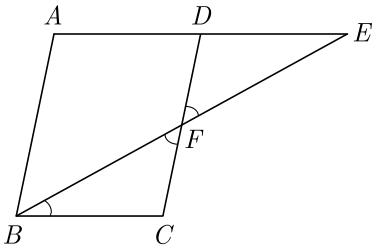
By AA criterion we have

$$\Delta ABE \sim \Delta CDE$$

Thus
$$\frac{AB}{CD} = \frac{ED + DB}{ED}$$
$$\frac{12}{1.5} = \frac{ED + 7.5}{ED}$$
$$\frac{12 - 1.5}{1.5} = \frac{ED + 7.5 - ED}{ED}$$
$$\frac{10.5}{1.5} = \frac{7.5}{ED}$$
$$\frac{10.5}{1.5} = \frac{7.5}{ED}$$
$$7 = \frac{7.5}{ED}$$
$$ED = \frac{7.5}{7} = 1.07 \text{ m}$$

OR

(B) As per question we have shown the figure below.



In ΔABE and ΔCFB

Due to opposite angles of parallelogram, we have

$$\angle BAE = \angle FCB$$

Since $AE \parallel BC$ and EB is transversal, we have

$$\angle AEB = \angle FBC$$

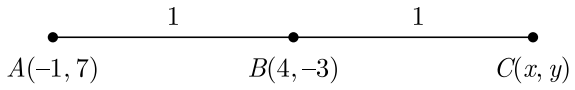
Thus due to AA symmetry we have

$$\Delta ABE \sim \Delta CFB$$

23. Find the coordinates of the point C which lies on the line AB produced such that $AC = 2BC$, where coordinates of points A and B are $(-1, 7)$ and $(4, -3)$ respectively.

Sol :

As per question we have shown the figure below.



Here $AC = 2BC$

$$\frac{BC}{AC} = \frac{1}{2}$$
$$\frac{BC}{AB + BC} = \frac{1}{2}$$
$$\frac{BC}{AB + BC - BC} = \frac{1}{2 - 1}$$
$$\frac{BC}{AB} = \frac{1}{1}$$
$$BC = AB$$

Thus B is mid-point of AC.

Now $(4, -3) = \left(\frac{-1 + 2}{2}, \frac{7 + y}{2}\right)$

$$\frac{-1 + x}{2} = 4 \Rightarrow x = 9$$

and $$\frac{7 + y}{2} = -3 \Rightarrow y = -13$$

Thus coordinates of C are $(9, -13)$

24. (A) Find the value of x for which

$$(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = x + \tan^2 A + \cot^2 A$$

OR

(B) Evaluate the following $\frac{3\sin 30^\circ - 4\sin^3 30^\circ}{2\sin^2 50^\circ + 2\cos^2 50^\circ}$

Sol :

We have

$$(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = x + \tan^2 A + \cot^2 A$$

$$\left(\sin A + \frac{1}{\sin A}\right)^2 + \left(\cos A + \frac{1}{\cos A}\right)^2 = x + \left(\frac{\sin A}{\cos A}\right)^2 + \left(\frac{\cos A}{\sin A}\right)^2$$

$$\sin^2 A + \frac{1}{\sin^2 A} + 2 + \cos^2 A + \frac{1}{\cos^2 A} + 2 = x + \frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A}$$

$$1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} + 4 = x + \frac{\sin^4 A + \cos^4 A}{\sin^2 A \cos^2 A}$$

$$1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} + 4 = x + \frac{(\sin^2 A + \cos^2 A)^2 - 2\sin^2 A \cos^2 A}{\sin^2 A \cos^2 A}$$

$$5 + \frac{1}{\sin^2 A \cos^2 A} = x + \frac{1 - 2\sin^2 A \cos^2 A}{\sin^2 A \cos^2 A}$$

$$5 + \frac{1}{\sin^2 A \cos^2 A} = x + \frac{1}{\sin^2 A \cos^2 A} - 2$$

$$5 = x - 2$$

$$x = 7$$

OR

$$\frac{3\sin 30^\circ - 4\sin^3 30^\circ}{2\sin^2 50^\circ + 2\cos^2 50^\circ} = \frac{3 \times \frac{1}{2} - 4\left(\frac{1}{2}\right)^3}{2(\sin^2 50^\circ + \cos^2 50^\circ)}$$
$$= \frac{\frac{3}{2} - \frac{1}{2}}{2}$$

25. Two friends Anil and Ashraf were born in the December month in the year 2010. Find the probability that

(i) they share same date of birth.

(ii) they have different dates of birth.

Sol :

(i) Share same date of birth :

Let's assume that Anil can be born on any of the 31 days. Ashraf is also randomly born on any of the 31 days. Since both birthdays are independent and equally likely over 31 days:

$$P(\text{same date}) = \frac{1}{31}$$

(ii) Different dates of birth

This is the complementary event of the first one.

$$P(\text{different date}) = 1 - P(\text{same date})$$
$$= 1 - \frac{1}{31} \quad \frac{30}{31}$$

Section - C

This section consists of 6 question of 3 marks each

26. (A) Prove that $\sqrt{2}$ is an irrational number

OR

(B) Let x and y be two distinct prime number and $p = x^2$ find the HCF and LCM of p,q and r further check if HCF (p,q,r) × LCM (p,q,r)= p × q × r or not

Sol :

Sol :

(A) Assume that $\sqrt{2}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Assume that $\sqrt{2}$ be a rational number then we have

$$\sqrt{2} = \frac{p}{q},$$

where p and q are co-primes and $q \neq 0$.

Now $$p = q\sqrt{2}$$

Squaring both the sides, we have

$$p^2 = 2q^2$$

Thus 2 is a factor of p^2 and in result 2 is also a factor of p.

Let $p = 2c$ where c is some integer, then we have

$$p^2 = 4c^2$$

Substituting $p = 2q^2$ we have

$$2q^2 = 4c^2$$

$$q^2 = 2c^2$$

Thus 2 is a factor of q^2 and in result 2 is also a factor of q.

Thus 2 is a common factor of p and q. But this contradicts the fact that p and q are co-primes. Thus, our assumption that $\sqrt{2}$ is rational number is wrong. Hence $\sqrt{2}$ is irrational.

(B)

We have $$p = x^2y^2$$

$$q = xy^4$$

$$r = x^5y^2$$

$$\text{HCF } (p,q,r) = xy^2$$

$$\text{LCM } (p,q,r) = x^5y^4$$

Now,

$$\text{HCF } (p,q,r) \times \text{LCM } (p,q,r) = xy^2 \times x^5y^4$$
$$= x^6y^6 \quad \text{...(i)}$$

$$p \times q \times r = x^2y^3 \times xy^4 \times x^5y^2$$
$$= x^8y^9 \quad \text{...(ii)}$$

From (i) and (ii),

$$\text{HCF } (p,q,r) \times \text{LCM } (p,q,r) = xy^2 \times x^5y^4$$
$$\neq p \times q \times r$$

27. The monthly incomes of two persons are in the ratio 9 : 7 and their monthly expenditures are in the ratio 4 : 3. If each saved ₹5,000, express the given situation algebraically as a system of linear equations in two variables. Hence, find their respective monthly incomes.

Sol :

Let the monthly incomes of two persons be $9x$ and $7x$ respectively.

Let monthly expenditures of two persons be $4y$ and $3y$ respectively.

Since each saved ₹5,000, we have

$$9x - 4y = 5000 \quad (1)$$

$$7x - 3y = 5000 \quad (2)$$

This is system of linear equations in two variables.

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Multiplying (1) by 3 and (2) by 4 we have

$$27x - 12y = 15000 \quad (3)$$

$$28x - 12y = 20000 \quad (4)$$

Subtracting (3) from (4) we have

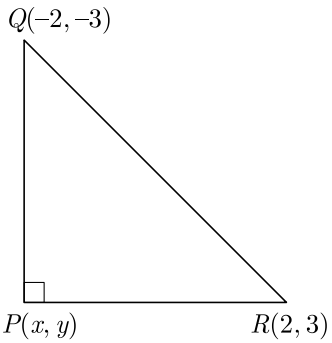
$$x = 5000 \quad (5)$$

Thus monthly incomes are $9 \times 5000 = ₹45000$ and $7 \times 5000 = ₹35000$

28. $P(x, y)$, $Q(-2, -3)$ and $R(2,3)$ are the vertices of a right triangle PQR right angled at P. find the relationship between x and y. Hence, find all the possible values of x for which $y = 2$.

Sol :

As per question we have shown in figure below.



From Pythagoras theorem we have

$$PQ^2 + PR^2 = QR^2$$

$$(x+2)^2 + (y+3)^2 + (x-2)^2 + (y-3)^2 = (-2-2)^2 + (-3-3)^2$$

$$x^2 + 4 + 4x + y^2 + 9 + 6y + x^2 - 4x + 4 + y^2 - 6y + 9 = 16 + 36$$

$$2(x^2 + y^2) + 2(4 + 9) = 52$$

$$(x^2 + y^2) + 13 = 26$$

Required relation, $x^2 + y^2 = 13$

For $y = 2$, $x^2 = 13 - 2^2 = 9$

29. (A) Prove that $\dfrac{\cos A + \sin A - 1}{\cos A - \sin A + 1} = \operatorname{cosec} A - \cot A$

OR

(B) if $\cot\theta + \cos\theta = p$ and $\cot\theta - \cos\theta = q$

Sol :

(A)

$$\text{LHS} = \frac{\cos A + \sin A - 1}{\cos A - \sin A + 1}$$

$$= \frac{\cos A + (\sin A - 1)}{\cos A - (\sin A + 1)} \times \frac{\cos A + (\sin A - 1)}{\cos A + (\sin A - 1)}$$

$$= \frac{\cos A + \sin A - 1^2}{\cos^2 A - (\sin A - 1)^2}$$

$$= \frac{\cos^2 A + \sin^2 A + 1 + 2\sin A\cos A - 2\cos A - \sin A}{\cos^2 A - \sin^2 A - 1 + 2\sin A}$$

$$= \frac{1 + 1 + 2\sin A\cos A - 2\cos A - 2\sin A}{2\sin A - (1 - \cos^2 A) - \sin^2 A}$$

$$= \frac{2 + 2\sin A\cos A - 2\cos A - 2\sin A}{2\sin A - 2\sin^2 A}$$

$$= \frac{2(1 - \sin A) - 2\cos A(1 - \sin A)}{2\sin A(1 - \sin A)}$$

$$= \frac{(1 - \sin A)(2 - 2\cos A)}{2\sin A(1 - \sin A)}$$

$$= \frac{2(1 - \cos A)}{2\sin A}$$

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$$= \operatorname{cosec} A - \cot A$$

$$= \text{RHS}$$

Hence, proved.

(B)

We have $\cot\theta + \cos\theta = p$

$\cot\theta - \cos\theta = q$

Squaring both side we get

$$(\cot\theta + \cos\theta)^2 = p^2$$

$$\cot^2\theta + \cos^2\theta + 2\cot\theta\cos\theta = p^2 \quad \ldots(i)$$

and $(\cot\theta - \cos\theta)^2 = q^2$

$$\cot^2\theta + \cos^2\theta - 2\cot\theta\cos\theta = q^2 \quad \ldots(ii)$$

Subtracting equation (ii) from equation (i), we get

$$4\cot\theta\cos\theta = p^2 - q^2 \quad \ldots(iii)$$

Now, $pq = (\cot\theta + \cos\theta)(\cot\theta - \cos\theta)$

$$= \cot^2\theta - \cos^2\theta$$

$$= \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$$

$$= \cos^2\theta(\operatorname{cosec}^2\theta - 1)$$

$$= \cos^2\theta\cot^2\theta \quad [\operatorname{cosec}^2\theta - 1 = \cot^2\theta]$$

$$4\sqrt{pq} = 4\cos\theta\cot\theta \quad \ldots(iv)$$

From equation (iii) and (iv), we get

$$4\sqrt{pq} = p^2 - q^2$$

30. α and β are zeros of a quadratic polynomial $px^2 + qx + 1$. Form a quadratic polynomial whose

zeroes are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$.

Sol :

Since α and β are zeros of the given polynomial px^2+qx+1, we have

$$\alpha+\beta = -\frac{q}{p} \text{ and } \alpha\beta = \frac{1}{p}$$

Since $\frac{2}{\alpha}$ and $\frac{2}{\beta}$ are the zeros of new polynomial then we have

Sum of zeros, $$S = \frac{2}{\alpha}+\frac{2}{\beta} = \frac{2(\beta+\alpha)}{\alpha\beta}$$

$$= \frac{-2\times\frac{q}{p}}{\frac{1}{p}} = -2q$$

Product of zeros, $P = \frac{2}{\alpha}\cdot\frac{2}{\beta} = \frac{4}{\alpha\beta} = \frac{4}{\frac{1}{p}} = 4p$

Required polynomial,

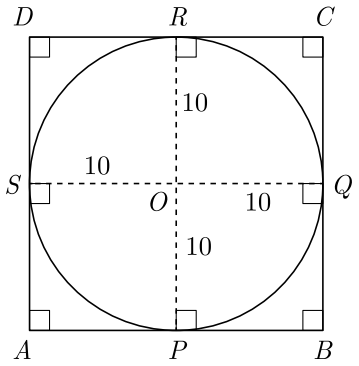
$$g(x) = x^2 - Sx + P$$

$$g(x) = x^2 + 2qx + 4p$$

So, the required polynomial is $k(x^2+2qx+4p)$

31. Rectangle $ABCD$ Circumscribes the circle of radius 10 cm. Prove that $ABCD$ is a square. Hence, find the perimeter of $ABCD$.

Sol :



Let O be centre of circle.

Due to tangents from same external point we get

$$AP = AS \quad (1)$$

$$PB = BQ \quad (2)$$

$$CR = CQ \quad (3)$$

$$DR = DS \quad (4)$$

Adding (1), (2), (3) and (4) we have

$$AP+PB+CR+DR = AS+BQ+CQ+DS$$

$$(AP+PB)+(CR+DR) = (AS+DS)+(BQ+CQ)$$

$$AB+CD = AD+BC$$

But $AB = CD$ and $AD = BC$, thus

$$2AB = 2AD$$

or $$AB = AD$$

$$AB = BC = CD = AD$$

Thus $ABCD$ is a square.

Perimeter of $ABCD = 4 \times 20 = 80$cm

Section - D

This section consists of 4 questions of 5 marks each.

32. (A) The sides of a right triangle are such that the longest side is 4 m more than the shortest side and the third side is 2 m less than the longest side. Find the length of each side of the triangle. Also, find the difference between the numerical values of the area and the perimeter of the given triangle

OR

(B) Express the equation $\frac{x-2}{x-3}+\frac{x-4}{x-5} = \frac{10}{3}$; $(x \neq 3, 5)$ as a quadratic equation in standard form. Hence, find the roots of the equation so formed.

Sol :

Let the shortest side of the right triangle be x. Then longest side will be $(4+x)$ and third side will be $(4+x-2)$ or $(x+2)$.

Now $$x^2+(x+2)^2 = (4+x)^2$$

$$x^2+x^2+4x+4 = x^2+8x+16$$

$$x^2-4x-12 = 0$$

$$x^2-6x+2x-12 = 0$$

$$x(x-6)+2(x-6) = 0$$

$$x(x-6)(x+2) = 0$$

$$x = 6, -2$$

Since side can't be negative, shortest side is 6 m and other sides of triangle are 8 m and 10 m.

Thus All sides of triangle are 6m, 8m and 10 m

Perimeter of triangle = (6 + 8 + 10)m = 24

and area of triangle $= \frac{1}{2}\times 6\times 8\,\text{m}^2 = 24$ m

Required difference $= 24-24 = 0$

OR

We have $$\frac{x-2}{x-3}+\frac{x-4}{x-5} = \frac{10}{3}$$

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$$\frac{(x-2)(x-5)+(x-3)(x-4)}{(x-3)(x-5)} = \frac{10}{3}$$

$$\frac{(x^2-7x+10)+(x^2-7x+12)}{x^2-8x+15} = \frac{10}{3}$$

$$\frac{2x^2-14x+22}{x^2-8x+15} = \frac{10}{3}$$

$$3(2x^2-14x+42) = 10(x^2-8x+15)$$

$$6x^2-42x+66 = 10x^2-80x+150$$

$$4x^2-38x+84 = 0$$

$$2x^2-19x+42 = 0$$

$$x = \frac{19 \pm \sqrt{(-19)^2-4(2)(42)}}{2\times 2}$$

$$= \frac{19 \pm \sqrt{25}}{4}$$

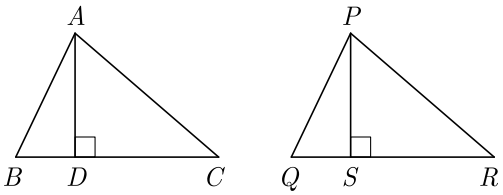
$$= \frac{19 \pm 5}{4}$$

$$= \frac{19+5}{4}, \frac{19-5}{4}$$

$$= \frac{24}{4}, \frac{14}{4}$$

$$= 6, \frac{7}{2}$$

33. (A) The corresponding sides of ΔABC and ΔPQR are in the ratio 3 : 5. $AD \perp BC$ and $PS \perp QR$ as shown in the following figures



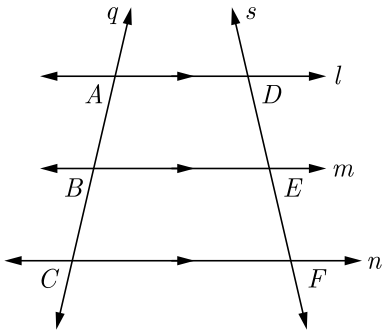
(i) Prove that $\Delta ADC \sim \Delta PSR$.

(ii) If $AD = 4$ cm, find the length of PS.

(iii) Using (ii) find ar (ΔABC) : ar (ΔPQR)

OR

(B) State basic proportionality theorem. Use it to prove the following

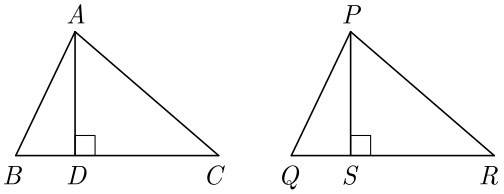


if three parallel lines i m n are intersected by transversals q and s as shown in the adjoining figure, then

$$\frac{AB}{BC} = \frac{DE}{EF}$$

Sol :

We have redrawn the figure as shown below.



Since corresponding sides of ΔABC and ΔPQR are in the same ratio,

$$\Delta ABC \sim \Delta\ PQR$$

Thus $\angle ACB = \angle PRQ$ ...(i)

(i) In ΔADC and ΔPSR

$\angle ACB = \angle PRQ$ from (i)

$\angle ADC = \angle PSR$ (90° each)

By AA criterion we get

$$\Delta ADC \sim \Delta PSR$$

(ii) $$\frac{AD}{PS} = \frac{AC}{PR} = \frac{3}{5}$$

$$\frac{4}{PS} = \frac{3}{5}$$

$$PS = \frac{20}{3} \text{ cm}$$

(iii) $$\frac{AD}{PS} = \frac{4}{\frac{20}{3}} = \frac{12}{20}$$

$$\frac{Ar(\Delta ABC)}{Ar(\Delta PQR)} = \frac{\frac{1}{2}\times AD \times BC}{\frac{1}{2}\times PS \times QR}$$

$$= \frac{AD}{PS}\times\frac{BC}{QR}$$

$$= \frac{12}{20}\times\frac{3}{5}$$

$$= \frac{36}{100} = \frac{9}{25}$$

OR

(B) Basic proportionality Theorem :

In a triangle, a line to one side of a triangle intersecting the other two side in district points, divides the other two sides in the same ratio.

In ΔABC, in which $DE \parallel BC$ then $\frac{AD}{DB} = \frac{AE}{EC}$

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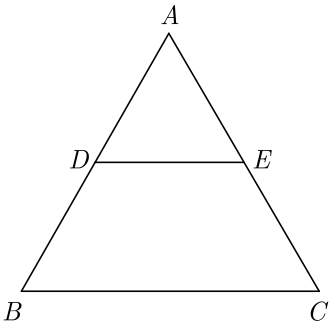
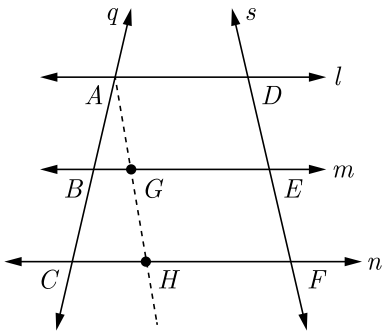


We have parallel lines l, m, n which are cut by the transversal q and s at A, B, C and D, E, F respectively shown below in figure.



We have drawn $AH \parallel DF$ meeting the lines m and n in G and H respectively.

Since $AH \parallel DF$ and $AG \parallel DE$, $AGED$ is a parallelogram

Thus $AG = DE$ ...(i)

Similarly $GHFE$ is also a parallelogram

$GH = EF$ ...(ii)

Now, in ΔACH, we have $BG \parallel CH$.

By B.P.T $\frac{AB}{BC} = \frac{DE}{EF}$ Hence Proved.

34. A wooden cubical die is formed by forming hemispherical depressions on each face of the cube such that face 1 has one depression, face 2 has two depressions and so on. The sum of number of hemispherical depressions on opposite faces is always 7. If the edge of the cubical die measures 5 cm and each hemispherical depression is of diameter 1.4 cm, find the total surface area of the die so formed.

Sol :

Number of depressions on each face are 1, 2, 3, 4, 5, 6 respectively, thus total 21 depressions.

Curved surface area of each depression

$$2\pi r^2 = 2 \times \frac{22}{7} \times 0.7 \times 0.7$$

$$= 44 \times 0.1 \times 0.7$$

$$= 3.08 \text{ cm}^2$$

Base area of hemisphere,

$$\pi r^2 = \frac{22}{7} \times 0.7 \times 0.7$$

$$= 1.54 \text{ cm}^2$$

Thus each depression reduce the surface area by 1.54 $\text{cm}^2$and increase the surface area by 3.08 cm^2. Thus net increase in surface area by each depression,

$$3.08 - 1.54 = 1.54 \text{ cm}^2$$

Total increase in surface area due to 21 depression

$$21 \times 1.54 = 32.34 \text{ cm}^2$$

$$\text{TSA of cube} = 6 \times 5^2$$

$$= 150 \text{ cm}^2$$

$$\text{TSA of the die} = 150 + 32.34$$

$$= 182.34 \text{ cm}^2$$

35. The following table shows the number of patients of different age group who were discharged from the hospital in a particular month:

Age (in years)	**Number of Patients Discharged**
5–15	6
15–25	11
25–35	21
35–45	23
45–55	14
55–65	5
Total	80

Find the mean and the mode of the above data.

Sol :

We make cumulative frequency table as follows :

Age (in years)	**Number of Patients Discharged** (f_i)	**Class Marks** x_i	$x_i f_i$
5–15	6	10	60
15–25	11	20	220
25–35	21	30	630
35–45	23	40	920
45–55	14	50	700
55–65	5	60	300
Total	80	0	$\Sigma x_i f_i$ =2830

35–45 is modal class.

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$$\text{Mean of Data} = = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{2830}{80}$$

$$= 35.375$$

$$\text{Mode} = l + \left[\frac{f_0 - f_1}{2f_0 - f_1 - f_2}\right] \times h,$$

where $l = 35$, $f_0 = 23$, $f_1 = 21$, $f_2 = 14$ and $h = 10$.

$$\text{Mode} = 35 + \left[\frac{23 - 21}{2 \times 23 - 21 - 14}\right] \times 10$$

$$= 35 + \left(\frac{2}{11}\right) \times 10$$

$$= 35 + \frac{20}{11}$$

$$= \frac{405}{11}$$

$$= 36.82 \text{ (approx.)}$$

Shaded Class is Modal Class.

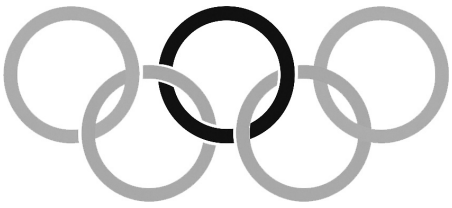
Section - E

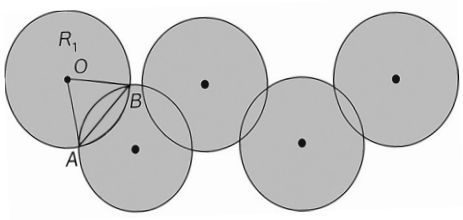
This section consists of 3 Case-Study Based Questions of 4 marks each.

36. The Olympic symbol comprising five interlocking rings represents the union of the five continents of the world and the meeting of athletes from all over the world at the Olympic games. In order to spread awareness about Olympic games, students of Class-X took part in various activities organised by the school. One such group of students made 5 circular rings in the school lawn with the help of ropes. Each circular ring required 44 m of rope.

Also, in the shaded region as shown in the figure, students made rangoli showcasing various sports and games.

It is given that ΔOAB is an equilateral triangle and all unshaded regions are congruent.





Based on above information, answer the following questions:

(i) Find the radius of each circular ring.

(ii) What is the measure of $\angle AOB$?

(iii) (a) Find the area of shaded region R_1.

OR

(iii) (b) Find the length of rope around the unshaded regions.

Sol :

(i) Circumference of each circular ring = 44 m

$$2\pi r = 44$$

$$r = \frac{44}{2\pi}$$

$$= \frac{44 \times 7}{2 \times 22}$$

$$= 7 \text{ m}$$

(ii) Since $\angle AOB$ is an equilateral triangle we have

$$\angle AOB = \angle OAB$$

$$= \angle OBA = 60^\circ$$

(iii)(a) Radius of a circle in rangoli = 7 m

Area of shaded region,

$$R_1 = \pi r^2 - 2 \text{ (Area of segment)}$$

$$= \pi r^2 - 2 \text{ (Area of Sector } - ar(\Delta AOB)]$$

$$= \pi r^2 - 2\left[\frac{\pi r^2 60^\circ}{360^\circ} - \frac{\sqrt{3}}{4} r^2\right]$$

$$= \pi r^2 - \frac{\pi r^2}{3} + \frac{\sqrt{3}}{2} r^2$$

$$= \frac{2\pi r^2}{3} + \frac{\sqrt{3}}{2} r^2$$

$$= \frac{22}{7} \times \frac{2}{3} \times 7^2 + \frac{\sqrt{3}}{2} \times 7 \times 7$$

$$= \frac{22 \times 2 \times 7}{3} + \frac{\sqrt{3} \times 7 \times 7}{2}$$

$$= \frac{308}{3} + 24.5 \times 1.732$$

$$= 102.67 + 42.434$$

$$= 145.104 \text{ m}^2$$

OR

(iii) (b) Length of rope around the unshaded regions

=8(Perimeter of arc with central angle $\angle AOB$)

$$= 8\left[2\pi r \times \frac{60^\circ}{360^\circ}\right]$$

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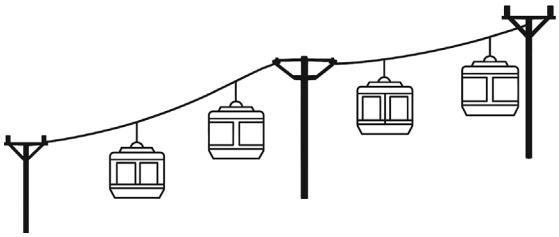
$$= \frac{8\pi r}{3} = \frac{8 \times 22 \times 7}{3 \times 7}$$

$$= \frac{176}{3} \text{ m}$$

$$= 58.67 \text{ m}$$

37. Cable cars at hill stations are one of the major tourist attractions. On a hill station, the length of cable car ride from base point to top most point on the hill is 5000 m. Poles are installed at equal intervals on the way to provide support to the cables on which car moves.



The distance of first pole from base point is 200 m and subsequent poles are installed at equal interval of 150 m.

Further, the distance of last pole from the top is 300 m.

Based on above information, answer the following questions using Arithmetic Progression:

(i) Find the distance of 10^{th} pole from the base.

(ii) Find the distance between 15^{th} pole and 25^{th} pole.

(iii) (a) Find the time taken by cable car to reach 15^{th} pole from the top if it is moving at the speed of 5 m/sec and coming from top.

OR

(b) Find the total number of poles installed along the entire journey.

Sol :

The distances of poles from base form the following AP 200, 200 + 150, 200 + 2(150), ...

(i)

$$a_{10} = a + (n-1)\,d$$

$$= 200 + 9(150)$$

$$= 1550 \text{ m}$$

(ii)

$$a_{25} - a_{15} = a + 24d - a - 14d)$$

$$= 10d$$

$$= 1500 \text{ m}$$

(iii) (a) Total number of poles be n

$$5000 - 300 = 200 + (n-1)(150)$$

$$4700 - 200 = (n-1)150$$

$$4500 = (n-1)150$$

$$30 = (n-1)$$

$$n = 31$$

15^{th} pole from top is $(31 - 15) + 1$ pole from bottom

$$a_{17} = 200 + 16(150)$$

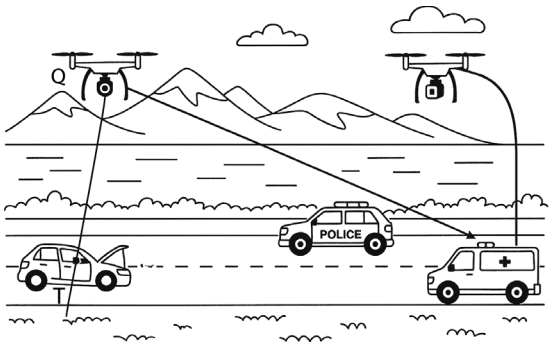
$$a_{17} = 2600$$

Time taken $= \frac{2600}{5} = 520$ seconds

OR

(b) Total number of poles has been already calculated as $n = 31$.

38. A drone was used to facilitate movement of an ambulance on the straight highway to a point P on the ground where there was an accident. The ambulance was travelling at the speed of 60 km/h. The drone stopped at a point Q, 100 m vertically above the point P. The angle of depression of the ambulance was found to be 30° at a particular instant.



Based on above information, answer the following questions:

(i) Represent the above situation with the help of a diagram.

(ii) Find the distance between the ambulance and the site of accident (P) at the particular instant.

(iii) (a) Find the time (in seconds) in which the angle of depression changes from 30° to 45°.

OR

(iii) (b) How long (in seconds) will the ambulance take to reach point P from a point T on the highway such that angle of depression of the ambulance at T is 60° from the drone?

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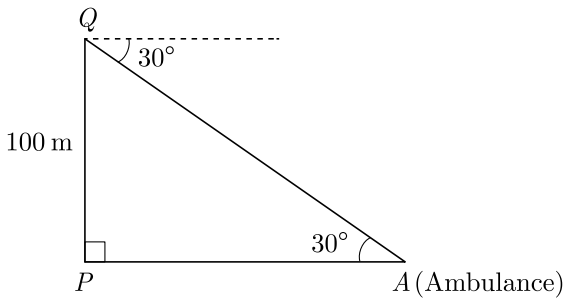
Sol :

(i) We have shown the figure below.



(ii) In ΔQPA,

$$\frac{PQ}{AP} = \tan 30^\circ$$

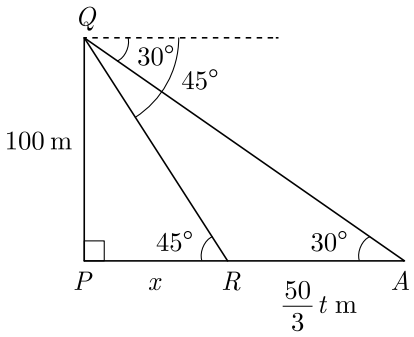
$$\frac{100}{AP} = \frac{1}{\sqrt{3}}$$

$$AP = 100\sqrt{3} \text{ m}$$

$$= 100 \times 1.73$$

(iii) As per question we have shown the figure below.

(a)



Let required time be t seconds

$$\text{Speed of Ambulance} = 60 \times \frac{5}{18} \text{ m/s}$$

$$= \frac{50}{3} \text{ m/s}$$

$$AR = \frac{50t}{3} \text{ m}$$

Let $PR = x$. In Δ QPR we have

$$\frac{100}{x} = \tan 45^\circ$$

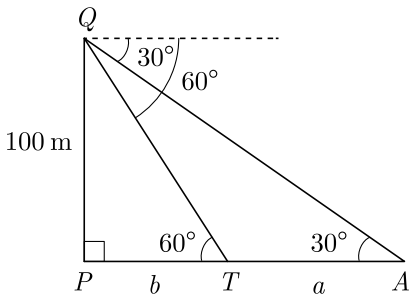
But $AP = 173$ m thus

$$x + \frac{50t}{3} = 173$$

$$t = \frac{73 \times 3}{50} \text{s}$$

$$= \frac{219}{50} \text{s} = 4.38 \text{ s}$$

(b) As per question we have shown the figure below.



Let $AT = a$ and $PT = b$

In Δ QPT we have

$$\frac{100}{b} = \tan 60^\circ$$

$$\frac{100}{b} = \sqrt{3}$$

$$b = \frac{100}{\sqrt{3}} \text{ m}$$

$$\text{Time} = \frac{100\sqrt{3}}{3} \times \frac{3}{50} = 2\sqrt{3} \text{ s}$$

$$= 3.46 \text{ s} \qquad [60 \text{ kmph} = \frac{50}{3} \text{ m/s}]$$

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CHAPTER 1

Real Numbers

OBJECTIVE QUESTIONS

1. If two positive integers p and q can be expressed as $p = 18a^2b^4$ and $q = 20a^3b^2$, where a and b are prime numbers, then LCM (p, q) is

(a) $2a^2b^2$ (b) $180a^2b^2$

(c) $12a^2b^2$ (d) $180a^3b^4$

Sol : [Board 2024 OD Standard]

LCM is the product of the greatest power of each prime factor involved in the numbers.

Writing p and q in prime factorization form

$$\begin{aligned} p &= 18ab^2 \\ &= 2 \times 3 \times 3 \times a \times a \times b \times b \times b \times b \\ q &= 20a^3b^2 \\ &= 2 \times 2 \times 5 \times a \times a \times a \times b \times b \\ \text{LCM}(p, q) &= \text{LCM}(18a^2b^4, 20a^3b^2) \\ &= 2 \times 2 \times 3 \times 3 \times 5 \times \\ &\quad \times a \times a \times a \times b \times b \times b \times b \\ &= 180a^3b^4 \end{aligned}$$

Thus (d) is correct option.

2. If $p^2 = \frac{32}{50}$ then p is a/an

(a) whole number

(b) integer

(c) rational number

(d) irrational number

Sol : [Board 2023 OD Standard]

We have $$p^2 = \frac{32}{50} = \frac{16}{25}$$

$$p = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Since p is in form of $\frac{p}{q}$ where $q \neq 0$, p is a rational number.

Thus (c) is correct option.

3. The sum of exponents of prime factors in the prime-factorisation of 196 is

(a) 3 (b) 4

(c) 5 (d) 2

Sol : [Board 2022 Term 1 Basic]

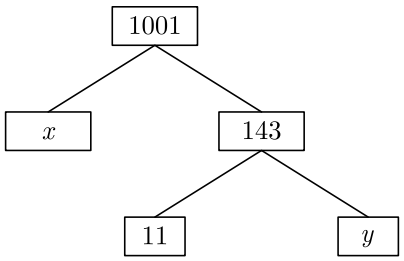
Prime factors of 196,

$$\begin{aligned} 196 &= 4 \times 49 \\ &= 2^2 \times 7^2 \end{aligned}$$

The sum of exponents of prime factor is $2 + 2 = 4$.

Thus (b) is correct option.

4. The values of x and y in the given figure are



(a) 7, 13 (b) 13, 7

(c) 9, 12 (d) 12, 9

Sol : [Board 2022 Term 1 Standard]

$$1001 = x \times 143 \Rightarrow x = 7$$

$$143 = y \times 11 \Rightarrow y = 13$$

Hence $x = 7,\ y = 13$

Thus (a) is correct option.

5. The total number of factors of prime number is

(a) 1 (b) 0

(c) 2 (d) 3

Sol : [Board 2020 Delhi Standard]

There are only two factors (1 and number itself) of any prime number.

Thus (c) is correct option.

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6. The HCF and the LCM of 12, 21, 15 respectively are

(a) 3, 140 (b) 12, 420

(c) 3, 420 (d) 420, 3

Sol : [Board 2020 Delhi Standard]

We have $12 = 2 \times 2 \times 3$

$21 = 3 \times 7$

$15 = 3 \times 5$

$\text{HCF}(12, 21, 15) = 3$

$\text{LCM}\ (12, 21, 15) = 2 \times 2 \times 3 \times 5 \times 7$

$= 420$

Thus (c) is correct option.

7. The LCM of smallest two digit composite number and smallest composite number is

(a) 12 (b) 4

(c) 20 (d) 44

Sol : [Board 2020 SQP Standard]

Smallest two digit composite number is 10 and smallest composite number is 4.

$\text{LCM}\ (10, 4) = 20$

Thus (c) is correct option.

8. HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, then the other number is

(a) 36 (b) 35

(c) 9 (d) 81

Sol : [Board 2020 OD Basic]

Let y be the second number.

Since, product of two numbers is equal to product of LCM and HCM,

$54 \times y = \text{LCM} \times \text{HCF}$

$54 \times y = 162 \times 27$

$y = \dfrac{162 \times 27}{54} = 81$

Thus (b) is correct option.

9. HCF of 144 and 198 is

(a) 9 (b) 18

(c) 6 (d) 12

Sol : [Board 2020 Delhi Basic]

Using prime factorization method,

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= 2^4 \times 3^2$

and $198 = 2 \times 3 \times 3 \times 11$

$= 2 \times 3^2 \times 11$

$\text{HCF}(144, 198) = 2 \times 3^2$

$= 2 \times 9 = 18$

Thus (b) is correct option.

10. 225 can be expressed as

(a) 5×3^2 (b) $5^2 \times 3$

(c) $5^2 \times 3^2$ (d) $5^3 \times 3$

Sol : [Board 2020 Delhi Basic]

By prime factorization of 225, we have

$225 = 3 \times 3 \times 5 \times 5$

$= 3^2 \times 5^2$ or $5^2 \times 3^2$

Thus (c) is correct option.

11. $2\sqrt{3}$ is

(a) an integer

(b) a rational number

(c) an irrational number

(d) a whole number

Sol : [Board 2020 OD Basic]

Let us assume that $2\sqrt{3}$ is a rational number.

Now $2\sqrt{3} = r$ where r is rational number

or $\sqrt{3} = \dfrac{r}{2}$

Now, we know that $\sqrt{3}$ is an irrational number, So, $\frac{r}{2}$ has to be irrational to make the equation true. This is a contradiction to our assumption. Thus, our assumption is wrong and $2\sqrt{3}$ is an irrational number.

Thus (c) is correct option.

12. The product of a non-zero rational and an irrational number is

(a) always irrational

(b) always rational

(c) rational or irrational

(d) one

Sol :

Product of a non-zero rational and an irrational number is always irrational i.e., $\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$ which is irrational.

Thus (a) is correct option.

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13. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then HCF (a, b) is

(a) xy (b) xy^2

(c) x^3y^3 (d) x^2y^2

Sol : [Board Term-1 2014, 2011]

We have $a = x^3y^2 = x \times x \times x \times y \times y$

$b = xy^3 = x \times y \times y \times y$

$\text{HCF}(a, b) = \text{HCF}(x^3y^3, xy^3)$

$= x \times y \times y = xy^2$

HCF is the product of the smallest power of each common prime factor involved in the numbers.

Thus (b) is correct option.

14. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; where a, b being prime numbers, then LCM (p, q) is equal to

(a) ab (b) a^2b^2

(c) a^3b^2 (d) a^3b^3

Sol : [Board 2010]

We have $p = ab^2 = a \times b \times b$

and $q = a^3b = a \times a \times a \times b$

$\text{LCM}(p, q) = \text{LCM}(ab^2, a^3b)$

$= a \times b \times b \times a \times a$

$= a^3b^2$

LCM is the product of the greatest power of each prime factor involved in the numbers.

Thus (c) is correct option.

15. The least number that is divisible by all the numbers from 1 to 10 (both inclusive)

(a) 10 (b) 100

(c) 504 (d) 2520

Sol : [Board Term-1 2016, 2015]

Factor of 1 to 10 numbers

$1 = 1$

$2 = 1 \times 2$

$3 = 1 \times 3$

$4 = 1 \times 2 \times 2$

$5 = 1 \times 5$

$6 = 1 \times 2 \times 3$

$7 = 1 \times 7$

$8 = 1 \times 2 \times 2 \times 2$

$9 = 1 \times 3 \times 3$

$10 = 1 \times 2 \times 5$

$\text{LCM}(1 \text{ to } 10) = \text{LCM}(1,2,3,4,5,6,7,8,9,10)$

$= 1 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 2520$

Thus (d) is correct option.

16. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is

(a) an even number

(b) an odd number

(c) an odd prime number

(d) a prime number

Sol :

$p_1^2 - p_2^2$ is an even number.

Let us take $p_1 = 5$

and $p_2 = 3$

Then, $p_1^2 - p_2^2 = 25 - 9 = 16$

16 is an even number.

Thus (a) is correct option.

17. The number $3^{13} - 3^{10}$ is divisible by

(a) 2 and 3 (b) 3 and 10

(c) 2, 3 and 10 (d) 2, 3 and 13

Sol :

$3^{13} - 3^{10} = 3^{10}(3^3 - 1) = 3^{10}(26)$

$= 2 \times 13 \times 3^{10}$

Hence, $3^{13} - 3^{10}$ is divisible by 2, 3 and 13.

Thus (d) is correct option.

18. 1. The L.C.M. of x and 18 is 36.

2. The H.C.F. of x and 18 is 2.

What is the number x?

(a) 1 (b) 2

(c) 3 (d) 4

Sol : [Board Term-1 2013]

LCM $\times$ HCF = First number $\times$ second number

Hence, required number $= \dfrac{36 \times 2}{18} = 4$

Thus (d) is correct option.

19. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and $\text{LCM}(a, b, c) = 2^3 \times 3^2 \times 5$, then n is

(a) 1 (b) 2

(c) 3 (d) 4

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Sol :

Value of n must be 2.

Thus (b) is correct option.

20. The least number which is a perfect square and is divisible by each of 16, 20 and 24 is

(a) 240 (b) 1600

(c) 2400 (d) 3600

Sol :

The LCM of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choices (a) and (c) since they are not perfect square number. 1600 is not multiple of 240.

Thus (d) is correct option.

21. **Assertion :** The HCF of two numbers is 5 and their product is 150, then their LCM is 30

Reason : For any two positive integers a and b, $\text{HCF}(a, b) + \text{LCM}(a, b) = a \times b$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have,

$$\begin{aligned} \text{LCM}(a, b) \times \text{HCF}(a, b) &= a \times b \\ \text{LCM} \times 5 &= 150 \\ \text{LCM} &= \frac{150}{5} = 30 \end{aligned}$$

Thus (c) is correct option.

ONE MARK QUESTIONS

22. What is the sum of exponents of prime factors in the prime-factorisation of 196 ?

Sol : [Board 2020 OD Standard]

Prime factors of 196,

$$\begin{aligned} 196 &= 4 \times 49 \\ &= 2^2 \times 7^2 \end{aligned}$$

The sum of exponents of prime factor is $2 + 2 = 4$.

PRACTICE

What is the sum of exponents of prime factors in the prime-factorisation of 1764 ?

[Board 2013 OD]

Ans : 6

23. Find the HCF and the LCM of 12, 21 and 15.

Sol : [Board 2020 Delhi Standard]

We have

$$\begin{aligned} 12 &= 2 \times 2 \times 3 \\ 21 &= 3 \times 7 \\ 15 &= 3 \times 5 \\ \text{HCF}(12, 21, 15) &= 3 \\ \text{LCM}(12, 21, 15) &= 2 \times 2 \times 3 \times 5 \times 7 \\ &= 420 \end{aligned}$$

PRACTICE

Find the HCF and the LCM of 36, 63, 90.

[Board 2014 Foreign]

Ans : 9 and 1260

24. Explain why 13233343563715 is a composite number?

Sol : [Board Term-1 2016]

The number 13233343563715 ends in 5. Hence it is a multiple of 5. Therefore it is a composite number.

25. Find the LCM of smallest two digit composite number and smallest composite number.

Sol : [Board 2020 SQP Standard]

Smallest two digit composite number is 10 and smallest composite number is 4.

$$\text{LCM}(10, 4) = 20$$

26. Express 225 in prime factorization.

Sol : [Board 2020 Delhi Basic]

By prime factorization of 225, we have

$$\begin{aligned} 225 &= 3 \times 3 \times 5 \times 5 \\ &= 3^2 \times 5^2 \text{ or } 5^2 \times 3^2 \end{aligned}$$

PRACTICE

Express 420 in prime factorization.

[Board 2009]

Ans : $2^2 \times 3 \times 5 \times 7$

27. HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, then what is the other number ?

NODIA

Sol : [Board 2020 OD Basic]

Let y be the second number. Since, product of two numbers is equal to product of LCM and HCM,

$$54 \times y = \text{LCM} \times \text{HCF}$$
$$54 \times y = 162 \times 27$$
$$y = \frac{162 \times 27}{54} = 81$$

PRACTICE

1. The L.C.M. of x and 18 is 36.
2. The H.C.F. of x and 18 is 2.

What is the number x?

[Board 2006, 2014 Foreign]

Ans : 4

28. Find the HCF and LCM of 90 and 144 by the method of prime factorization.

Sol : [Board Term-1 2012]

We have $90 = 9 \times 10 = 9 \times 2 \times 5$
$= 2 \times 3^2 \times 5$

and $144 = 16 \times 9$
$= 2^4 \times 3^2$

$\text{HCF} = 2 \times 3^2 = 18$

$\text{LCM} = 2^4 \times 3^2 \times 5 = 720$

PRACTICE

Find HCF of 144 and 198.

[Board 2020 Delhi Basic]

Ans : 18

29. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then find HCF (a, b).

Sol : [Board Term -1 2014]

We have $a = x^3y^2 = x \times x \times x \times y \times y$

$b = xy^3 = x \times y \times y \times y$

$\text{HCF}(a, b) = \text{HCF}(x^3y^2, xy^3)$
$= x \times y \times y = xy^2$

HCF is the product of the smallest power of each common prime factor involved in the numbers.

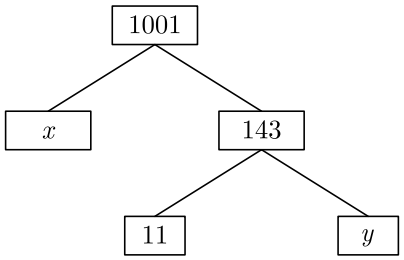
PRACTICE

If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; where a, b being prime numbers, then what is the LCM of (p, q) ?

[Board Term -1 2014]

Ans : a^3b^2

30. What are the values of x and y in the given figure?



Sol : [Board Term -1 2012]

We have $1001 = x \times 143 \Rightarrow x = 7$

$143 = y \times 11 \Rightarrow y = 13$

Hence $x = 7,\ y = 13$

31. If $\text{HCF}(336, 54) = 6$, find $\text{LCM}(336, 54)$.

Sol : [Board 2019 OD]

$\text{HCF} \times \text{LCM} = \text{Product of number}$

$6 \times \text{LCM} = 336 \times 54$

$\text{LCM} = \frac{336 \times 54}{6}$

$= 56 \times 54$

$= 3024$

Thus LCM of 336 and 54 is 3024.

32. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then calculate the least prime factor of $(a + b)$.

Sol : [Board Term-1 2014]

Here a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. The least prime factor of $(a + b)$ would be 2.

33. What is the HCF of the smallest composite number and the smallest prime number?

Sol : [Board Term-1 OD 2018]

The smallest prime number is 2 and the smallest composite number is $4 = 2^2$.

Hence, required HCF is $(2^2, 2) = 2$.

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34. Calculate the HCF of $3^3\times5$ and $3^2\times5^2$.

Sol : [Board 2007]

We have $3^3\times5 = 3^2\times5\times3$

$3^2\times5^2 = 3^2\times5\times5$

HCF $(3^3\times5,\ 3^2\times5^2) = 3^2\times5 = 9\times5 = 45$

35. If HCF $(a,b)=12$ and $a\times b=1{,}800$, then find LCM (a,b).

Sol :

We know that

$$\text{HCF}(a,b)\times\text{LCM}(a,b) = a\times b$$

Substituting the values we have

$$12\times\text{LCM}(a,b) = 1800$$

or, $$\text{LCM}(a,\ b) = \frac{1{,}800}{12} = 150$$

36. Find the smallest natural number by which 1200 should be multiplied so that the square root of the product is a rational number.

Sol : [Board Term-1 2016, 2015]

We have $1200 = 12\times100$

$= 4\times3\times4\times25$

$= 4^2\times3\times5^2$

Here if we multiply by 3, then its square root will be $4\times3\times5$ which is a rational number. Thus the required smallest natural number is 3.

37. Can two numbers have 15 as their HCF and 175 as their LCM? Give reasons.

Sol : [Board Term-1 2012]

LCM of two numbers should be exactly divisible by their HCF. Since, 15 does not divide 175, two numbers cannot have their HCF as 15 and LCM as 175.

38. Find the least number that is divisible by all numbers between 1 and 10 (both inclusive).

Sol : [Board 2010]

The required number is the LCM of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,

$\text{LCM} = 2\times2\times3\times2\times3\times5\times7$

$= 2520$

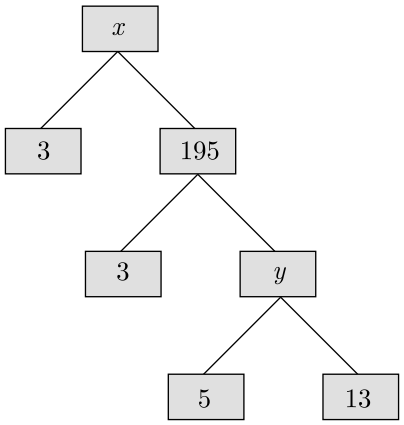
39. Find HCF of the numbers given below:

$k, 2k, 3k, 4k$ and $5k$,, where k is a positive integer.

Sol : [Board Term-1 2015]

Here we can see easily that k is common factor between all and this is highest factor Thus HCF of $k, 2k, 3k, 4k$ and $5k$, is k.

40. Complete the following factor tree and find the composite number x.



Sol : [Board Term-1 2015]

We have $y = 5\times13 = 65$

and $x = 3\times195 = 585$

TWO MARKS QUESTIONS

41. Prove that $5-2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number.

Sol : [Board 2024 OD Standard]

Assume that $5-2\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q\neq0$.

Now $$5-2\sqrt{3} = \frac{p}{q}$$

where $q\neq0$ and p and q are integers.

Rewriting the above expression as,

$$2\sqrt{3} = 5-\frac{p}{q}$$

$$\sqrt{3} = \frac{5q-p}{2q}$$

Here $\frac{5q-p}{2q}$ is rational because p and q are co-prime integers, thus $\sqrt{3}$ should be a rational number. But $\sqrt{3}$ is irrational. This contradicts the given fact that $\sqrt{3}$ is irrational. Hence $5-2\sqrt{3}$ is an irrational number.

42. Show that the number $5\times11\times17+3\times11$ is a composite number.

Sol : [Board 2024 OD Standard]

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$$\begin{aligned} 5 \times 11 \times 17 + 3 \times 11 &= 11(5 \times 17 + 3) \\ &= 11 \times (85 + 3) \\ &= 11 \times 88 \\ &= 11 \times 8 \times 11 \\ &= 2^3 \times 11^2 \end{aligned}$$

As we can see $5 \times 11 \times 17 + 3 \times 11$ can be factorised as $2^3 \times 11^2$. It means it has factors other than 1. Thus $5 \times 11 \times 17 + 3 \times 11$ is a composite number

43. Show that 6^n cannot end with digit 0 for any natural number n.

Sol : [Board 2023 OD Standard]

If the number 6^n for any n, were to end with the digit five, then it would be divisible by 5.

That is, the prime factorization of 6^n would contain the prime 5. This is not possible because the only prime in the factorization of $6^n = (2 \times 3)^n$ are 2 and 3. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 6^n. Since there is no prime factor 5, 6^n cannot end with the digit five.

44. Find the HCF and LCM of 72 and 120.

Sol : [Board 2023 OD Standard]

By Prime Factorisation, we get

$$\begin{aligned} 72 &= 2^3 \times 3^2 \\ 120 &= 2^3 \times 3^1 \times 5^1 \\ \text{HCF } (72, 120) &= 2^3 \times 3^1 \\ &= 8 \times 3 = 24 \\ \text{LCM } (72, 120) &= 2^3 \times 3^2 \times 5 \\ &= 8 \times 9 \times 5 = 360 \end{aligned}$$

Thus, HCF and LCM of 72 and 120 are 24 and 360 respectively.

45. Find the HCF and LCM of 26, 65 and 117, using prime factorisation.

Sol : [Board 2023 OD Standard]

Using prime factorization,

$$\begin{aligned} 26 &= 2 \times 13 \\ 65 &= 5 \times 13 \\ 117 &= 3^2 \times 13 \\ \text{HCF}(26, 65, 117) &= 13 \\ \text{LCM } (26, 65, 117) &= 2 \times 5 \times 3^2 \times 13 \\ &= 1170 \end{aligned}$$

NODIA

46. Explain why $(7 \times 13 \times 11) + 11$ and $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3$ are composite numbers.

Sol : [Board Term-1 2012]

$$\begin{aligned} (7 \times 13 \times 11) + 11 &= 11 \times (7 \times 13 + 1) \\ &= 11 \times (91 + 1) \\ &= 11 \times 92 \end{aligned}$$

and $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3$

$$\begin{aligned} &= 3(7 \times 6 \times 5 \times 4 \times 2 \times 1 + 1) \\ &= 3 \times (1681) = 3 \times 41 \times 41 \end{aligned}$$

Since given numbers have more than two prime factors, both number are composite.

47. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number.

Sol : [Board Term-1 2017]

A prime number (or a prime) is a natural number greater than 1 that cannot be formed by multiplying two smaller natural numbers. A natural number greater than 1 that is not prime is called a composite number. For example, 5 is prime because the only ways of writing it as a product, 1×5 or 5×1, involve 5 itself. However, 6 is composite because it is the product of two numbers (2×3) that are both smaller than 6. Every composite number can be written as the product of two or more (not necessarily distinct) primes.

$$\begin{aligned} 3 \times 12 \times 101 + 4 &= 4(3 \times 3 \times 101 + 1) \\ &= 4(909 + 1) \\ &= 4(910) \\ &= 2 \times 2 \times (10 \times 7 \times 13) \\ &= 2 \times 2 \times 2 \times 5 \times 7 \times 13 \\ &= \text{a composite number} \end{aligned}$$

48. Given that HCF $(306, 1314) = 18$. Find LCM $(306, 1314)$

Sol : [Board Term-1 2013]

We have HCF $(306, 1314) = 18$

LCM $(306, 1314) = ?$

Let $a = 306$ and $b = 1314$, then we have

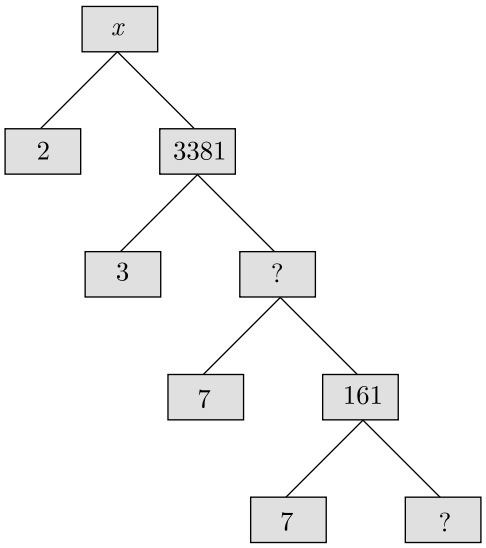
$$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b$$

Substituting values we have

$$\begin{aligned} \text{LCM}(a, b) \times 18 &= 306 \times 1314 \\ \text{LCM } (a, b) &= \frac{306 \times 1314}{18} \end{aligned}$$

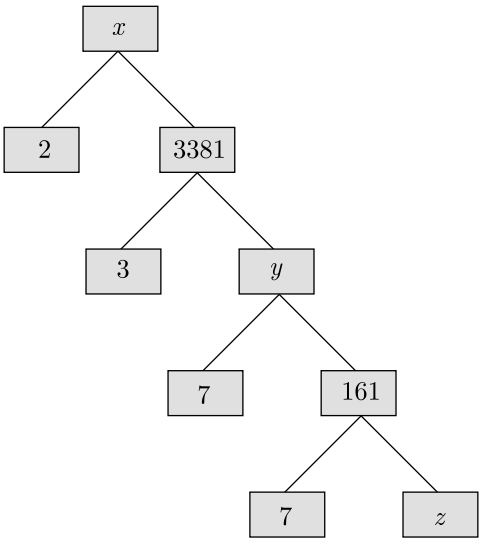
$$\text{LCM } (306, 1314) = 22,338$$

49. Complete the following factor tree and find the composite number x.



Sol : [Board Term-1 2015, 2014]

We complete the given factor tree writing variable y and z as following.



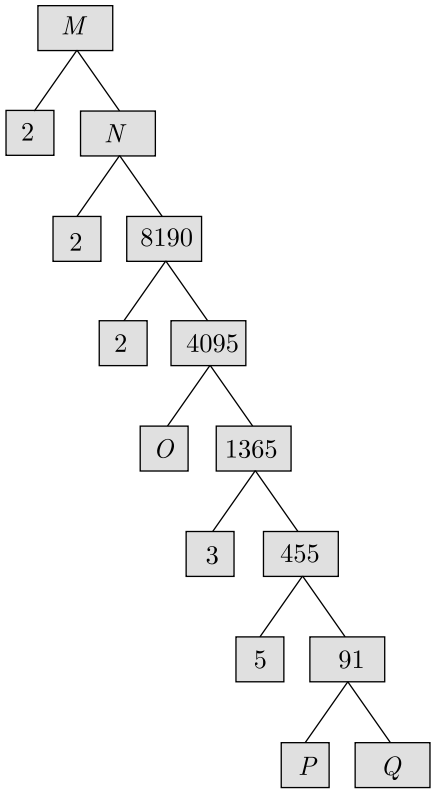
We have $z = \frac{161}{7} = 23$

$y = 7 \times 161 = 1127$

Composite number, $x = 2 \times 3381 = 6762$

PRACTICE

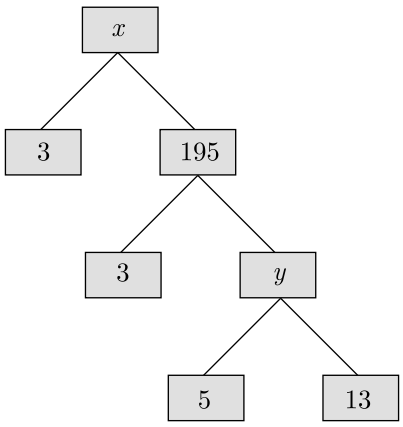
✍ Complete the factor-tree and find the composite number M.



[Board Term-1 2013]

Ans : $M = 32760, N = 16380, O = 3, P = 7$ and $Q = 13$

✍ Complete the following factor tree and find the composite number x.



[Board Term-1 2015]

Ans : $x = 585$ and $y = 65$

✍ Complete the following factor tree and find the composite number x

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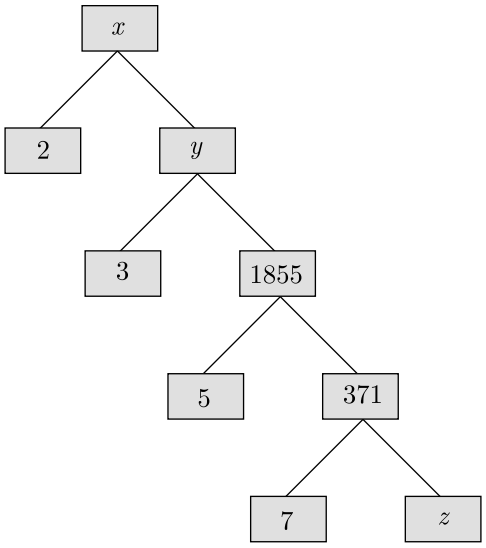
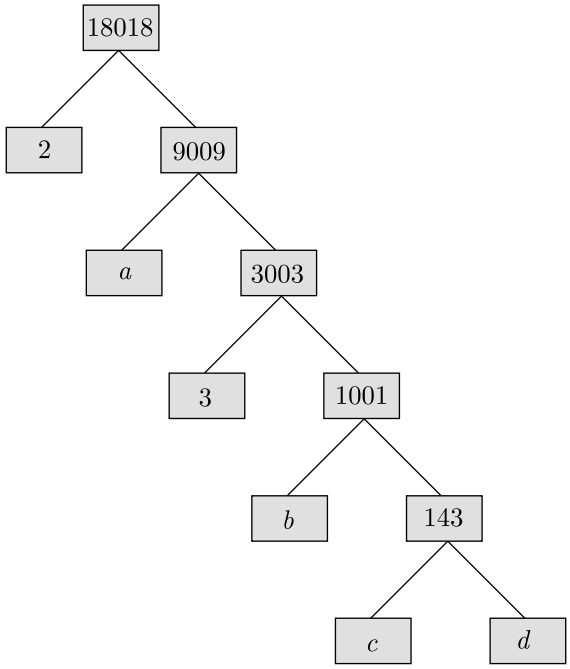


[Board Term-1 2015]

Ans : $x = 11130, y = 5565$ and $z = 53$

✍ Find the missing numbers a, b, c and d in the given factor tree:



[Board Term-1 2012]

Ans : $c = 11$, $b = 7$, $c = 13$ and $d = 11$.

50. Check whether 4^n can end with the digit 0 for any natural number n.

Sol : [Board Term-1 2015]

If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2. That is, the prime factorization of 4^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $4^n = 2^{2n}$ is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero. Hence 4^n cannot end with the digit zero.

PRACTICE

✍ Check whether $(15)^n$ can end with digit 0 for any $n \in N$.

[Board Term-1 2012]

Ans : No

✍ Show that 7^n cannot end with the digit zero, for any natural number n.

[Board Term-1 2012]

Ans : Proof

✍ Show that numbers 8^n can never end with digit 0 of any natural number n.

[Board Term-1 2015]

Ans : Proof

NODIA

✍ Show that $5\sqrt{6}$ is an irrational number.

Sol : [Board Term-1 2015]

Let $5\sqrt{6}$ be a rational number, which can be expressed as $\frac{a}{b}$, where $b \neq 0$; a and b are co-primes.

Now $$5\sqrt{6} = \frac{a}{b}$$

$$\sqrt{6} = \frac{a}{5b}$$

or, $$\sqrt{6} = \text{rational}$$

But, $\sqrt{6}$ is an irrational number. Thus, our assumption is wrong. Hence, $5\sqrt{6}$ is an irrational number.

52. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Sol : [Board Term-1 2013]

We have $\sqrt{2} = \sqrt{\frac{200}{100}}$ and $\sqrt{3} = \sqrt{\frac{300}{100}}$

We need to find a rational number x such that

$$\frac{1}{10}\sqrt{200} < x < \frac{1}{10}\sqrt{300}$$

Choosing any perfect square such as 225 or 256 in between 200 and 300, we have

$$x = \sqrt{\frac{225}{100}} = \frac{15}{10} = \frac{5}{3}$$

Similarly if we choose 256, then we have

$$x = \sqrt{\frac{256}{100}} = \frac{16}{10} = \frac{8}{5}$$

53. Show that 571 is a prime number.

Sol :

Let $x = 571$

$$\sqrt{x} = \sqrt{571}$$

Now 571 lies between the perfect squares of $(23)^2 = 529$ and $(24)^2 = 576$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Here 571 is not divisible by any of the above numbers, thus 571 is a prime number.

54. If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$, where a and b are prime numbers than verify $\text{LCM}(p,q) \times \text{HCF}(q,q) = pq$

Sol : [Sample Paper 2017]

We have $p = a^2b^3 = a \times a \times b \times b \times b$

and $q = a^3b = a \times a \times a \times b$

Now $\text{LCM}(p,q) = a \times a \times a \times b \times b \times b$

$= a^3b^3$

and $\text{HCF}(p,q) = a \times a \times b$

$= a^2b$

$$\text{LCM}(p,q) \times \text{HCF}(p,q) = a^3b^3 \times a^2b$$
$$= a^5b^4$$
$$= a^2b^3 \times a^3b$$
$$= pq$$

55. Prove that $3+\sqrt{5}$ is an irrational number.

Sol : [Board 2010]

Assume that $3+\sqrt{5}$ is a rational number, then we have

$$3+\sqrt{5} = \frac{p}{q}, \quad q \neq 0$$

$$\sqrt{5} = \frac{p}{q} - 3$$

$$\sqrt{5} = \frac{p-3q}{q}$$

Here $\sqrt{5}$ is irrational and $\frac{p-3q}{q}$ is rational. But rational number cannot be equal to an irrational number. Hence $3+\sqrt{5}$ is an irrational number.

56. Show that any positive even integer can be written in the form $6q, 6q+2$ or $6q+4$, where q is an integer.

Sol : [Board Term1, 2016]

Let a be any positive integer, then by Euclid's division algorithm, a can be written as

$$= = bq + r$$

Take $b = 6$, then $0 \leq r < 6$ because $0 \leq r < b$,

Thus $a = 6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5$

Here $6q$, , $6q+2$ and $6q+4$ are divisible by 2 and so $6q$, $6q+2$ and $6q+4$ are even positive integers.

Hence a is always an even integer if

$$a = 6q,\ 6q+2,\ 6q+4$$

PRACTICE

✎ Show that any positive odd integer is of the form $4q+1$ or $4q+3$, where q is some integer. [Board 2019, 2017]

Ans : Proof

THREE MARKS QUESTIONS

57. Prove that $\sqrt{2}$ is an irrational number.

Sol : [Board 2023 OD Standard]

Assume that $\sqrt{2}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Assume that $\sqrt{2}$ be a rational number then we have

$$\sqrt{2} = \frac{p}{q},$$

where p and q are co-primes and $q \neq 0$.

Now $p = q\sqrt{2}$

Squaring both the sides, we have

$$p^2 = 2q^2$$

Thus 2 is a factor of p^2 and in result 2 is also a factor of p.

Let $p = 2c$ where c is some integer, then we have

$$p^2 = 4c^2$$

Substituting $p = 2q^2$ we have

$$2q^2 = 4c^2$$

$$q^2 = 2c^2$$

Thus 2 is a factor of q^2 and in result 2 is also a factor of q.

Thus 2 is a common factor of p and q. But this contradicts the fact that p and q are co-primes. Thus, our assumption that $\sqrt{2}$ is rational number is wrong. Hence $\sqrt{2}$ is irrational.

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58. In a teachers' workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.

Sol : [Board 2024 OD Standard]

The number of rooms will be minimum if every room has maximum number of participants.

Number of teachers in each room must be HCF of 48, 80 and 144.

$$48 = 2^4 \times 3$$
$$80 = 2^4 \times 5$$
$$144 = 2^4 \times 3^2$$

HCF $(48, 80, 144) = 2^4 = 16$

In each room, 16 teachers can be seated.

Minimum number of rooms required

$$= \frac{48 + 80 + 144}{16}$$
$$= 17$$

Thus minimum number of rooms are 17.

59. Given that $\sqrt{5}$ is irrational, prove that $2\sqrt{5} - 3$ is an irrational number.

Sol : [Board 2020 SQP Standard]

Assume that $2\sqrt{5} - 3$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Now $$2\sqrt{5} - 3 = \frac{p}{q}$$

where $q \neq 0$ and p and q are co-prime integers.

Rewriting the above expression as,

$$2\sqrt{5} = \frac{p}{q} + 3$$
$$\sqrt{5} = \frac{p + 3q}{2q}$$

Here $\frac{p+3q}{2q}$ is rational because p and q are co-prime integers, thus $\sqrt{5}$ should be a rational number. But $\sqrt{5}$ is irrational. This contradicts the given fact that $\sqrt{5}$ is irrational. Hence $2\sqrt{5} - 3$ is an irrational number.

PRACTICE

- Given that $\sqrt{3}$ is irrational, prove that $2\sqrt{3} - 5$ is an irrational number.
  [Board 2008, 2015]
  Ans : Proof

60. Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number.

Sol : [Board 2018]

Assume that $(5 + 3\sqrt{2})$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Now $$5 + 3\sqrt{2} = \frac{p}{q}$$

where $q \neq 0$ and p and q are integers.

Rewriting the above expression as,

$$3\sqrt{2} = \frac{p}{q} - 5$$
$$\sqrt{2} = \frac{p - 5q}{3q}$$

Here $\frac{p-5q}{3q}$ is rational because p and q are co-prime integers, thus $\sqrt{2}$ should be a rational number. But $\sqrt{2}$ is irrational. This contradicts the given fact that $\sqrt{2}$ is irrational. Hence $(5 + 3\sqrt{2})$ is an irrational number.

PRACTICE

- Given that $\sqrt{3}$ is irrational, prove that $(5 + 2\sqrt{3})$ is an irrational number.
  [Board 2020 Delhi Basic]
  Ans : Proof

- Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.
  [Board 2019 OD]
  Ans : Proof

- Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.
  [Board 2019 Delhi]
  Ans : Proof

61. Write the smallest number which is divisible by both 306 and 657.

Sol : [Board 2019 OD]

The smallest number that is divisible by two numbers is obtained by finding the LCM of these numbers Here, the given numbers are 306 and 657.

$$306 = 6 \times 51 = 3 \times 2 \times 3 \times 17$$
$$657 = 9 \times 73 = 3 \times 3 \times 73$$
$$\text{LCM}(306, 657) = 2 \times 3 \times 3 \times 17 \times 73$$
$$= 22338$$

Hence, the smallest number which is divisible by 306 and 657 is 22338.

NODIA

62. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Sol : [Board Term-1 2011]

The required answer will be HCF of 144 and 90.

$$144 = 2^4 \times 3^2$$
$$90 = 2 \times 3^2 \times 5$$
$$\text{HCF}(144, 90) = 2 \times 3^2 = 18$$

Thus each stack would have 18 cartons.

63. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?

Sol : [Board Term-1 2011]

The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$$9 = 3 \times 3 = 3^2$$
$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$
$$15 = 3 \times 5$$
$$\text{LCM}(9, 12, 15) = 2^2 \times 3^2 \times 5$$
$$= 150 \text{ minutes}$$

The bells will toll next together after 180 minutes.

64. Find HCF and LCM of 16 and 36 by prime factorization and check your answer.

Sol : [Board 2009]

Finding prime factor of given number we have,

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$
$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$
$$\text{HCF}(16, 36) = 2 \times 2 = 4$$
$$\text{LCM}(16, 36) = 2^4 \times 3^2$$
$$= 16 \times 9 = 144$$

Check :

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$
or, $$4 \times 144 = 16 \times 36$$
$$576 = 576$$
Thus $$\text{LHS} = \text{RHS}$$

PRACTICE

✎ Find the HCF and LCM of 510 and 92 and verify that HCF × LCM = Product of two given numbers.

[Board Term-1 2011]

Ans : 2, 23460

✎ Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.

[Board 2018]

Ans : 4, 9696

65. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

Sol : [Board Term-1 2016]

Here we have to determine the HCF of all length which can measure all dimension.

$$\text{Length, } l = 8 \text{ m } 50 \text{ cm} = 850 \text{ cm}$$
$$= 50 \times 17 = 2 \times 5^2 \times 17$$
$$\text{Breadth, } b = 6 \text{ m } 25 \text{ cm} = 625 \text{ cm}$$
$$= 25 \times 25 = 5^2 \times 5^2$$
$$\text{Height, } h = 4 \text{ m } 75 \text{ cm} = 475 \text{ cm}$$
$$= 25 \times 19 = 5^2 \times 19$$
$$\text{HCF}(l, b, h) = \text{HCF } (850, 625, 475)$$
$$= \text{HCF } (2 \times 5^2 \times 17,\ 5^2,\ 5^2 \times 19)$$
$$= 5^2 = 25 \text{ cm}$$

Thus 25 cm rod can measure the dimensions of the room exactly. This is longest rod that can measure exactly.

FIVE MARKS QUESTIONS

66. If p is prime number, then prove that $\sqrt{p}$ is an irrational.

Sol : [Board Term-1 2013]

Let p be a prime number and if possible, let $\sqrt{p}$ be rational

Thus $$\sqrt{p} = \frac{m}{n},$$

where m and n are co-primes and $n \neq 0$.

Squaring on both sides, we get

$$p = \frac{m^2}{n^2}$$
or, $$pn^2 = m^2 \quad \ldots(1)$$

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Here p divides pn^2. Thus p divides m^2 and in result p also divides m.

Let $m = pq$ for some integer q and putting $m = pq$ in eq. (1), we have

$$pn^2 = p^2q^2$$

or, $$n^2 = pq^2$$

Here p divides pq^2.Thus p divides n^2 and in result p also divides n.

[p is prime and p divides $n^2 \Rightarrow p$ divides n]

Thus p is a common factor of m and n but this contradicts the fact that m and n are primes. The contradiction arises by assuming that $\sqrt{p}$ is rational. Hence, $\sqrt{p}$ is irrational.

67. Prove that $\sqrt{3}$ is an irrational number.

Sol : [Board 2020 OD Basic]

Assume that $\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{a}{b}$ where a and b are co-prime integers and $q \neq 0$.

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b},$$

where a and b are co-primes and $b \neq 0$.

Now $$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 3b^2$ we have

$$3b^2 = 9c^2$$

$$b^2 = 3c^2$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

✎ Prove that $\sqrt{5}$ is an irrational number.

[Board 2020 OD Standard]

Ans : Proof

✎ Prove that $\sqrt{2}$ is an irrational number.

[Board Term-1 2011]

Ans : Proof

68. Prove that $\sqrt{5}$ is an irrational number and hence show that $2 - \sqrt{5}$ is also an irrational number.

Sol : [Board Term-1 2011]

Assume that $\sqrt{5}$ be a rational number then we have

$$\sqrt{5} = \frac{a}{b}, \ (a,b \text{ are co-primes and } b \neq 0)$$

$$a = b\sqrt{5}$$

Squaring both the sides, we have

$$a^2 = 5b^2$$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let $a = 5c$ where c is some integer, then we have

$$a^2 = 25c^2$$

Substituting $a^2 = 5b^2$ we have

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

Let us assume that $2 - \sqrt{5}$ be rational equal to a , then we have

$$2 - \sqrt{5} = a$$

$$2 - a = \sqrt{5}$$

Since we have assume $2 - a$ is rational, but $\sqrt{5}$ is not rational. Rational number cannot be equal to an irrational number. Thus $2 - \sqrt{5}$ is irrational.

✎ Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7 - 2\sqrt{3}$ is also an irrational number.

[Board Term-1 2012]

Ans : Proof

69. Find HCF and LCM of 378, 180 and 420 by prime factorization method. Is HCF × LCM of these numbers equal to the product of the given three numbers?

Sol : [Board 2009]

Finding prime factor of given number we have,

$$378 = 2 \times 3^3 \times 7$$

$$180 = 2^2 \times 3^2 \times 5$$

$$420 = 2^2 \times 3 \times 7 \times 5$$

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$$\text{HCF}(378, 180, 420) = 2 \times 3 = 6$$

$$\text{LCM}(378, 180, 420) = 2^2 \times 3^3 \times 5 \times 7$$

$$= 2^2 \times 3^3 \times 5 \times 7 = 3780$$

$$\text{HCF} \times \text{LCM} = 6 \times 3780 = 22680$$

Product of given numbers

$$= 378 \times 180 \times 420$$

$$= 28576800$$

Hence, HCF $\times$ LCM $\neq$ Product of three numbers.

70. State Fundamental theorem of Arithmetic. Find LCM of numbers 2520 and 10530 by prime factorization.

Sol : [Board Term-1 2016]

The fundamental theorem of arithmetic (FTA), also called the unique factorization theorem or the unique-prime-factorization theorem, states that every integer greater than 1 either is prime itself or is the product of a unique combination of prime numbers.

OR

Every composite number can be expressed as the product powers of primes and this factorization is unique.

Finding prime factor of given number we have,

$$2520 = 20 \times 126 = 20 \times 6 \times 21$$

$$= 2^3 \times 3^2 \times 5 \times 7$$

$$10530 = 30 \times 351 = 30 \times 9 \times 39$$

$$= 30 \times 9 \times 3 \times 13$$

$$= 2 \times 3^4 \times 5 \times 13$$

$$\text{LCM}(2520, 10530) = 2^3 \times 3^4 \times 5 \times 7 \times 13$$

$$= 294840$$

71. Can the number 6^n, n being a natural number, end with the digit 5 ? Give reasons.

Sol : [Board Term-1 2015]

If the number 6^n for any n, were to end with the digit five, then it would be divisible by 5.

That is, the prime factorization of 6^n would contain the prime 5. This is not possible because the only prime in the factorization of $6^n = (2 \times 3)^n$ are 2 and 3. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 6^n. Since there is no prime factor 5, 6^n cannot end with the digit five.

72. State Fundamental theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

Sol : [Board Term-1 2015]

Fundamental theorem of Arithmetic : Every integer greater than one ither is prime itself or is the product of prime numbers and that this product is unique. Up to the order of the factors.

LCM of two numbers should be exactly divisible by their HCF. In other words LCM is always a multiple of HCF. Since, 24 does not divide 540 two numbers cannot have their HCF as 24 and LCM as 540.

$$\text{HCF} = 24$$

$$\text{LCM} = 540$$

$$\frac{LCM}{HCF} = \frac{540}{24} = 22.5 \text{ not an integer}$$

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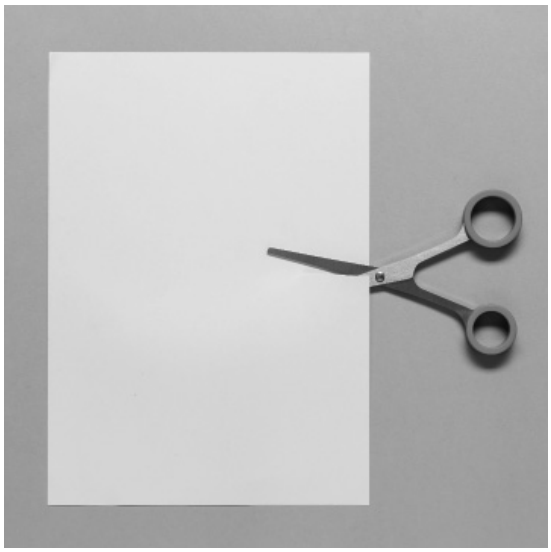
COMPETENCEY BASED QUESTIONS

73. You have a piece of construction paper that measures 32 cm by 48 cm. You want to cut it into squares of equal size.

(i) What will be the dimensions of the largest possible square?

(ii) How many squares will you have?



Sol :

Here we have to find HCF of 32 and 48.

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{HCF}\ (32, 48) = 2 \times 2 \times 2 \times 2 = 16$$

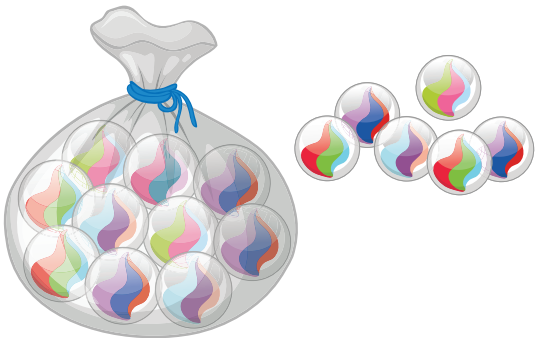
Area of construction paper $= 32 \times 48$ cm^2

Area of square $= 16 \times 16$ cm^2

74. When the marbles in a bag are divided evenly between two friends, there is one marble left over. When the same marbles are divided evenly among three friends, there is one marble left over. When the marbles are divided evenly among five friends, there is one marble left over.

(i) What is the least possible number of marbles in the bag?

(ii) What is another possible number of marbles in the bag?



Sol :

(i) In all three case one marble is left after division. Thus total marble will be one more than LCM of numbers.

$$\text{LCM}(2, 3, 5) = 2 \times 3 \times 5 = 30$$

Thus 31 marbles are in bag.

(ii) If we add one in multiple of 30, we will get another possible number of marble. These are 61, 91, 121....

75. An online shopping website sells 10 types of items which are packed into various sizes of cartons which are given below.

Carton type	Inner Dimensions (L × W)cm^2
Small	6 × 8
Medium	12 × 24
Large	24 × 36
Extra large	36 × 48
XXL	48 × 96

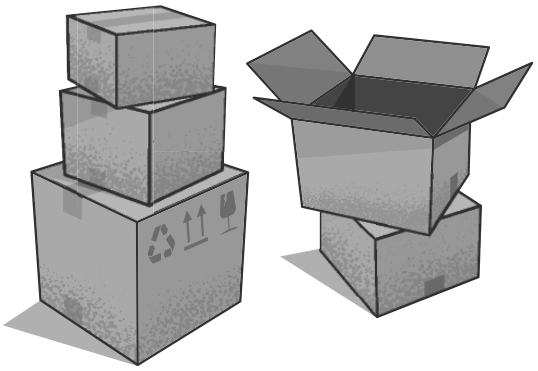
The company places supporting thermocol sheets inside every package along the edges. The company thought of procuring same sized sheets for all types of cartons.

(i) What should be the maximum size of the sheet that fits into all type of cartons?

(ii) How many such sheet sizes are possible?

(iii) The company later introduced a new size of carton called semi large whose measurements are 14 × 15. Whether the existing maximum size sheet fits this shape?

(iv) What should have been the size of the semi large carton (which is larger than medium carton but smaller than large carton) so that the maximum sized sheet remains same?



Sol :

(i) To find dimension of maximum size sheet which can be fitted in all carton, it is required that we should find HCF of length of all different sized cartons that is HCF(6, 12, 24, 36, 48) = 6 and HCF of width of all cartons that is HCF(8, 24, 36, 48, 96) = 4.

Thus maximum size of the sheet is 6 by 4.

(ii) Because HCF of certain numbers is always unique so only one sized sheet is possible.

(iii) 14 is not multiple of 6 and 15 is not multiple of 4 so it is not possible to have a carton with dimension 14 × 15.

(iv) 18 is the only multiple of 6 between 12 and 24 for length of semi sized carton and there are choices for width of semi sized cartons from 28 and 32, so possible answers are 18 × 28 and 18 × 32.

76. Two oil tankers contain 825 litres and 675 litres of kerosene oil respectively.

(i) Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.

(ii) How many times we have to use container for both tanker to fill ?

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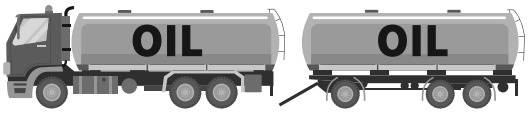

Sol :

The maximum capacity of the required container has to measure both the tankers in a way that the count is an exact number of times. So its capacity is exactly divisible by both the tankers. So we have to find the HCF of 825 and 675.

First we find prime factorization of 825 and 675.

$$675 = 3\times3\times3\times5\times5$$
$$825 = 3\times5\times5\times11$$
$$\text{HCF}(675, 825) = 3\times5\times5$$
$$= 75$$

(i) Thus the maximum capacity of the required container is 75 litres.

(ii) Therefore, the first tanker will require $\frac{825}{75} = 11$ times to fill it and 2nd tanker will require $\frac{675}{75} = 9$ times to fill it.

77. Last year my grand mother was admitted to Fortis hospital due to a small accident. She was prescribed a pain medication to be given every 4 hr and an antibiotic to be given every 5 hr. Bandages applied to the my grand mother's external injuries needed changing every 12 hr. The nurse changed the bandages and gave my grand mother both medications at 6:00 AM Monday morning.

(i) How many hours will pass before the grand mother is given both medications and has her bandages changed at the same time?

(ii) What day and time will this be?

Sol :

Here we have to find LCM of 4, 5, and 12. First we have to find prime factorization of 4, 5, and 12.

$$4 = 2\times2$$
$$5 = 5$$
$$12 = 2\times2\times3$$
$$\text{LCM}(4, 5, 12) = 2\times2\times5\times3$$
$$= 60$$

(i) The shortest length of before all are done at the same time is 60 hours i.e. 2 days 12 hours,

(ii) 2 days 12 hours after 6:00 AM. Monday morning. Thus Wednesday at 6 PM.

Number of square in construction paper $= \frac{32\times48}{16\times16} = 6$

(i) The dimension of largest possible square is 16 cm

(ii) 6 square will be possible in construction paper.

78. Tina has 39 pairs of headphones and 13 music players. Tina wants to sell all of the headphones and music players in identical packages. What is the greatest number of packages Tina can make?

Sol :

In order to know how many packages Tina can make, we need a number that is a factor of 39 and 13, so that the 39 pairs of headphones and the 13 music players can be divided up evenly. To find the greatest number of identical packages, we have to find the HCF of 39 and 13

First we have to find prime factorization of 13 and 39.

$$13 = 13$$
$$39 = 3\times13$$
$$\text{HCF } (13, 39) = 13$$

Tina can make 13 identical package.

79. Taniya have 54 football cards, 72 volleyball cards, and 63 basketball cards and she want to put them in a binder. Each page of the binder should have cards from a single sport, and there should be the same number of cards on each page.

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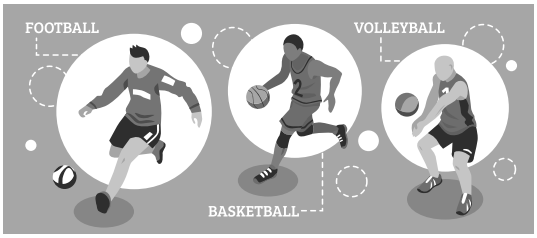
(i) What is the greatest number of cards, Taniya can put on a page?

(ii) How many pages will Taniya need for each sport?



Sol :

Each page of the binder should have cards from a single sport, and there should be the same number of cards on each page. Here we have to find HCF of 54, 63 and 72.

$$54 = 2\times3\times3\times3$$
$$63 = 3\times3\times7$$
$$72 = 2\times2\times2\times3\times3$$
$$\text{HCF}\ (54, 63, 72) = 3\times3 = 9$$

(i) Thus Taniya can put 9 card on a page.

(ii) For football, $\frac{54}{9} = 6$, for volleyball $\frac{72}{9} = 8$ and for basketball $\frac{63}{9} = 7$ pages are needed.

80. A tile floor is to be made from 10 inch, 12 inch, and 15 inch square tiles. A design is made by alternating rows with different size tiles. The first row uses only 10 inch tiles, the second row uses only 12 inch tiles, and the third row uses only 15 inch tiles. Neglecting the grout seams, what is the shortest length of floor space that can be covered evenly by each row?

Sol :

In order to know the shortest length of floor space that can be covered evenly by each row, we have to find LCM of 10, 12, and 15. First we have to find prime factorization of 10, 12, and 15.

$$10 = 2\times5$$
$$12 = 2\times2\times3$$
$$15 = 3\times5$$
$$\text{LCM}(10, 12, 15) = 2\times2\times3\times5 = 60$$

The shortest length of floor space is 60 inch.

81. In a morning walk, three persons step off together. Their steps measure 75 cm, 80 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

Sol :

The distance required by each of them is same as well as minimum, thus we have to find the least common multiple (LCM). First we find prime factorization of 75, 80 and 90.

Now
$$75 = 3\times5\times5$$
$$80 = 2\times2\times2\times2\times5$$
$$90 = 2\times3\times3\times5$$
$$\text{LCM}(75, 80, 90) = 2\times2\times2\times2\times3\times3\times5\times5$$
$$= 3600\ \text{cm}$$

So the required minimum distance is 3600 cm.

82. Four satellites revolve around the earth once every 6, 8, 10, and 15 hr, respectively. If the satellites are initially lined up, how many hours must pass before they will again be lined up?

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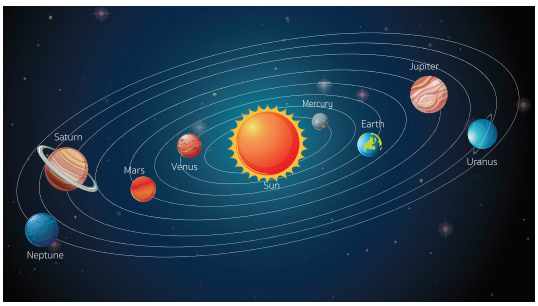
Sol :

In order to know the minimum number of hours required for them to be aligned again, we have to find LCM of 6, 8, 10, and 15. First we find prime factorization of 6, 8, 10, and 15.

$$\begin{aligned} \text{Now} \qquad 6 &= 2\times 3 \\ 8 &= 2\times 2\times 2 \\ 10 &= 2\times 5 \\ 15 &= 3\times 5 \\ \text{LCM}(6, 8, 10, 15) &= 2\times 2\times 2\times 3\times 5 \\ &= 120 \text{ hours} \end{aligned}$$

It will take 120 hours or 5 days for the satellites to be aligned again.

83. Mercury, Venus, and Earth revolve around the Sun approximately once every 3 months, 7 months, and 12 months, respectively. If the planets begin lined up, what is the minimum number of months required for them to be aligned again? (Assume that the planets lie roughly in the same plane.)



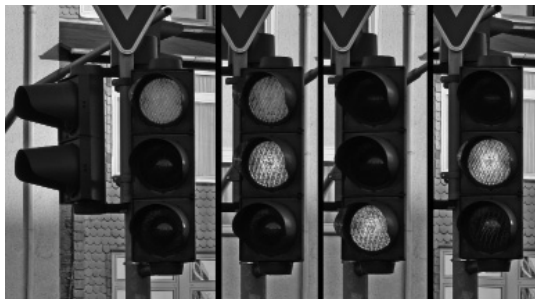
Sol :

In order to know the minimum number of months required for them to be aligned again, we have to find LCM of 3, 7, and 12. First we have to find prime factorization of 3, 7, and 12.

$$\begin{aligned} 3 &= 3 \\ 7 &= 7 \\ 12 &= 2\times 2\times 3 \\ \text{LCM}(2, 7, 12) &= 3\times 7\times 2\times 2 \\ &= 84 \end{aligned}$$

It will take 84 months or 7 years for the planets to be aligned again.

84. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 AM, at what time will they change simultaneously again?



Sol :

The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. As these lights change simultaneously at 7 AM. So the minimum time required to change these lights again. As there is a word simultaneously so you have to find the LCM of 48, 72 and 108. First we find prime factorization of 48, 72 and 108.

$$\begin{aligned} 48 &= 2\times 2\times 2\times 2\times 3 \\ 72 &= 2\times 2\times 2\times 3\times 3 \\ 108 &= 2\times 2\times 3\times 3\times 3 \\ \text{LCM}(48, 72, 108) &= 2\times 2\times 2\times 2\times 3\times 3\times 3 \\ &= 432 \text{ second} \end{aligned}$$

So after 432 seconds or 7 min 12 seconds, all the traffic lights will change simultaneously 7 min 12 seconds past 7 AM.

85. Lina is preparing dinner plates. She has 12 pieces of chicken and 16 rolls. If she wants to make all the plates identical without any food left over, what is the greatest number of plates Lina can prepare ?

Sol :

To make all the plates identical and find the greatest number of plates, we have to find the greatest number that can divide 12 and 16 evenly.

NODIA

That is the highest common factor of 12 and 16.

First we have to find prime factorization of 12 and 16.

$$12 = 2 \times 2 \times 3$$
$$16 = 2 \times 2 \times 2 \times 2$$
$$\text{HCF } (12, 16) = 2 \times 2 = 4$$

Hence, Lina can prepare is 4 plates maximum.

86. At a train station, the blue line has a train leaving every 15 minutes, the green line has a train leaving every 24 minutes, and the red line every 10 minutes. If the first train on each line leaves at the same time, how often will there be trains on all three lines departing the train station at the same time?

Sol :

In order to know how often will there be trains on all three lines departing the train station at the same time, we need a number that is a factor of 10, 15 and 24. Here we have to find LCM of 10, 15, and 24. First we have to find prime factorization of 10, 15, and 24.

$$10 = 2 \times 5$$
$$15 = 3 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3$$
$$\text{LCM}(10, 15, 24) = 2 \times 2 \times 2 \times 3 \times 5$$
$$= 120$$

Train will depart on all three lines at the same time after every 120 minutes or 2 hours.

87. Jasmin is completing an art project. She has two pieces of construction paper. The first piece is 44 centimeters wide and the second piece is 33 centimeters wide. Jasmin wants to cut the paper into strips that are equal in width and are as wide as possible. How wide should Jasmin cut each strip?

Sol :

In order to know how wide Jasmin should cut each strip, we need a number that is a factor of 44 and 33, so that the piece that is 44 centimeters wide and the piece that is 33 centimeters wide can be cut into equal strips. To find the greatest width possible, we want to find the HCF of 44 and 33.

First we have to find prime factorization of 33 and 44.

$$33 = 3 \times 11$$
$$44 = 2 \times 2 \times 11$$
$$\text{HCF } (33, 44) = 11$$

Jasmin should cut 11 cm wide strip.

88. Shalvi is a tuition teacher and teaches mathematics to some kids at her home. She is very innovative and always plan new games to make her students learn concepts.

Today, she has planned a prime number game. She announce the number 2 in her class and asked the first student to multiply it by a prime number and then pass it to second student. Second student

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also multiplied it by a prime number and passed it to third student. In this way by multiplying to a prime number the last student got 173250. He told this number to Shalvi in class. Now she asked some questions to the students as given below.

(i) How many students are in the class?

(ii) What is the highest prime number used by student?

(iii) What is the least prime number used by students ?

(iv) Which prime number has been used maximum times ?

(v) Which prime number has been used minimum times ?

Sol :

(i) Prime factorization of 173250,

$173250 = 2\times3\times3\times5\times5\times5\times7\times11$

It includes 8 numbers. Number 2 has been used by Shalvi. Remaining 7 numbers have been by 7 students.

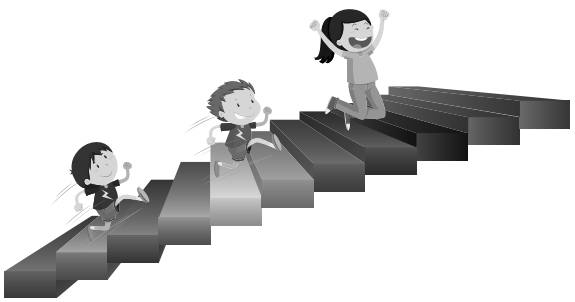
(ii) Highest prime factor included in factorization of 173250 is 11.

(iii) Least prime factor included in factorization of 173250 is 2. But 2 is used by Shalvi, thus next least prime number used by students is 3.

(iv) Number 5 has been used 3 times which is maximum.

(v) Number 7 has been used only one time.

89. Amar, Akbar and Anthony are playing a game. Amar climbs 5 stairs and gets down 2 stairs in one turn. Akbar goes up by 7 stairs and comes down by 2 stairs every time. Anthony goes 10 stairs up and 3 stairs down each time.



Doing this they have to reach to the nearest point of 100th stairs and they will stop once they find it impossible to go forward. They can not cross 100th stair in anyway.

(i) Who reaches the nearest point?

(ii) How many times can they meet in between on same stair ?

(iii) Who takes least number of steps to reach near hundred?

(iv) What is the first stair where any two out of three will meet together?

(v) What is the second stair where any two out of three will meet together?

Sol :

(i) Amar will reach up to 93th stairs then he will go for 5 stairs up and 2 stairs down hence covering 96 stairs. Since 100th stair is final, so he will not cover more stairs. Akbar will reach up to 95th stair, since 100th stair is final, so he will not cover more stairs. Anthony will reach up to 91th stairs, since 100th stair is final, so she will not cover more stairs. Thus amar reaches the nearest point.

(ii) We find the LCM of 3, 5, and 7.

LCM (3, 5, 7) =105th stair.

Since, total stairs are 100, they all cannot meet in between on same stair.

(iii) Amar will take $(\frac{100}{3} = 33.33)$ 32 steps to reach to 96th stair, Akbar will take $(\frac{100}{5} = 20)$ 19 steps to reach to 9th stairs and Anthony will take $(\frac{100}{7} = 14.22)$ 13 steps to reach 91th stairs.

(iv) Since LCM(3, 5)=15 ; LCM(5, 7)=35 ; LCM(3, 7)=21. Since, 15 is the smallest so Amar and Akbar will meet for the first time on 15th stair.

(v) As already calculated in (iii), LCM(3, 7) =21

90. Lavanya wants to organize her birthday party. She is very happy on her birthday. She is very health conscious, thus she decided to serve fruits only in her birthday party.

NODIA

She has 36 apples and 60 bananas at home and decided to serve them. She wants to distribute fruits among guests. She does not want to discriminate among guests, so she decided to distribute fruits equally among all.

(i) How many maximum guests Lavanya can invite?

(ii) How many apples and bananas will each guest get?

(iii) Lavanya decide to add 42 mangoes also. In this case how many maximum guests Lavanya can invite ?

(iv) How many total fruits will each guest get?

(v) If Lavanya decide to add 3 more mangoes and remove 6 apple in total fruits, in this case how many maximum guests Lavanya can invite ?

Sol :

(i) In this case we need to calculate HCF (36, 60) =12.

Thus fruits will be equally distributed among 12 guests.

(ii) Out of 36 apples, each guest will get $\frac{36}{12} = 3$ apples and out of 60 bananas, each guest will get $\frac{60}{5} = 12$ bananas.

(iii) In this case we need to calculate HCF (36, 42, 60) =6.

Thus fruits will be equally distributed among 6 guests.

(iv) Out of 36 apples, each guest will get $\frac{36}{6} = 6$ apples and out of 42 mangoes, each guest will get $\frac{42}{6} = 7$ mangoes, out of 60 bananas, each guest will get $\frac{60}{6} = 10$ bananas. Thus each guest will get $6 + 7 + 12 = 25$ fruits.

(v) Now Lavanya has 30 apples, 60 bananas, and 45 mangoes. HCF (30, 45, 60) = 15. Thus Lavanya can invite 15 guest.

91. The Republic Day parade, first held in 1950, has been a yearly ritual since. The parade marches from the Rashtrapati Bhawan along the Rajpath in New Delhi. Several regiments of the army, navy, and air force, along with their bands, march to India Gate. The parade is presided over by the President of India, who is the Commander-in-Chief of the Indian Armed Forces. As he unfurls the tricolour, the national anthem is played. The regiments of the Armed Forces then start their march past. Prestigious awards like Kirti Chakra, Ashok Chakra, Paramvir Chakra and Vir Chakra are given out by the President. Nine to twelve different regiments of the Indian Army, in addition to the Navy and Air Force march toward India Gate along with their bands. Contingents of paramilitary forces and other civil forces also participate in the parade.

On 71th republic day parade, captain RS Meel is planing for parade of following two group:

(a) First group of Army troops of 624 members behind an army band of 32 members.

(b) Second group of CRPF troops with 468 soldiers behind the 228 members of bikers.

These two groups are to march in the same number of columns. This sequence of soldiers is followed by different states Jhanki which are showing the culture of the respective states.

(i) What is the maximum number of columns in which the army troop can march?

(ii) What is the maximum number of columns in which the CRPF troop can march?

(iii) What is the maximum number of columns in which total army troop and CRPF troop together can march past?

(iv) What should be subtracted with the numbers of CRPF soldiers and the number of bikers so that their maximum number of column is equal to the maximum number of column of army troop?

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(v) What should be added with the numbers of CRPF soldiers and the number of bikers so that their maximum number of column is equal to the maximum number of column of army troop?

Sol :

(i) We will find the HCF (624, 32) = 16

(ii) We will find the HCF (228, 468) = 12.

According to the question, we have to find out

HCF(624, 32, 228, 468) = 4

(iii) Alternatively we can find,

HCF (16, 12) = 4

(iv) Maximum number of column of army troop is 16. But 228 and 468 are not divisible by 16. If we subtract 4 from 228 and 468, both(224 and 464) are divisible by 16.

(v) Maximum number of column of army troop is 16. But 228 and 468 are not divisible by 16. If we add 12 in 228 and 468, both(240 and 480) are divisible by 16.

92. Ashish supplies bread and jams to a hospital and a school. Bread and jam are supplied in equal number of pieces. Bread comes in a packet of 8 pieces and Jam comes in a pack of 6 pieces.

On a particular day, Ashish has supplied x packets of bread and y packets of jam to the school. On the same day, Ashish has supplied $3x$ packets of bread along with sufficient packets of jam to hospital. It is known that the number of students in the school are between 500 and 550.

(i) How many students are there in school?

(ii) How many packets of bread are supplied in the school?

(iii) How many packets of jams are supplied in the school?

(iv) How many packets of bread are supplied in the hospital?

(v) How many packets of jams are supplied in the hospital?

Sol :

(i) First we will find LCM (8, 6) =24. Now we will find a multiple of 24 in between 500 and 550 i.e., 504 or 528. Thus there 504 students in school.

(ii) For equal distribution of bread among each student, we need 504 pieces of bread. Hence, we need $\frac{504}{8} = 63$ i.e. 63 packets of bread.

(iii) For equal distribution of jam pieces among each student, we need 504 pieces of jam. Hence, we need $\frac{504}{6} = 84$ i.e. 84 packets of jam.

(iv) For hospital, we need $3x$ packets of bread i.e. $3 \times 63 = 189$ packets of bread.

(v) Since, number of bread pieces are $189 \times 8 = 1512$, so we need same number of jam pieces. Hence $\frac{1512}{6} = 252$ packets of jam are distributed in the hospital.

93. Mahesh works as a manager in a hotel. He has to arrange chairs in hall for a function. The hall has a certain number of chairs. Guests want to sit in different groups like in pairs, triplets, quadruplets, fives and sixes etc. Mahesh want to arrange chairs in such a way that there are no chair left after arrangement.

When Mahesh arranges chairs in such pattern like in 2's, 3's, 4's 5's and 6's then 1, 2, 3, 4 and 5 chairs are left respectively. But when he arranges in 11's, no chair will be left.

(i) In the hall, how many chairs are available?

(ii) If one chair is removed, which arrangement is possible now?

(iii) If one chair is added to the total number of chairs, how many chairs will be left when arranged in 11's.

(iv) How many chairs will be left in original arrangement if same number of chairs are arranged in 7's?

(v) How many chairs will be left in original arrangement if same number of chairs will be arranged in 9's?

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Sol :

(i) By dividing all the options by 2, 3, 4, 5, 6 and 11, we will get that 539 is the only option which leaves remainder 1, 2, 3, 4, 5, 0 respectively.

(ii) After removing 1 chair, we are left with 538 chairs. On arranging chairs in pair of 3's, 4's, 5's, 6's, 11's ; 1, 2, 3 ,4, 10 chairs are left. So, only pair of 2 chairs is possible now.

(iii) 539 chairs are already arranged in pair of 11's. On adding 1 extra chair, that 1 chair will be left only.

(iv) 539 is divisible by 7 and remainder is zero, so arranging chairs in pair of 7's, no chair will be left.

(v) If 539 is divided by 9, remainder is 8, so arranging chairs in pair of 9's, 8 chair will be left.

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NODIA

CHAPTER 2

Polynomials

OBJECTIVE QUESTIONS

1. The zeroes of a polynomial x^2+px+q are twice the zeroes of the polynomial $4x^2-5x-6$. The value of p is

(a) $-\frac{5}{2}$ (b) $\frac{5}{2}$

(c) -5 (d) 10

Sol : [Board 2024 OD Standard]

Let α and β are zeroes of $4x^2-5x-6$, then zeroes of x^2+px+q will be 2α and 2β

From $4x^2-5x-6$ we have,

Now $\alpha+\beta = -\frac{b}{a} = -\frac{(-5)}{4} = \frac{5}{4}$

and $\alpha\beta = \frac{c}{a} = \frac{-6}{4}$

From x^2+px+q we have,

Now $2\alpha+2\beta = -\frac{b}{a} = -p$

$2(\alpha+\beta) = -p$

Submitting $\alpha+\beta = \frac{5}{4}$ we have

$2\times\frac{5}{4} = -p$

$p = -\frac{5}{2}$

Thus (a) is correct option.

2. If α, β are the zeroes of the polynomial $6x^2-5x-4$, then $\frac{1}{\alpha}+\frac{1}{\beta}$ is equal to

(a) $\frac{5}{4}$ (b) $-\frac{5}{4}$

(c) $\frac{4}{5}$ (d) $\frac{5}{24}$

Sol : [Board 2024 OD Standard]

We have $p(x) = 6x^2-5x-4$

If α and β are zeroes of $p(x) = 6x^2-5x-4$, then

$\alpha+\beta = -\frac{b}{a} = -\frac{(-5)}{6} = \frac{5}{6}$

and $\alpha\beta = \frac{c}{a} = \frac{-4}{6}$

$\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{5}{6}}{-\frac{4}{6}} = -\frac{5}{4}$

Thus (b) is correct option.

3. The number of quadratic polynomials having zeroes -5 and -3 is

(a) 1 (b) 2

(c) 3 (d) more than 3

Sol : [Board 2023 OD Standard]

Let the zeroes of polynomial be $\alpha = -5$ and $\beta = -3$

The general form of polynomial with α and β as the zeroes is given by

$p(x) = k\left[x^2-(\alpha+\beta)x+\alpha\beta\right]$

where k is any real number.

Now $p(x) = k\left[x^2-(\alpha+\beta)x+\alpha\beta\right]$

$= k[x^2-(-8)x+(15)]$

$= k(x^2+8x+15)$

Here k can have any value. Hence, more than 3 polynomials can have the zeroes – 5 and – 3.

Thus (d) is correct option.

4. If α and β are the zeroes of the polynomial x^2-1, then the value of $(\alpha+\beta)$ is

(a) 2 (b) 1

(c) –1 (d) 0

Sol : [Board 2023 OD Standard]

Comparing x^2-1, with ax^2+bx+c we get $b=0$.

Thus $\alpha+\beta = -\frac{b}{c} = 0$

Thus (d) is correct option.

5. If the zeroes of the quadratic polynomial $x^2+(a+1)x+b$ are 2 and -3, then

(a) $a=-7$, $b=-1$ (b) $a=5$, $b=-1$

(c) $a=2$, $b=-6$ (d) $a=0$, $b=-6$

Sol : [Board 2023 OD Standard]

We have $p(x) = x^2+(a+1)x+b$ ...(i)

Quadratic polynomial having zeros 2 and -3, will

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be

$$p(x) = x^2 - (2-3)x + (2)(-3)$$
$$= x^2 + x - 6$$

Comparing with $x^2 + (a+1)x + b$ we get

$$a + 1 = 1 \Rightarrow a = 0$$
$$b = -6$$

Thus (d) is correct option.

6. If the sum of the zeroes of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to their product, then k equals

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

Sol : [Board 2020 OD Basic]

We have $p(x) = kx^2 + 2x + 3k$

Comparing it by $ax^2 + bx + c$, we get $a = k$, $b = 2$ and $c = 3k$.

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\frac{2}{k}$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{3k}{k} = 3$

According to question, we have

$$\alpha + \beta = \alpha\beta$$
$$-\frac{2}{k} = 3 \Rightarrow k = -\frac{2}{3}$$

Thus (d) is correct option.

7. If α and β are the zeroes of the polynomial $x^2 + 2x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

(a) -2 (b) 2

(c) 0

Sol : [Board 2020 Delhi Basic]

Since α and β are the zeros of polynomial $x^2 + 2x + 1$,

Sum of zeroes, $\alpha + \beta = -\frac{2}{1} = -2$

and product of zeroes, $\alpha\beta = \frac{1}{1} = 1$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = -\frac{2}{1} = -2$

Thus (a) is correct option.

8. If α and β are the zeroes of the polynomial $2x^2 - 13x + 6$, then $\alpha + \beta$ is equal to

(a) -3 (b) 3

(c) $\frac{13}{2}$ (d) $-\frac{13}{2}$

Sol : [Board 2020 Delhi Basic]

We have $p(x) = 2x^2 - 13x + 6$

Comparing it with $ax^2 + bx + c$ we get $a = 2$, $b = -13$ and $c = 6$

Sum of zeroes $\alpha + \beta = -\frac{b}{a}$

$$= -\frac{(-13)}{2} = \frac{13}{2}$$

Thus (c) is correct option.

9. If one zero of a quadratic polynomial $(kx^2 + 3x + k)$ is 2, then the value of k is

(a) $\frac{5}{6}$ (b) $-\frac{5}{6}$

(c) $\frac{6}{5}$ (d) $-\frac{6}{5}$

Sol : [Board 2020 Delhi Basic]

We have $p(x) = kx^2 + 3x + k$

Since, 2 is a zero of the quadratic polynomial

$$p(2) = 0$$
$$k(2)^2 + 3(2) + k = 0$$
$$4k + 6 + k = 0$$
$$5k = -6 \Rightarrow k = -\frac{6}{5}$$

Thus (d) is correct option.

10. The maximum number of zeroes a cubic polynomial can have, is

(a) 1 (b) 4

(c) 2 (d) 3

Sol : [Board 2020 OD Basic]

A cubic polynomial has maximum 3 zeroes because its degree is 3.

Thus (d) is correct option.

11. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) -10

(c) -7 (d) -2

Sol : [Board 2020 Delhi Standard]

We have $p(x) = x^2 + 3x + k$

If 2 is a zero of $p(x)$, then we have

$$p(2) = 0$$
$$(2)^2 + 3(2) + k = 0$$
$$4 + 6 + k = 0$$
$$10 + k = 0 \Rightarrow k = -10$$

Thus (b) is correct option.

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12. The quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is

(a) x^2+5x+6 (b) x^2-5x+6

(c) x^2-5x-6 (d) $-x^2+5x+6$

Sol : [Board 2020 Delhi Standard]

Let α and β be the zeroes of the quadratic polynomial, then we have

$$\alpha+\beta = -5$$

and $$\alpha\beta = 6$$

Now $$p(x) = x^2-(\alpha+\beta)x+\alpha\beta$$
$$= x^2-(-5)x+6$$
$$= x^2+5x+6$$

Thus (a) is correct option.

13. If one zero of the polynomial $(3x^2+8x+k)$ is the reciprocal of the other, then value of k is

(a) 3 (b) -3

(c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

Sol : [Board 2020 OD Basic]

Let the zeroes be α and $\frac{1}{\alpha}$.

Product of zeroes, $\alpha\cdot\frac{1}{\alpha} = \frac{\text{constant}}{\text{coefficient of } x^2}$

$$1 = \frac{k}{3} \Rightarrow k = 3$$

Thus (a) is correct option.

14. The zeroes of the polynomial $x^2-3x-m(m+3)$ are

(a) $m, m+3$ (b) $-m, m+3$

(c) $m, -(m+3)$ (d) $-m, -(m+3)$

Sol : [Board 2020 OD Standard]

We have $p(x) = x^2-3x-m(m+3)$

Substituting $x = -m$ in $p(x)$ we have

$$p(-m) = (-m)^2-3(-m)-m(m+3)$$
$$= m^2+3m-m^2-3m = 0$$

Thus $x=-m$ is a zero of given polynomial.

Now substituting $x = m+3$ in given polynomial we have

$$p(x) = (m+3)^2-3(m+3)-m(m+3)$$
$$= (m+3)[m+3-3-m]$$
$$= (m+3)[0] = 0$$

Thus $x=m+3$ is also a zero of given polynomial.

Hence, $-m$ and $m+3$ are the zeroes of given polynomial.

Thus (b) is correct option.

15. The value of x, for which the polynomials x^2-1 and x^2-2x+1 vanish simultaneously, is

(a) 2 (b) -2

(c) -1 (d) 1

Sol :

Both expression $(x-1)(x+1)$ and $(x-1)(x-1)$ have 1 as zero. This both vanish if $x=1$.

Thus (d) is correct option.

16. If α and β are zeroes and the quadratic polynomial $f(x)=x^2-x-4$, then the value of $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$ is

(a) $\frac{15}{4}$ (b) $\frac{-15}{4}$

(c) 4 (d) 15

Sol :

We have $f(x) = x^2-x-4$

$$\alpha+\beta = -\frac{-1}{1} = 1 \text{ and } \alpha\beta = \frac{-4}{1}-4$$

Now $$\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta = \frac{\alpha+\beta}{\alpha\beta}-\alpha\beta$$
$$= -\frac{1}{4}+4 = \frac{15}{4}$$

Thus (a) is correct option.

17. The value of the polynomial $x^8-x^5+x^2-x+1$ is

(a) positive for all the real numbers

(b) negative for all the real numbers

(c) 0

(d) depends on value of x

Sol :

We have $f(x) = x^8-x^5+x^2-x+1$

$f(x)$ is always positive for all $x>1$

For $x=1$ or 0, $f(x) = 1>0$

For $x<0$ each term of $f(x)$ is positive, thus $f(x)>0$. Hence, $f(x)$ is positive for all real x.

Thus (a) is correct option.

18. Lowest value of x^2+4x+2 is

(a) 0 (b) -2

(c) 2 (d) 4

Sol :

$$x^2+4x+2 = (x^2+4x+4)-2$$
$$= (x+2)^2-2$$

Here $(x+2)^2$ is always positive and its lowest value is zero. Thus lowest value of $(x+2)^2-2$ is

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-2 when $x+2=0$.

Thus (b) is correct option.

19. If the sum of the zeroes of the polynomial $f(x)=2x^3-3kx^2+4x-5$ is 6, then the value of k is

(a) 2 (b) -2

(c) 4 (d) -4

Sol :

Sum of the zeroes, $6=\dfrac{3k}{2}$

$$k=\frac{12}{3}=4$$

Thus (c) is correct option.

20. If the square of difference of the zeroes of the quadratic polynomial $x^2+px+45$ is equal to 144, then the value of p is

(a) ± 9 (b) ± 12

(c) ± 15 (d) ± 18

Sol :

We have $f(x)=x^2+px+45$

Then, $\alpha+\beta=\dfrac{-p}{1}=-p$

and $\alpha\beta=\dfrac{45}{1}=45$

According to given condition, we have

$$\begin{aligned}(\alpha-\beta)^2&=144\\(\alpha+\beta)^2-4\alpha\beta&=144\\(-p)^2-4(45)&=144\\p^2&=144+180\\&=324\Rightarrow p=\pm 18\end{aligned}$$

21. If one of the zeroes of the quadratic polynomial $(k-1)x^2+kx+1$ is -3, then the value of k is

(a) $\dfrac{4}{3}$ (b) $\dfrac{-4}{3}$

(c) $\dfrac{2}{3}$ (d) $-\dfrac{2}{3}$

Sol :

If a is zero of quadratic polynomial $f(x)$, then

$$f(a)=0$$

So, $f(-3)=(k-1)(-3)^2+(-3)k+1$

$$\begin{aligned}0&=(k-1)(9)-3k+1\\0&=9k-9-3k+1\\0&=6k-8\\k&=\frac{8}{6}=\frac{4}{3}\end{aligned}$$

Thus (a) is correct option.

22. A quadratic polynomial, whose zeroes are -3 and 4, is

(a) x^2-x+12 (b) x^2+x+12

(c) $\dfrac{x^2}{2}-\dfrac{x}{2}-6$ (d) $2x^2+2x-24$

Sol :

We have $\alpha=-3$ and $\beta=4$.

Sum of zeros $\alpha+\beta=-3+4=1$

Product of zeros, $\alpha\cdot\beta=-3\times 4=-12$

So, the quadratic polynomial is

$$\begin{aligned}x^2-(\alpha+\beta)x+\alpha\beta&=x^2-1\times x+(-12)\\&=x^2-x-12\\&=\frac{x^2}{2}-\frac{x}{2}-6\end{aligned}$$

Thus (c) is correct option.

23. If the zeroes of the quadratic polynomial $x^2+(a+1)x+b$ are 2 and -3, then

(a) $a=-7,b=-1$ (b) $a=5,b=-1$

(c) $a=2,b=-6$ (d) $a=0,b=-6$

Sol :

If a is zero of the polynomial, then $f(a)=0$.

Here, 2 and -3 are zeroes of the polynomial $x^2+(a+1)x+b$

So, $f(2)=(2)^2+(a+1)(-3)+b=0$

$$\begin{aligned}4+2a+2+b&=0\\6+2a+b&=0\\2a+b&=-6 \qquad ...(1)\end{aligned}$$

Again, $f(-3)=(-3)^2+(a+1)2+b=0$

$$\begin{aligned}9-3(a+1)+b&=0\\9-3a-3+b&=0\\6-3a+b&=0\\-3a+b&=-6\\3a-b&=6 \qquad ...(2)\end{aligned}$$

Adding equations (1) and (2), we get

$$5a=0\Rightarrow a=0$$

Substituting value of a in equation (1), we get

$$b=-6$$

Hence, $a=0$ and $b=-6$.

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Thus (d) is correct option.

24. The zeroes of the quadratic polynomial $x^2+99x+127$ are

(a) both positive

(b) both negative

(c) one positive and one negative

(d) both equal

Sol :

Let $f(x) = x^2+99x+127$

Comparing the given polynomial with ax^2+bx+c, we get $a=1$, $b=99$ and $c=127$.

Sum of zeroes $\alpha+\beta = \frac{-b}{a} = -99$

Product of zeroes $\alpha\beta = \frac{c}{a} = 127$

Now, product is positive and the sum is negative, so both of the numbers must be negative.

Alternative Method :

Let $f(x) = x^2+99x+127$

Comparing the given polynomial with ax^2+bx+c, we get $a=1$, $b=99$ and $c=127$.

Now by discriminant rule,

$$D = \sqrt{b^2-4ac}$$
$$= \sqrt{(99)^2-4\times1\times127}$$
$$= \sqrt{9801-508} = \sqrt{9293}$$
$$= 96.4$$

So, the zeroes of given polynomial,

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-99\pm\sqrt{96.4}}{2}$$

Now,as $99 > 96.4$

So, both zeroes are negative.

Thus (b) is correct option.

25. The zeroes of the quadratic polynomial x^2+kx+k where $k\neq0$,

(a) cannot both be positive

(b) cannot both be negative

(c) are always unequal

(d) are always equal

Sol :

Let $f(x) = x^2+kx+k,\ k\neq0$

Comparing the given polynomial with ax^2+bx+c, we get $a=1$, $b=k$ and $c=k$.

Again, let if α,β be the zeroes of given polynomial then,

$$\alpha+\beta = -k$$
$$\alpha\beta = k$$

Case 1: If k is negative, then $\alpha\beta$ is negative. It means α and β are of opposite sign.

Case 2: If k is positive, then $\alpha+\beta$ must be negative and $\alpha\beta$ must be positive and α and β both negative.

Hence, α and β cannot both positive.

Thus (a) is correct option.

26. If the zeroes of the quadratic polynomial ax^2+bx+c, where $c\neq0$, are equal, then

(a) c and a have opposite signs

(b) c and b have opposite signs

(c) c and a have same sign

(d) c and b have the same sign

Sol :

Let $f(x) = ax^2+bx+c$

Let α and β are zeroes of this polynomial

Then, $\alpha+\beta = -\frac{b}{a}$

and $\alpha\beta = \frac{c}{a}$

Since $\alpha=\beta$, then α and β must be of same sign i.e. either both are positive or both are negative. In both case

$$\alpha\beta > 0$$
$$\frac{c}{a} > 0$$

Both c and a are of same sign.

Thus (c) is correct option.

27. If one of the zeroes of a quadratic polynomial of the form x^2+ax+b is the negative of the other, then it

(a) has no linear term and the constant term is negative.

(b) has no linear term and the constant term is positive.

(c) can have a linear term but the constant term is negative.

(d) can have a linear term but the constant term is positive.

Sol :

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Let $f(x) = x^2 + ax + b$

and let the zeroes of $f(x)$ are α and β,

As one of zeroes is negative of other,

sum of zeroes $\alpha + \beta = \alpha + (-\alpha)$

$= 0$ ...(1)

and $\alpha\beta = \alpha \cdot (-\alpha)$

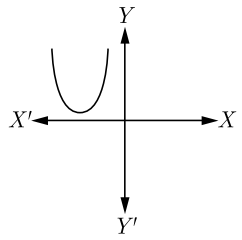
$= -\alpha^2$ ...(2)

Hence, the given quadratic polynomial has no linear term and the constant term is negative.

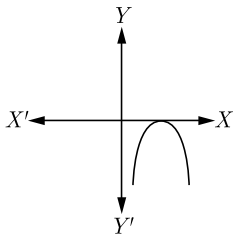
Thus (a) is correct option.

28. Which of the following is not the graph of a quadratic polynomial?

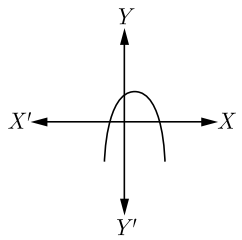
(a)



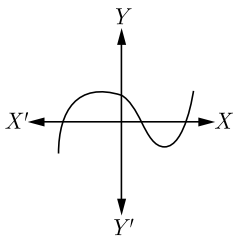
(b)



(c)



(d)



Sol :

As the graph of option (d) cuts x-axis at three points.

So, it does not represent the graph of quadratic polynomial.

Thus (d) is correct option.

29. **Assertion (A):** If the graph of a polynomial touches x-axis at only one point, then the polynomial cannot be a quadratic polynomial.

Reason (R): A polynomial of degree $n(n > 1)$ can have at most n zeroes.

(a) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A)

(b) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A)

(c) Assertion (A) is true but Reason (R) is false

(d) Assertion (A) is false but Reason (R) is true

Sol : [Board 2024 OD Standard]

The quadratic polynomials have at most two zeros but not exactly two zeroes. When the graph line of a polynomial intersects the x axis at only one point, it implies that the polynomial has two equal roots where discriminant = 0. Therefore, the graph can represent a quadratic polynomial. Hence assertion is not true but reason is correct statement.

Thus (d) is correct option.

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30. **Assertion :** $p(x) = 14x^3 - 2x^2 + 8x^4 + 7x - 8$ is a polynomial of degree 3.

Reason : The highest power of x in the polynomial $p(x)$ is the degree of the polynomial.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

The highest power of x in the polynomial $p(x) = 14x^3 - 2x^2 + 8x^4 + 7x - 8$ is 4. Degree of $p(x)$ is 4. So, A is incorrect but R is correct.

Thus (d) is correct option.

31. **Assertion :** $(2 - \sqrt{3})$ is one zero of the quadratic polynomial then other zero will be $(2 + \sqrt{3})$.

Reason : Irrational zeros (roots) always occurs in pairs.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

As irrational roots/zeros always occurs in pairs therefore, when one zero is $(2 - \sqrt{3})$ then other will be $2 + \sqrt{3}$. So, both A and R are correct and R explains A.

Thus (a) is correct option.

32. **Assertion :** If one zero of poly-nominal $p(x)=(k^2+4)x^2+13x+4k$ is reciprocal of other, then $k=2$.

Reason : If $(x-\alpha)$ is a factor of $p(x)$, then $p(\alpha)=0$ i.e. α is a zero of $p(x)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Let $\alpha, \frac{1}{\alpha}$ be the zeroes of $p(x)$, then

$$\alpha \cdot \frac{1}{\alpha} = \frac{4k}{k^2+4}$$

$$1 = \frac{4k}{k^2+4}$$

$$k^2-4k+4 = 0$$

$$(k-2)^2 = 0 \Rightarrow k = 2$$

Thus (b) is correct option.

33. **Assertion :** x^3+x has only one real zero.

Reason : A polynomial of n th degree must have n real zeroes.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

A polynomial of n th degree at most can have n real zeroes. Thus reason is not true.

Again, $x^3+x = x(x^2+1)$

which has only one real zero because $x^2+1 \neq 0$ for all $x \in R$.

Assertion (A) is true but reason (R) is false.

Thus (c) is correct option.

34. **Assertion :** If both zeros of the quadratic polynomial $x^2-2kx+2$ are equal in magnitude but opposite in sign then value of k is ½.

Reason : Sum of zeros of a quadratic polynomial ax^2+bx+c is $\frac{-b}{a}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

As the polynomial is $x^2-2kx+2$ and its zeros are equal but opposition sign, sum of zeroes must be zero.

$$\text{sum of zeros} = 0$$

$$\frac{-(-2k)}{1} = 0 \Rightarrow k = 0$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

ONE MARK QUESTIONS

35. If the sum and product of the zeroes of a quadratic polynomial are 3 and -10 respectively, find the quadratic polynomial.

Sol : [Board 2020 Delhi Basic]

Sum of zeroes, $\alpha+\beta = 3$

and product of zeroes, $\alpha\beta = -10$

Quadratic polynomial,

$$p(x) = x^2-(\alpha+\beta)+\alpha\beta$$

$$= x^2-3x-10$$

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PRACTICE

Find the quadratic polynomial, the sum of whose zeroes is -5 and their product is 6.

[Board 2020 Delhi Standard]

Ans : x^2+5x+6

36. If the sum of the zeroes of the quadratic polynomial $kx^2+2x+3k$ is equal to their product, then what is the value of k?

Sol : [Board 2020 OD Basic]

We have $p(x) = kx^2+2x+3k$

Comparing it by ax^2+bx+c, we get $a = k$, $b = 2$ and $c = 3k$.

Sum of zeroes, $\alpha+\beta = -\frac{b}{a} = -\frac{2}{k}$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{3k}{k} = 3$

According to question, we have

$$\alpha + \beta = \alpha\beta$$

$$-\frac{2}{k} = 3 \Rightarrow k = -\frac{2}{3}$$

37. If α and β are the zeroes of the polynomial $2x^2 - 13x + 6$, then what is the value of $\alpha + \beta$?

Sol : [Board 2020 Delhi Basic]

We have $p(x) = 2x^2 - 13x + 6$

Comparing it with $ax^2 + bx + c$ we get $a = 2$, $b = -13$ and $a = 6$.

Sum of zeroes $\alpha + \beta = -\frac{b}{a}$

$$= -\frac{(-13)}{2} = \frac{13}{2}$$

38. If one zero of a quadratic polynomial $(kx^2 + 3x + k)$ is 2, then the what is the value of k ?

Sol : [Board 2020 Delhi Basic]

We have $p(x) = kx^2 + 3x + k$

Since, 2 is a zero of the quadratic polynomial

$$p(2) = 0$$

$$k(2)^2 + 3(2) + k = 0$$

$$4k + 6 + k = 0$$

$$5k = -6 \Rightarrow k = -\frac{6}{5}$$

PRACTICE

✍ If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the what is the value of k?

[Board 2020 Delhi Standard]

Ans : -10

39. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then what is the value of k ?

Sol : [Board Term-1 Delhi 2013]

If a is zero of quadratic polynomial $f(x)$, then

$$f(a) = 0$$

So, $f(-3) = (k-1)(-3)^2 + (-3)k + 1$

$$0 = (k-1)(9) - 3k + 1$$

$$0 = 9k - 9 - 3k + 1$$

$$0 = 6k - 8$$

$$k = \frac{8}{6} = \frac{4}{3}$$

PRACTICE

✍ Find the value of k if -1 is a zero of the polynomial $p(x) = kx^2 - 4x + k$.

[Board Term-1 2012]

Ans : -2

40. If one zero of the polynomial $(3x^2 + 8x + k)$ is the reciprocal of the other, then what is the value of k?

Sol : [Board 2020 OD Basic]

Let the zeroes be α and $\frac{1}{\alpha}$.

Product of zeroes,

$$\alpha \cdot \frac{1}{\alpha} = \frac{\text{constant}}{\text{coefficient of } x^2}$$

$$1 = \frac{k}{3} \Rightarrow k = 3$$

41. What is the value of x, for which the polynomials $x^2 - 1$ and $x^2 - 2x + 1$ vanish simultaneously?

Sol : [Board Term-1 OD 2011]

Both expression $(x-1)(x+1)$ and $(x-1)(x-1)$ have 1 as zero. This both vanish if $x = 1$.

42. If the square of difference of the zeroes of the quadratic polynomial $x^2 + px + 45$ is equal to 144, then what is the value of p ?

Sol : [Board Term-1 Foreign 2014]

We have $f(x) = x^2 + px + 45$

Then, $\alpha + \beta = \frac{-p}{1} = -p$

and $\alpha\beta = \frac{45}{1} = 45$

According to given condition, we have

$$(\alpha - \beta)^2 = 144$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$

$$(-p)^2 - 4(45) = 144$$

$$p^2 = 144 + 180$$

$$= 324 \Rightarrow p = \pm 18$$

43. Find a quadratic polynomial, whose zeroes are -3 and 4 ?

Sol : [Board 2010]

We have $\alpha = -3$ and $\beta = 4$.

Sum of zeros $\alpha + \beta = -3 + 4 = 1$

Product of zeros, $\alpha \cdot \beta = -3 \times 4 = -12$

So, the quadratic polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 1 \times x + (-12)$$

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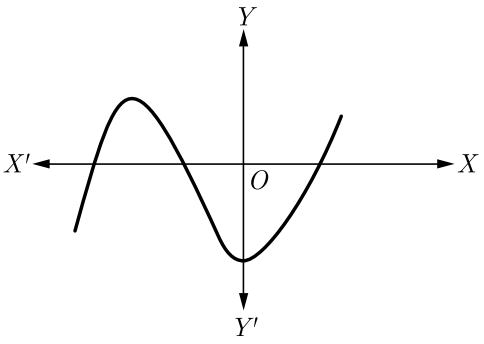
$$= x^2 - x - 12$$

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

44. The graph of a polynomial is shown in Figure. What is the number of its zeroes?

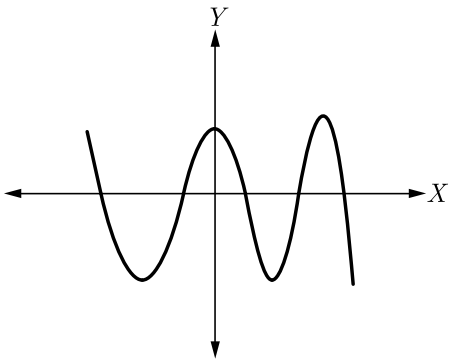


Sol : [Board 2020 Delhi Basic]

Since, the graph cuts the x-axis at 3 points, the number of zeroes of polynomial $p(x)$ is 3.

PRACTICE

✍ The graph of $y = p(x)$, where $p(x)$ is a polynomial in variable x, is as follows.

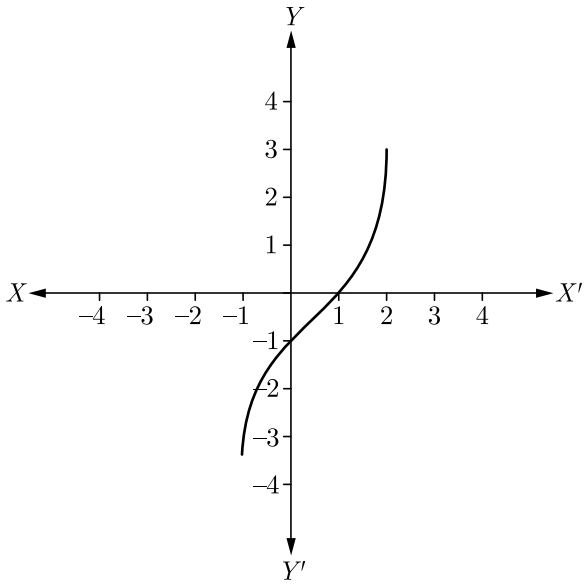


The number of zeroes of $p(x)$ is ..... .

[Board 2020 SQP Standard]

Ans : 5

✍ In given figure, the graph of a polynomial $p(x)$ is shown. Calculate the number of zeroes of $p(x)$.



[Board Term-1 2013]

Ans : 1

45. If α and β are zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, then what is the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$?

Sol : [Board Term-1 Delhi 2017]

We have $f(x) = x^2 - x - 4$

$$\alpha + \beta = -\frac{-1}{1} = 1 \text{ and } \alpha\beta = \frac{-4}{1} - 4$$

Now $$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha + \beta}{\alpha\beta} - \alpha\beta$$

$$= -\frac{1}{4} + 4 = \frac{15}{4}$$

PRACTICE

✍ If α and β are the zeroes of the polynomial $x^2 + 2x + 1$, then what is the value of $\frac{1}{\alpha} + \frac{1}{\beta}$?

[Board 2020 Delhi Basic]

Ans : -2

46. What is the lowest value of $x^2 + 4x + 2$?

Sol : [Board Term-1 OD 2013]

We have $$x^2 + 4x + 2 = (x^2 + 4x + 4) - 2$$

$$= (x + 2)^2 - 2$$

Here $(x+2)^2$ is always positive and its lowest value is zero. Thus lowest value of $(x+2)^2 - 2$ is -2 when $x + 2 = 0$.

47. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then what is the value of k.

Sol : [Board Term-1 2016]

Sum of the zeroes, $6 = \frac{3k}{2}$

$$k = \frac{12}{3} = 4$$

48. Calculate the zeroes of the polynomial $p(x) = 4x^2 - 12x + 9$.

Sol : [Board Term-1 2010]

$$\begin{aligned} p(x) &= 4x^2 - 12x + 9 \\ &= 4x^2 - 6x - 6x + 9 \\ &= 2x(2x-3) - 3(2x-3) \\ &= (2x-3)(2x-3) \end{aligned}$$

Substituting $p(x) = 0$, and solving we get $x = \frac{3}{2}, \frac{3}{2}$

$x = \frac{3}{2}, \frac{3}{2}$

Hence, zeroes of the polynomial are $\frac{3}{2}, \frac{3}{2}$.

49. If sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

Sol : [Board 2009]

We have $p(x) = 3x^2 - kx - 6$

$$\text{Sum of the zeroes} = 3 = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Thus $3 = -\frac{(-k)}{3} \Rightarrow k = 9$

50. If -1 is a zero of the polynomial $f(x) = x^2 - 7x - 8$, then calculate the other zero.

Sol : [Board Term-1 Foreign 2015]

We have $f(x) = x^2 - 7x - 8$

Let other zero be k, then we have

Sum of zeroes, $-1 + k = -\left(\frac{-7}{1}\right) = 7$

or $k = 8$

51. If zeroes of the polynomial $x^2 + 4x + 2a$ are a and $\frac{2}{a}$, then find the value of a.

Sol : [Board Term-1 2016]

Product of (zeroes) roots,

$$\frac{c}{a} = \frac{2a}{1} = \alpha \times \frac{2}{\alpha} = 2$$

or, $2a = 2$

Thus $a = 1$

52. Find all the zeroes of $f(x) = x^2 - 2x$.

Sol : [Board Term-1 2013]

We have $f(x) = x^2 - 2x$

$$= x(x-2)$$

Substituting $f(x) = 0$, and solving we get $x = 0, 2$

Hence, zeroes are 0 and 2.

53. Find the condition that zeroes of polynomial $p(x) = ax^2 + bx + c$ are reciprocal of each other.

Sol : [Board Term-1 2012]

We have $p(x) = ax^2 + bx + c$

Let α and $\frac{1}{\alpha}$ be the zeroes of $p(x)$, then

Product of zeroes,

$$\frac{c}{a} = \alpha \times \frac{1}{\alpha} = 1 \text{ or } \frac{c}{a} = 1$$

So, required condition is, $c = a$

54. Find the values of a and b, if they are the zeroes of polynomial $x^2 + ax + b$.

Sol : [Board Term-1 2013]

We have $p(x) = x^2 + ax + b$

Since a and b, are the zeroes of polynomial, we get,

Product of zeroes, $ab = b \Rightarrow a = 1$

Sum of zeroes, $a + b = -a$

$\Rightarrow b = -2a = -2$

55. What are the zeroes of the polynomial $x^2 - 3x - m(m+3)$?

Sol : [Board 2020 OD Standard]

We have $p(x) = x^2 - 3x - m(m+3)$

Substituting $x = -m$ in $p(x)$ we have

$$\begin{aligned} p(-m) &= (-m)^2 - 3(-m) - m(m+3) \\ &= m^2 + 3m - m^2 - 3m = 0 \end{aligned}$$

Thus $x = -m$ is a zero of given polynomial.

Now substituting $x = m+3$ in given polynomial we have

$$\begin{aligned} p(x) &= (m+3)^2 - 3(m+3) - m(m+3) \\ &= (m+3)[m+3-3-m] \\ &= (m+3)[0] = 0 \end{aligned}$$

Thus $x = m+3$ is also a zero of given polynomial.

Hence, $-m$ and $m+3$ are the zeroes of given polynomial.

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TWO MARKS QUESTIONS

56. Find the zeroes of the quadratic polynomial $\sqrt{3}\,x^2 - 8x + 4\sqrt{3}$.

Sol : [Board Term-1 2013]

We have $p(x) = \sqrt{3}\,x^2 - 8x + 4\sqrt{3}$

$= \sqrt{3}\,x^2 - 6x - 2x + 4\sqrt{3}$

$= \sqrt{3}\,x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$

$= (\sqrt{3}\,x - 2)(x - 2\sqrt{3})$

Substituting $p(x) = 0$, we have

$(\sqrt{3}\,x - 2)(x - 2\sqrt{3})\ p(x) = 0$

Solving we get $x = \frac{2}{\sqrt{3}},\ 2\sqrt{3}$

Hence, zeroes are $\frac{2}{\sqrt{3}}$ and $2\sqrt{3}$.

57. Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively. Hence find the zeroes.

Sol : [Board Term-1 2016]

Sum of zeroes, $\alpha + \beta = 6$

Product of zeroes $\alpha\beta = 9$

Now $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

Thus $= x^2 - 6x + 9$

Thus quadratic polynomial is $x^2 - 6x + 9$.

Now $p(x) = x^2 - 6x + 9$

$= (x - 3)(x - 3)$

Substituting $p(x) = 0$, we get $x = 3, 3$

Hence zeroes are 3, 3.

58. Form a quadratic polynomial $p(x)$ with 3 and $-\frac{2}{5}$ as sum and product of its zeroes, respectively.

Sol : [Board Term-1 2012]

Sum of zeroes, $\alpha + \beta = 3$

Product of zeroes $\alpha\beta = -\frac{2}{5}$

Now $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 3x - \frac{2}{5}$

$= \frac{1}{5}(5x^2 - 15x - 2)$

The required quadratic polynomial is $\frac{1}{5}(5x^2 - 15x - 2)$

PRACTICE

✍ Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.

[Board Term-1 2012]

Ans : $\frac{1}{16}(16x^2 - 42x + 5)$

59. If α and β are the zeroes of the polynomial $f(x) = 5x^2 - 7x + 1$ then find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$

Sol : [Board 2020 OD Basic]

Since, α and β are the zeroes of the quadratic polynomial $f(x) = 5x^2 - 7x + 1$,

Sum of zeros, $\alpha + \beta = -\left(\frac{-7}{5}\right) = \frac{7}{5}$ ...(1)

Product of zeros, $\alpha\beta = \frac{1}{5}$ ...(2)

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{7}{5}\right)^2 - 2 \times \frac{1}{5}}{\frac{1}{5}}$

$= \frac{49 - 2 \times 5}{5} = \frac{39}{5}$

PRACTICE

✍ If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

[Board Term-1 2012]

Ans : $-\frac{145}{12}$

✍ If p and q are the zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

[Board Term-1 2012]

Ans : $\frac{37}{4}$

60. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}\,x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

Sol : [Board Term-1 2015]

We have $p(x) = x^2 - 4\sqrt{3}\,x + 3$

If α and β are the zeroes of $x^2 - 4\sqrt{3}\,x + 3$, then

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\frac{(-4\sqrt{3})}{1}$

or, $\alpha + \beta = 4\sqrt{3}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{3}{1}$

or, $\alpha\beta = 3$

Now $\alpha + \beta - \alpha\beta = 4\sqrt{3} - 3$.

NODIA

61. If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find the value of 'k'.

Sol : [Board Term-1 2012]

We have $f(x) = 14x^2 - 42k^2x - 9$

Let one zero be α, then other zero will be $-\alpha$.

Sum of zeroes $\alpha + (-\alpha) = 0$.

Thus sum of zero will be 0.

$$\text{Sum of zeroes} \quad 0 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$0 = -\frac{42k^2}{14} = -3k^2$$

Thus $k = 0$.

62. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and the other zero.

Sol : [Board Term-1 2012]

Let, the zero of $2x^2 + 3x + \lambda$ be $\frac{1}{2}$ and β.

Product of zeroes $\frac{c}{a}$, $\quad \frac{1}{2}\beta = \frac{\lambda}{2}$

or, $\quad \beta = \lambda$

and sum of zeroes $-\frac{b}{a}, \frac{1}{2} + \beta = -\frac{3}{2}$

or $\quad \beta = -\frac{3}{2} - \frac{1}{2} = -2$

Hence $\quad \lambda = \beta = -2$

Thus other zero is -2.

63. Find the value of k such that the polynomial $x^2 - (k+6)x + 2(2k+1)$ has sum of its zeros equal to half of their product.

Sol : [Board 2019 Delhi]

Let α and β be the roots of given quadratic equation

$$x^2 - (k+6)x + 2(2k+1) = 0$$

Now sum of roots,. $\quad \alpha + \beta = -\frac{-(k+6)}{1}$

$$= k + 6$$

Product of roots, $\quad \alpha\beta = \frac{2(2k+1)}{1}$

$$= 2(2k+1)$$

According to given condition,

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$$k + 6 = \frac{1}{2}[2(2k+1)]$$

$$k + 6 = 2k + 1 \Rightarrow k = 5$$

Hence, the value of k is 5.

64. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

Sol : [Board Term-1 2012]

We have $f(x) = 2x^2 - 5x - 3$

Let the zeroes of polynomial be α and β, then

Sum of zeroes $\quad \alpha + \beta = \frac{5}{2}$

Product of zeroes $\quad \alpha\beta = -\frac{3}{2}$

According to the question, zeroes of $x^2 + px + q$ are 2α and 2β.

Sum of zeros, $\quad 2\alpha + 2\beta = \frac{-p}{1}$

$$2(\alpha + \beta) = -p$$

Substituting $\alpha + \beta = \frac{5}{2}$ we have

$$2 \times \frac{5}{2} = -p$$

or $\quad p = -5$

Product of zeroes, $\quad 2\alpha 2\beta = \frac{q}{1}$

$$4\alpha\beta = q$$

Substituting $\alpha\beta = -\frac{3}{2}$ we have

$$4 \times \frac{-3}{2} = q$$

$$-6 = q$$

Thus $p = -5$ and $q = -6$.

65. If α and β are zeroes of $x^2 - (k-6)x + 2(2k-1)$, find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$.

Sol : [Board Term-1 2013]

We have $p(x) = x^2 - (k-6)x + 2(2k-1)$

Since α, β are the zeroes of polynomial $p(x)$, we get

$$\alpha + \beta = -[-(k-6)] = k - 6$$

$$\alpha\beta = 2(2k-1)$$

Now $\quad \alpha + \beta = \frac{1}{2}\alpha\beta$

Thus $\quad k + 6 = \frac{2(2k-1)}{2}$

or, $\quad k - 6 = 2k - 1$

$$k = -5$$

Hence the value of k is -5.

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PRACTICE

If α and β are the zeroes of the polynomial $f(x)=x^2-6x+k$, find the value of k, such that $\alpha^2+\beta^2=40$.

[Board Term-1 2015]

Ans : -2

THREE MARKS QUESTIONS

66. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x)=ax^2+bx+c,\ a\neq 0,\ c\neq 0$.

Sol : [Board 2020 Delhi Standard]

Let α and β be zeros of the given polynomial ax^2+bx+c.

$$\alpha+\beta=-\frac{b}{a} \text{ and } \alpha\beta=\frac{c}{a}$$

Let $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ be the zeros of new polynomial then we have

Sum of zeros, $s=\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$

$$=\frac{-\frac{b}{a}}{\frac{c}{a}}=\frac{-b}{c}$$

Product of zeros, $p=\frac{1}{\alpha}\cdot\frac{1}{\beta}=\frac{1}{\alpha\beta}=\frac{a}{c}$

Required polynomial,

$$g(x)=x^2-sx+p$$

$$g(x)=x^2+\frac{b}{c}x+\frac{a}{c}$$

$$cg(x)=cx^2+bx+a$$

$$g'(x)=cx^2+bx+a$$

67. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x)=2x^3-11x^2+17x-6$.

Sol : [Board Term-1 2013]

If 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x)=0$

(1) 2, $p(x)=2x^2-11x^2+17x-6$

$$p(2)=2(2)^3-11(2)^2+17(2)-6$$

$$=16-44+34-6$$

$$=50-50$$

or $p(2)=0$

(2) 3, $p(3)=2(3)^3-11(3)^2+17(3)-6$

$$=54-99+51-6=105-105$$

or $p(3)=0$

(3) $\frac{1}{2}$ $p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3-11\left(\frac{1}{2}\right)^2+17\left(\frac{1}{2}\right)-6$

$$=\frac{1}{4}-\frac{11}{4}+\frac{17}{2}-6$$

or $p\left(\frac{1}{2}\right)=0$

Hence, 2, 3, and $\frac{1}{2}$ are the zeroes of $p(x)$.

68. If the sum and product of the zeroes of the polynomial ax^2-5x+c are equal to 10 each, find the value of 'a' and 'c'.

Sol : [Board Term-1 2011]

We have $f(x)=ax^2-5x+c$

Let the zeroes of $f(x)$ be α and β, then,

Sum of zeroes $\alpha+\beta=-\frac{-5}{a}=\frac{5}{a}$

Product of zeroes $\alpha\beta=\frac{c}{a}$

According to question, the sum and product of the zeroes of the polynomial $f(x)$ are equal to 10 each.

Thus $\frac{5}{a}=10$ ...(1)

and $\frac{c}{a}=10$ ...(2)

Dividing (2) by eq. (1) we have

$$\frac{c}{5}=1\Rightarrow c=5$$

Substituting $c=5$ in (2) we get $a=\frac{1}{2}$

Hence $a=\frac{1}{2}$ and $c=5$.

69. If one the zero of a polynomial $3x^2-8x+2k+1$ is seven times the other, find the value of k.

Sol : [Board Term-1 2011]

We have $f(x)=3x^2-8x+2k+1$

Let α and β be the zeroes of the polynomial, then

$$\beta=7\alpha$$

Sum of zeroes, $\alpha+\beta=-\left(-\frac{8}{3}\right)$

$$\alpha+7\alpha=8\alpha=\frac{8}{3}$$

So $\alpha=\frac{1}{3}$

Product of zeroes, $\alpha\times 7\alpha=\frac{2k+1}{3}$

$$7\alpha^2=\frac{2k+1}{3}$$

$$7\left(\frac{1}{3}\right)^2=\frac{2k+1}{3}$$

$$7\times\frac{1}{9}=\frac{2k+1}{1}$$

$$\frac{7}{3}-1=2k$$

NODIA

$$\frac{4}{3} = 2k \Rightarrow k = \frac{2}{3}$$

70. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Sol : [Board Term-1 2015]

We have $f(x) = 2x^2 - 3x + 1$

If α and β are the zeroes of $2x^2 - 3x + 1$, then

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{1}{2}$

New quadratic polynomial whose zeroes are 3α and 3β is,

$$\begin{aligned} p(x) &= x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta \\ &= x^2 - 3(\alpha + \beta)x + 9\alpha\beta \\ &= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right) \\ &= x^2 - \frac{9}{2}x + \frac{9}{2} \\ &= \frac{1}{2}(2x^2 - 9x + 9) \end{aligned}$$

Hence, required quadratic polynomial is $\frac{1}{2}(2x^2 - 9x + 9)$

71. If α and β are the zeroes of the polynomial $f(x) = x^2 - 4x - 5$ then find the value of $\alpha^2 + \beta^2$

Sol : [Board 2020 Delhi Basic]

We have $p(x) = x^2 - 4x - 5$

Comparing it by $ax^2 + bx + c$, we get $a = 1$, $b = -4$ and $c = -5$

Since, given α and β are the zeroes of the polynomial,

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{1} = 4$

and product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{-5}{1} = -5$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (4)^2 - 2(-5)$

$= 16 + 10 = 26$

72. Find the quadratic polynomial, the sum and product of whose zeroes are -3 and 2 respectively. Hence find the zeroes.

Sol : [Board 2020 OD Basic]

Sum of zeroes $\alpha + \beta = -3$ ...(1)

and product of zeroes $\alpha\beta = 2$

Thus quadratic equation is

$$\begin{aligned} x^2 - (\alpha + \beta)x + \alpha\beta &= 0 \\ x^2 - (-3)x + 2 &= 0 \\ x^2 + 3x + 2 &= 0 \end{aligned}$$

Thus quadratic equation is $x^2 + 3x + 2 = 0$.

Now above equation can be written as

$$\begin{aligned} x^2 + 2x + x + 2 &= 0 \\ x(x+2) + (x+2) &= 0 \\ (x+2)(x+1) &= 0 \end{aligned}$$

Hence, zeroes are -2 and -1.

73. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.

Sol : [Board 2020 Delhi Basic]

We have $p(x) = 6x^2 - 3 - 7x$

For zeroes of polynomial, $p(x) = 0$,

$$\begin{aligned} 6x^2 - 7x - 3 &= 0 \\ 6x^2 - 9x + 2x - 3 &= 0 \\ 3x(2x-3) + 1(2x-3) &= 0 \\ (2x-3)(3x+1) &= 0 \end{aligned}$$

Thus $2x - 3 = 0$ and $3x + 1 = 0$

Hence $x = \frac{3}{2}$ and $x = -\frac{1}{3}$

Therefore $\alpha = \frac{3}{2}$ and $\beta = -\frac{1}{3}$ are the zeroes of the given polynomial.

Verification :

Sum of zeroes, $\alpha + \beta = \frac{3}{2} + \left(-\frac{1}{3}\right) = \frac{3}{2} - \frac{1}{3} = \frac{7}{6}$

$$= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and product of zeroes

$$\alpha\beta = \left(\frac{3}{2}\right)\left(-\frac{1}{3}\right) = -\frac{1}{2}$$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

PRACTICE

Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

[Board 2020 Delhi Basic]

Ans : -2 and -5

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14. If the zeroes of the quadratic polynomial $x^2+(a+1)x+b$ are 2 and -3, then find the value of a and b.

Sol : [Board Term-1 2013]

If a is zero of the polynomial, then $f(a)=0$.

Here, 2 and -3 are zeroes of the polynomial $x^2+(a+1)x+b$

So, $f(2)=(2)^2+(a+1)(-3)+b=0$

$4+2a+2+b=0$

$6+2a+b=0$

$2a+b=-6$ ...(1)

Again, $f(-3)=(-3)^2+(a+1)2+b=0$

$9-3(a+1)+b=0$

$9-3a-3+b=0$

$6-3a+b=0$

$-3a+b=-6$

$3a-b=6$ ...(2)

Adding equations (1) and (2), we get

$5a=0 \Rightarrow a=0$

Substituting value of a in equation (1), we get

$b=-6$

Hence, $a=0$ and $b=-6$.

15. If α and β are the zeroes of the polynomial $6y^2-7y+2$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Sol : [Board Term-1 2011]

We have $p(y)=6y^2-7y+2$

Sum of zeroes $\alpha+\beta=-\left(-\frac{7}{6}\right)=\frac{7}{6}$

Product of zeroes $\alpha\beta=\frac{2}{6}=\frac{1}{3}$

Sum of zeroes of new polynomial $g(y)$

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{\frac{7}{6}}{\frac{2}{6}}=\frac{7}{2}$$

and product of zeroes of new polynomial $g(y)$,

$$\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{\alpha\beta}=\frac{1}{\frac{1}{3}}=3$$

The required polynomial is

$$g(x)=y^2-\frac{7}{2}y+3=\frac{1}{2}[2y^2-7y+6]$$

16. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2+4x-3$ and verify relationship between zeroes and coefficients of the polynomial.

Sol : [Board Term-1 2011]

We have $p(x)=4x^2+4x-3$

If $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x)=0$

$$p\left(\frac{1}{2}\right)=4\left(\frac{1}{4}\right)+4\left(\frac{1}{2}\right)-3$$
$$=1+2-3=0$$

and $$p\left(-\frac{3}{2}\right)=4\left(\frac{9}{2}\right)+4\left(-\frac{3}{2}\right)-3$$
$$=9-6-3=0$$

Thus $\frac{1}{2},-\frac{3}{2}$ are zeroes of polynomial $4x^2+4x-3$.

$$\text{Sum of zeroes}=\frac{1}{2}-\frac{3}{2}=-1=\frac{-4}{4}$$
$$=-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$
$$\text{Product of zeroes}=\left(\frac{1}{2}\right)\left(-\frac{3}{2}\right)=\frac{-3}{4}$$
$$=\frac{\text{Constan term}}{\text{Coefficient of } x^2}$$ Verified

17. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students :

$2x+3$, $3x^2+7x+2$, $4x^3+3x^2+2$, $x^3+\sqrt{3x}+7$, $7x+\sqrt{7}$, $5x^3-7x+2$, $2x^2+3-\frac{5}{x}$, $5x-\frac{1}{2}$, ax^3+bx^2+cx+d, $x+\frac{1}{x}$.

Answer the following question :

(i) How many of the above ten, are not polynomials?

(ii) How many of the above ten, are quadratic polynomials?

Sol : [Board 2020 OD Standard]

(i) $x^3+\sqrt{3x}+7, 2x^2+3-\frac{5}{x}$ and $x+\frac{1}{x}$ are not polynomials.

(ii) $3x^2+7x+2$ is only one quadratic polynomial.

18. Find the zeroes of the quadratic polynomial $5x^2+8x-4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Sol : [Board Term-1 2013]

We have $p(x)=5x^2+8x-4=0$

$=5x^2+10x-2x-4=0$

$=5x(x+2)-2(x+2)=0$

$=(x+2)(5x-2)$

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Substituting $p(x)=0$ we get zeroes as -2 and $\frac{2}{5}$.

Verification :

$$\text{Sum of zeroes} = -2+\frac{2}{5}=\frac{-8}{5}$$

$$\text{Product of zeroes} = (-2)\times\left(\frac{2}{5}\right)=\frac{-4}{5}$$

Now from polynomial we have

$$\text{Sum of zeroes} \quad -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=\frac{-8}{5}$$

$$\text{Product of zeroes} \quad \frac{c}{a} = \frac{\text{Constan term}}{\text{Coefficient of } x^2} = -\frac{4}{5}$$

Hence Verified.

PRACTICE

Find the zeroes of the quadratic polynomial $x^2-2\sqrt{2}\,x$ and verify the relationship between the zeroes and the coefficients of the polynomial.

[Board Term-1 2015]

Ans : 0 and $2\sqrt{2}$

79. If α and β are the zeroes of a quadratic polynomial such that $\alpha+\beta=24$ and $\alpha-\beta=8$. Find the quadratic polynomial having α and β as its zeroes.

Sol : [Board 2009]

We have $\alpha+\beta = 24$ ...(1)

$\alpha-\beta = 8$ ...(2)

Adding equations (1) and (2) we have

$$2\alpha = 32 \Rightarrow \alpha = 16$$

Subtracting (1) from (2) we have

$$2\beta = 16 \Rightarrow \beta = 8$$

Hence, the quadratic polynomial

$$\begin{aligned} p(x) &= x^2-(\alpha+\beta)x+\alpha\beta \\ &= x^2-(16+8)x+(16)(8) \\ &= x^2-24x+128 \end{aligned}$$

80. If α,β and γ are zeroes of the polynomial $6x^3+3x^2-5x+1$, then find the value of $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$.

Sol : [Board 2010]

We have $p(x) = 6x^3+3x^2-5x+1$

Since α,β and γ are zeroes polynomial $p(x)$, we have

$$\alpha+\beta+\gamma = -\frac{b}{c}=-\frac{3}{6}=-\frac{1}{2}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}=-\frac{5}{6}$$

and $$\alpha\beta\gamma = -\frac{d}{a}=-\frac{1}{6}$$

Now $$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$$

$$= \frac{-\frac{5}{6}}{-\frac{1}{6}}=\frac{-5}{6}\times\frac{6}{-1}=5$$

Hence $\alpha^{-1}+\beta^{-1}+\gamma^{-1}=5$.

81. When $p(x)=x^2+7x+9$ is divisible by $g(x)$, we get $(x+2)$ and -1 as the quotient and remainder respectively, find $g(x)$.

Sol : [Board Term-1 2011]

We have $p(x) = x^2+7x+9$

$q(x) = x+2$

$r(x) = -1$

Now $p(x) = g(x)q(x)+r(x)$

$x^2+7x+9 = g(x)(x+2)-1$

or, $$g(x) = \frac{x^2+7x+10}{x+2}$$

$$= \frac{(x+2)(x+5)}{(x+2)} = x+5$$

Thus $g(x) = x+5$

82. Find the value for k for which $x^4+10x^3+25x^2+15x+k$ is exactly divisible by $x+7$.

Sol : [Board Term 2010]

We have $f(x) = x^4+10x^3+25x^2+15x+k$

If $x+7$ is a factor then -7 is a zero of $f(x)$ and $x=-7$ satisfy $f(x)=0$.

Thus substituting $x=-7$ in $f(x)$ and equating to zero we have,

$$(-7)^4+10(-7)^3+25(-7)^2+15(-7)+k=0$$

$$2401-3430+1225-105+k = 0$$

$$3626-3535+k = 0$$

$$91+k = 0$$

$$k = -91$$

83. On dividing the polynomial $4x^4-5x^3-39x^2-46x-2$ by the polynomial $g(x)$, the quotient is x^2-3x-5 and the remainder is $-5x+8$. Find the polynomial $g(x)$.

Sol : [Board 2009]

Dividend $=$ (Divisor $\times$ Quotient) $+$ Remainder

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$$4x^4 - 5x^3 - 39x^3 - 46x - 2$$
$$= g(x)(x^2 - 3x - 5) + (-5x + 8)$$
$$4x^2 - 5x^3 - 39x^2 - 46x - 2 + 5x - 8$$
$$= g(x)(x^2 - 3x - 5)$$
$$4x^4 - 5x^3 - 39x^2 - 41x - 10 = g(x)(x^2 - 3x - 5)$$
$$g(x) = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{(x^2 - 3x - 5)}$$
Hence, $g(x) = 4x^2 + 7x + 2$

84. If the squared difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Sol : [Board 2008]

We have $f(x) = x^2 + px + 45$

Let α and β be the zeroes of the given quadratic polynomial.

Sum of zeroes, $\alpha + \beta = -p$

Product of zeroes $\alpha\beta = 45$

Given, $(\alpha - \beta)^2 = 144$

$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$

Substituting value of $\alpha + \beta$ and $\alpha\beta$ we get

$$(-p)^2 - 4 \times 45 = 144$$
$$p^2 - 180 = 144$$
$$p^2 = 144 + 180 = 324$$

Thus $p = \pm\sqrt{324} = \pm 18$

Hence, the value of p is ± 18.

FIVE MARKS QUESTIONS

85. Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then find the value of p and q.

Sol : [Board Term-1 2015]

We have $f(x) = x^4 + 7x^3 + 7x^2 + px + q$

Now $x^2 + 7x + 12 = 0$

$$x^2 + 4x + 3x + 12 = 0$$
$$x(x + 4) + 3(x + 4) = 0$$
$$(x + 4)(x + 3) = 0$$
$$x = -4, -3$$

Since $f(x) = x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then $x = -4$ and $x = -3$ must be its zeroes and these must satisfy $f(x) = 0$

So putting $x = -4$ and $x = -3$ in $f(x)$ and equating to zero we get

$$f(-4) : (-4)^4 + 7(-4)^3 + 7(-4)^2 + p(-4) + q = 0$$
$$256 - 448 + 112 - 4p + q = 0$$
$$-4p + q - 80 = 0$$
$$4p - q = -80 \quad ...(1)$$
$$f(-3) : (-3)^4 + 7(-3)^3 + 7(-3)^2 + p(-3) + q = 0$$
$$81 - 189 + 63 - 3p + q = 0$$
$$-3p + q - 45 = 0$$
$$3p - q = -45 \quad ...(2)$$

Subtracting equation (2) from (1) we have

$$p = -35$$

Substituting the value of p in equation (1) we have

$$4(-35) - q = -80$$
$$-140 - q = -80$$
$$-q = 140 - 80$$

or $-q = 60$

$$q = -60$$

Hence, $p = -35$ and $q = -60$.

86. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k.

Sol : [Board Term-1 2012]

We have $p(x) = 2x^2 + 5x + k$

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\left(\frac{5}{2}\right)$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{k}{2}$

According to the question,

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$$
$$\alpha^2 + \beta^2 + 2\alpha\beta - \alpha\beta = \frac{21}{4}$$
$$(\alpha + b)^2 - \alpha\beta = \frac{21}{4}$$

Substituting values we have

$$\left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$
$$\frac{k}{2} = \frac{25}{4} - \frac{21}{4}$$
$$\frac{k}{2} = \frac{4}{4} = 1$$

Hence, $k = 2$

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87. Find the zeroes of the quadratic polynomial $7y^2-\frac{11}{3}y-\frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

Sol : [Board 2019 OD]

We have $7y^2-\frac{11}{3}y-\frac{2}{3}=0$

$$21y^2-11y-2=0 \quad ...(1)$$

$$21y^2-14y+3y-2=0$$

$$7y(3y-2)+(3y-2)=0$$

$$(3y-2)(7y+1)=0$$

$$y=\frac{2}{3},\ \frac{-1}{7}$$

Hence, zeros of given polynomial are,

$$y=\frac{2}{3} \text{ and } y=\frac{-1}{7}$$

Comparing the given equation with $ax^2+bx+c=0$ we get $a=21$, $b=-11$ and $c=-2$

Now, sum of roots, $\alpha+\beta=\frac{2}{3}+\left(-\frac{1}{7}\right)$

$$=\frac{2}{3}-\frac{1}{7}=\frac{11}{21}$$

Thus $\alpha+\beta=-\frac{b}{a}$ Hence verified

and product of roots, $\alpha\beta=\frac{2}{3}\times\left(-\frac{1}{7}\right)=\frac{-2}{21}$

Thus $\alpha\beta=\frac{c}{a}$ Hence verified

88. If α and β are the zeroes of polynomial $p(x)=3x^2+2x+1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

Sol : [Board Term-1 2010, 2012]

We have $p(x)=3x^2+2x+1$

Since α and β are the zeroes of polynomial $3x^2+2x+1$, we have

$$\alpha+\beta=-\frac{2}{3}$$

and $\alpha\beta=\frac{1}{3}$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1+\beta_1=\frac{1-\alpha}{1+\alpha}+\frac{1-\beta}{1+\beta}$$

$$=\frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$$

$$=\frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta}=\frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$$=\frac{\frac{4}{3}}{\frac{2}{3}}=2$$

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1=\left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$$

$$=\frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$

$$=\frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta}$$

$$=\frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$=\frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}}=\frac{\frac{6}{3}}{\frac{2}{3}}=3$$

Hence, Required polynomial

$$q(x)=x^2-(\alpha_1+\beta_1)2x+\alpha_1\beta_1$$

$$=x^2-2x+3$$

PRACTICE

If α and β are the zeroes of the polynomial x^2+4x+3, find the polynomial whose zeroes are $1+\frac{\beta}{\alpha}$ and $1+\frac{\alpha}{\beta}$.

[Board Term-1 2013]

Ans : $\frac{1}{3}(3x^2-16x+16)$

89. If β and $\frac{1}{\beta}$ are zeroes of the polynomial $(a^2+a)x^2+61x+6a$. Find the value of β and α.

Sol :

We have $p(x)=(a^2+a)x^2+61x+6$

Since β and $\frac{1}{\beta}$ are the zeroes of polynomial, $p(x)$

Sum of zeroes, $\beta+\frac{1}{\beta}=-\frac{61}{a^2+a}$

or, $\frac{\beta^2+1}{\beta}=\frac{-61}{a^2+a}$ ...(1)

Product of zeroes $\beta\frac{1}{\beta}=\frac{6a}{a^2+a}$

or, $1=\frac{6}{a+1}$

$$a+1=6$$

$$a=5$$

Substituting this value of a in (1) we get

$$\frac{\beta^2+1}{\beta}=\frac{-61}{5^2+5}=-\frac{61}{30}$$

$$30\beta^2+30=-61\beta$$

$$30\beta^2+61\beta+30=0$$

Now $\beta\ \frac{-61\pm\sqrt{(-61)^2-4\times30\times30}}{2\times30}$

$$=\frac{-61\pm\sqrt{3721-3600}}{60}$$

$$\frac{-61\mp11}{60}$$

Thus $\beta = \frac{-5}{6}$ or $\frac{-6}{5}$

Hence, $\alpha = 5, \beta = \frac{-5}{6}, \frac{-6}{5}$

90. If α and β are the zeroes the polynomial $2x^2 - 4x + 5$, find the values of

(i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

(iii) $(\alpha - \beta)^2$ (iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(v) $\alpha^2 + \beta^2$

Sol : [Board 2007]

We have $p(x) = 2x^2 - 4x + 5$

If α and β are then zeroes of $p(x) = 2x^2 - 4x + 5$, then

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{2} = 2$$

and $$\alpha\beta = \frac{c}{a} = \frac{5}{2}$$

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 2^2 - 2 \times \frac{5}{2}$

$= 4 - 5 = -1$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{2}{\frac{5}{2}} = \frac{4}{5}$

(iii) $(\alpha - \beta)^2 = (\alpha - \beta)^2 - 4\alpha\beta$

$= 2^2 - \frac{4 \times 5}{2}$

$4 - 10 = -6$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{-1}{(\frac{5}{2})^2} = \frac{-4}{25}$

(v) $(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= 2^3 - 3 \times \frac{5}{2} \times 2 = 8 - 15 = -7$

91. If α and β are zeroes of the polynomial $p(x) = 6x^2 - 5x + k$ such that $\alpha - \beta = \frac{1}{6}$, Find the value of k.

Sol : [Board 2007]

We have $p(x) = 6x^2 - 5x + k$

Since α and β are zeroes of

$p(x) = 6x^2 - 5x + k$,

Sum of zeroes, $\alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ ...(1)

Product of zeroes $\alpha\beta = \frac{k}{6}$ ...(2)

Given $\alpha - \beta = \frac{1}{6}$ ...(3)

Solving (1) and (3) we get $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ and substituting the values of (2) we have

$$\alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3}$$

Hence, $k = 1$.

PRACTICE

If α and β are zeroes of the polynomial $f(x) = x^2 - x - k$, such that $\alpha - \beta = 9$, find k.

[Board Term-1 2013]

Ans : 20

COMPETENCEY BASED QUESTIONS

92. Box : For the box to satisfy certain requirements, its length must be three unit greater than the width, and its height must be two unit less than the width.

(i) If width is taken as x, find the polynomial that represent volume of box.

(ii) Find the polynomial that represent the area of paper sheet used to make box.

(iii) If it must have a volume of 18 unit, what must be its length and height ?

(iv) If box is made of a paper sheet which cost is Rs 100 per square unit, what is the cost of paper?

Sol :

(i) If x represent width, length and height will be $x + 3$ and $x - 2$.

Volume, $V(x) = WLH$

$= x(x + 3)(x - 2)$

$= x(x^2 + x - 6)$

$= x^3 + x^2 - 6x$

(ii) Surface Area,

$S(x) = 2(LW + WH + HL)$

$= 2[x(x + 3) + (x + 3)(x - 2) + x(x - 2)]$

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$$= 2[x^2+3x+x^2+x-6+x^2-2x]$$
$$= 2(3x^2+2x-6) = 6x^2+4x-12$$

(iii) We have $V(x) = x^3+x^2-6$

$$18 = x^3+x^2-6x$$
$$x^3+x^2-6x-18 = 0$$

There is only one positive zero to this equation, because there is one change of sign.

Substituting $x=3$ in above equation we have

$$3^3+3^2-6\times3-18 = 0$$
$$27+9-18-18 = 0$$
$$36-36 = 0$$

Thus by hit and trial we get that 3 is zero of above polynomial. Thus width is 3 unit.

$$\text{Length} = x+3 = 6 \text{ unit}$$
$$\text{Height} = x-2 = 3-2 = 1 \text{ unit}$$

(iv) $S(x) = 6x^2+4x-12$

$$= 6\times3\times3+4\times3-12 \text{ square unit}$$
$$= 54 \text{ square unit}$$
$$C = 100\times54 = 5400 ₹$$

93. Cubic Coating : Frozen specimens are stored in a cubic metal box that is x inches on each side. The box is surrounded by a 2 inch thick layer of foam insulation.

(i) Find a polynomial function $V(x)$ that gives the total volume in cubic inches for the box and insulation.

(ii) Find the total volume if x is 10 inches.

(iii) Use the remainder theorem to find the total volume when x is 10 inches.

Sol :

(i) Since 2 inches of foam is added all around the box, the sides are now $x+4$ inches each.

Volume of a cube of side $x+4$,

$$V(x) = (x+4)^3 = (x+4)(x^2+8x+16)$$
$$= x(x^2+8x+16)+4(x^2+8x+16)$$
$$V(x) = x^3+12x^2+48x+64$$

(ii) The total volume if x is 10 inches,

$$V(10) = 10^3+12\times10^2+48\times10+64$$
$$= 2744 \text{ in}^3$$

94. Swimming Pool : The volume of water in a rectangular, in-ground, swimming pool is given by $V(x) = x^3+11x^2+24x$ where $V(x)$ is the volume in cubic feet when the water is x ft high.

(i) Find the dimension of base of pool.

(ii) Use the remainder theorem to find the volume when $x=3$ ft.

(iii) If the volume is 100 ft^3 of water, what is the height x ?

(iv) If the maximum capacity of the pool is 520 ft^3 what is the maximum depth?

Sol :

(i) We have $V(x) = x^3+11x^2+24x$

$$= x(x^2+11x+24)$$
$$= x(x+3)(x+8)$$

If x represent height, the dimension of base will be $x+3$ and $x+8$.

(ii) Here we have to find $V(3)$ using the remainder theorem. we divide $V(x) = x^3+11x^2+24x$ by $x-3$. Remainder will be value of $V(3)$.

$$\begin{array}{r|l} & x^2+14x+66 \\ \hline x-3 & x^3+11x^2+24x \\ & \underline{x^3-3x^2} \\ & \quad 14x^2+24x \\ & \quad \underline{14x^2-42x} \\ & \quad\quad 66x \\ & \quad\quad \underline{66x-198} \\ & \quad\quad\quad 198 \end{array}$$

Here remainder is 198. Thus $V(3) = 198$

Hence volume is 198 ft^3 at $x=3$ ft.

(iii) If the volume is 100 ft^3 of water,

$$100 = x^3+11x^2+24x$$
$$0 = x^3+11x^2+24x-100$$

There is only one positive solution to this equation, because there is one change of sign. By hit and trial we find the $x=2$ is the only positive solution.

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$$
\begin{array}{r}
x^2+13x+50 \\
x-2\overline{)\,x^3+11x^2+24x-100} \\
\underline{x^3-2x^2} \\
13x^2+24x-100 \\
\underline{13x^2-26x} \\
50x-100 \\
\underline{50x-100} \\
0
\end{array}
$$

Thus height is 2 ft.

(iv) If the maximum capacity of the pool is 520 ft^3,

$$520 = x^3+11x^2+24x$$
$$0 = x^3+11x^2+24x-520$$

There is only one positive zero to this equation, because there is one change of sign. By inspection we find the $x=5$ may be the only positive solution.

$$
\begin{array}{r}
x^2+16x+104 \\
x-5\overline{)\,x^3+11x^2+24x-520} \\
\underline{x^3-5x^2} \\
16x^2+24x-520 \\
\underline{16x^2-80x} \\
104x-520 \\
\underline{104x-520} \\
0
\end{array}
$$

(iii) Here we have to find $V(10)$ using the remainder theorem. we divide $V(x)=x^3+12x^2+48x+64$ by $x-10$. Remainder will be value of $V(10)$.

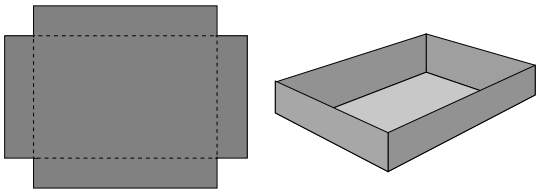
$$
\begin{array}{r}
x^2+22x+268 \\
x-10\overline{)\,x^3+12x^2+48x+64} \\
\underline{x^3-10x^2} \\
22x^2+48x+64 \\
\underline{22x^2-220x} \\
268x+64 \\
\underline{268x-2680} \\
2744
\end{array}
$$

Remainder is 2744, thus $V(10) = 2744$ in.^{3}

95. Overflow Pan : A metalworker makes an overflow pan by cutting equal squares with sides of length x from the corners of a 30 cm by 20 cm piece of aluminium, as shown in the figure. The sides are then folded up and the corners sealed.

(i) Find a polynomial function $V(x)$ that gives the volume of the pan.

(ii) Find the volume of the pan if the height is 6 cm. Use remainder theorem.



Sol :

(i) Since x is taken off of both sides of the rectangle, the sides are $20-2x$ and $30-2x$.

$$V(x) = x(20-2x)(30-2x)$$
$$= x(4x^2-100x+600)$$
$$V(x) = 4x^3-100x^2+600x$$

(ii) Here we have to find $V(6)$ using the remainder theorem. we divide $V(x)=4x^3-100x^2+600x$ by $x-6$. Remainder will be value of $V(6)$.

$$
\begin{array}{r}
4x^2-76x+144 \\
x-6\overline{)\,4x^3-100x^2+600x} \\
\underline{4x^3-24x^2} \\
-76x^2+600x \\
\underline{-76x^2+456x} \\
144x \\
\underline{144x-864} \\
864
\end{array}
$$

Here remainder is 864. Hence volume is 864 cm^3 at $x=6$ cm.

96. Square Pan : Suppose that the pan in previous example is formed from a square piece of aluminium that is 30 cm on each side.

(i) Find a polynomial function $V(x)$ that gives the volume in cubic cm.

(ii) The cost is Rs 5 per square cm of aluminium used in the finished pan. Find a polynomial function $C(x)$ that gives the cost.

Sol :

(i) Since x is taken off of both sides of the square, the sides are $30-2x$ and $30-2x$.

$$V(x) = x(30-2x)(30-2x)$$
$$= x(4x^2-120x+900)$$
$$V(x) = 4x^3-120x^2+900x$$

(ii) The area of the bottom of the pan is $(30-2x)(30-2x)$ or $4x^2-120x+900$. The area of the 4 sides is $4(x)(30-2x)$ or $120x-8x^2$. So the total area is $-4x^2+900$ square cm. Multiply

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by Rs 5 per square cm to get the function $C(x) = -20x^2 + 4500$, which gives the cost in rupees.

97. Volume of a Bird Cage. A company makes rectangular shaped bird cages with height b inches and square bottoms. The volume of these cages is given by the function $V = b^3 - 6b^2 + 9b$.

(i) Find an expression for the length of each side of the square bottom.

(ii) Use the function to find the volume of a cage with a height of 18 inches.

(iii) Use the remainder theorem to find the volume of a cage with a height of 15 inches.

(iv) Verify the result of (iii) using function ?

Sol :

(i) $V = b^3 - 6b^2 + 9b$

$= b(b^2 - 6b + 9)$

$= b(b-3)(b-3)$

If b is height, $(b-3)$ is the length of each side of the square bottom.

(ii) Volume of a cage with a height of 18 inches,

$V(18) = b^3 - 6b^2 + 9b$

$= b(b-3)(b-3)$

$= 18(18-3)(18-3)$

$= 18 \times 15 \times 15 = 4050$ in^2

(iii) Here we have to find $V(15)$ using the remainder theorem. we divide $b^3 - 6b^2 + 9b$ by $b - 15$. Remainder will be value of $V(15)$.

$$
\begin{array}{r|l}
b-15 & b^2+9b+144 \\
\hline
 & b^3-6b^2+9b \\
 & b^3-15b^2 \\
\hline
 & 9b^2+9b \\
 & 9b^2-135b \\
\hline
 & 144b \\
 & 144b-2160 \\
\hline
 & 2160
\end{array}
$$

Here remainder is 2160. Thus $V(15) = 2160$ in^3

(iv) $V(15) = b^3 - 6b^2 + 9b$

$= b(b-3)(b-3)$

$= 15(15-3)(15-3)$

$= 15 \times 12 \times 12 = 2160$ in^2

98. Propylene Storage Tank : R K Agrawal is designing a propylene tank in the shape of a cylinder with hemispherical ends. If the length of the cylinder is to be 20 unit larger than its radius and the volume is to be 3321π cubic unit, then what is the radius?

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Sol :

Let x be the radius of cylindrical part and hemispheres part because both are common. The volume of a hemisphere with radius x is $\frac{1}{2} \cdot \frac{4}{3}\pi x^3$ and the volume of a cylinder with radius x and height $x+20$ is $\pi x^2(x+20)$. Since the tank consists of 2 hemispheres and one cylinder with a total volume of 3321π cubic unit, we can write the following equation

$\frac{4}{3}\pi x^3 + \pi x^2(x+20) = 3321\pi$

$4x^3 + 3x^2(x+20) = 9963$

$$7x^3+60x^2-9963 = 0$$

There is only one positive zero to this polynomial, because there is one change of sign. By hit and trial we find the $x=9$ is the only positive zero.

$$\begin{array}{r|l} & 7x^2+123x+1107 \\ \hline x-9 & 7x^3+60x^2-9963 \\ & \underline{7x^3-63x^2} \\ & \quad 123x^2-9963 \\ & \quad \underline{123x^2-1107x} \\ & \qquad 1107x-9963 \\ & \qquad \underline{1107x-9963} \\ & \qquad 0 \end{array}$$

The radius is 9 unit.

99. Cereal Box : An independent marketing research agency has determined that the best box for breakfast cereal has a height that is 6 inches larger than its thickness and a width that is 5 inches larger than its thickness.

(i) If we let the thickness be x inches, find a polynomial function $V(x)$ that gives the total volume.

(ii) If such a box is to have a volume of 112 inches^3, then what should the thickness be?

Sol :

(i) If we let the thickness be x inches, then the height is $x+6$ inches and the width is $x+5$ inches.

Volume, $V(x) = x(x+6)(x+5)$

$$= x(x^2+11x+30)$$

$$= x^3+11x^2+30x$$

(ii) Since the volume is 112 cubic inches, we can write the following equation.

$$112 = x^3+11x^2+30x$$

$$x^3+11x^2+30x-112 = 0$$

There is only one positive zero to this equation, because there is one change of sign. By hit and trial we find the $x=2$ is the only positive zero.

$$\begin{array}{r|l} & x^2+13x+56 \\ \hline x-2 & x^3+11x^2+30x-112 \\ & \underline{x^3-2x^2} \\ & \quad 13x^2+30x-112 \\ & \quad \underline{13x^2-26x} \\ & \qquad 56x-112 \\ & \qquad \underline{56x-112} \\ & \qquad 0 \end{array}$$

Thus thickness should be 2 inch.

100. A bulk dog food storage bin with dimensions 2 feet by 3 feet by 4 feet needs to be increased in size to hold five times as much food as the current bin. Assume each dimension is increased by the same amount.

(i) Let x be the amount by which dimension is increased. Write a function that represents the volume of the new bin.

(ii) Find the dimensions of the new bin.

Sol :

(i) Let all dimension is increased by x. The new dimension of box are $x+2$, $x+3$ and $x+4$. The volume of box is,

$$\begin{aligned} V(x) &= (x+2)(x+3)(x+4) \\ &= (x+2)(x^2+7x+12) \\ &= x^3+9x^2+26x+24 \end{aligned}$$

(ii) Original volume of bin,

$$V = 2\times 3\times 4 = 24$$

Since volume of new bin must be 5 times of old bin, we have

$$\begin{aligned} 24\times 5 &= x^3+9x^2+26x+24 \\ 120 &= x^3+9x^2+26x+24 \\ 0 &= x^3+9x^2+26x-96 \end{aligned}$$

There is only one positive solution to this equation, because there is one change of sign. By hit and trial we find the $x=2$ is the only positive solution.

$$\begin{array}{r|l} x-2 & x^2+11x+48 \\ \hline & x^3+9x^2+26x-96 \\ & \underline{x^3-2x^2} \\ & \quad 11x^2+26x-96 \\ & \quad \underline{11x^2-22x} \\ & \qquad 48x-96 \\ & \qquad \underline{48x-96} \\ & \qquad 0 \end{array}$$

Thus $x=2$ feet is increased in all dimensions. New dimension of bin will be 4 feet, 5 feet and 6 feet.

101. The discharge rate of a river is a measure of the river's water flow as it empties into a lake, sea, or ocean. The rate depends on many factors, but is primarily influenced by the precipitation in the surrounding area and is often seasonal.

Suppose the discharge rate of the Brhamputra River was modelled by

$$D(m) = -m^4+22m^3-147m^2+317m+150$$

where $D(m)$ represents the discharge rate in thousands of cubic meters of water per second in month m.

$(m=1\to \text{Jan}, m=2\to \text{Feb})$

(i) What was the discharge rate in June (summer heat)?

(ii) Is the discharge rate higher in June or October?

Sol :

(i) To find the discharge rate in June, we find D at $m=6$ using remainder theorem. For this we divide $-m^4+22m^3-147m^2+317m+150$ by $m-6$.

$$\begin{array}{r|l} m-6 & -m^3+16m^2-51m+11 \\ \hline & -m^4+22m^3-147m^2+317m+150 \\ & \underline{-m^4+6m^3} \\ & \quad 16m^3-147m^2+317m+150 \\ & \quad \underline{16m^3-96m^2} \\ & \qquad -51m^2+317m+150 \\ & \qquad \underline{-51m^2+306m} \\ & \qquad\quad 11m+150 \\ & \qquad\quad \underline{11m-66} \\ & \qquad\quad 216 \end{array}$$

Here remainder is 216, thus $D(6)$ is 216 thousand cubic meter per second.

(ii) To find the discharge rate in October, we find D at $m=10$ using remainder theorem. For this we divide $-m^4+22m^3-147m^2+317m+150$ by $m-10$.

$$\begin{array}{r|l} m-10 & -m^3+12m^2-27m+47 \\ \hline & -m^4+22m^3-147m^2+317m+150 \\ & \underline{-m^4+10m^3} \\ & \quad 12m^3-147m^2+317m+150 \\ & \quad \underline{12m^3-120m^2} \\ & \qquad -27m^2+317m+150 \\ & \qquad \underline{-27m^2+270m} \\ & \qquad\quad 47m+150 \\ & \qquad\quad \underline{47m-470} \\ & \qquad\quad 620 \end{array}$$

Here remainder is 620, thus $D(10)$ is 620 thousand cubic meter per second. The discharge rate during the fall rains in October is much higher.

102. Cost of Production : The cost to produce bottled spring water is given by $C(x)=16x-63$ where x is the number of thousands of bottles. The total

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income (revenue) from the sale of these bottles is given by the function $R(x) = -x^2 + 326x - 7463$.

(i) Since Profit = Revenue − Cost, find the profit function.

(ii) How many bottles sold will produce the maximum profit?

(iii) What is the maximum profit?

(iv) Find the profit when 245 thousand bottles are sold. Use remainder theorem

Sol :

Since Profit = Revenue − Cost, we have

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= -x^2 + 326x - 7463 - 16x - 63 \\ &= -x^2 + 310x - 7400 \end{aligned}$$

(i) Thus profit function is $P(x) = -x^2 + 310x - 7400$

Now $$\begin{aligned} P(x) &= -x^2 + 310x - 7400 \\ &= -x^2 + 310x - 155^2 + 155^2 - 7400 \\ &= -(x^2 - 310x + 155^2) + 16625 \\ &= -(x - 155)^2 + 16625 \end{aligned}$$

From above equation it is clear that $P(x)$ is maximum at $x = 155$ and this minimum value is 16625.

(ii) For maximum profit 155 thousand bottle must be sold.

(iii) The maximum profit is Rs 16625.

(iv) To find profit at $x = 245$ using remainder theorem we divide $P(x) = -x^2 + 310x - 7400$ by $x - 245$.

$$\begin{array}{r|l} & -x + 65 \\ \hline x - 245 & -x^2 + 310x - 7400 \\ & -x^2 + 245x \\ \hline & 65x - 7400 \\ & 65x - 15925 \\ \hline & 8525 \end{array}$$

When 245 thousand bottles are sold, profit is Rs 8525.

103. Selection of Bridesmaids : A bride-to-be has many girlfriends, but she has decided to have only five bridesmaids, including the maid of honour. The number of different ways n girlfriends can be chosen and assigned a position, such as maid of honour, first bridesmaid, second bridesmaid, and so on, is given by the polynomial function

$S(n) = n^5 - 10n^4 + 35n^3 - 50n^2 + 8n \quad n \geq 5$

(i) Use the Remainder Theorem to determine the number of ways the bride can select her bridesmaids if she chooses from $n = 5$ girlfriends.

(ii) Evaluate $S(n)$ for $n = 5$ by substituting 5 for n. How does this result compare with the result obtained in (i)?

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Sol :

To find $S(5)$ using remainder theorem we divide $S(n) = n^5 - 10n^4 + 35n^3 - 50n^2 + 8n$ by $n - 5$.

$$\begin{array}{r|l} & n^4 - 5n^3 + 10n^2 + 8 \\ \hline n - 5 & n^5 - 10n^4 + 35n^3 - 50n^2 + 8n \\ & n^5 - 5n^4 \\ \hline & -5n^4 + 35n^3 - 50n^2 + 8n \\ & -5n^4 + 25n^3 \\ \hline & 10n^3 - 50n^2 + 8n \\ & 10n^3 - 50n^2 \\ \hline & 8n \\ & 8n - 40 \\ \hline & 40 \end{array}$$

Since remainder is 40, $S(5) = 40$. Thus bride can select her bridesmaids in 40 ways.

(ii) $$\begin{aligned} S(n) &= n^5 - 10n^4 + 35n^3 - 50n^2 + 8n \\ &= 5^5 - 10 \times 5^4 + 35 \times 5^3 - 50 \times 5^2 + 8 \times 5 \end{aligned}$$

$$= 5^5 - 2\times 5^5 + 7\times 5^4 - 2\times 5^4 + 8\times 5$$
$$= -5^5 + 5\times 5^4 + 8\times 5$$
$$= -5^5 + 5^5 + 8\times 5$$
$$= 8\times 5 = 40$$

104. Display of Soda Cans : The number of soda cans S needed to build a square pyramid display with n levels is given by the function $L(n) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$. Use the Remainder Theorem to determine the number of soda cans needed to build a square pyramid display with $n = 6$ levels.

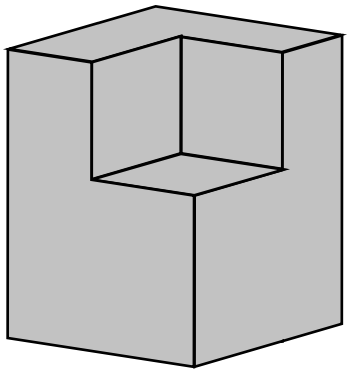
Sol :

To find $L(6)$ using remainder theorem we divide $L(n) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$ by $n-6$.

$$\begin{array}{r|l} & \frac{1}{3}n^2 + 2\frac{1}{2}n + 15\frac{1}{6} \\ \hline n-6 & \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n \\ & \underline{\frac{1}{3}n^3 - 2n^2} \\ & \quad 2\frac{1}{2}n^2 + \frac{1}{6}n \\ & \quad \underline{2\frac{1}{2}n^2 - 15n} \\ & \qquad 15\frac{1}{6}n \\ & \qquad \underline{15\frac{1}{6}n - 91} \\ & \qquad\quad 91 \end{array}$$

Here remainder is 91, thus $L(6) = 91$.

105. Volume of Solid : A cuboidal solid of base x by $x+1$ is shown in figure. Height of original solid is $x+2$. A small cuboidal solid of base $x-2$ by $x-2$ and height 2 is cut from this solid as shown in figure.



(i) Find the polynomial for the volume of remaining solid.

(ii) Use the remainder theorem to determine the volume of remaining solid at $x = 8$ inch.

(iii) Use the polynomial to determine the volume at $x = 8$ inch and verify the result in (ii).

(iv) Use the remainder theorem to determine the volume at $x = 10$ inch.

Sol :

(i) Volume of original solid,

$$V_o(x) = x(x+1)(x+2)$$
$$= x(x^2 + 3x + 2)$$
$$= x^3 + 3x^2 + 2x$$

Volume of solid to be cut,

$$V_s(x) = 2(x-2)(x-2)$$
$$= 2(x^2 - 4x + 4)$$
$$= 2x^2 - 8x + 8$$

Volume of remaining solid,

$$V(x) = V_o(x) - V_s(x)$$
$$= x^3 + 3x^2 + 2x - 2x^2 + 8x - 8$$
$$= x^3 + x^2 + 10x - 8$$

(ii) To find $V(8)$ using remainder theorem we divide $V(x) = x^3 + x^2 + 10x - 8$ by $x-8$.

$$\begin{array}{r|l} & x^2 + 9x + 82 \\ \hline x-8 & x^3 + x^2 + 10x - 8 \\ & \underline{x^3 - 8x^2} \\ & \quad 9x^2 + 10x - 8 \\ & \quad \underline{9x^2 - 72x} \\ & \qquad 82x - 8 \\ & \qquad \underline{82x - 656} \\ & \qquad\quad 648 \end{array}$$

Since remainder is 648, $V(8) = 648$. Thus volume of remaining solid is 648 cubic inch at $x = 8$ inch.

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(iii) $V(8) = 8^3 + 8^2 + 10 \times 8 - 8$

$= 8^2(8+1) + 80 - 8$

$= 8^2 \times 9 + 72$

$= 72(8+1)$

$= 72 \times 9 = 648$ cubic inch

(iv) To find $V(10)$ using remainder theorem we divide $V(x) = x^3 + x^2 + 10x - 8$ by $x - 10$.

$$\begin{array}{r|l} & x^2 + 11x + 120 \\ \hline x - 10 & x^3 + x^2 + 10x - 8 \\ & x^3 - 10x^2 \\ \hline & 11x^2 + 10x - 8 \\ & 11x^2 - 110x \\ \hline & 120x - 8 \\ & 120x - 1200 \\ \hline & 1192 \end{array}$$

Since remainder is 1192, $V(8) = 1192$. Thus volume of remaining solid is 1192 cubic inch at $x = 10$ inch.

Both results are same.

106. The Prime Minister's Citizen Assistance and Relief in Emergency Situations Fund was created on 28 March 2020, following the COVID-19 pandemic in India. The fund will be used for combating, and containment and relief efforts against the coronavirus outbreak and similar pandemic like situations in the future.

The allotment officer is trying to come up with a method to calculate fair division of funds across various affected families so that the fund amount and amount received per family can be easily adjusted based on daily revised numbers. The total fund allotted for a village is $x^3 + 6x^2 + 20x + 9$. The officer has divided the fund equally among families of the village and each family receives an amount of $x^2 + 2x + 2$. After distribution, some amount is left.

(i) How many families are there in the village?

(ii) If an amount of ₹ 1911 is left after distribution, what is value of x?

(iii) How much amount does each family receive?

(iv) What is the amount of fund allocated?

(v) How many families are there in the village?

Sol :

(i) To get number of families we divide $x^3 + 6x^2 + 20x + 9$ by $x^2 + 2x + 2$.

$$\begin{array}{r|l} & x + 4 \\ \hline x^2 + 2x + 2 & x^3 + 6x^2 + 20x + 9 \\ & x^3 + 2x^2 + 2x \\ \hline & 4x^2 + 18x + 9 \\ & 4x^2 + 8x + 8 \\ \hline & 10x + 1 \end{array}$$

Number of families are $x + 4$.

(ii) Amount left $= 10x + 1$

$10x + 1 = 1911$

$x = \frac{1910}{10} = 191$

(iii) Since, $x = 191$, amount received by each family is

$x^2 + 2x + 2 = (191)^2 + 2(191) + 2$

$= 36865$

(iv) Since $x = 191$, allotted fund,

$x^3 + 6x^2 + 20x + 9$

$= (x^2 + 2x + 2)(x + 4) + 10x + 1$

$= 36865(191 + 4) + 1911$

$= 69,72,846$

(v) No. of families $= x + 4 = 191 + 4 = 195$

107. Pyramid, in architecture, a monumental structure constructed of or faced with stone or brick and having a rectangular base and four sloping triangular sides meeting at an apex. Pyramids have been built at various times in Egypt, Sudan, Ethiopia, western Asia, Greece, Cyprus, Italy, India, Thailand, Mexico, South America, and on some islands of the Pacific Ocean. Those of Egypt and of Central and South America are the best known.

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The volume and surface area of a pyramid with a square base of area a^2 and height h is given by

$$V = \frac{ha^2}{3} \text{ and } S = a^2 + 2a\sqrt{\left(\frac{a}{2}\right)^2 + h^2}$$

A pyramid has a square base and a volume of $3y^3 + 18y^2 + 27y$ cubic units.

(i) If its height is y, then what polynomial represents the length of a side of the square base ?

(ii) If area of base is 576 metre, what is the side of base?

(iii) What is the height of pyramid at above area of base ?

(iv) What is the ratio of length of side to the height ?

(v) What is surface area of pyramid ?

Sol :

(i) $V(y) = 3y^3 + 18y^2 + 27y$

$= 3y(y^2 + 6y + 9) = 3y(y+3)^2$

If y represent height, then comparing its volume with standard volume, we have

$$h\frac{a^2}{3} = 3y(y+3)^2$$

$$y\frac{a^2}{3} = 3y(y+3)^2$$

$$a^2 = 9(y+3)^2$$

$$a = 3(y+3)$$

(ii) $a^2 = 576 \Rightarrow a = 24$ unit

(iii) At $a = 24$ meter,

$$24 = 3(y+3)$$

$$8 = y+3$$

$$y = 5 \text{ metre}$$

(iv) We have $a = 24$ and $y = 5$.

$$\frac{a}{y} = \frac{24}{5}$$

(v) We have $S = a^2 + 2a\sqrt{\left(\frac{a}{2}\right)^2 + h^2}$

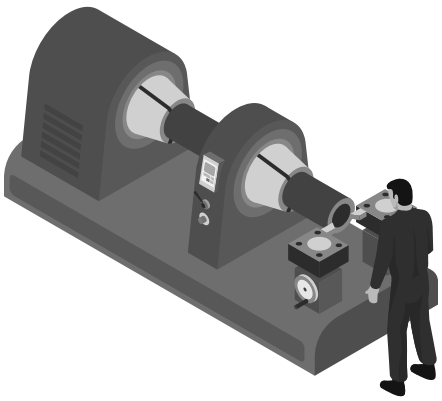
We have $a = 24$ and $y = 5$.

Thus $S = 24^2 + 2 \times 24\sqrt{\left(\frac{24}{2}\right)^2 + 5^2}$

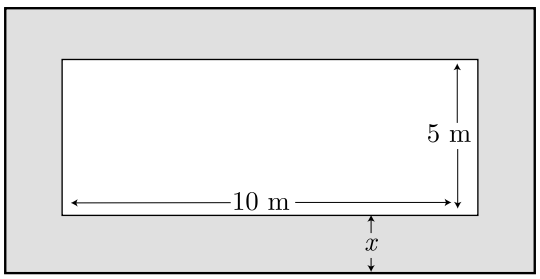
$= 2 \times 24(12 + \sqrt{12^2 + 5^2})$

$= 48(12 + 13) = 1200 \text{ m}^2$

108. SBL Fabricators has got a order for making a frame for machine of their client. For which, they are using a AutoCAD software to create a constructible model that includes the relevant information such as dimensions of the frame and materials needed.



The frame will have a solid base and will be cut out of a piece of steel. The final area of the frame should be 54 sq m. The digram of frame is shown below.



In order to input the right values in the AutoCAD software, the engineer needs to calculate some basic values.

(i) What are the dimensions of the outer frame ?

(ii) A metal sheet of minimum area is used to make the frame. What should be the minimum area of metal sheet before cutting ?

(iii) What is the area of required final metal frame?

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(iv) If the area of the frame is 54 sq m, what is the value of x?

(v) What is the perimeter of the frame?

Sol :

(i) Length $=(10+x+x)=(10+2x)$

Breadth $=(5+x+x)=(5+2x)$ cm

(ii) Length of steel plate, $l=(10+2x)$

Breadth of steel plate, $b=(5+2x)$

Area of steel plate, $A=lb$

$$=(10+2x)(5+2x)$$
$$=50+10x+20x+4x^2$$
$$=50+30x+4x^2$$
$$A=4x^2+30x+50$$

(iii) Area of frame to be cut $=10\times 5=50$ m^2

Area of frame left $=4x^2+30x+50-50$

$$=4x^2+30x \text{ m}^2$$

(iv) Here, area of frame $=54$ m^2

$$4x^2+30x=54$$
$$2x^2+15x-27=0$$
$$2x^2+18x-3x-27=0$$
$$(x+9)(2x-3)=0$$
$$x=1.5 \text{ or } -9$$

(v) Perimeter of frame,

$=$Perimeter of Outside Rectangle

$=2(10+2x+5+2x)=2(15+4x)$

$=2(15+4\times 1.5)=42$ m

109. An barrels manufacturer can produce up to 300 barrels per day. The profit made from the sale of these barrels can be modelled by the function $P(x)=-10x^2+3500x-66000$ where $P(x)$ is the profit in rupees and x is the number of barrels made and sold.

Based on this model answer the following questions:

(i) When no barrels are produce what is a profit loss?

(ii) What is the break even point ? (Zero profit point is called break even)

(iii) What is the profit/loss if 175 barrels are produced

(iv) What is the profit/loss if 400 barrels are produced

(v) What is the maximum profit which can manufacturer earn?

Sol :

(i) When no barrels are produced, $x=0$

$$P(x)=0+0-66000$$
$$P(x)=-66000 \text{ Rs}$$

(ii) At break-even point $P(x)=0$, thus

$$0=-10x^2+3500x-66000$$
$$x^2+350x+6600=0$$
$$x^2-330x-20x+6600=0$$
$$x(x-330)-20(x+330)=0$$
$$(x-330)(x-20)=0$$
$$x=20, 330$$

(iii) $P(175)=-10(175)^2+3500(175)-66000$

$$=240250$$

(iv) $P(400)=-10(400)^2+3500(400)-66000$

$$=-266000 \text{ Rs}$$

(v) Rearranging the given equation we have

$$P(x)=-10x^2+3500x-66000$$
$$=-10(x^2-350x+6600)$$
$$=-10[(x-175)^2-30625+6600]$$
$$=-10[(x-175)^2-24025]$$
$$=-10(x-175)^2+240250$$

From above equation it is clear that maximum value of $P(x)$ is Rs 240250.

110. Lavanya throws a ball upwards, from a rooftop, which is 20 m above from ground. It will reach a maximum height and then fall back to the ground. The height of the ball from the ground at time t is h, which is given by $h=-4t^2+16t+20$.

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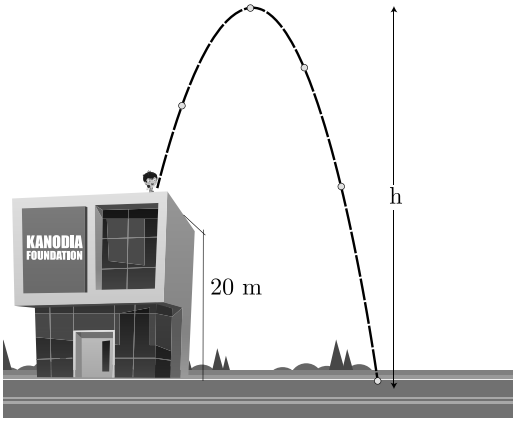

(i) What is the height reached by the ball after 1 second?

(ii) What is the maximum height reached by the ball?

(iii) How long will the ball take to hit the ground?

(iv) What are the two possible times to reach the ball at the same height of 32 m?

(v) Where is the ball after 5 seconds ?

Sol :

(i) Height is given by,

$$h = -4t^2 + 16t + 20$$

At $t = 1$ second,

$$h = -4(1)^2 + 16(1) + 20 = 32 \text{ m}$$

(ii) Rearranging the given equation, by completing the square,

$$\begin{aligned} h &= -4(t^2 - 4t - 5) \\ &= -4(t^2 - 4t + 4 - 4 - 5) \\ &= -4[(t-2)^2 - 9] \\ &= -4(t-2)^2 + 36 \end{aligned}$$

Height is maximum, at $t = 2$, thus

$$h_{\max} = 0 + 36 = 36 \text{ m}$$

(iii) When ball hits the ground, $h = 0$, thus

$$\begin{aligned} -4t^2 + 16t + 20 &= 0 \\ t^2 - 4t - 5 &= 0 \\ (t-5)(t+1) &= 0 \end{aligned}$$

Thus $t = 5$ or $t = -1$. Since, time cannot be negative, the $t = 5$ seconds is correct answer.

(iv) Since,

$$\begin{aligned} h &= -4t^2 + 16t^2 + 20 \\ 32 &= -4t^2 + 16t^2 + 20 \\ 8 &= -t^2 + 4t^2 + 5 \\ t^2 - 4t + 3 &= 0 \\ t^2 + 3t - t + 3 &= 0 \\ (t-1)(t-3) &= 0 \Rightarrow t = 3, 1 \end{aligned}$$

(v) From (iii) at $t = 5$ we have $h = 0$. Thus it will hit ground, then after that ball will rebound.

111. Underground water tank is very popular in India. It is usually used for large water tank storage and can be built cheaply using cement-like materials. Underground water tank are typically chosen by people who want to save space. The water in the underground water tank is not affected by extreme weather conditions. The underground water tank maintain cool temperatures in both winter and summer.

A builder wants to build a tank to store water in an apartment. The volume of the rectangular tank will be modelled by $V(x) = x^3 + x^2 - 4x - 4$.

(i) He planned in such a way that its base dimensions are $(x+1)$ and $(x+2)$. How much he has to dig?

(ii) If $x = 4$ meter, what is the volume of the water tank?

(iii) If $x = 4$ and the builder wants to paint the entire inner portion on the water tank, what is the total area to be painted ?

(iv) If the cost of paint is Rs. 25/ per square metre, what is the cost of painting ?

(v) What is the storage capacity of this water tank ?

Sol :

(i) We have,

$$V(x) = x^3 + x^2 - 4x - 4$$

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$$= x^2(x+1) - 4(x+1)$$
$$= (x+1)(x^2-4)$$
$$= (x+1)(x-2)(x+2)$$

If $(x+1)$ and $(x+2)$ are two dimension, 3rd dimension will be $(x-2)$. Thus he has to dig $(x-2)$.

(ii) $V(x) = (x+1)(x-2)(x+2)$

$V(4) = (4+1)(4-2)(4+2)$

$= 5 \times 2 \times 6 = 60\,\text{m}^3$

(iii) Three dimension of tank are

$x+1 = 4+1 = 5$

$x+2 = 4+2 = 6$

$x-2 = 4-2 = 2$

$S = 2(5 \times 2 + 2 \times 6 + 6 \times 5)$

$= 2(10+12+30)$

$= 2(52) = 104\,\text{m}^2$

(iv) $C = 104 \times 25 = 2600$ ₹

(v) 1 m^3 can store 1000 litre, thus 60 m^3 can store 60000 litre.

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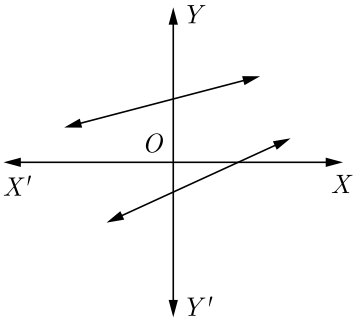
CHAPTER 3

Pair of Linear Equation in Two Variables

OBJECTIVE QUESTIONS

1. In the given figure, graphs of two linear equations are shown. The pair of these linear equations is:



(a) consistent with unique solution.

(b) consistent with infinitely many solutions.

(c) inconsistent.

(d) inconsistent but can be made consistent by extending these lines.

Sol : [Board 2024 OD Standard]

These two lines are not parallel and will cut on a point. Thus pair of these linear equations is consistent with unique solution.

Thus (a) is correct option.

2. 3 chairs and 1 table cost ₹900; whereas 5 chairs and 3 tables cost ₹2,100. If the cost of 1 chair is ₹x and the cost of 1 table is ₹y, then the situation can be represented algebraically as

(a) $3x+y=900,\ 3x+5y=2100$

(b) $x+3y=900,\ 3x+5y=2100$

(c) $3x+y=900,\ 5x+3y=2100$

(d) $x+3y=900,\ 5x+3y=2100$

Sol : [Board 2023 OD Standard]

Cost of one chair is x and cost of one table is y.

3 chairs and 1 table cost ₹900, thus

$$3x+y = ₹900$$

5 chairs and 3 tables cost ₹2,100

$$5x+3y = ₹2100$$

Thus (c) is correct option.

3. The 2 digit number which becomes $\frac{5}{6}$th of itself when its digits are reversed. The difference in the digits of the number being 1, then the two digits number is

(a) 45 (b) 54

(c) 36 (d) None of these

Sol : [Board 2022 Term-1 SQP STD]

If the two digits are x and y, then the number is $10x+y$.

Now $$\frac{5}{6}(10x+y) = 10y+x$$

Solving, we get $44x+55y$

$$\frac{x}{y} = \frac{5}{4}$$

Also $x-y=1$. Solving them, we get $x=5$ and $y=4$. Therefore, number is 54.

Thus (b) is correct option.

4. In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. The number is

(a) 36 (b) 63

(c) 48 (d) 84

Sol : [Board 2022 Term-1 SQP Basic]

Let x be units digit and y be tens digit, then number will be $10y+x$

Then, $$x = 2y \quad ...(1)$$

If 36 be added to the number, the digits are reversed, i.e. number will be $10x+y$.

$$10y+x+36 = 10x+y$$

$$9x-9y = 36$$

$$x-y = 4 \quad ...(2)$$

Solving (1) and (2) we have $x=8$ and $y=4$.

Thus number is 48.

Thus (c) is correct option.

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5. If $3x+4y:x+2y=9:4$, then $3x+5y:3x-y$ is equal to

(a) 4 : 1 (b) 1 : 4

(c) 7 : 1 (d) 1 : 7

Sol : [Board 2022 Term-1 STD]

$$\frac{3x+4y}{x+2y} = \frac{9}{4}$$

Hence, $12x+16y = 9x+18y$

or $3x = 2y$

$$x = \frac{2}{3}y$$

Substituting $x=\frac{2}{3}y$ in the required expression we have

$$\frac{3x\frac{2}{3}y+5y}{3x\frac{2}{3}y-y} = \frac{7y}{y} = \frac{7}{1} = 7:1$$

Thus (c) is correct option.

6. The pair of equations $3^{x+y}=81$, $81^{x-y}=3$ has

(a) no solution

(b) unique solution

(c) infinitely many solutions

(d) $x=2\frac{1}{8}, y=1\frac{7}{8}$

Sol : [Board 2022 Term-1 Basic]

Given, $3^{x+y} = 81$

$3^{x+y} = 3^4$

$x+y = 4$ ...(1)

and $81^{x-y} = 3$

$3^{4(x-y)} = 3^1$

$4(x-y) = 1$

$x-y = \frac{1}{4}$ ...(2)

Adding equation (1) and (2), we get

$$2x = 4+\frac{1}{4} = \frac{17}{4}$$

$$x = \frac{17}{8} = 2\frac{1}{8}$$

From equation (1), we get

$$y = \frac{15}{8} = 1\frac{7}{8}$$

Thus (d) is correct option.

7. A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is

(a) 2 (b) 3

(c) 5 (d) 15

Sol : [Board 2022 Term-1 Basic]

Let the fraction be $\frac{x}{y}$,

$$\frac{x+1}{y+1} = 4 \Rightarrow x = 4y+3 \quad ...(1)$$

and $$\frac{x-1}{y-1} = 7 \Rightarrow x = 7y-6 \quad ...(2)$$

Solving (1) and (2), we have $x = 15$, $y = 3$,

Thus (d) is correct option.

8. For which value(s) of p, will the lines represented by the following pair of linear equations be parallel

$3x-y-5 = 0$ and $6x-2y-p = 0$

(a) all real values except 10

(b) 10

(c) 5/2

(d) 1/2

Sol : [Board Term-1 Delhi 2012]

We have, $3x-y-5 = 0$

and $6x-2y-p = 0$

Here, $a_1 = 3$, $b_1 = -1$, $c_1 = -5$

and $a_2 = 6$, $b_2 = -2$, $c_2 = -p$

Since given lines are parallel,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{-1}{-2} \neq \frac{-5}{-p}$$

$$p \neq 5 \times 2 \Rightarrow p \neq 10$$

9. x and y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then value of $x+y$ is

(a) 10 (b) 11

(c) 12 (d) 13

Sol : [Board 2022 Term-1 STD]

The numbers that can be formed are xy and yx . Hence, $(10x+y)+(10y+x) = 11(x+y)$. If this is a perfect square than $x+y = 11$.

10. The value of k for which the system of linear equations $x+2y=3$, $5x+ky+7=0$ is inconsistent is

(a) $-\frac{14}{3}$ (b) $\frac{2}{5}$

(c) 5 (d) 10

Sol : [Board 2020 OD Standard]

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We have $x+2y-3=0$

and $5x+ky+7=0$

If system is inconsistent, then

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

From first two orders, we have

$$\frac{1}{5}=\frac{2}{k}\Rightarrow k=10$$

Thus (d) is correct option.

11. The value of k for which the system of equations $x+y-4=0$ and $2x+ky=3$, has no solution, is

(a) -2 (b) $\neq 2$

(c) 3 (d) 2

Sol : [Board 2020 Delhi Standard]

We have $x+y-4=0$

and $2x+ky-3=0$

Here, $\frac{a_1}{a_1}=\frac{1}{2}$, $\frac{b_1}{b_2}=\frac{1}{k}$ and $\frac{c_1}{c_2}=\frac{-4}{-3}=\frac{4}{3}$

Since system has no solution, we have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{1}{2}=\frac{1}{k}\neq\frac{4}{3}$$

$$k=2 \text{ and } k\neq\frac{3}{4}$$

Thus (d) is correct option.

12. The pair of linear equations $2kx+5y=7$, $6x-5y=11$ has a unique solution, if

(a) $k\neq -3$ (b) $k\neq\frac{2}{3}$

(c) $k\neq 5$ (d) $k\neq\frac{2}{9}$

Sol : [Board Term-1 SQP 2012]

Given the pair of linear equations are

$2kx+5y-7=0$

and $6x-5y-11=0$

On comparing with

$a_1x+b_1y+c_1=0$

and $a_2x+b_2y+c_2=0$

we get, $a_1=2k$, $b_1=5$, $c_1=-7$

and $a_2=6$, $b_2=-5$, $c_2=-11$

For unique solution,

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

$$\frac{2k}{6}\neq\frac{5}{-5}$$

$$\frac{k}{3}=\neq -1$$

$$k\neq -3$$

Thus (a) is correct option.

13. The pair of equations $x+2y+5=0$ and $-3x-6y+1=0$ has

(a) a unique solution

(b) exactly two solutions

(c) infinitely many solutions

(d) no solution

Sol : [Board Term-1 Foreign 2012]

Given,equations are

$x+2y+5=0$

and $-3x-6y+1=0$

Here, $a_1=1, b_1=2, c_1=5$

and $a_2=-3, b_2=-6, c_2=1$

Now $\frac{a_1}{a_2}=-\frac{1}{3}$,

$\frac{b_1}{b_2}=-\frac{2}{6}=-\frac{1}{3}$, $\frac{c_1}{c_2}=\frac{5}{1}$

Now, we observe that

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Hence, the pair of equations has no solution.

Thus (d) is correct option.

14. If a pair of linear equations is consistent, then the lines will be

(a) parallel

(b) always coincident

(c) intersecting or coincident

(d) always intersecting

Sol : [Board Term-1 OD 2013]

Condition for a consistent pair of linear equations

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

[intersecting lines having unique solution]

and $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ [coincident or dependent]

Thus (c) is correct option.

15. The pair of equations $y=0$ and $y=-7$ has

(a) one solution

(b) two solutions

(c) infinitely many solutions

(d) no solution

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Sol : [Board Term-1 SQP 2017]

The pair of both equations are parallel to x-axis and we know that parallel lines never intersects. So, there is no solution of these lines.

Thus (d) is correct option.

16. For what value of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines ?

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 2 (d) -2

Sol : [Board Term-1 Foreign 2016]

Given, equations,

$$3x - y + 8 = 0$$

and $$6x - ky + 16 = 0$$

Condition for coincident lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad ...(1)$$

Here, $a_1 = 3, b_1 = -1, c_1 = 8$

and $a_2 = 6, b_2 = -k, c_2 = 16$

From equation (1),

$$\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$$

$$\frac{1}{k} = \frac{1}{2} \quad \left[\text{since } \frac{3}{6} = \frac{8}{16} = \frac{1}{2}\right]$$

$$k = 2$$

Thus (c) is correct option.

17. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(a) parallel

(b) intersecting at (b, a)

(c) coincident

(d) intersecting at (a, b)

Sol : [Board Term-1 Foreign 2011]

The pair of equations

$$x = a$$

and $$y = b$$

Graphically represents lines which are intersecting at (a, b).

Thus (d) is correct option.

18. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(a) $-\frac{5}{4}$ (b) $\frac{2}{5}$

(c) $\frac{15}{4}$ (d) $\frac{3}{2}$

Sol : [Board Term-1 OD 2011]

We have $3x + 2ky - 2 = 0$

and $2x + 5y + 1 = 0$

Condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad ...(i)$$

Here, $a_1 = 3, b_1 = 2k, c_1 = -2$

and $a_2 = 2, b_2 = 5, c_2 = 1$

From equation (i), we have

$$\frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$$

Considering, $$\frac{3}{2} = \frac{2k}{5} \quad \left[\frac{3}{2} \neq \frac{-2}{1} \text{ in any case}\right]$$

$$k = \frac{15}{4}$$

Thus (c) is correct option.

19. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have is

(a) 3 (b) -3

(c) -12 (d) no value

Sol : [Board Term-1 SQP 2014]

The given lines are, $cx - y = 2$

and $$6x - 2y = 3$$

Condition for infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad ...(i)$$

Here, $a_1 = c,\ b_1 = -1,\ c_1 = -2$

and $a_2 = 6,\ b_2 = -2,\ c_2 = -3$

From equation (i), $\frac{c}{6} = \frac{-1}{-2} = \frac{-2}{-3}$

Here, $$\frac{c}{6} = \frac{1}{2}$$

and $$\frac{c}{6} = \frac{2}{3}$$

$$c = 3 \text{ and } c = 4$$

Since, c has different values.

Hence, for no value of c the pair of equations will have infinitely many solutions.

Thus (d) is correct option.

20. One equation of a pair of dependent linear equations $-5x + 7y = 2$ The second equation can be

(a) $10x + 14y + 4 = 0$

(b) $-10x - 14y + 4 = 0$

(c) $-10x + 14y + 4 = 0$

(d) $10x - 14y = -4$

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Sol : [Board Term-1 OD 2013]

For dependent linear equation,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Checking for option (a):

$$\frac{-5}{10} \neq \frac{7}{14}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, option (a) is rejected.

Checking for option (b):

$$\frac{-5}{-10} \neq \frac{7}{-14}$$

So, option (b) is also rejected.

Checking for option (c):

$$\frac{-5}{-10} = \frac{7}{14} \neq \frac{-2}{4}$$

So, option (b) is also rejected

Checking for option (d):

$$\frac{-5}{10} = \frac{7}{-14} = \frac{-2}{4}$$

Thus (d) is correct option.

21. If $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are, respectively

(a) 3 and 5 (b) 5 and 3

(c) 3 and 1 (d) -1 and -3

Sol : [Board Term-1 OD 2015]

Since, $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then these values will satisfy that equation

$$a - b = 2 \quad ...(1)$$

and $$a + b = 4 \quad ...(2)$$

Adding equations (1) and (2), we get

$$2a = 6$$

$$a = 3$$

Substituting $a = 3$ in equation (2),we have

$$3 + b = 4 \Rightarrow b = 1$$

Thus $a = 3$ and $b = 1$.

Thus (c) is correct option.

22. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15 (b) 35 and 20

(c) 15 and 35 (d) 25 and 25

Sol : [Board Term-1 OD 2016]

Let number of ₹ 1 coins $= x$

and number of ₹ 2 coins $= y$

Now, by given conditions,

$$x + y = 50 \quad ...(1)$$

Also, $x \times 1 + y \times 2 = 75$

$$x + 2y = 75 \quad ...(2)$$

Subtracting equation (1) form equation (2), we get

$$(x + 2y) - (x + y) = 75 - 50$$

$$y = 25$$

From equation (i), $x = 75 - 2x(25)$

Then, $x = 25$

Thus (d) is correct option.

23. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in year) of the son and the father are, respectively.

(a) 4 and 24

(b) 5 and 30

(c) 6 and 36

(d) 3 and 24

Sol : [Board Term-1 Delhi 2016]

Let the present age of father $= x$ years

and the present age of son $= y$ years

Four years hence, it has relation by given condition

$$(x + 4) = 4(y + 4)$$

$$x - 4y = 12 \quad ...(1)$$

As the father's age is six times his son's age, so we have

$$x = 6y \quad ...(2)$$

Putting the value of x from equation (2) in equation (1), we get

$$6x - 4y = 12$$

$$2y = 12$$

$$y = 6$$

From equation (1), $x = 6 \times 6$

Then, $x = 36$

Hence, present age of father is 36 year and age of son is 6 year.

Thus (c) is correct option.

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24. **Assertion :** Pair of linear equations : $9x+3y+12=0,\ 8x+6y+24=0$ have infinitely many solutions.

Reason : Pair of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ have infinitely many solutions, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

From the given equations, we have

$$\frac{9}{18}=\frac{3}{6}=\frac{12}{24}$$

$$\frac{1}{2}=\frac{1}{2}=\frac{1}{2} \text{ i.e., } \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

25. **Assertion :** $x+y-4=0$ and $2x+ky-3=0$ has no solution if $k=2$.

Reason : $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are consistent if $\frac{a_1}{a_2}\neq\frac{k_1}{k_2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

For assertion, given equation has no solution if

$$\frac{1}{2}=\frac{1}{k}\neq\frac{-4}{-3} \text{ i.e. } \frac{4}{3}$$

$$k=2\left[\frac{1}{2}\neq\frac{4}{3}\text{holds}\right]$$

Assertion is true.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

ONE MARK QUESTIONS

26. Find the value of k for which the system of linear equations $x+2y=3,\ 5x+ky+7=0$ is inconsistent.

Sol : [Board 2020 OD Standard]

We have $x+2y-3=0$

and $5x+ky+7=0$

If system is inconsistent, then

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

From first two orders, we have

$$\frac{1}{5}=\frac{2}{k}\Rightarrow k=10$$

27. Find the value of k for which the system of equations $x+y-4=0$ and $2x+ky=3$, has no solution.

Sol : [Board 2020 Delhi Standard]

We have $x+y-4=0$

and $2x+ky-3=0$

Here, $\frac{a_1}{a_1}=\frac{1}{2},\ \frac{b_1}{b_2}=\frac{1}{k}$ and $\frac{c_1}{c_2}=\frac{-4}{-3}=\frac{4}{3}$

Since system has no solution, we have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{1}{2}=\frac{1}{k}\neq\frac{4}{3}$$

$$k=2 \text{ and } k\neq\frac{3}{4}$$

PRACTICE

For what value of k, the system of equations $kx+3y=1,\ 12x+ky=2$ has no solution.

[Board Term-1 2011]

Ans : $k=-6$

28. For which value(s) of p, will the lines represented by the following pair of linear equations be parallel ?

$$3x-y-5=0,\quad 6x-2y-p=0$$

Sol : [Board Term-1 OD 2017]

We have, $3x-y-5=0$

and $6x-2y-p=0$

Here, $a_1=3,\ b_1=-1,\ c_1=-5$

and $a_2=6,\ b_2=-2,\ c_2=-p$

Since given lines are parallel,

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$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$
$$\frac{-1}{-2} \neq \frac{-5}{-p}$$
$$p \neq 5 \times 2 \Rightarrow p \neq 10$$

29. The 2 digit number which becomes $\frac{5}{6}$th of itself when its digits are reversed. If the difference in the digits of the number being 1, what is the two digits number?

Sol : [Board Term-1 Delhi 2011]

If the two digits are x and y, then the number is $10x+y$.

Now $\frac{5}{6}(10x+y) = 10y+x$

Solving, we get $44x+55y$

$$\frac{x}{y} = \frac{5}{4}$$

Also $x-y=1$. Solving them, we get $x=5$ and $y=4$. Therefore, number is 54.

30. In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. What is the number ?

Sol : [Board Term-1 Delhi 2016]

Let x be units digit and y be tens digit, then number will be $10y+x$

Then, $x = 2y$ ...(1)

If 36 be added to the number, the digits are reversed, i.e. number will be $10x+y$.

$$10y+x+36 = 10x+y$$
$$9x-9y = 36$$
$$x-y = 4 \quad ...(2)$$

Solving (1) and (2) we have $x=8$ and $y=4$.

Thus number is 48.

31. If $3x+4y : x+2y = 9:4$, then find the value of $3x+5y : 3x-y$.

Sol : [Board Term-1 Foreign 2012]

We have $\frac{3x+4y}{x+2y} = \frac{9}{4}$

Hence, $12x+16y = 9x+18y$

or $3x = 2y$

$$x = \frac{2}{3}y$$

Substituting $x = \frac{2}{3}y$ in the required expression we have

$$\frac{3x\frac{2}{3}y+5y}{3x\frac{2}{3}y-y} = \frac{7y}{y} = \frac{7}{1} = 7:1$$

32. A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. What is the numerator of the given fraction ?

Sol : [Board Term-1 Foreign 2016]

Let the fraction be $\frac{x}{y}$,

$$\frac{x+1}{y+1} = 4 \Rightarrow x = 4y+3 \quad ...(1)$$

and $\frac{x-1}{y-1} = 7 \Rightarrow x = 7y-6 \quad ...(2)$

Solving (1) and (2), we have $x=15$, $y=3$,

33. x and y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then what is the value of $x+y$?

Sol : [Board Term-1 OD 2013]

The numbers that can be formed are xy and yx . Hence, $(10x+y)+(10y+x) = 11(x+y)$. If this is a perfect square than $x+y=11$.

34. If a pair of linear equations is consistent, then the lines will be intersecting or coincident. Justify.

Sol : [Board 2008]

Condition for a consistent pair of linear equations

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

[intersecting lines having unique solution]

and $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ [coincident or dependent]

35. The pair of equations $y=0$ and $y=-7$ has no solution. Justify.

Sol : [Board Term-1 Foreign 2014]

The given pair of equations are

$$y = 0 \quad y = -7$$

The pair of both equations are parallel to x-axis and we know that parallel lines never intersects. So, there is no solution of these lines.

36. If the equations $kx-2y=3$ and $3x+y=5$ represent two intersecting lines at unique point, then the value of k is ..... .

Sol : [Board Term-1 2011]

For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

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Here, $a_1 = k$, $b_1 = -2$, $a_2 = 3$ and $b_2 = 1$

Now $\frac{k}{3} \neq -\frac{2}{1}$

or, $k \neq -6$

37. Find whether the pair of linear equations $y = 0$ and $y = -5$ has no solution, unique solution or infinitely many solutions.

Sol : [Board Term-1 OD 2011]

The given variable y has different values. Therefore the pair of equations $y = 0$ and $y = -5$ has no solution.

38. If $am = bl$, then find whether the pair of linear equations $ax + by = c$ and $lx + my = n$ has no solution, unique solution or infinitely many solutions.

Sol : [Board Term-1 OD 2015]

Since, $am = bl$, we have

$$\frac{a}{1} = \frac{b}{m} \neq \frac{c}{n}$$

Thus, $ax + by = c$ and $lx + my = n$ has no solution.

39. If $ad \neq bc$, then find whether the pair of linear equations $ax + by = p$ and $cx + dy = q$ has no solution, unique solution or infinitely many solutions.

Sol : [Board Term-1 Delhi 2015]

Since $ad \neq bc$ or $\frac{a}{c} \neq \frac{b}{d}$

Hence, the pair of given linear equations has unique solution.

40. Two lines are given to be parallel. The equation of one of the lines is $4x + 3y = 14$, then find the equation of the second line.

Sol : [Board 2007]

The equation of one line is $4x + 3y = 14$. We know that if two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or $\frac{4}{a_2} = \frac{3}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{a_2}{b_2} = \frac{4}{3} = \frac{12}{9}$

Hence, one of the possible, second parallel line is $12x + 9y = 5$.

41. Find whether the lines represented by $2x + y = 3$ and $4x + 2y = 6$ are parallel, coincident or intersecting.

Sol : [Board Term-1 Delhi 2016]

Here $a_1 = 2, b_1 = 1, c_1 = -3$ and $a_1 = 4, b_2 = 2, c_2 = -6$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

then the lines are parallel.

Clearly $\frac{2}{4} = \frac{1}{2} = \frac{3}{6}$

Hence lines are coincident.

42. Given the linear equation $3x + 4y = 9$. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is:

(1) intersecting lines

(2) coincident lines.

Sol : [Board Term-1 2016]

(1) For intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, one of the possible equation $3x - 5y = 10$

(2) For coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, one of the possible equation $6x + 8y = 18$

43. Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.

Sol : [Board 2019 OD]

We have $x + 2y - 5 = 0$ ...(1)

and $3x + ky + 15 = 0$ ...(2)

Comparing equation (1) with $a_1x + b_1y + c_1 = 0$, and equation (2) with $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 1$, $a_2 = 3$, $b_1 = 2$, $b_2 = k$, $c_1 = -5$ and $c_2 = 15$

Since, given equations have unique solution, So,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

i.e. $\frac{1}{3} \neq \frac{2}{k}$

$$k \neq 6$$

Hence, for all values of k except 6, the given pair of equations have unique solution.

PRACTICE

For what value of p does the pair of linear equations given below has unique solution ?
$4x + py + 8 = 0$ and $2x + 2y + 2 = 0$.

[Board Term-1 2012]

Ans : $p \neq 4$

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44. If $2x+y=23$ and $4x-y=19$, find the value of $(5y-2x)$ and $(\frac{y}{x}-2)$.

Sol : [Board 2020 OD Standard]

We have $2x+y = 23$ ...(1)

$4x-y = 19$ ...(2)

Adding equation (1) and (2), we have

$6x = 42 \Rightarrow x = 7$

Substituting the value of x in equation (1), we get

$14+y = 23$

$y = 23-14 = 9$

Hence, $5y-2x = 5\times 9-2\times 7$

$= 45-14 = 31$

and $\frac{y}{x}-2 = \frac{9}{7}-2 = \frac{9-14}{7} = \frac{-5}{7}$

45. Find whether the following pair of linear equation is consistent or inconsistent:

$3x+2y=8,\ 6x-4y=9$

Sol : [Board Term-1 2016]

We have $\frac{3}{6} \neq \frac{2}{-4}$

i.e., $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the pair of linear equation is consistent.

46. Is the system of linear equations $2x+3y-9=0$ and $4x+6y-18=0$ consistent? Justify your answer.

Sol : [Board Term-1 2012]

For the equation $2x+3y-9=0$ we have

$a_2 = 2, b_1 = 3$ and $c_1 = -9$

and for the equation, $4x+6y-18=0$ we have

$a_2 = 4, b_2 = 6$ and $c_2 = -18$

Here $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

and $\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$

Thus $\frac{c_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, system is consistent and dependent.

47. For what value of k, the pair of linear equations $kx-4y=3,\ 6x-12y=9$ has an infinite number of solutions ?

Sol : [Board Term-1 2012]

We have $kx-4y-3 = 0$

and $6x-12y-9 = 0$

where, $a_1 = k, b_1 = 4, c_1 = -3$

$a_2 = 6, b_2 = -12, c_2 - 9$

Condition for infinite solutions:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{k}{6} = \frac{-4}{-12} = \frac{3}{9}$

Hence, $k = 2$

PRACTICE

- For what value of k, $2x+3y=4$ and $(k+2)x+6y=3k+2$ will have infinitely many solutions ?
  [Board Term-1 2012]
  Ans : $k=2$

- Find the value(s) of k for which the pair of linear equations $kx+y=k^2$ and $x+ky=1$ have infinitely many solutions.
  [Board Term-1 2017]
  Ans : $k=\pm 1$

48. Solve the following pair of linear equations by substitution method:

$3x+2y-7 = 0,\ 4x+y-6 = 0$

Sol : [Board Term-1 2015]

We have $3x+2y-7 = 0$ ...(1)

$4x+y-6 = 0$ ...(2)

From equation (2), $y = 6-4x$ ...(3)

Putting this value of y in equation (1) we have

$3x+2(6-4x)-7 = 0$

$3x+12-8x-7 = 0$

$5-5x = 0$

$5x = 5$

Thus $x = 1$

Substituting this value of x in (2), we obtain,

$y = 6-4\times 1 = 2$

Hence, values of x and y are 1 and 2 respectively.

PRACTICE

- Solve the following system of linear equations by substitution method:

$2x-y = 2$

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$$x+3y = 15$$

[Board Term-1 2012]

Ans : $x=3$ and $y=4$

49. Solve : $99x+101y = 499, \quad 101x+99y = 501$

Sol : [Board Term-1 2012]

We have $99x+101y = 499$ ...(1)

$101x+99y = 501$ ...(2)

Adding equation (1) and (2), we have

$200x+200y = 1000$

$x+y = 5$ ...(3)

Subtracting equation (2) from equation (1), we get

$-2x+2y = -2$

$x-y = 1$ ...(4)

Adding equations (3) and (4), we have

$2x = 6 \Rightarrow x = 3$

Substituting the value of x in equation (3), we get

$3+y = 5 \Rightarrow y = 2$

THREE MARKS QUESTIONS

50. Solve the following system of linear equations algebraically :

$2x+5y=-4;\ 4x-3y=5$

Sol : [Board 2024 OD Standard]

We have $2x+5y = -4$ ...(1)

$4x-3y = 5$ ...(2)

Multiplying equation (1) by 2, we get

$4x+10y = -8$ ...(3)

Now subtracting (2) from (3), we get

$10y+3y = -13$

$y = -1$

Submitting the value of y in equation (1), we get

$2x+5(-1) = -4$

$2x = 1$

$x = \frac{1}{2}$

51. The sum of the digits of a 2-digit number is 14. The number obtained by interchanging its digits exceeds the given number by 18. Find the number.

Sol : [Board 2024 OD Standard]

Let the tenth digit be x and ones digit be y. The number will $10x+y$. Since sum of digits of two digit number is 14, we have

$x+y = 14$ ...(1)

Number obtained by interchanging its digits exceeds the given number by 18. Thus

$10y+x = 10x+y+18$

$9y = 9x+18$

$y = x+2$ (2)

Substituting the value of y in equation (1), we get

$x+(x+2) = 14$

$2x+2 = 14$

$2x = 12$

$x = 6$

Substituting the value of x in equation (2), we get

$y = 6+2 = 8$

Required number $= 10x+y$

$= 10(6)+8$

$= 68$

52. Solve graphically : $2x-3y+13=0;\ 3x-2y+12=0$

Sol : [Board 2020 OD Basic]

We have $2x-3y+13 = 0$

and $3x-2y+12 = 0$

Now $2x-3y = -13$

$y = \frac{2x+13}{3}$

x	0	-6.5	1
y	4.3	0	5

and $3x-2y = -12$

$y = \frac{3x+12}{2}$

x	0	-4	-2
y	6	0	3

These lines intersect each other at point $(-2,3)$ Hence, $x=-2$ and $y=3$.

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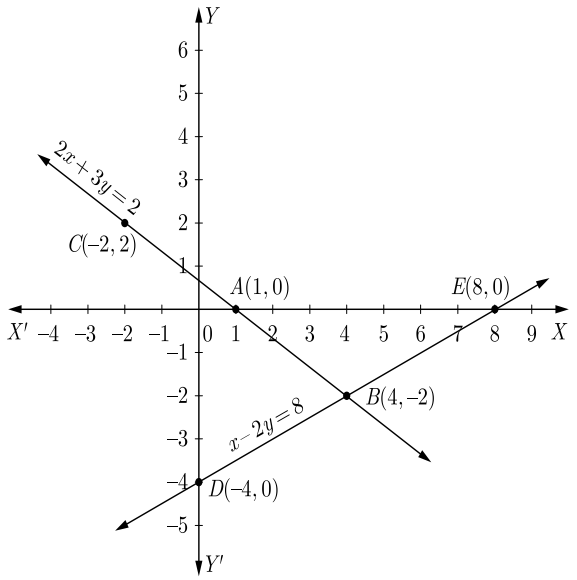

PRACTICE

Solve the pair of equations graphically :
$4x - y = 4$ and $3x + 2y = 14$
[Board Term-1 2014]
Ans : $x = 2$ and $y = 4$.

53. Solve graphically : $2x + 3y = 2,\ x - 2y = 8$

Sol : [Board 2020 Delhi Basic]

We have $2x + 3y = 2$ ...(1)

and $x - 2y = 8$ ...(2)

Taking equation (1), $y = \dfrac{2 - 2x}{3}$

x	1	4	-2
y	0	-2	2

and $x - 2y = 8$

Taking equation (2), $y = \dfrac{x - 8}{2}$

x	0	8	4
y	-4	0	-2

Plotting the above points and drawing the lines joining them, we get the graph of above equations.
Two obtained lines intersect at point $P(4, -2)$.
Hence, Solution of the given equation is $x = 4$, $y = -2$

54. A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator- Find the fraction.

Sol : [Board 2019 Delhi]

Let the fraction be $\dfrac{x}{y}$. According to the first condition,

$$\frac{x-2}{y} = \frac{1}{3}$$
$$3x - 6 = y$$
$$y = 3x - 6 \quad ...(1)$$

According to the second condition,

$$\frac{x}{y-1} = \frac{1}{2}$$
$$2x = y - 1$$
$$y = 2x + 1 \quad ...(2)$$

From equation (1) and (2), we have

$$3x - 6 = 2x + 1 \Rightarrow x = 7$$

Substitute value of x in equation (1), we get

$$y = 3(7) - 6$$
$$= 21 - 6 = 15$$

Hence, fraction is $\dfrac{7}{15}$.

55. Represent the following pair of linear equations graphically and hence comment on the condition of consistency of this pair.
$x - 5y = 6$ and $2x - 10y = 12$.

Sol : [Board Term-1 2011]

We have $x - 5y = 6$ or $x = 5y + 6$

x	6	1	-4
y	0	-1	-2

and $2x - 10y = 12$ or $x = 5y + 6$

x	6	1	-4
y	0	-1	-2

Plotting the above points and drawing lines joining them, we get the following graph.
Since the lines are coincident, so the system of linear equations is consistent with infinite many solutions.

56. Determine the values of m and n so that the following system of linear equation have infinite number of solutions :
$(2m - 1)x + 3y - 5 = 0$
$3x + (n - 1)y - 2 = 0$

Sol : [Board Term-1 2013]

We have $(2m - 1)x + 3y - 5 = 0$ ...(1)
Here $a_1 = 2m - 1, b_1 = 3, c_1 = -5$

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$$3x+(n-1)y-2=0 \quad ...(2)$$

Here $a_2=3, b_2=(n-1), c_2=-2$

For a pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2m-1}{3}=\frac{3}{n-1}=\frac{5}{2}$$

or $2(2m-1)=15$ and $5(n-1)=6$

Hence, $m=\frac{17}{4}, n=\frac{11}{5}$

PRACTICE

Find the values of α and β for which the following pair of linear equations has infinite number of solutions : $2x+3y=7;\ 2\alpha x+(\alpha+\beta)y=28$.

[Board Term-1 2011]

Ans : $\alpha=4$, and $\beta=8$

57. For what value of p will the following system of equations have no solution ?

$$(2p-1)x+(p-1)y=2p+1;\ y+3x-1=0$$

Sol : [Board Term-1 2011]

We have $(2p-1)x+(p-1)y-(2p+1)=0$

Here $a_1=2p-1, b_1=p-1$ and $c_1=-(2p+1)$

Also $3x+y-1=0$

Here $a_2=3, b_2=1$ and $c_2=-1$

The condition for no solution is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{2p-1}{3}=\frac{p-1}{1}\neq\frac{2p+1}{1}$$

From $\frac{2p-1}{3}=\frac{p-1}{1}$ we have

$$3p-3=2p-1$$

$$3p-2p=3-1$$

$$p=2$$

From $\frac{p-1}{1}\neq\frac{2p+1}{1}$ we have

$$p-1\neq 2p+1$$

or $2p-p\neq -1-1$

$$p\neq -2$$

From $\frac{2p-1}{3}\neq\frac{2p+1}{1}$ we have

$$2p-1\neq 6p+3$$

$$4p\neq -4$$

$$p\neq -1$$

Hence, system has no solution when $p=2$

PRACTICE

Find the value of k for which the following pair of equations has no solution :
$x+2y=3, (k-1)x+(k+1)y=(k+2)$.

[Board Term-1 2011]

Ans : $k=3$

58. Solve for x and y :

$$\frac{x}{2}+\frac{2y}{3}=-1 \text{ and } x-\frac{y}{3}=3$$

Sol : [Board Term-1 2015]

We have $\frac{x}{2}+\frac{2y}{3}=-1$

$$3x+4y=-6 \quad ...(1)$$

and $\frac{x}{1}-\frac{y}{3}=3$

$$3x-y=9 \quad ...(2)$$

Subtracting equation (2) from equation (1), we have

$$5y=-15 \Rightarrow y=-1$$

Substituting $y=-3$ in eq. (1), we get

$$3x+4(-3)=-6$$

$$3x-12=-6$$

$$3x=12-6 \Rightarrow x=2$$

Hence $x=2$ and $y=-3$.

59. Solve the following pair of linear equations :

$$8x+5y=9, \quad 3x+2y=4$$

Sol : [Board Term-1 2015]

From equation (2), we have

$$3x=4-2y$$

or, $x=\frac{4-2y}{3}$ ...(3)

Substituting this value of y in equation (3) in (1), we get

$$8\left(\frac{4-2y}{3}\right)+5y=9$$

$$32-16y+15y=27$$

$$-y=27-32$$

Thus $y=5$

Substituting this value of y in equation (3)

$$x=\frac{4-2(5)}{3}=\frac{4-10}{3}=-2$$

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Hence, $x = -2$ and $y = 5$.

60. Solve for x and y :

$$\frac{x+1}{2}+\frac{y-1}{3}=9;\quad \frac{x-1}{3}+\frac{y+1}{2}=8.$$

Sol : [Board Term-1 Delhi 2011]

We have $\frac{x+1}{2}+\frac{y-1}{3} = 9$

$$3(x+1)+2(y-1) = 54$$
$$3x+3+2y-2 = 54$$
$$3x+2y = 53 \quad (1)$$

and $\frac{x-1}{3}+\frac{y+1}{2} = 8$

$$2(x-1)+3(y+1) = 48$$
$$2x-2+3y+3 = 48$$
$$2x+3y = 47 \quad (2)$$

Multiplying equation (1) by 3 we have

$$9x+6y = 159 \quad (3)$$

Multiplying equation (2) by 2 we have

$$4x+6y = 94 \quad (4)$$

Subtracting equation (4) from (3) we have

$$5x = 65$$

or $x = 13$

Substitute the value of x in equation (2),

$$2(13)+3y = 47$$
$$3y = 47-26 = 21$$
$$y = \frac{21}{3} = 7$$

Hence, $x = 13$ and $y = 7$

61. Given the linear equation $2x+3y-8=0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(a) intersecting lines

(b) parallel lines

(c) coincident lines.

Sol : [Board Term-1 2014]

Given, linear equation is $2x+3y-8=0$ ...(1)

(a) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

To get its parallel line one of the possible equation may be taken as

$$5x+2y-9 = 0 \quad (2)$$

(b) For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

One of the possible line parallel to equation (1) may be taken as

$$6x+9y+7 = 0$$

(c) For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

To get its coincident line, one of the possible equation may be taken as

$$4x+6y-16 = 0$$

62. Solve for x and y :

$$ax+by = \frac{a+b}{2} \text{ and } 3x+5y = 4$$

Sol : [Board Term-1 2011]

We have $ax+by = \frac{a+b}{2}$

or $2ax+2by = a+b$ ...(1)

and $3x+5y = 4$ ...(2)

Multiplying equation (1) by 5 we have

$$10ax+10by = 5a+5b \quad ...(3)$$

Multiplying equation (2) by $2b$, we have

$$6bx+10by = 8b \quad ...(4)$$

Subtracting (4) from (3) we have

$$(10a-6b)x = 5a-3b$$

or $x = \frac{5a-3b}{10a-6b} = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ in equation (2), we get

$$3\times\frac{1}{2}+5y = 4$$
$$5y = 4-\frac{3}{2} = \frac{5}{2}$$
$$y = \frac{5}{2\times 5} = \frac{1}{2}$$

Hence $x = \frac{1}{2}$ and $y = \frac{1}{2}$.

63. Find whether the following pair of linear equations has a unique solutions. If yes, find the solution :

$$7x-4y = 49, 5x-6y = 57.$$

Sol : [Board Term-1 2017]

We have $7x-4y = 49$ (1)

$$5x-6y = 57 \quad (2)$$

Comparing with the equation $a_1x+b_1y=c_1$,

$$a_1 = 7, b_1 = -4, c_1 = 49$$
$$a_2 = 5, b_2 = -6, c_2 = 57$$

Since, $\frac{a_1}{a_2} = \frac{7}{5}$ and $\frac{b_1}{b_2} = \frac{4}{6}$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, system has a unique solution.

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Multiply equation (1) by 5 we get

$$35x-20y = 245 \quad (3)$$

Multiply equation (2) by 7 we get

$$35x-42y = 399 \quad (4)$$

Subtracting (4) by (3) we have

$$22y = -154$$

$$y = -7$$

Putting the value of y in equation (2),

$$5x-6(-7) = 57$$

$$5x = 57-42 = 15$$

$$x = 3$$

Hence $x = 3$ and $y = -7$.

FIVE MARKS QUESTIONS

64. For what value of k, which the following pair of linear equations have infinitely many solutions:
$2x+3y = 7$ and $(k+1)x+(2k-1)y = 4k+1$

Sol : [Board 2019 Delhi]

We have $2x+3y = 7$

and $(k+1)x+(2k-1)y = 4k+1$

Here $\frac{a_1}{a_2} = \frac{2}{k+1}, \frac{b_1}{b_2} = \frac{3}{(2k-1)}$

and $\frac{c_1}{c_2} = \frac{-7}{-(4k+1)} = \frac{7}{(4k+1)}$

For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

For $\frac{a_1}{a_2} = \frac{c_1}{c_2}$ we have

$$\frac{2}{k+1} = \frac{7}{4k+1}$$

$$2(4k+1) = 7(k+1)$$

$$8k+2 = 7k+7$$

$$k = 5$$

Hence, the value of k is 5, for which the given equation have infinitely many solutions.

PRACTICE

For what value of k, will the following pair of equations have infinitely many solutions:
$2x+3y = 7$ and $(k+2)x-3(1-k)y = 5k+1$
[Board 2019 OD]
Ans : $k = 4$

65. Find c if the system of equations $cx+3y+(3-c) = 0; 12x+cy-c = 0$ has infinitely many solutions?

Sol : [Board 2019 Delhi]

We have $cx+3y+(3-c) = 0$

$12x+cy-c = 0$

Here, $\frac{a_1}{a_2} = \frac{c}{12}, \frac{b_1}{b_2} = \frac{3}{c}, \frac{c_1}{c_2} = \frac{3-c}{-c}$

For infinite many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

For $\frac{a_1}{a_2} = \frac{c_1}{c_2}$ we have,

$$\frac{c}{12} = \frac{3-c}{-c}$$

$$-c^2 = 36-12c$$

$$-c^2+12c-36 = 0$$

$$c^2-12c+36 = 0$$

$$c^2-6c-6c+36 = 0$$

$$c(c-6)-6(c-6) = 0$$

$$(c-6)(c-6) = 0 \Rightarrow c = 6$$

and for $\frac{b_1}{b_2} = \frac{c_1}{c_2}$,

$$\frac{3}{c} = \frac{3-c}{-c}$$

$$-3c = 3c-c^2$$

$$c^2-6c = 0$$

$$c(c-6) = 0 \Rightarrow c = 6 \text{ or } c \neq 0$$

Hence, the value of c is 6, for which the given equations have infinitely many solutions.

66. Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are given by $2y-x = 8$, $5y-x = 14$ and $y-2x = 1$.

Sol : [Board 2020 Delhi Standard]

We have $2y-x = 8$

L_1 : $x = 2y-8$

y	0	4	5
$x = 2y-8$	-8	0	2

$5y-x = 14$

L_2 : $x = 5y-14$

y	3	4	2
$x = 5y-14$	1	6	-4

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and $y-2x=1$

L_3 : $y=1+2x$

x	0	1	2
$y=1+2x$	1	3	5

Plotting the above points and drawing lines joining them, we get the graphical representation:
Hence, the coordinates of the vertices of a triangle ABC are $A(1,3)$, $B(2,5)$ and $C(-4,2)$.

67. Determine graphically whether the following pair of linear equations :

$3x-y=7$ and $2x+5y+1=0$ has :

(a) unique solution

(b) infinitely many solutions or

(c) no solution.

Sol : [Board Term-1 2015]

We have $3x-y=7$

or $3x-y-7=0$ (1)

Here $a_1=3, b_1=1, c_1=-7$

$2x+5y+1=0$ (2)

Here $a_2=2, b_2=5, c_2=1$

Now $\frac{a_1}{a_2}=\frac{3}{2}, \frac{b_1}{b_2}=\frac{-1}{5}$

Since $\frac{3}{2}\neq\frac{-1}{5}$, thus $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

Hence, given pair of linear equations has a unique solution.

Now line (1) $y=3x-7$

x	0	2	3
y	-7	-1	2

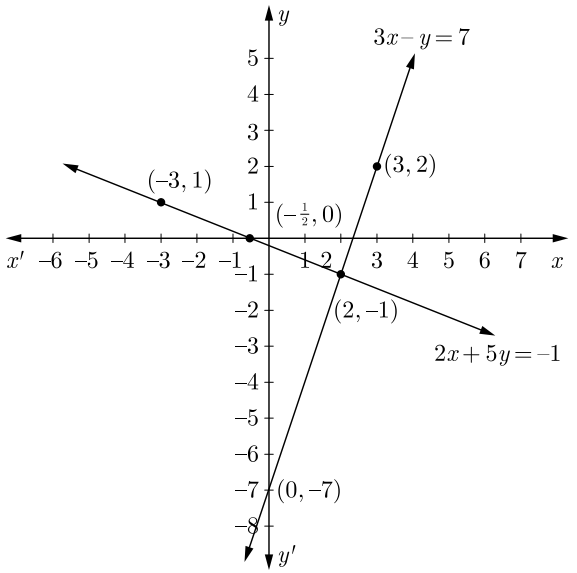
and line (2)

$2x+5y+1=0$

or, $y=\frac{-1-2x}{5}$

x	2	-3
y	-1	1

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(2,-1)$.
Hence $x=2$ and $y=-1$

68. Aftab tells his daughter, '7 years ago, I was seven times as old as you were then. Also, 3 years from now, I shall be three times as old as you will be.' Represent this situation algebraically and graphically.

Sol : [Board Term-1 2015]

Let the present age of Aftab be x years and the age of daughter be y years.

7 years ago father's(Aftab) age $=(x-7)$years

7 years ago daughter's age $=(y-7)$ years

According to the question,

$(x-7)=7(y-7)$

or, $(x-7y)=-42$ (1)

After 3 years father's(Aftab) age $=(x+3)$ years

After 3 years daughter's age $=(y+3)$ years

According to the condition,

$x+3=3(y+3)$

or, $x-3y=6$ (2)

From equation(1) $x-7y=-42$

x	0	7	14
$y=\frac{x+42}{7}$	6	7	8

From equation (2) $x-3y=6$

x	6	12	18
$y=\frac{x-6}{3}$	0	2	4

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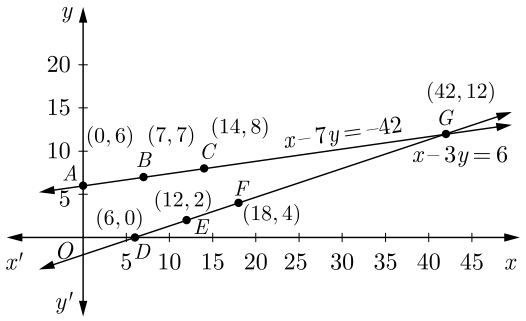
Plotting the above points and drawing lines joining them, we get the following graph.



Two lines obtained intersect each other at $(42, 12)$

Hence, father's age $= 42$ years

and daughter's age $= 12$ years

69. For Uttarakhand flood victims two sections A and B of class contributed Rs. 1,500. If the contribution of X-A was Rs. 100 less than that of X-B, find graphically the amounts contributed by both the sections.

Sol : [Board Term-1 2016]

Let amount contributed by two sections X-A and X-B be Rs. x and Rs. y.

$$x + y = 1{,}500 \quad \ldots(1)$$

$$y - x = 100 \quad \ldots(2)$$

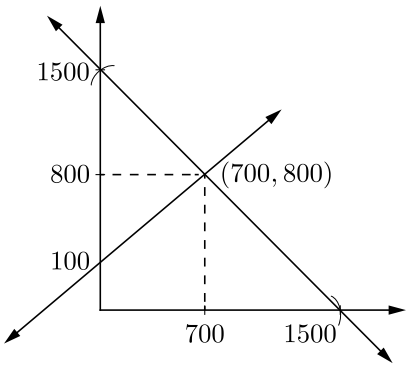
From (1) $y = 1500 - x$

x	0	700	1,500
y	1,500	800	0

From (2) $y = 100 + x$

x	0	700
y	100	800

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point (700, 800)

Hence X-A contributes 700 Rs and X-B contributes 800 Rs.

70. Solve graphically the pair of linear equations :

$3x - 4y + 3 = 0$ and $3x + 4y - 21 = 0$

Find the co-ordinates of the vertices of the triangular region formed by these lines and $x-$axis. Also, calculate the area of this triangle.

Sol : [Board Term-1 2015]

We have $3x - 4y + 3 = 0$

or, $y = \dfrac{3x+3}{4}$

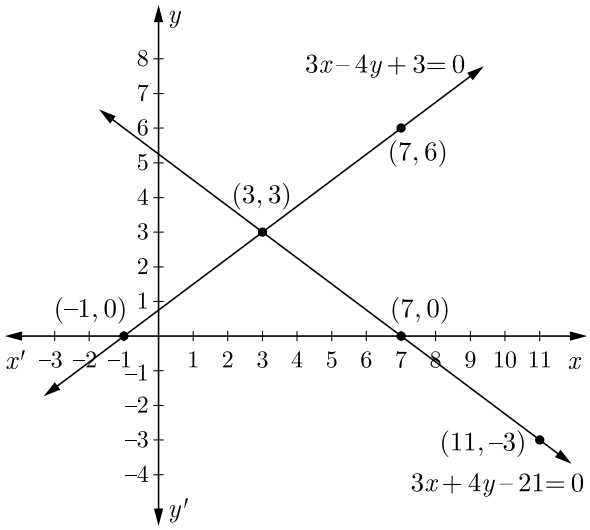
x	3	7	-1
y	3	6	0

and $3x + 4y - 21 = 0$

or, $y = \dfrac{21 - 3x}{4}$

x	3	7	11
y	3	0	-2

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(3, 3)$.

(a) These lines intersect each other at point $(3, 3)$.

Hence $x = 3$ and $y = 3$

(b) The vertices of triangular region are $(3, 3)$, $(-1, 0)$ and $(7, 0)$.

(c) Area of $\Delta = \frac{1}{2} \times 8 \times 3 = 12$

Hence, Area of obtained Δ is 12 sq unit.

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71. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs. 160. After a month, the cost of 4kg of apples and 2kg of grapes is Rs. 300. Represent the situations algebraically and geometrically.

Sol : [Board Term-1 2013]

Let the cost of 1 kg of apples be Rs.x and cost of 1 kg of grapes be Rs. y.

The given conditions can be represented given by the following equations :

$$2x+y = 160 \quad ...(1)$$

$$4x+2y = 300 \quad ...(2)$$

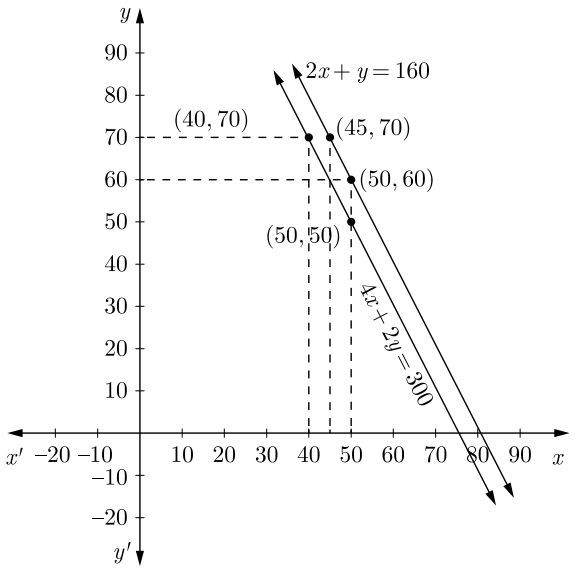
From equation (1) $y = 160-2x$

x	50	45
y	60	70

From equation (2) $y = 150-2x$

x	50	40
y	50	70

Plotting these points on graph, we get two parallel line as shown below.



72. Solve for x and y :

$2x-y+3$ and $3x-5y+1 = 0$

Sol : [Board Term-1 2015]

We have $2x-y+3 = 0 \quad ...(1)$

$$3x-5y+1 = 0 \quad ...(2)$$

Multiplying equation (1) by 5,and subtracting (2) from it we have

$$7x = -14$$

$$x = \frac{-14}{7} = -2$$

Substituting the value of x in equation (1) we get

$$2x-y+3 = 0$$
$$2(-2)-y+3 = 0$$
$$-4-y+3 = 0$$
$$-y-1 = 0$$
$$y = -1$$

Hence, $x = -2$ and $y = -1$.

73. Draw the graphs of the equations $x-y+1=0$ and $3x+2y-12=0$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

Sol : [Board Term-1 2013]

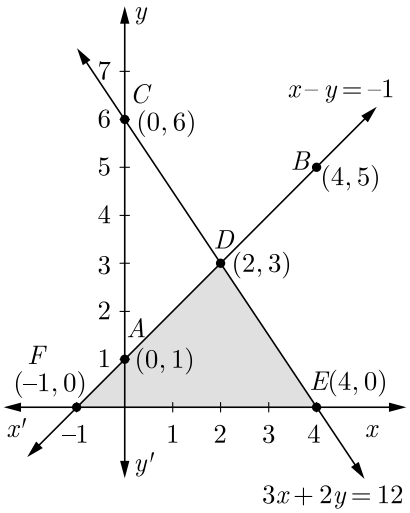
We have $x-y+1 = 0 \quad ...(1)$

x	0	4	2
$y = x+1$	1	5	3

and $3x+2y-12 = 0 \quad ...(2)$

x	0	2	4
$y = \frac{12-3x}{2}$	6	3	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $D(2,3)$. Hence, $x=2$ and $y=3$ is the solution of the given pair of equations. The line CD intersects the x-axis at the point $E(4,0)$ and the line AB intersects the x-axis at the points $F(-1,0)$. Hence, the co-ordinates of the vertices of the

triangle are $D(2,3)$, $E(4,0)$ and $F(-1,0)$.

14. Solve the following pair of linear equations graphically:

$x+3y=12,\ 2x-3y=12$

Also shade the region bounded by the line $2x-3y=2$ and both the co-ordinate axes.

Sol : [Board Term-1 2013, 2012]

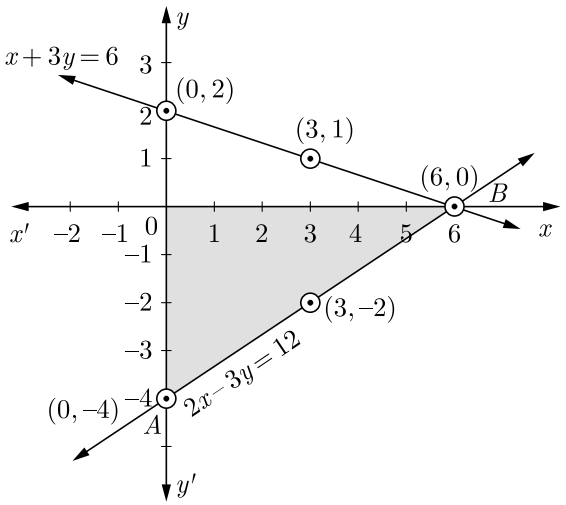
We have $x+3y=6 \Rightarrow y=\dfrac{6-x}{3}$ ...(1)

x	3	6	0
y	1	0	2

and $2x-3y=12 \Rightarrow y=\dfrac{2x-12}{3}$

x	0	6	3
y	-4	0	-2

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $B(6,0)$.

Hence, $x=6$ and $y=0$

Again ΔOAB is the region bounded by the line $2x-3y=12$ and both the co-ordinate axes.

15. Solve the following pair of linear equations graphically:

$x-y=1,\ 2x+y=8$

Also find the co-ordinates of the points where the lines represented by the above equation intersect $y-$ axis.

Sol : [Board Term-1 Delhi 2012]

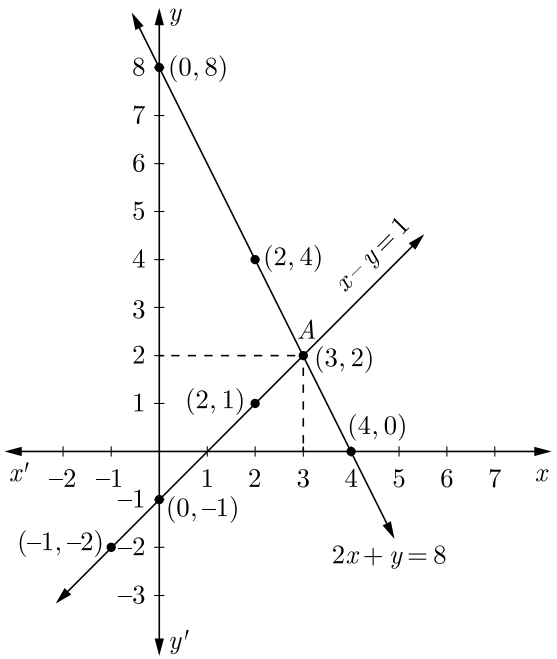
We have $x-y=1 \Rightarrow y=x-1$

x	2	3	-1
y	1	2	-2

and $2x+y=8 \Rightarrow y=8-2x$

x	2	4	0
y	4	0	8

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $A(3,2)$. Thus solution of given equations is $x=3, y=2$.

Again, $x-y=1$ intersects $y-$ axis at $(0,-1)$

and $2x+y=8$ $y-$ axis at $(0,8)$.

PRACTICE

Draw the graphs of the pair of linear equations :

$x+2y=5$ and $2x-3y=-4$

Also find the points where the lines meet the $x-$axis.

[Board Term-1 2015]

Ans : $(5,0)$ and $(-2,0)$

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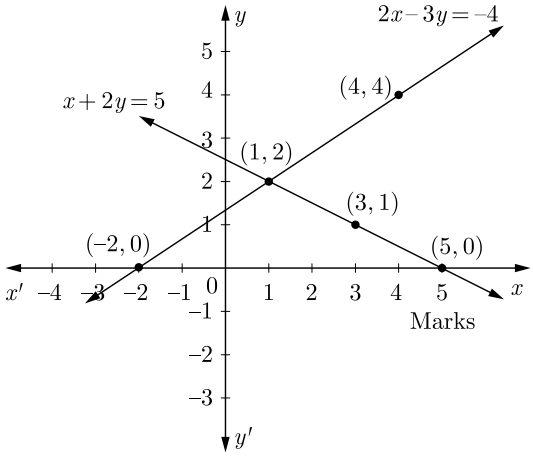


16. Draw the graph of the following equations:

$2x - y = 1, \quad x + 2y = 13$

Find the solution of the equations from the graph and shade the triangular region formed by the lines and the y-axis.

Sol : [Board Term-1 OD 2012]

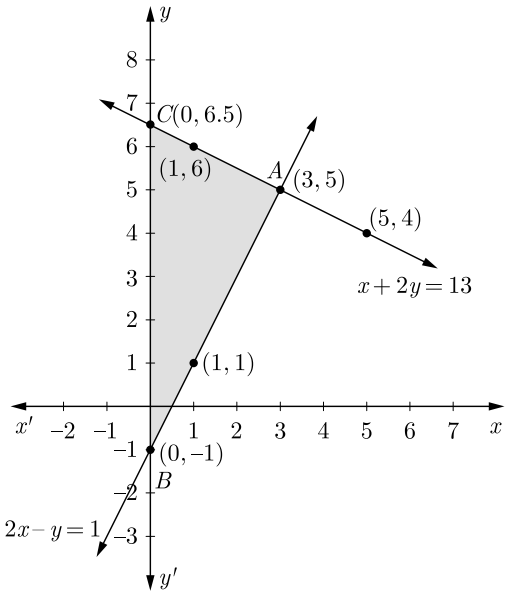
We have $2x - y = 1 \Rightarrow y = 2x - 1$

x	0	1	3
y	-1	1	5

and $x + 2y = 13 \Rightarrow y = \dfrac{13 - x}{2}$

x	1	3	5
y	6	5	4

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly two obtained lines intersect at point $A(3,5)$.

Hence, $x = 3$ and $y = 5$

ABC is the triangular shaded region formed by the obtained lines with the y-axis.

PRACTICE

Solve the following pair of linear equations graphically:
$2x + 3y = 12$ and $x - y = 1$
Find the area of the region bounded by the two lines representing the above equations and y-axis.

[Board Term-1 2012]

Ans : $x = 3$ and $y = 2$, 7.5 square unit

17. Solve the following pair of equations graphically:

$2x + 3y = 12, \; x - y - 1 = 0.$

Shade the region between the two lines represented by the above equations and the X-axis.

Sol : [Board Term-1 2013]

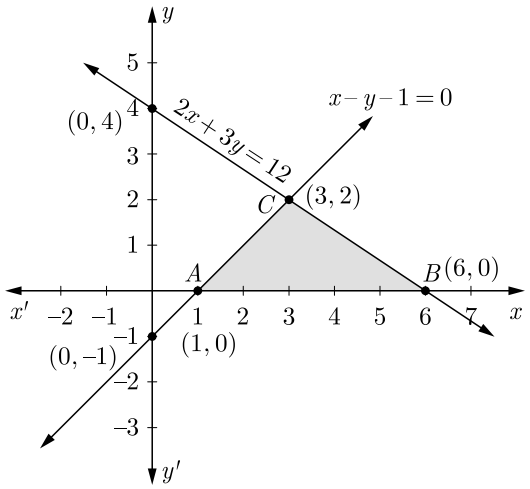
We have $2x + 3y = 12 \Rightarrow y = \dfrac{12 - 2x}{3}$

x	0	6	3
y	4	0	2

also $x - y = 1 \Rightarrow y = x - 1$

x	0	1	3
y	-1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $(3, 2)$,
Hence, $x = 3$ and $y = 2$.

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ΔABC is the region between the two lines represented by the given equations and the x -axis.

78. Solve $x+y=5$ and $2x-3y=4$ by elimination method and the substitution method.

Sol : [Board Term-1 2015]

By Elimination Method :

We have, $x+y=5$ ...(1)

and $2x-3y=4$ ...(2)

Multiplying equation (1) by 3 and adding in (2) we have

$$3(x+y)+(2x-3y)=3\times 5+4$$

or, $$3x+3y+2x-3y=15+4$$

$$5x=19 \Rightarrow x=\frac{19}{5}$$

Substituting $x=\frac{19}{5}$ in equation (1),

$$\frac{19}{5}+y=5$$

$$y=5-\frac{19}{5}=\frac{25-19}{5}=\frac{6}{5}$$

Hence, $x=\frac{19}{5}$ and $y=\frac{6}{5}$

By Substituting Method :

We have, $x+y=5$ ...(1)

and $2x-3y=4$ ...(2)

From equation (1), $y=5-x$ ...(3)

Substituting the value of y from equation (3) in equation (2),

$$2x-3(5-x)=4$$

$$2x-15+3x=4$$

$$5x=19$$

$$x=\frac{19}{5}$$

Substituting this value of x in equation (3), we get

$$y=5-\frac{19}{5}=\frac{6}{5}$$

Hence $x=\frac{19}{5}$ and $y=\frac{6}{5}$

PRACTICE

✍ Solve $3x-5y-4=0$ and $9x=2y+7$ by elimination method and the substitution method.

[Board Term-1 OD 2012]

Ans : $x=\frac{9}{13}$ and $y=\frac{-5}{13}$

✍ Solve for x and y :

$$3x+4y=10$$

$$2x-2y=2$$

[Board Term-1 2015]

Ans : $x=2$ and $y=1$

79. For what values of a and b does the following pair of linear equations have infinite number of solution ?

$2x+3y=7, a(x+y)-b(x-y)=3a+b-2$

Sol : [Board Term-1 2015]

We have $2x+3y-7=0$

Here $a_1=2, b_1=3, c_1=-7$

and $$a(x+y)-b(x-y)=3a+b-2$$

$$ax+ay-bx+by=3a+b-2$$

$$(a-b)x+(a+b)y-(3a+b-2)=0$$

Here $a_2=a-b, b_2=a+b, c_2=-(3a+b-2)$

For infinite many solutions

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{a-b}=\frac{3}{a+b}=\frac{-7}{(3a+b-2)}$$

From $\frac{2}{a-b}=\frac{7}{3a+b-2}$ we have

$$2(3a+b-2)=7(a-b)$$

$$6a+2b-4=7a-7b$$

$$a-9b=-4 \quad ...(1)$$

From $\frac{3}{a+b}=\frac{7}{3a+b-2}$ we have

$$3(3a+b-2)=7(a+b)$$

$$9a+3b-6=7a+7b$$

$$2a-4b=6$$

$$a-2b=3 \quad ...(2)$$

Subtracting equation (1) from (2),

$$-7b=-7$$

$$b=1$$

Substituting the value of b in equation (1),

$$a=5$$

Hence, $a=5, b=1$.

80. Find the value of p and q for which the system of equations represent coincident lines $2x+3y=7$, $(p+q+1)x+(p+2q+2)y=4(p+q)+1$

Sol : [Board Term-1 Delhi 2012]

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We have $2x+3y = 7$

$(p+q+1)x+(p+2q+2)y = 4(p+q)+1$

Comparing given equation to $ab+by+c=0$ we have

$a_1 = 2, b_1 = 3, c_1 = -7$

$a_2 = p+q+1, b_2 = p+2q+2, c_2 = -4(p+q)-1$

For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{p+q+1} = \frac{3}{p+2q+2} = \frac{7}{4(p+q)+1}$$

From $\frac{3}{p+2q+2} = \frac{7}{4(p+q)+1}$ we have

$$7p+14q+14 = 12p+12q+3$$

$$5p-2q-11 = 0 \quad ...(1)$$

From $\frac{2}{p+q+1} = \frac{7}{4(p+q)+1}$ we have

$$8(p+q)+2 = 7p+7q+7$$

$$8p+8q+2 = 7p+7q+7$$

$$p+q-5 = 0 \quad ...(2)$$

Multiplying equation (2) by 5 we have

$$5p+5q-25 = 0 \quad ...(3)$$

Subtracting equation (1) from (3) we get

$$7q = 14$$

$$q = 2$$

Hence, $p = 3$ and $q = 2$.

WORD PROBLEMS

81. In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes and this later number was equal to twice his majority over B. If there were 1,8000 persons on the electoral roll. How many votes for B.

Sol : [Board Term-1 2012]

Let x and y be the no. of votes for A and B respectively.

The no. of persons who did not vote is $18000-x-y$.

We have $x = 2(18000-x-y)$

$$3x+2y = 36000 \quad ...(1)$$

and $(18000-x-y) = 2(x-y)$

or $3x-y = 18000 \quad ...(2)$

Subtracting equation (2) from equation (1),

$$3y = 18000$$

$$y = 6000$$

Hence vote for B is 6000.

82. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

Sol : [Board Term-1 2015]

Let age of father and son be x and y respectively.

$$x+y = 40 \quad ...(1)$$

$$x = 3y \quad ...(2)$$

Solving equations (1) and (2), we get

$$x = 30 \text{ and } y = 10$$

Ages are 30 years and 10 years.

83. Half the perimeter of a rectangular garden, whose length is 4 m more then its width, is 36 m. Find the dimensions of garden.

Sol : [Board Term-1 2013]

Let the length of the garden be x m and its width be y m.

Perimeter of rectangular garden

$$p = 2(x+y)$$

Since half perimeter is given as 36 m,

$$x+y = 36 \quad ...(1)$$

Also, $x = y+4$

or $x-y = 4 \quad ...(2)$

Now adding eq (1) and (2) we have

$$2x = 40 \Rightarrow x = \frac{40}{2} = 20$$

Subtracting eq (2) from (1) we have

$$2y = 32 \Rightarrow x = \frac{32}{2} = 16$$

Hence, length is 20 m and width is 16 m.

84. A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay Rs. 3,000 as hostel charges whereas Mansi who takes food for 25 days Rs. 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

Sol : [Board Term-1 2016, 2015]

NODIA

Let fixed charge be x and per day food cost be y

$$x+20y = 3000 \quad ...(1)$$

$$x+25y = 3500 \quad ...(2)$$

Subtracting (1) from (2) we have

$$5y = 500 \Rightarrow y = 100$$

Substituting this value of y in (1), we get

$$x+20(100) = 3000$$

$$x = 1000$$

Thus $x = 1000$ and $y = 100$

Fixed charge and cost of food per day are Rs. 1,000 and Rs. 100.

85. The present age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Sol : [Board Term-1 2012]

Let the sum of the ages of the 2 children be x and the age of the father be y years.

Now $y = 2x$

$$2x-y = 0 \quad ...(1)$$

and $20+y = x+40$

$$x-y = -20 \quad ...(2)$$

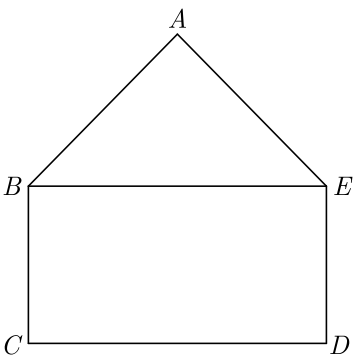
Subtracting (2) from (1), we get

$$x = 20$$

From(1), $y = 2x = 2 \times 20 = 40$

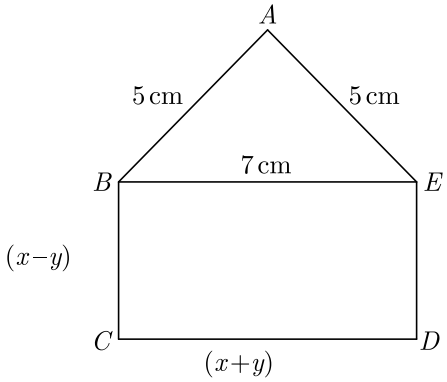
Hence, the age of the father is 40 years.

86. In the figure, $ABCDE$ is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to CD. $AB = 5$ cm, $AE = 5$ cm, $BE = 7$ cm, $BC = x-y$ and $CD = x+y$. If the perimeter of $ABCDE$ is 27 cm. Find the value of x and y, given $x, y \neq 0$.



Sol : [Board 2020 SQP Standard]

We have redrawn the given figure as shown below.



We have $CD = BE$

$$x+y = 7 \quad ...(1)$$

Also, perimeter of $ABCDE$ is 27 cm, thus

$$AB+BC+CD+DE+AE = 27$$

$$5+(x-y)+(x+y)+(x-y)+5 = 27$$

$$3x-y = 17 \quad ...(2)$$

Adding equation (1) and (2) we have

$$4x = 24 \Rightarrow x = 6$$

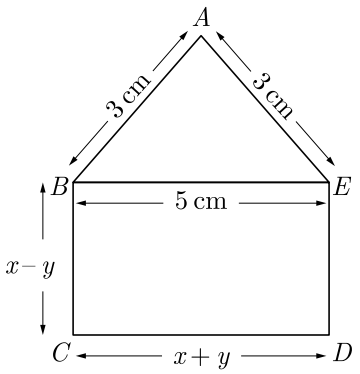
Substituting $x = 6$ in equation (1) we obtain

$$y = 7-x = 7-6 = 1$$

Thus $x = 6$ and $y = 1$.

PRACTICE

In the figure below $ABCDE$ is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to DC. If the perimeter of $ABCDE$ is 21 cm, find the values of x and y.



[Board 2010]

Ans : $x = 5$ and $y = 0$.

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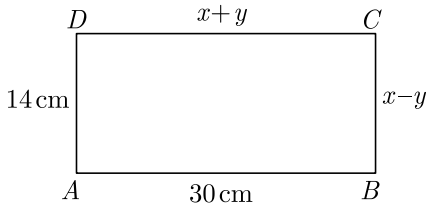
87. In Figure, $ABCD$ is a rectangle. Find the values of x and y.



Sol : [Board 2018]

Since $ABCD$ is a rectangle, we have

$$AB = CD \text{ and } BC = AD$$

Now $$x + y = 30 \quad ...(1)$$

$$x - y = 14 \quad ...(2)$$

Adding equation (1) and (3) we obtain,

$$2x = 44 \Rightarrow x = \frac{44}{2} = 22$$

Substituting value of x in equation (1) we have

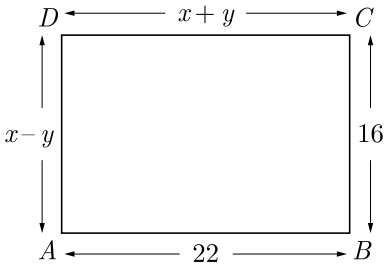
$$22 + y = 30$$

$$y = 30 - 22 = 8$$

$$x = 22\,\text{cm} \text{ and } y = 8\,\text{cm}$$

PRACTICE

In the figure given below, $ABCD$ is a rectangle. Find the values of x and y.



[Board Term-1 2012]

Ans : $x = 19$ and $y = 3$.

88. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number.

Sol : [Board Term-1 2017]

Let the ten's and unit digit by y and x respectively,

So the number is $10y + x$

The number when digits are reversed becomes $10x + y$

Thus $$7(10y + x) = 4(10x + y)$$

$$70y + 7x = 40x + 4y$$

$$70y - 4y = 40x - 7x$$

$$2y = x \quad ...(1)$$

or $$x - y = 3 \quad ...(2)$$

From (1) and (2) we get

$$y = 3 \text{ and } x = 6$$

Hence the number is 36.

89. 4 chairs and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs 1750. Find the cost of one chair and one table separately.

Sol : [Board Term-1 2015]

Let cost of 1 chair be Rs x and cost of 1 table be Rs y According to the question,

$$4x + 3y = 2100 \quad ...(1)$$

$$5x + 2y = 1750 \quad ...(2)$$

Multiplying equation (1) by 2 and equation (2) by 3,

$$8x + 6y = 4200 \quad ...(3)$$

$$15x + 6y = 5250 \quad ...(iv)$$

Subtracting equation (3) from (4) we have

$$7x = 1050$$

$$x = 150$$

Substituting the value of x in (1), $y = 500$

Thus cost of chair and table is Rs 150, Rs 500 respectively.

90. A chemist has one solution which is 50 % acid and a second which is 25 % acid. How much of each should be mixed to make 10 litre of 40 % acid solution.

Sol : [Board Term-1 2015]

Let 50 % acids in the solution be x and 25 % of other solution be y.

Total volume in the mixture

$$x + y = 10 \quad ...(1)\ 1$$

and $$\frac{50}{100}x + \frac{25}{100}y = \frac{40}{100} \times 10$$

$$2x + y = 16 \quad ...(2)\ 1$$

Subtracting equation (1) from (2) we have

$$x = 6$$

Substituting this value of x in equation (1) we get

$$6 + y = 10$$

NODIA

$$y = 4$$

Hence, $x = 6$ and $y = 4$.

91. It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for four hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. How long would it take for each pipe to fill the pool separately?

Sol : [Board 2020 OD Standard]

Let x be time taken to fill the pool by the larger diameter pipe and y be the time taken to fill the pool by the smaller diameter pipe.

According to question,

$$\frac{1}{x}+\frac{1}{y} = \frac{1}{12} \quad ...(1)$$

and $$\frac{4}{x}+\frac{9}{y} = \frac{1}{2} \quad ...(2)$$

Multiplying equation (1) by 9 and subtracting from equation (2), we get

$$\frac{5}{x} = \frac{9}{12}-\frac{1}{2} = \frac{1}{4}$$

$$x = 20$$

Substituting the value of x in equation (1), we have

$$\frac{1}{20}+\frac{1}{y} = \frac{1}{12}$$

$$\frac{1}{y} = \frac{1}{12}-\frac{1}{20} = \frac{5-3}{60}$$

$$\frac{1}{y} = \frac{2}{60} = \frac{1}{30} \Rightarrow y = 30$$

Hence, time taken to fill the pool by the larger and smaller diameter pipe are 20 hrs and 30 hrs respectively.

92. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream.

Sol : [Board 2020 Delhi Standard]

Let x be the speed of the boat in still water and y be the speed of the stream.

Relative Speed of boat in upstream will be $(x-y)$ and relative speed of boat in downstream will be $(x+y)$.

According to question, we have

$$\frac{20}{x+y} = 2$$

$$x+y = 10 \quad ...(1)$$

and $$\frac{4}{x-y} = 2$$

$$x-y = 2 \quad ...(2)$$

Adding equation (1) and (2), we have

$$2x = 12 \Rightarrow x = 6 \text{ km/hr}$$

Substituting the value of x is equation (1) we have,

$$6+y = 10$$

$$\Rightarrow \quad y = 10-6 = 4 \text{ km/hr}$$

Thus speed of a boat in still water is 6 km/hr and speed of the stream 4 km/hr.

93. A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hr more than the scheduled time. Find the distance covered by the train.

Sol : [Board Term-1 Delhi 2012]

Let the actual speed of the train be s and actual time taken t.

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$= st \text{ km}$$

According to the given condition, we have

$$st = (s+10)(t-2)$$

$$st = st-2s+10t-20$$

$$2s-10t+20 = 0$$

$$s-5t = -10 \quad (1)$$

and $$st = (s-10)(t+3)$$

$$st = st+3s-10t-30$$

$$3s-10t = 30 \quad ...(2)$$

Multiplying equation (1) by 3 and subtracting equation (2) from equation (1),

$$3\times(s-5t)-(3s-10t) = -3\times 10-30$$

$$-5t = -60$$

$$t = 12$$

Substituting value of t equation (1),

$$s-5\times 12 = -10$$

$$s = -10+60 = 50$$

Hence, the distance covered by the train

$$= 50\times 12 = 600 \text{ km}.$$

94. The ratio of incomes of two persons is 11:7 and the ratio of their expenditures is 9:5. If each of them manages to save Rs 400 per month, find their monthly incomes.

Sol : [Board Term-1 2012]

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Let the incomes of two persons be $11x$ and $7x$.

Also the expenditures of two persons be $9y$ and $5y$.

$$11x-9y = 400 \quad ...(1)$$

and $$7x-5y = 400 \quad ...(2)$$

Multiplying equation (1) by 5 and equation (2) by 9 we have

$$55x-45y = 2000 \quad ...(3)$$

and $$63x-45y = 3600 \quad ...(4)$$

Subtracting, above equation we have

$$-8x = -1600$$

or, $$x = \frac{-1,600}{-8} = 200$$

Hence Their monthly incomes are $11 \times 200 =$ Rs 2200 and $7 \times 200 =$ Rs 1400.

95. A and B are two points 150 km apart on a highway. Two cars start A and B at the same time. If they move in the same direction they meet in 15 hours. But if they move in the opposite direction, they meet in 1 hours. Find their speeds.

Sol : [Board Term-1 2012]

Let the speed of the car I from A be x and speed of the car II from B be y.

Same Direction :

Distance covered by car I

$= 150 +$(distance covered by car II)

$$15x = 150 + 15y$$

$$15x-15y = 150$$

$$x-y = 10 \quad ...(1)$$

Opposite Direction :

Distance covered by car I + distance covered by car II

$$= 150 \text{ km}$$

$$x+y = 150 \quad ...(2)$$

Adding equation (1) and (2), we have $x = 80$.

Substituting $x = 80$ in equation (1), we have $y = 70$.

Speed of the car I from $A = 80$ km/hr and speed of the car II from $B = 70$ km/hr.

96. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes $\frac{1}{2}$, but when 4 is added to the numerator and 3 is subtracted from the denominator, it becomes $\frac{3}{2}$. Find the fraction.

Sol : [Board Term-1 2012]

Let the fraction be $\frac{x}{y}$ then we have

$$\frac{x-2}{y+1} = \frac{1}{2}$$

$$2x-4 = y+1$$

$$2x-y = 5 \quad ...(1)1$$

Also, $$\frac{x+4}{y-3} = \frac{3}{2}$$

$$2x+8 = 3y-9]$$

$$2x-3y = -17 \quad ...(2)\ 1$$

Subtracting equation (2) from equation (1),

$$2y = 22 \Rightarrow y = 11$$

Substituting this value of y in equation (1) we have,

$$2x-11 = 5$$

$$x = 8$$

Hence, Fraction $= \frac{8}{11}$

PRACTICE

A fraction become $\frac{9}{11}$ if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction.

[Board Term-1 2012]

Ans : $\frac{7}{9}$

97. If a bag containing red and white balls, half the number of white balls is equal to one-third the number of red balls. Thrice the total number of balls exceeds seven times the number of white balls by 6. How many balls of each colour does the bag contain ?

Sol : [Board Term-1 2012]

Let the number of red balls be x and white balls be y.

According to the question,

$$\frac{y}{2} = \frac{1}{3}x \text{ or } 2x-3y = 0 \quad ...(1)$$

and $$3(x+y)-7y = 6$$

or $$3x-4y = 6 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by we have

$$6x-9y = 0 \quad ...(3)$$

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$$6x-8y = 12 \quad ...(4)$$

Subtracting equation (3) from (4) we have

$$y = 12$$

Substituting $y=12$ in equation (1),

$$2x-36 = 0$$

$$x = 18$$

Hence, number of red balls $= 18$

and number of white balls $= 12$

98. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and the breadth is increased by 3 units. The area is increased by 67 square units if length is increased by 3 units and breadth is increased by 2 units. Find the perimeter of the rectangle.

Sol : [Board Term-1 Delhi 2012]

Let length of given rectangle be x and breadth be y, then area of rectangle will be xy.

According to the first condition we have

$$(x-5)(y+3) = xy-9$$

or, $$3x-5y = 6 \quad ...(1)$$

According to the second condition, we have

$$(x+3)(y+2) = xy-67$$

or, $$2x+5y = 61 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by 5 and then adding,

$$9x-15y = 18$$

$$10x+15y = 305$$

$$x = \frac{323}{19} = 17$$

Substituting this value of x in equation (1),

$$3(17)-5y = 6$$

$$5y = 51-6$$

$$y = 9$$

Hence, perimeter $= 2(x+y) = 2(17+9) = 52$ units.

99. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Sol : [Board Term-1 2012]

Let the number of students in a row be x and the number of rows be y. Thus total will be xy.

Now $$(x+3)(y-1) = xy$$

$$xy+3y-x-3 = xy$$

$$-x+3y-3 = 0 \quad ...(1)$$

and $$(x-3)(y+2) = xy$$

$$xy-3y+2x-6 = xy$$

$$2x-3y-6 = 0 \quad ...(2)$$

Multiply equation (1) 2 we have

$$-2x+6y-6 = 0 \quad ...(3)$$

Adding equation (2) and (3) we have

$$3y-12 = 0$$

$$y = 4$$

Substitute $y=4$ in equation (1)

$$-x+12-3 = 0$$

$$x = 9$$

Total students $xy = 9 \times 4 = 36$

Total students in the class is 36.

100. The ages of two friends ani and Biju differ by 3 years. Ani's father Dharam is twice as old as ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 year. Find the ages of Ani and Biju.

Sol : [Board Term-1 2012]

Let the ages of Ani and Biju be x and y, respectively.

According to the given condition,

$$x-y = \pm 3 \quad ...(1)$$

Also, age of Ani's father Dharam $= 2x$ years

And age of Biju's sister $= \frac{y}{2}$ years

According to the given condition,

$$2x-\frac{y}{2} = 30$$

$$4x-y = 60 \quad ...(2)$$

Case I : When $x-y = 3 \quad ...(3)$

Subtracting equation (3) from equation (2),

$$3x = 57$$

$$x = 19 \text{ years}$$

Putting $x=19$ in equation (3),

$$19-y = 3$$

$$y = 16 \text{ years}$$

Case II : When $x-y = -3 \quad ...(4)$

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Subtracting equation (iv) from equation (2),

$$3x = 60 + 3$$
$$3x = 63$$
$$x = 21 \text{ years}$$

Subtracting equation (4), we get

$$21 - y = -3$$
$$y = 24 \text{ years}$$

Hence, Ani's age = 19 years or 21 years Biju age = 16 years or 24 years.

101. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital.

Sol : [Board Term-1 2012]

Let the amount of their respective capitals be x and y.

According to the given condition,

$$x + 100 = 2(y - 100)$$
$$x - 2y = -300 \quad ...(1)$$

and
$$6(x - 10) = y + 10$$
$$6x - y = 70 \quad ...(2)$$

Multiplying equation (2) by 2 we have

$$12x - 2y = 140 \quad ...(3)$$

Subtracting (1) from equation (3) we have

$$11x = 440$$
$$x = 40$$

Substituting $x = 40$ in equation (1),

$$40 - 2y = -300$$

or,
$$2y = 340$$
$$y = 170$$

Hence, the amount of their respective capitals are 40 and 170.

102. At a certain time in a deer, the number of heads and the number of legs of deer and human visitors were counted and it was found that there were 39 heads and 132 legs.

Find the number of deer and human visitors in the park.

Sol : [Board Term-1 2015]

Let the no. of deer be x and no. of human be y.

According to the question,

$$x + y = 39 \quad ...(1)$$

and
$$4x + 2y = 132 \quad ...(2)$$

Multiply equation (1) from by 2,

$$2x + 2y = 78 \quad ...(3)$$

Subtract equation (3) from (2),

$$2x = 54$$
$$x = 27$$

Substituting this value of x in equation (1)

$$27 + y = 39$$
$$y = 12$$

So, No. of deer = 27 and No. of human = 12

103. The length of the sides of a triangle are $2x + \frac{y}{2}$, $\frac{5x}{3} + y + \frac{1}{2}$ and $\frac{2}{3}x + 2y + \frac{5}{2}$. If the triangle is equilateral , find its perimeter.

Sol : [Board Term-1 2012]

For an equilateral Δ,

$$2x + \frac{y}{2} = \frac{5x}{3} + y + \frac{1}{2}$$
$$= \frac{1}{2}x + 2y + \frac{5}{2}$$

Now
$$\frac{4x + y}{2} = \frac{10x + 6y + 3}{6}$$
$$12x + 3y = 10x + 6y + 3$$
$$2x - 3y = 3 \quad ...(1)$$

Again,
$$2x + \frac{y}{2} = \frac{2}{3}x + 2y + \frac{5}{2}$$
$$\frac{4x + y}{2} = \frac{4x + 12y + 15}{6}$$
$$12x + 3y = 4x + 12y + 15$$
$$8x - 9y = 15 \quad ...(2)$$

Multiplying equation (1) by 3 we have

$$6x - 9y = 9 \quad ...(1)$$

Subtracting it from (2) we get

$$2x = 6 \Rightarrow x = 3$$

Substituting this value of x into (1), we get

$$2 \times 3 - 3y = 3$$

or,
$$3y = 3 \Rightarrow y = 1$$

Now substituting these value of x and y

$$2x + \frac{y}{2} = 2 \times 3 + \frac{1}{2} = 6.5$$

The perimeter of equilateral triangle = side $\times$ 3

$$= 6.5 \times 3 = 19.5 \text{ cm}$$

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Hence, the perimeter of $\Delta = 19.5$ m.

104. When 6 boys were admitted and 6 girls left, the percentage of boys increased from 60% to 75%. Find the original no. of boys and girls in the class.

Sol : [Board Term-1 2015]

Let the no. of boys be x and no. of girls be y.

No. of students $= x+y$

Now $$\frac{x}{x+y} = \frac{60}{100} \quad ...(1)$$

and $$\frac{x+6}{(x+6)+(y-6)} = \frac{75}{100} \quad ...(2)$$

From (1), we have

$$\begin{aligned} 100x &= 60x+60y \\ 40x-60y &= 0 \\ 2x-3y &= 0 \\ 2x &= 3y \quad (3) \end{aligned}$$

From (2) we have

$$\begin{aligned} 100x+600 &= 75x+75y \\ 25x-75y &= -600 \\ x-3y &= -24 \quad ...(4) \end{aligned}$$

Substituting the value of $3y$ from (3) in to (4) we have,

$$\begin{aligned} x-2x &= -24 \Rightarrow x = 24 \\ 3y &= 24 \times 2 \\ y &= 16 \end{aligned}$$

Hence, no. of boys is 24 and no. of girls is 16.

105. A cyclist, after riding a certain distance, stopped for half an hour to repair his bicycle, after which he completes the whole journey of 30 km at half speed in 5 hours. If the breakdown had occurred 10 km farther off, he would have done the whole journey in 4 hours. Find where the breakdown occurred and his original speed.

Sol : [Board Term-1 2013]

Let x be the distance of the place where breakdown occurred and y be the original speed,

$$\frac{x}{y} + \frac{30-x}{\frac{y}{2}} = 5$$

or $$\frac{x}{y} + \frac{60-2x}{y} = 5$$

$$\begin{aligned} x+60-2x &= 5y \\ x+5y &= 60 \quad ...(1) \end{aligned}$$

and $$\frac{x+10}{y} + \frac{30-(x+10)}{\frac{y}{2}} = 4$$

$$\begin{aligned} \frac{x+10}{y} + \frac{60-2(x+10)}{y} &= 4 \\ x+10+60-2x-20 &= 4y \\ -x+50 &= 4y \\ x+4y &= 50 \quad (2) \end{aligned}$$

Subtract equation (2) from (1), $y = 10$ km/hr.

Now from (2), $x+40 = 50 \Rightarrow x = 10$ km

Break down occurred at 10 km and original speed was 10 km/hr.

106. The population of a village is 5000. If in a year, the number of males were to increase by 5% and that of a female by 3% annually, the population would grow to 5202 at the end of the year. Find the number of males and females in the village.

Sol : [Board Term-1 2012]

Let the number of males be x and females be y

Now $$x+y = 5,000 \quad ...(1)$$

and $$x+\frac{5}{100}x+y+\frac{3y}{100} = 5202$$

$$\frac{5x+3y}{100} + 5000 = 5202$$

$$5x+3y = (5202-5000) \times 100$$

$$5x+3y = 20200 \quad (2)$$

Multiply (1) by 3 we have

$$3x+3y = 15,000 \quad ...(3)$$

Subtracting (2) from (3) we have

$$2x = 5200 \Rightarrow x = 2600$$

Substituting value of x in (1) we have

$$2600-y = 5000 \Rightarrow y = 2400$$

Thus no. of males is 2600 and no. of females is 2400.

107. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

Sol : [Board 2019 Delhi]

Let x be the age of father and y be the sum of the ages of his children.

After 5 years,

Father's age $= (x+5)$ years

Sum of ages of his children $= (y+10)$ years

According to the given condition,

$$x = 3y \quad ...(1)$$

and $$x+5 = 2(y+10)$$

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or, $x-2y = 15$ ...(2)

Solving equation (1) and (2), we have

$$3y-2y = 15 \Rightarrow y = 15$$

Substituting value of y in equation (1), we get

$$x = 3 \times 15 = 45$$

Hence, father's present age is 45,

108. Sumit is 3 times as old as his son. Five years later he shall be two and a half times as old as his son. How old is Sumit at present?

Sol : [Board 2019 OD]

Let x be Sumit's present age and y be his son's present age.

According to given condition,

$$x = 3y$$

After five years, ...(1)

$$\text{Sumit's age} = x+5$$

and $\text{His son's age} = y+5$

Now, again according to given condition,

$$x+5 = 2\frac{1}{2}(y+5)$$

$$x+5 = \frac{5}{2}(y+5)$$

$$2(x+5) = 5(y+5)$$

$$2x+10 = 5y+25$$

$$2x = 5y+15$$

$$2(3y) = 5y+15 \quad [\text{from eq (1)}]$$

$$6y = 5y+15$$

$$y = 15$$

Again, from eq (1)

$$x = 3y$$

$$= 3 \times 15 = 45$$

Hence, Sumit's present age is 45 years.

COMPETENCEY BASED QUESTIONS

109. A coaching institute of Mathematics conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, there are 20 poor and 5 rich children, whereas in batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch I is ₹9000 and from batch II is ₹26,000. Assume that each poor child pays ₹x per month and each rich child pays ₹y per month.

Based on the above information, answer the following questions:

(i) Represent the information given above in terms of x and y.

(ii) Find the monthly fee paid by a poor child.

(iii) Find the difference in the monthly fee paid by a poor child and a rich child.

(iv) If there are 10 poor and 20 rich children in batch II, what is the total monthly collection of fees from batch II?

Sol : [Board 2023 OD Standard]

(i) Representation in Equation

Monthly fees paid by each poor children = ₹ x

Monthly fees paid by each rich children = ₹y

In batch I, there are 20 poor and 5 rich children.

Fees paid by batch I

$$20x+5y = 9000 \quad ...(1)$$

In batch II, there are 5 poor and 25 rich children.

Fees paid by batch II

$$5x+25y = 26000 \quad ...(2)$$

(ii) Monthly fee paid by a poor child

Multiply equation (1) by 5 we get

$$100x+25y = 45000 \quad ...(3)$$

Subtracting (2) from (3) we have

$$95x = 19000$$

$$x = \frac{19000}{95} = 200$$

Thus, monthly fee paid by poor child is ₹200.

(iii) Difference in the monthly fee paid by a poor child and a rich child,

Substitute value of x in equation (1) we have

$$20 \times 200 + 5y = 9000$$

$$5y = 9000 - 4000$$

$$y = \frac{5000}{5} = 1000$$

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Monthly fee paid by Rich child is ₹1000.

Difference in monthly fee paid by poor child and a rich child $= 1000 - 200 =$ ₹800

(iv) Total monthly collection of fees from batch II,

Poor children $= 10$

Rich children $= 20$

Total monthly collection of fees from batch II

$= 10 \times 200 + 200 \times 800$

$= 2000 + 16000 =$ ₹18000

110. Lawn Service : Nitin and his sons run a lawn service, which includes mowing, edging, trimming, and aerating a lawn. His fixed cost includes insurance, his salary, and monthly payments on equipment, and amounts to Rs 4000 per month. The variable costs include gas, oil, hourly wages for his employees, and miscellaneous expenses, which run about Rs 75 per lawn. The average charge for full service lawn care is Rs 115 per visit.

(i) How many lawns Nitin must service each month to break even ?

(ii) What is the revenue required to break even ?

(iii) What is the revenue if they get 90 services ?

Sol :

Let x represent the number of service. Break even point occur when cost is equal to revenue. Let y represent both cost and revenue at break even since they are equal.

(i) Fixed cost $= 4000$

Variable cost for each service $= 75$

Total cost for x service,

$y = 75x + 4000$

Charge for per visit $= 115$

Total revenue for x service,

$y = 115x$

At breakeven revenue must be equal to cost.

Thus, $115x = 75x + 4000$

$40x = 4000$

$x = 100$

Thus, 100 lawns/month are required for breakeven.

(ii) Revenue required to break even,

$y = 115 \times 100$

$= 11500$ Rs per month

(iii) Revenue for 90 services,

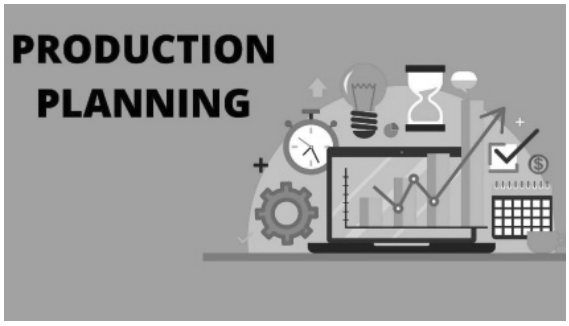
$y = 115x$

$= 115 \times 90 = 1015$ Rs

111. Production of Frying Pan : Due to high market demand, a manufacturer decides to introduce a new line of frying pan. By using existing factory space and retraining some employees, fixed costs are estimated at Rs 84000/mo. The components to assemble and test each frying pan are expected to run Rs 450 per unit. If market research shows consumers are willing to pay at least Rs 690 for this product, find

(i) How many units must be made and sold each month to break even ?

(ii) What is the revenue required to break even ?



Sol :

Let x represent the number of units made. Break even point occur when cost is equal to revenue. Let y represent both cost and revenue at break even since they are equal.

(i) Fixed cost $= 84000$ Rs

Variable cost for each unit $= 450$ Rs

Total cost, for making in x unit,

$y = 450x + 84000$

Projected Revenue, $y = 690x$

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At breakeven revenue must be equal to cost. Thus

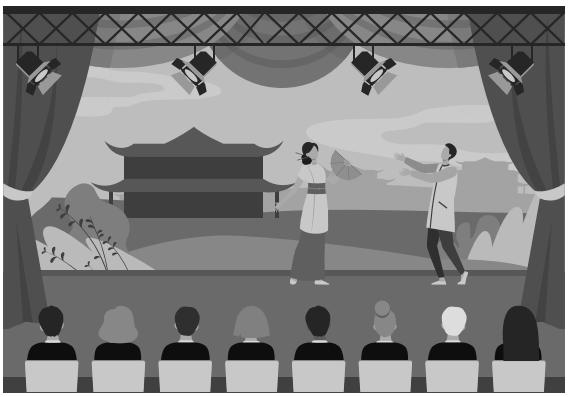
$$450x + 84000 = 690x$$
$$45x + 8400 = 69x$$
$$8400 = 24x \Rightarrow x = 350$$

Thus, 350 units/month are required for breakeven.

(ii) Revenue required to break even,

$$y = 690 \times 350$$
$$= 241500 \text{ Rs per month}$$

112. Theatre Productions : A play is a work of drama, usually consisting mostly of dialogue between characters and intended for theatrical performance rather than just reading. Comedies are plays which are designed to be humorous. Comedies are often filled with witty remarks, unusual characters, and strange circumstances. Certain comedies are geared toward different age groups.



At a recent production of a comedy drama, the Ravindra Rangmanch Theater brought in a total of Rs 304950 in revenue. If adult tickets were Rs 90 and children's tickets were Rs 65, how many tickets of each type were sold if 3800 tickets in all were sold?

Sol :

Let a represent the number of adult tickets sold and c represent the number of child tickets sold.

Since adult tickets were Rs 90 and children's tickets were Rs 65, revenue is Rs 304950, thus

$$90a + 65c = 304950 \quad (1)$$
$$18a + 13c = 60990 \quad (2)$$

If 3800 tickets in all were sold,

$$a + c = 3800 \quad (3)$$

Multiply the equation (2) by 13 we have

$$13a + 13c = 49400 \quad (4)$$

Subtracting (4) from (2) we have

$$5a = 11590$$
$$a = \frac{11590}{5} = 2318$$

Substituting $a = 2318$ in (3) we have

$$2318 + c = 3800$$
$$a = 3800 - 2318 = 1482$$

Thus 2318 adult tickets and 1482 child tickets were sold.

113. Alumni Contributions : Alumni can help college sustain through their donations and voluntary help. Alumni can also be helpful in providing valuable financial, intellectual and human resource. If a big chunk of money that institutes require comes from alumni, it will help those institutes remain competitive.

Alumni association of NIT Kuruskhstra donated Rs 100,000 to his alma mater. The college used the funds to make a loan to a science student at 7% interest and a loan to a engineering student at 6% interest. That year the college earned Rs 6350 in interest.

(i) How much was loaned to engineering student?

(ii) How much was loaned to science student?

Sol :

Let s represent the loan made to the science student and n represent the loan made to the engineering student. College earned total Rs 6350 interest, 7% interest from science student and 6% interest from engineering student and. Thus

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$$\frac{7}{100}s+\frac{6}{100}n = 6350$$
$$0.07s+0.06n = 6350 \quad (1)$$

Since total amount was Rs 100,000, we have

$$s+n = 100000 \quad (2)$$

Substituting $s = 100000-n$ from (2) in (1) we have

$$0.07(100000-n)+0.06n = 6350$$
$$7000-0.07n+0.06n = 6350$$
$$-0.01n = 6350-7000$$
$$-0.01n = -650$$
$$n = \frac{-650}{-0.01} = 65000$$

Now substituting $n = 65000$ in (ii) we have

$$s = 100000-65000 = 35000$$

(i) Rs 65000 was loaned to the engineering student.

(ii) Rs 35000 was loaned to the science student.

114. Airport Walkways : A moving walkway, also known as an autowalk, is a slow-moving conveyor mechanism that transports people across a horizontal or inclined plane over a short to medium distance. Moving walkways can be used by standing or walking on them. They are often installed in pairs, one for each direction.

As part of an algebra field trip, Jenish takes his class to the airport to use their moving walkways for a demonstration. The class measures the longest walkway, which turns out to be 256 ft long. Using a stop watch, Jenish shows it takes him just 32 sec to complete the walk going in the same direction as the walkway. Walking in a direction opposite the walkway, it takes him 320 sec (10 times as long!). The next day in class, Jenish hands out a two question quiz:

(i) What is my (Jenish's) normal walking speed?

(ii) What was the speed of the walkway in feet per second?

Sol :

Let w represent the speed of the walkway and j represent Jenish's walking speed.

(i) In the direction of walkway,

$$32(j+w) = 256$$
$$j+w = 8 \quad (1)$$

Opposite direction of walkway,

$$320(j-w) = 256$$
$$10j-10w = 8 \quad (2)$$

Multiplying eq (1) with 10 we have

$$10j+10w = 80 \quad (3)$$

Adding eq (2) and (3) we have

$$20j = 88 \Rightarrow j = \frac{88}{20} = 4.4 \text{ ft/sec}$$

(ii) Substituting above value of j in (1) we have

$$4.4+w = 8$$
$$w = 8-4.4 = 3.6 \text{ ft/sec}$$

115. Nutrition : Shalvi wants to use milk and orange juice to increase the amount of calcium and vitamin A in her daily diet. An ounce of milk contains 38 milligrams of calcium and 56 micrograms of vitamin A. An ounce of orange juice contains 5 milligrams of calcium and 60 micrograms of vitamin A. How many ounces of milk and orange juice should she drink each day to provide exactly 550 milligrams of calcium and 1,200 micrograms of vitamin A?

Sol :

Let x represent the number of ounces of milk and y represent number of ounces of orange juice.

Next we summarize the given information in table.

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	Milk	Orange Juice	Total Needed
Calcium	$38x$	$5y$	550
Vitamin A	$56x$	$60y$	1,200

For calcium, $38x+5y = 550$ (1)

For Vitamin A, $56x+60y = 1200$ (2)

Multiplying eq (1) by 12 we have

$$456x+60y = 6600 \quad (3)$$

Subtracting eq (2) from (iii) we have

$$400x = 5400$$

$$x = \frac{5400}{400} = 13.5$$

Substituting $x = 13.5$ in eq (1) we have

$$38 \times 13.5 + 5y = 550$$

$$513 + 5y = 550$$

$$5y = 550 - 513 = 37$$

$$y = \frac{37}{5} = 7.2$$

Thus Shalvi should drink 13.5 ounce of milk and 7.2 ounce of orange juices

116. CHEMISTRY : When you mix two or more substances with different levels of concentration, the final solution does not simply equate to the combined concentration levels of the original ingredients. It depends on the concentration of each solution.

Rahman works as a chemist in Biolab Pvt Ltd at Jaipur. He has two solutions of hydrochloric acid in stock: a 50% solution and an 80% solution. He want to make 100 milliliters of a 68% solution? How much of each should be used to obtain 100 milliliters of a 68% solution?

Sol :

Let x represent the amount of 50% solution and y represent amount of 80% solution.

Since 100 millilitres are required, so

$$x+y = 100 \quad (1)$$

Since 68% of the 100 millilitres must be acid, so

$$0.50x+0.80y = 0.68(100)$$

$$0.50x+0.80y = 68 \quad (2)$$

Multiplying eq (2) by 2 we have

$$x+1.60y = 136 \quad (3)$$

Subtracting eq (1) from (iii) we have

$$0.60y = 36 \Rightarrow y = \frac{36}{0.60} = 60 \text{ millilitres}$$

Substituting $y = 60$ in (i) we get $x = 40$ millilitres

Thus chemist must use 40 ml of 50% solution and 60 ml of 80% solution.

117. Nutrition : Pathmeda village near Sanchore has Gopal Govardhan Gaushala, the largest Gaushala in India, spread over 200 acres. The gaushala takes care of more than 18,000 cattle.

Cows of Pathmeda gaushala in an experiment are to be kept on a strict diet. Each cow is to receive, among other things, 20 grams of protein and 6 grams of fat.

The laboratory technician is able to purchase two food mixes of the following compositions:

Mix A has 10% protein and 6% fat,

Mix B has 20% protein and 2% fat.

How many grams of each mix should be used to obtain the right diet for a single cow ?

Sol :

Let x represent number of grams of Mix A and y represent number of grams of Mix B. Now we

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prepare the following table.

	Mix A	Mix B	Total Nutrition
Protein	$0.10x$	$0.20y$	20
Fat	$0.06x$	$0.02y$	6

For protein requirement,

$$0.1x+0.2y = 20$$
$$x+2y = 200 \quad (1)$$

For fat requirement,

$$0.06x+0.02y = 6$$
$$6x+2y = 600 \quad (2)$$

Subtracting eq (1) from (2) we have

$$5x = 400 \Rightarrow x = 80$$

Substituting $x = 80$ in eq (1) we have

$$80+2y = 200$$
$$2y = 200-80 = 120$$
$$y = \frac{120}{2} = 60$$

Thus 80 grams Mix A and 60 grams Mix B should be used.

118. Gold Mixing : A jeweller has two bars of gold alloy in stock, one of 12 carats and the other of 18 carats (24 carat gold is pure gold, 12 carat is $\frac{12}{24}$ pure, 18 carat gold is $\frac{18}{24}$ pure, and so on). How many grams of each alloy must be mixed to obtain 10 grams of 14 carat gold?

Sol :

Let x represent the grams of 12 carat pure gold and y represent the grams of 18 carat pure gold.

Since 10 gram of gold is required, so

$$x+y = 10 \quad (1)$$

Since purity of gold must be 14 carat, so

$$\frac{12}{24}x+\frac{18}{24}y = \frac{14}{24}\times 10$$
$$12x+18y = 140 \quad (2)$$

Multiplying eq (1) by 18 and subtracting (ii) from it we have

$$6x = 40 \Rightarrow x = \frac{40}{6} = 6\frac{2}{3}$$

Now substituting $x = 6\frac{2}{3}$ in eq (1) we have

$$6\tfrac{2}{3}+y = 10$$
$$y = 10-6\tfrac{2}{3} = 3\tfrac{1}{3}$$

Thus $6\frac{2}{3}$ grams of 12 carat gold and $3\frac{1}{3}$ grams of 18 carat gold is required to make 10 gram of 14 carat gold.

119. BREAK-EVEN ANALYSIS : It costs a small recording company Rs 176, 800 to prepare a compact disc. This is a one-time fixed cost that covers recording, package design, and so on. Variable costs, including such things as manufacturing, marketing, and royalties, are Rs 46 per CD.

(i) If the CD is sold to music shops for Rs 80 each, how many must be sold for the company to break even?

(ii) What is the break even revenue ?

Sol :

Let x represent the number of CD made. Break even point occur when cost is equal to revenue. Let y represent both cost and revenue at break even since they are equal.

$$\text{Fixed cost} = 176800 \text{ Rs}$$

Variable cost for each CD $= 46$ Rs

Total cost, for making in x CD,

$$y = 46x+176800$$

Projected Revenue, $y = 80x$

At breakeven revenue must be equal to cost. Thus

$$46x+176800 = 80x$$
$$176800 = 34x$$

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$$x = \frac{176800}{34} = 5,200$$

Now substituting $x = 5200$ in $y = 80x$ we have

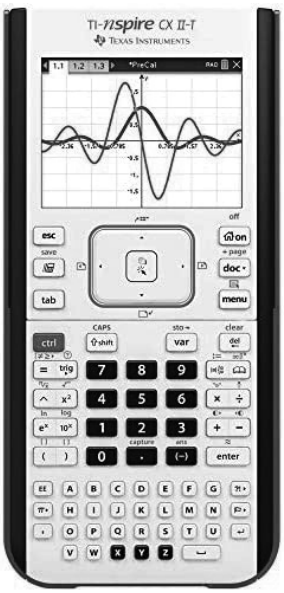
$$y = 80 \times 5200 = 416000 \text{ Rs}$$

(i) 5200 CDs must be sold for the company to break even.

(ii) Break even revenue is Rs 416000.

120. Production : Ridhima Electronics Pvt . Ltd is main supplier for CASIO for electronics component. They manufactures keyboards and screens for graphing calculators at plants in Bangalore and Bhiwadi. The hourly production rates at each plant are given in the table. How many hours should each plant be operated to fill an order for exactly 4,000 keyboards and exactly 4,000 screens?

Plant	Keyboards	Screens
Bangalore	40	32
Bhiwadi	20	32



Sol :

Let x represent number of hours Bangalore plant is operated and y represent number of hours Bhiwadi plant is operated.

We summarized production in following table.

	Bangalore	Bhiwadi	Requirement
Keyboards	$40x$	$20y$	4000
Screens	$32x$	$32y$	4000

Since we require 4000 keyboard, thus

$$40x + 20y = 4000$$
$$2x + y = 200 \quad (1)$$

Since we require 4000 screens, thus

$$32x + 32y = 4000$$
$$x + y = 125 \quad (2)$$

Subtracting eq (2) from (1) we have

$$x = 75 \text{ hours}$$

Substituting above value of x in (ii) we have

$$75 + y = 125$$
$$y = 125 - 75 = 50 \text{ hours}$$

Thus Bangalore plant should be operated for 75 hours and Bhiwadi plant should be operated for 50 hours.

121. NUTRITION : Orange trees thrive in warm, Mediterranean climates where there is no threat of frost. This full-sun plant produces the best fruit when provided with optimal moisture, light and nutrition, in the form of fertilization. Orange trees require fertilization three times per year. You need to increase fertilizer amounts as the tree ages and becomes established. Complete nutrition is essential for a healthy tree.

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A fruit grower can use two types of fertilizer in an orange grove, brand A and brand B. Each bag of brand A contains 8 pounds of nitrogen and 4 pounds of phosphoric acid. Each bag of brand B contains 7 pounds of nitrogen and 7 pounds of phosphoric acid. Tests indicate that the grove needs 720 pounds of nitrogen and 500 pounds of phosphoric acid.

(i) How many bags of brand A should be used to provide the required amounts of nitrogen and phosphoric acid?

(ii) How many bags of brand B should be used to provide the required amounts of nitrogen and phosphoric acid?

Sol :

Let a represent requirement of bag of brand A and b represent requirement of bag of brand B.

Each bag of brand A contains 8 pounds of nitrogen and 4 pounds of phosphoric acid. Each bag of brand B contains 7 pounds of nitrogen and 7 pounds of phosphoric acid.

We summarized production in following table.

	Brand A	Brand B	Requirement
Nitrogen	$8a$	$7b$	720
Phosphoric acid	$4a$	$7b$	500

(i) Since grove needs 720 pounds of nitrogen, we have

$$8a+7b = 720 \quad (1)$$

Since grove needs 500 pounds of phosphoric acid, we have

$$4a+7b = 500 \quad (2)$$

Subtracting eq (2) from eq (1) we have

$$4a = 220$$

$$a = 55$$

Thus 55 bags of brand A is required.

(ii) Multiplying eq (2) by 2 we have

$$8a+14b = 1000 \quad (3)$$

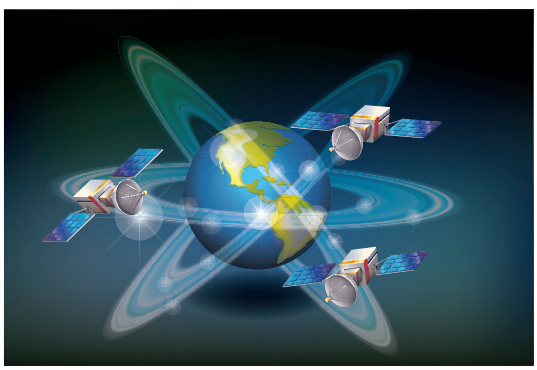
Subtracting eq (1) from eq (iii) we have

$$7b = 280$$

$$\Rightarrow \quad b = \frac{280}{7} = 40$$

Thus 40 bags of brand B is required.

122. Orbital Launches : In 2013 there was a total of 81 commercial and noncommercial orbital launches worldwide. In addition, the number of noncommercial orbital launches was twelve more than twice the number of commercial orbital launches. Determine the number of commercial and noncommercial orbital launches in 2013.



Sol :

Let x and y be the number of commercial and non-commercial orbital launches.

In 2013 there was a total of 81 commercial and noncommercial orbital launches worldwide. Thus

$$x+y = 81 \quad ...(1)$$

The number of noncommercial orbital launches was twelve more than twice the number of commercial orbital launches. Thus

$$y-2x = 12 \quad ...(2)$$

Subtracting eq (2) from eq (1), we get

$$x+y-y+2x = 81-12$$

$$3x = 69 \Rightarrow x = 23$$

Substituting $x=23$ in eq (1), we get

$$y = 81-x$$

$$= 81-23$$

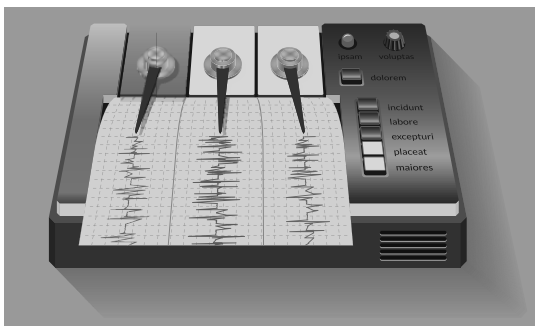
$$= 58$$

So, the number of commercial and non-commercial orbital launches are 23 and 58 respectively.

123. EARTH SCIENCE : An earthquake emits a primary wave and a secondary wave. Near the surface of the Earth the primary wave travels at about 5 miles per second and the secondary wave at about 3 miles per second. From the time lag between the two waves arriving at a given receiving station, it is possible to estimate the distance to the quake. (The epicenter can be located by obtaining distance bearings at three or more stations.) Suppose a station measured a time difference of 16 seconds between the arrival of the two waves.

(i) How long did each wave travel ?

(ii) How far was the earthquake from the station?



Sol :

Let p represent time of primary wave and s represent time for secondary wave

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Since time difference is 16 second

$$s - p = 16 \quad (1)$$

To find a second equation, we have to use

$$\text{Distance} = \text{Rate} \times \text{Time}$$

Distance for primary wave $= 5p$

Distance for secondary wave $= 3s$

Since these distances are equal, so

$$5p = 3s \quad (2)$$

From (1) $s = p + 16$, substituting in eq (2) we have

$$5p = 3(p + 16)$$

$$5p = 3p + 48$$

$$2p = 48 \Rightarrow p = 24 \text{ seconds}$$

Now $s = 24 + 16 = 40$ seconds

The distance travelled is $5p = 120$ miles.

(i) Primary wave travelled 24 second and secondary wave travelled 40 second.

(ii) Earthquake is 120 mile from station.

124. Architectural Wonders : When it was first constructed in 1889, the Eiffel Tower in Paris, France, was the tallest structure in the world. In 1975, the CN Tower in Toronto, Canada, became the world's tallest structure. The CN Tower is 153 ft less than twice the height of the Eiffel Tower, and the sum of their heights is 2799 ft.

(i) How tall is CN tower?

(ii) How tall is Eiffel tower?

Sol :

Let the heights of Eiffel tower and CN tower be x and y respectively.

The CN Tower is 153 ft less than twice the height of the Eiffel Tower, thus

$$2x - y = 153 \quad ...(1)$$

The sum of their heights is 2799 ft, thus

$$x + y = 2799 \quad ...(2)$$

Adding eqn, (1) and (2), we get

$$3x = 2952 \Rightarrow x = 984$$

Substituting $x = 984$ in eq (2), we get

$$y = 2799 - 984 = 1815$$

(i) The height of CN tower is 1815 ft.

(ii) The height of Eiffel tower is 984 ft.

125. TOWER OF PISA : To prove that objects of different weights fall at the same rate, Galileo dropped two objects with different weights from the Leaning Tower of Pisa in Italy. The objects hit the ground at the same time.

An object dropped off the top of Leaning Tower of Pisa falls vertically with constant acceleration. If s is the distance of the object above the ground (in feet) t seconds after its release, then s and t are related by an equation of the form $s = a + bt^2$ where a and b are constants. Suppose the object is 180 feet above the ground 1 second after its release and 132 feet above the ground 2 seconds after its release.

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(i) Find the constants a and b.

(ii) How high is the Leaning Tower of Pisa?

(iii) How long does the object fall?

Sol :

We have $s = a + bt^2$

(i) When $t = 1$, $s = 180$

Substituting these values in the given equation, we have

$$180 = a + b(1)^2$$

$$180 = a + b \quad (1)$$

When $t = 2$, $s = 132$

Substituting these values in the given equation, we have

$$132 = a + b(2)^2$$

$$132 = a + 4b \quad (2)$$

Subtracting eq (1) from (2) we have

$$-48 = 3b \Rightarrow b = -16$$

Now substituting above value of b in (i) we have

$$180 = a - 16 \Rightarrow a = 196$$

Thus $a = 196$ and $b = -16$

(ii) The height of the building is represented by s, the distance of the object above the ground, when $t = 0$, Since from part (i),we now know

$$s = 196 - 16t^2$$

From above equation, when $t = 0$, $s = 196$ feet is the height of the building.

(iii) The object falls until s, its distance above the ground, is zero. Since

$$s = 196 - 16t^2$$

Substituting $s = 0$ we have

$$0 = 196 - 16t^2$$

$$16t^2 = 196$$

$$t^2 = \frac{196}{16} \Rightarrow t = \frac{14}{4} = 3.5 \text{ sec}$$

126. Jyoti Kumari is an Indian student from Sirhulli in the rural Darbhanga district of Bihar. She came to notice after she bicycled some 1,200 km with her injured father to reach their home village during COVID-19 lockdowns in India. This act of bravery was praised by the Senior Advisor to the President of the United States, Ivanka Trump, and Prime Minister Narendra Modi. She was given a national award, and a Bollywood film was proposed to record her story.

Jyoti travelled 90 km every day to reach her home town in Harbin. One day, when she started, after riding a certain distance, she stopped for some time to repair his bicycle. After which she completes the whole journey of 90 km at half speed in 12 hours. If the breakdown had occurred 10 km farther off, she would have done the whole journey in 11 hours.

(i) Find where the breakdown occurred.

(ii) Find her original speed.

Sol :

Let x be the distance of the place where breakdown occurred and y be the original speed.

Now $$\frac{x}{y} + \frac{90 - x}{\frac{y}{2}} = 12$$

or $$\frac{x}{y} + \frac{180 - 2x}{y} = 12$$

$$x + 180 - 2x = 12y$$

$$x + 12y = 180 \quad ...(1)$$

and $$\frac{x+10}{y} + \frac{90 - (x+10)}{\frac{y}{2}} = 11$$

$$\frac{x+10}{y} + \frac{180 - 2(x+10)}{y} = 11$$

$$x + 10 + 180 - 2x - 20 = 11y$$

$$-x + 170 = 11y$$

$$x + 11y = 170 \quad (2)$$

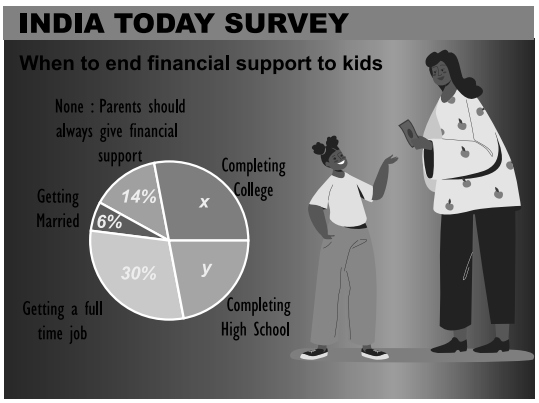
Subtract equation (2) from (1), $y = 10$ km/hr.

Now from (2), $x + 11 \times 10 = 170 \Rightarrow x = 60$ km

(i) Break down occurred at 60 km.

(ii) Her original speed was 10 km/hr.

127. At some point, it's time to kick, or gently ease, kids off the parental gravy train. The circle graph shows the percentage of parents who think significant financial support should end at various milestones.



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The difference in the percentage who would end this support after completing college and after completing high school is 6 %.

(i) Find the percentage of parents who would end financial support after a child completes college.

(ii) Find the percentage of parents who would end financial support after a child completes high school.

Sol :

According to the question, we have

$$x+y+30+6+14 = 100$$

$$x+y+50 = 100$$

or $$x+y = 50 \quad ...(1)$$

The difference in the percentage who would end this support after completing college and after completing high school is 6 %. Thus

$$x-y = 6 \quad ...(2)$$

Adding eq (1) and (2), we get

$$2x = 56 \Rightarrow x = 28$$

Substituting $x = 28$ in eq (1), we get

$$y = 50 - x$$

$$= 50 - 28 = 22$$

(i) 28 % parents would end financial support after a child completes college.

(ii) 22 % parents would end financial support after a child completes high school.

128. Cash Register Malfunction : You are the manager of a shoe store. On Sunday morning you are going over the receipts for the previous week's sales. A total of 320 pairs of cross-training shoes were sold. One style sold for Rs 1135 and the other sold for Rs 1495. The total receipts were Rs 420,480. The cash register that was supposed to keep track of the number of each type of shoe sold malfunctioned. Can you recover the information? If so, how many of each type were sold?

Sol :

Let x and y be the number of pairs of shoes of both style sold.

Since total of 320 pairs of cross-training shoes were sold, we have

$$x+y = 320 \quad ...(1)$$

One style sold for Rs 1139 and the other sold for Rs 1459. The total receipts were Rs 420,480. Thus

$$1459x+1139y = 420480 \quad ...(2)$$

Substitution the value of $y = 320 - x$ from (1) in (2), we get

$$1139x+1459(320-x) = 420480$$

$$1139x+466880-1459x = 420480$$

$$-320x = -46400 \Rightarrow x = 145$$

Substitution $x = 145$ in eq (1), we get

$$y = 320 - x$$

$$= 320 - 145 = 175$$

So, 145 and 175 pairs of shoes of style one and two were sold.

129. Hostel Life : Banasthali Vidyapith, is a fully residential women's university offering courses from primary to Ph.D. level. It offers a number of UG, PG, and Doctoral level Programs under various Departments. Admission to the same is done on the basis of merit scored in qualifying examination, however, for some courses, an aptitude test is also conducted at the university level.

Swati is doing MSc. in biotechnology from Banastli Vidyapith and lives in university hostel. A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 18 days, she has to pay Rs. 5160 as hostel charges whereas Taniya who takes food for 23 days Rs. 5760 as hostel charges.

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(i) Find the fixed charges of hostel.

(ii) Find the cost of food per day.

Sol :

Let fixed charge be x and per day food cost be y.

$$x+18y = 5160 \quad ...(1)$$

$$x+23y = 5760 \quad ...(2)$$

Subtracting (1) from (2) we have

$$5y = 600 \Rightarrow y = 120$$

Substituting this value of y in (1), we get

$$x+18 \times 120 = 5160$$

$$x = 5160 - 2160 = 3000$$

Thus $x = 3000$ and $y = 120$

(i) Fixed charges of hostel is Rs 3000.

(ii) Cost of food per day is Rs 120.

130. Uniform motion with current :

$$(R+C)t = d \quad \text{With the current}$$

$$(R-C)t = d \quad \text{Against the current}$$

The formula shown can be used to solve uniform motion problems involving a current, where d represents distance travelled, R is the rate of the object with no current, C is the speed of the current, and t is the time. Vibhur rows 9 km up river (against the current) in 3 hr. It only took him 1 hr to row 5 km downstream (with the current).

(i) How fast was the current?

(ii) How fast can he row in still water?

Sol :

We have $(R+C)t = d$ With the current

$(R-C)t = d$ Against the current

Vibhur rows 9 km up river (against the current) in 3 hr.

Thus $(R-C)3 = 9$

$$R-C = 3 \quad (1)$$

It took him 1 hr to row 5 km downstream (with the current).

Thus $(R+C)1 = 5$

$$R+C = 5 \quad (2)$$

Adding eq (1) and (2) we have

$$2R = 8 \Rightarrow R = 4$$

Substituting $R = 4$ in (ii) we get $C = 1$

The current was 1 km/h. He can row 4 km/h in still water.

131. Canoeing on a stream : On a recent camping trip, it took Mohinder and Aslam 2 hr to row 4 mi upstream from the drop in point to the camp site. After a leisurely weekend of camping, fishing, and relaxation, they rowed back downstream to the drop in point in just 30 min. Use this information to find

(i) Find the speed of the current.

(ii) Find the speed Mohinder and Aslam would be rowing in still water.

Sol :

Let c represent the speed of the current and s represent the speed of the boat in still water.

To the drop point :

$$(s-c)2 = 4$$

$$s-c = 2 \quad (1)$$

Return to the drop point :

$$(s+c)\frac{1}{2} = 4$$

$$s+c = 8 \quad (2)$$

Adding eq (1) and (2) we have

$$2s = 10 \Rightarrow s = 5$$

Substituting $s = 5$ in (ii) we get $c = 3$

(i) Speed of current is 3 mph

(ii) Speed of boat in still water is 5 mph

132. Luxury Cruise : As India's first domestic cruise liner, Angriya has made many voyages on the Mumbai-Goa sea route, along the pristine Konkan Coast. It has given India and Indians a sense of pride and happiness, while introducing the travelers to coral diversity and royal sea forts along the way.

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Last year we enjoyed our summer vacation at Angariya cruise. From Mumbai to the Goa, with the current the trip took 70 hr. After a few days of fun in the sun, the ship leaves for Mumbai, against the current with the return trip taking 82 hr.

(i) Find the speed of the current.

(ii) Find the cruising speed of the ship.

Sol :

Let c represent the speed of the current and s represent the cruising speed of the ship.

Trip to the Goa,

$$(s+c)70 = 1435$$
$$2s+2c = 41 \quad (1)$$

Return to Mumbai,

$$(s-c)82 = 1435$$
$$2s-2c = 35 \quad (2)$$

Adding eq (1) and (2) we have

$$4s = 76 \Rightarrow s = 19 \text{ kmph}$$

Subtracting eq (2) from (1) we have

$$4c = 6 \Rightarrow s = 1.5 \text{ kmph}$$

133. Point of No Return : The point during a flight at which an aircraft is no longer capable of returning to the airfield from which it took off due to fuel considerations. Beyond this point the aircraft must proceed to some other destination.

A plane carries enough fuel for 20 hours of flight at an airspeed of 150 miles per hour. How far can it fly into a 30 mph headwind and still have enough fuel to return to its starting point?

Sol :

Let t_1 represent the time taken in fly. Distance travelled in fly is $(150-30)\,t_1$.

Let t_2 represent time taken in return. Distance travelled in return fly is $(150+30)\,t_2$.

Since these both distance is equal,

$$(150-30)\,t_1 = (150+30)\,t_2$$
$$120t_1 = 180t_2$$
$$2t_1 = 3t_2$$
$$t_1 = 1.5t_2$$

Since there is enough fuel for total fly of 20 hours,

$$t_1 + t_2 = 20$$
$$1.5t_2 + t_2 = 20$$
$$2.5t_2 = 20 \Rightarrow t_2 = \frac{20}{2.5} = 8 \text{ hours}$$

Distance travelled in 8 hour in return fly

$$= (150+30) \times 8$$
$$= 1,440 \text{ miles}$$

134. Computing a Refund : The grocery store we use does not mark prices on its goods. My wife went to this store, purchased three 1-kg packages of almond and two 500-gram packages cashew, and paid a total of Rs 1345. Not knowing that she went to the store, I also went to the same store, purchased two 1-kg packages of almond and three 500-gram packages cashew, and paid a total of Rs 1145. Now we want to return two 1-kg packages of almond and two 500-gram packages cashew. How much will be refunded?

Sol :

Let x and y be the price of 1-kg package of almond and one 500-gram packages cashew.

Since my wife purchased three 1-kg packages of almond and two 500-gram packages cashew, and paid a total of Rs 1345, we have

$$3x+2y = 1345 \quad ...(1)$$

Since I purchased two 1-kg packages of almond and three 500-gram packages cashew, and paid a total of Rs 1145, thus

$$2x+3y = 1145 \quad ...(2)$$

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Multiplying eq (1) by 3 and eq (2) by 2, we get

$$9x+6y = 4035 \quad ...(3)$$

$$4x+6y = 2290 \quad ...(4)$$

Subtracting eq (4) from eq (3), we get

$$5x = 1745 \Rightarrow x = 349$$

Substituting $x = 34.9$ in eq (2), we get

$$3 \times 349 + 2y = 1345$$

$$2y = 298 \Rightarrow y = 149$$

Thus price of one 1-kg packages of almond is Rs 349 and price of one 500-gram packages cashew is Rs 149.

Since we want to return two 1-kg packages of almond and two 500-gram packages cashew, refunded amount,

$$2x+2y = 2 \times 349 + 2 \times 149 = 996 \text{ Rs}$$

135. Financial Planning : Planning for retirement starts with thinking about your retirement goals and how long you have to meet them. Then you need to look at the types of retirement accounts that can help you raise the money to fund your future. As you save that money, you have to invest it to enable it to grow.

A recently retired couple needs Rs 120,000 per year to supplement their Social Security. They have Rs 1,500,000 to invest to obtain this income. They have decided on two investment options: AA bonds yielding 10% per annum and a fixed deposit yielding 5%.

(i) How much should be invested in each to realize exactly Rs 120,000?

(ii) If, after 2 years, the couple requires Rs 140,000 per year in income, how should they reallocate their investment to achieve the new amount?

Sol :

Let x and y be the amount invested in AA bonds and fixed deposit respectively.

Since they have Rs 1,500,000 or 1500 thousand to invest, we have

$$x+y = 1500 \quad ...(1)$$

(i) AA bonds yielding 10% per annum and a fixed deposit yielding 5% giving Rs 120,000 or Rs 120 thousand exactly, thus

$$\frac{10}{100}x+\frac{5}{100}y = 120$$

$$2x+y = 2400 \quad ...(2)$$

Subtracting eq (1) from (2) we have

$$x = 900$$

Substituting $x = 900$ in eq (1), we get

$$y = 1500 - 900 = 600$$

So, amount invested in AA bond and fixed deposit are Rs 900 thousand and Rs 600 thousand respectively.

(ii) Now the couple requires Rs 140,000 or Rs 140 thousand year in income, thus

$$\frac{10}{100}x+\frac{5}{100}y = 140$$

$$2x+y = 2800 \quad ...(3)$$

and rewriting $x+y = 1500 \quad ...(1)$

Subtracting eq (3) from (4) we have

$$x = 1300$$

Substituting $x = 1300$ in eq (3), we get

$$y = 1500 - 1300 = 200$$

So, amount invested in AA bond and fixed deposit are Rs 1300 thousand and Rs 200 thousand respectively.

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136. Actual Number of Calories : University of Arkansas researchers discovered that we underestimate the number of calories in restaurant meals. The next time you eat out, take the number of calories you think you ate and double it.

The researchers concluded that this number should be a more accurate estimate. The actual number of calories in one portion of hamburger and fries and two portions of pizza is 4240. The actual number of calories in two portions of hamburger and fries and one portion of pizza is 3980.

(i) Find the actual number of calories in one portions of hamburger and fries.

(ii) Find the actual number of calories in one portions of pizza.

Sol :

Let number of calories in one portion of hamburger and fries be x and in one portion of pizza be y.

The actual number of calories in one portion of hamburger and fries and two portions of pizza is 4240. Thus

$$x+2y = 4240 \qquad ...(1)$$

The actual number of calories in two portions of hamburger and fries and one portion of pizza is 3980. Thus

$$2x+y = 3980 \qquad ...(2)$$

Multiplying equation (1) by 2 and then subtracting (2) from (1) we have

$$4y-y = 2\times 4240 - 3980$$

$$3y = 4500 \Rightarrow y = 1500$$

Substituting the value of y in eq (2), we get

$$2x+1500 = 3980$$

$$2x = 3980 - 1500 = 2480$$

$$x = 1240$$

(i) The number of calories in one portion of hamburger and fries is 1240.

(ii) The number of calories in one portion of pizza is 1500.

137. Presale Order : A wireless store owner takes presale orders for a new smartphone and tablet. He gets 340 preorders for the smartphone and 250 preorders for the tablet. The combined value of the preorders is Rs 27,050,000. The price of a smartphone and tablet together is Rs 96500.

(i) How much does smartphone cost?

(ii) How much does tablet cost?

Sol :

Let x and y be the price of smart phone and tablet. Then, according to the question, we have

Combined value of 340 smartphone and 250 tablet is Rs 27,050,000. Thus

$$340x+250y = 27050000 \qquad ...(1)$$

If the price of a smartphone and tablet together is Rs 96500, thus

$$x+y = 96500 \qquad ...(2)$$

Substituting the value of y from eq (2) in eq (1), we get

$$340x+250(96500-x) = 27050000$$

$$340x+24125000-250x = 27050000$$

$$90x = 2925000 \Rightarrow x = 32500$$

Substituting $x=32500$ in eq (2), we get

$$y = 96500 - x$$

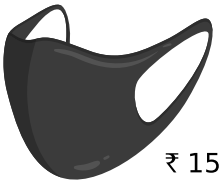
$$= 96500 - 32500 = 64000$$

(i) Cost of a smart phone is Rs 32500.

(ii) Cost of a tablet is Rs 64000.

138. MASK : Masks are an additional step to help prevent people from getting and spreading COVID-19. They provide a barrier that keeps respiratory droplets from spreading. Wear a mask and take every day preventive actions in public settings.

Due to ongoing Corona virus outbreak, Wellness Medical store has started selling masks of decent quality. The store is selling two types of masks currently type A and type B.



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The cost of type A mask is Rs. 15 and of type B mask is Rs. 20. In the month of April, 2020, the store sold 100 masks for total sales of Rs. 1650.

(i) How many masks of each type were sold in the month of April?

(ii) If the store had sold 50 masks of each type, what would be its sales in the month of April?

(iii) Due to great demand and short supply, the store has increased the price of each type by Rs. 5 from May 1, 2020. In the month of May, 2020, the store sold 310 masks for total sales of Rs. 6875. How many masks of each type were sold in the month of May?

(iv) What percent of masks of each type sale was increased in the month of May, compared with the sale of month April?

(v) What extra profit did store earn by increasing price in May month.

Sol :

(i) Let x be the mask of type A sold and y be the type of mask B sold in April.

Now $x+y = 100$ ...(1)

and $15x+20y = 1650$ ...(2)

Multiplying equation (1) by 15 and subtracting from (2) we obtain,

$$5y = 150 \Rightarrow y = 30$$

$$x = 100 - 30 = 70$$

Hence 70 masks of type A, and 30 masks of type B were sold.

(ii) Total Sales $= 50 \times 15 + 50 \times 20 = 1750$

(iii) Let x be the mask of type A sold and y be the type of mask B sold in April.

Now, $x+y = 310$ ...(1)

and $20x+25y = 6875$ ...(ii)

Multiplying equation (1) by 20 and subtracting it from equation (2), we obtain

$$5y = 675 \Rightarrow y = 135$$

$$x = 310 - 135 = 175$$

(iv) Increase in type $A = \frac{175-70}{70} \times 100 = 150\,\%$

Increase in type $B = \frac{105-30}{30} \times 100 = 350\,\%$

(v) Total sale value in May at old price

$$= 175 \times 15 + 135 \times 20 = 5325$$

Total sale value in May at new price $= 6875$

$$\text{Extra Profit} = 6875 - 5325 = 1550$$

Alternative :

Since extra profit is Rs 5 on per mask and total mask sold are 310, thus extra profit $= 310 \times 5 = 1550$.

139. Wilt Chamberlain : Wilton Norman "Wilt" Chamberlain was an American basketball player, and played in the NBA during the 1960s. At 7 feet 1 inch, he was the tallest and heaviest player in the league for most of his career, and he was one of the most famous people in the game for many years. He is the first and only basketball player to score 100 points in an NBA game.

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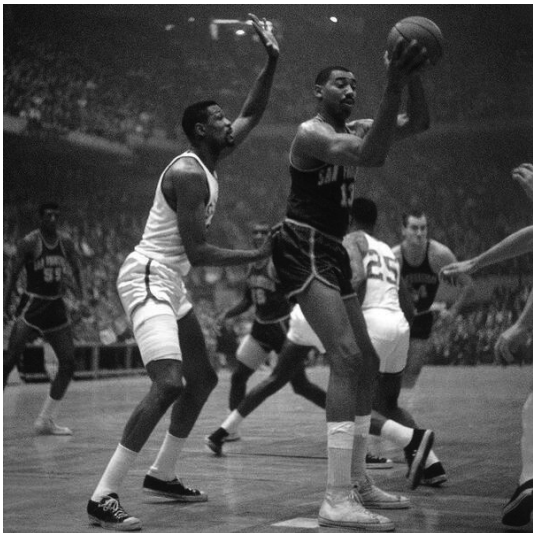


In the 1961–1962 NBA basketball season, Wilt Chamberlain of the Philadelphia Warriors made 30 baskets. Some of the baskets were free throws (worth 1 point each) and some were field goals (worth 2 points each). The number of field goals was 10 more than the number of free throws.

(i) How many field goals did he make ?

(ii) How many free throws did he make?

(iii) What was the total number of points scored?

(iv) If Wilt Chamberlain played 5 games during this season, what was the average number of points per game?

(v) If Wilt Chamberlain played 10 games during this season, what was the average number of points per game?

Sol :

(i) Let x be the free throw and y be the fixed goal.

As per question

$$x+y = 30$$

$$y = x + 10$$

Solving $x = 10,\ y = 20$

Thus he made 20 fixed goal.

(ii) Free throw $x = 10$

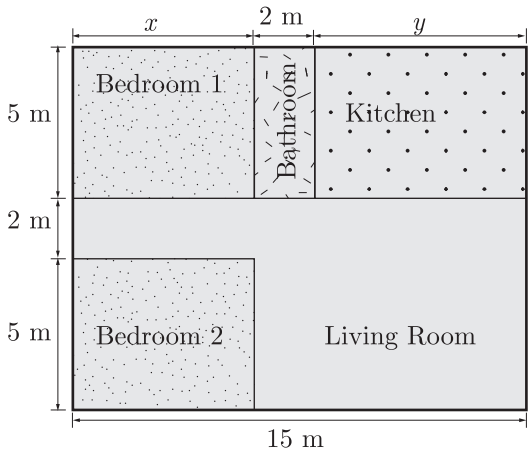
(iii) Point scored $= 10 + 2 \times 20 = 50$

(iv) Average point $\frac{50}{5} = 10$

(v) Average point $\frac{50}{10} = 5$

140. Architect : An architect is a skilled professional who plans and designs buildings and generally plays a key role in their construction. Architects are highly trained in the art and science of building design. Since they bear responsibility for the safety of their buildings' occupants, architects must be professionally licensed.

Varsha is a licensed architect and design very innovative house. She has made a house layout for her client which is given below. In the layout, the design and measurements has been made such that area of two bedrooms and kitchen together is 95 sq. m.



(i) Which pair of linear equations does describe this situation ?

(ii) What is the length of the outer boundary of the layout.

(iii) What is the area of bedroom 1 ?

(iv) What is the area of living room in the layout?

(v) What is the cost of laying tiles in Kitchen at the rate of Rs. 50 per sq. m ?

Sol :

(i) Area of two bedrooms $= 5x + 5x = 10x$ m^2

Area of kitchen $= 5y$ m^2

Thus $10x + 5y = 95 \Rightarrow 2x + y = 19$

Also from figure, we have,

$x + 2 + y = 15 \Rightarrow x + y = 13$

(ii) Length of outer boundary

$$= 2(5 + 2 + 5 + 15) = 54 \text{ m}$$

(iii) Solving and $2x + y = 19$ and $x + y = 13$ we get $x = 6$ m and $y = 7$ m.

Area of bedroom $= 5 \times 6 = 30$ sq. m

Area of kitchen $= 5 \times 7 = 35$ sq. m

(iv) Area of living room

$$= (15 \times 7) - 30 = 105 - 30 = 75 \text{ } m^2$$

(v) Area of kitchen $= 7 \times 5 = 35$ sq m

Cost of laying tiles in kitchen = Rs. 50 per m^2

Total cost of laying tiles in kitchen

$$= 50 \times 35 = 1750 \text{ Rs}$$

141. Mr. RK Agrawal is owner of a famous amusement park in Delhi. The ticket charge for the park is Rs 150 for children and Rs 400 for adult.

Generally he does not go to park and it is managed by team of staff. One day Mr Agrawal decided to random check the park and went there. When he checked the cash counter, he found that 480 tickets were sold and Rs 134500 was collected.

(i) Let the number of children visited be x and the number of adults visited be y. Which of the following is the correct system of equations that model the problem ?

(ii) How many children visited the park ?

(iii) How many adults visited the park?

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(iv) How much amount collected if 300 children and 350 adults visited the park?

(v) One day total visited children and adults together is 750 and the total amount collected is Rs 212500. What are the number of children and adults visited the park ?

Sol :

(i) Since 480 people visited, we obtain $x+y=480$

Collected amount is Rs 134500 thus

$150x+400y=134500 \Rightarrow 3x+8y=2690$

(ii) Solving the equations $x+y=480$ and $3x+8y=2690$ we get $x=230$ and $y=250$

Number of children attended $=230$

Number of adults attended $=250$

(iii) Number of adults visited the park $=250$

(iv) Amount $=150\times300+400\times350=185000$ Rs

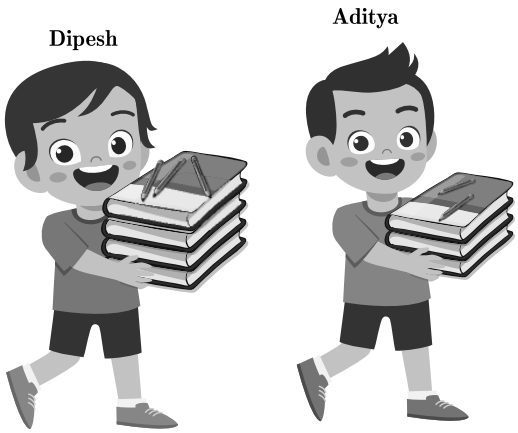
(v) Solving the equations $x+y=750$ and $150x+400y=212500 \Rightarrow 3x+8y=4250$ we have

$x=350$ and $y=400$

i.e Number of children $=350$

Number of adults $=400$.

142. Dipesh bought 3 notebooks and 2 pens for Rs. 80. His friend Ramesh said that price of each notebook could be Rs. 25. Then three notebooks would cost Rs.75, the two pens would cost Rs. 5 and each pen could be for Rs. 2.50. Another friend Amar felt that Rs. 2.50 for one pen was too little. It should be at least Rs. 16. Then the price of each notebook would also be Rs.16.



Aditya also bought the same types of notebooks and pens as Dipesh. He paid 110 for 4 notebooks and 3 pens.

(i) Whether the estimation of Ramesh and Amar is applicable for Aditya?

(ii) Let the cost of one notebook be x and that of pen be y. Which of the following set describe the given problem ?

(iii) What is the exact cost of the notebook?

(iv) What is the exact cost of the pen?

(v) What is the total cost if they purchase the same type of 15 notebooks and 12 pens.

Sol :

(i) Consider the prices mentioned by Ramesh.

If the price of one notebook is Rs. 25 and the price of one pen is Rs. 2.50 then,

The cost of 4 notebooks would be : $4\times25=100$ Rs

And the cost for 3 pens would be : $3\times2.5=7.5$ Rs

Aditya should have paid $100+7.5=107.5$ Rs.

But he paid Rs. 110, thus Ramesh's estimation is wrong.

Now, consider the prices mentioned by Amar.

The cost of 4 notebooks, if one is for Rs.16, would be : $4\times16=64$ Rs

And the cost for 3 pens, if one is for Rs. 16, would be : $3\times16=64$ Rs

Aditya should have paid $64+48=112$ Rs but this is more than the price he paid.

Therefore, Amar's estimation is also wrong.

(ii) According to the statement, we have

$3x+2y=80$ and $4x+3y=110$

(iii) Solving $3x+2y=80$ and $4x+3y=110$ we get

$x=20$ and $y=10$

Thus cost of 1 notebook is 20 Rs and cost of 1 pen is 10 Rs

(iv) Cost of 1 pen $=$ Rs. 10

(v) Total cost $15\times20+12\times10=420$ Rs

143. Jodhpur is the second-largest city in the Indian state of Rajasthan and officially the second metropolitan city of the state. Jodhpur was historically the capital of the Kingdom of Marwar, which is now part of Rajasthan. Jodhpur is a popular tourist destination, featuring many palaces, forts, and temples, set in the stark landscape of the Thar Desert. It is popularly known as the "Blue City" among people of Rajasthan and all over India. The old city circles the Mehrangarh Fort and is bounded by a wall with several gates. The city has expanded greatly outside the wall, though, over the past several

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decades. Jodhpur is also known for the rare breed of horses known as Marwari or Malani, which are only found here.

Last year we visited Jodhpur in a group of 25 friends. When we went Mehrangarh fort we found following fare for ride :

Ride	Normal Hours Fare	Peak Hours Fare
Horse	Rs 50	3 Times
Elephant	Rs 100	2 Times

Some people choose to ride on horse and rest choose to ride on elephant.

(i) First day we rode in normal hours and we paid Rs 1950 for ride. Let x be the number of horses hired and y be the number elephants hired. Which of the following is the correct system of equation that model the problem ?

(ii) How many horses were hired ?

(iii) How many elephant were hired ?

(iv) Next day we rode in peak hours, then how much total fare was paid by our group?

(v) What was the increase in total fare because of peak hours ride ?

Sol :

(i) Let x be the number of horses hired and y be the number of elephant hired, then we have $x+y=25$ and $50x+100y=1950 \Rightarrow x+2y=39$

(ii) Solving equations $x+y=25$ and $x+2y=39$ we get $x=11$ and $y=14$.

Number of horses hired $= 11$

(iii) Number of elephant hired $= 14$.

(iv) For horse riding fare $= 3\times 50\times 11 = 1650$ Rs.

For elephant ride fare $= 2\times 100\times 14 = 2800$ Rs

Total fare $= 1650+2800 = 4450$ Rs

(v) Total fare in normal hour $= 1950$

Total fare in peak hour $= 4450$

Extra fare $= 4450-1950 = 2500$

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CHAPTER 4

Quadratic Equations

OBJECTIVE QUESTIONS

1. If the roots of equation $ax^2+bx+c=0,\ a\neq 0$ are real and equal, then which of the following relation is true?

(a) $a=\frac{b^2}{c}$ (b) $b^2=ac$

(c) $ac=\frac{b^2}{4}$ (d) $c=\frac{b^2}{a}$

Sol : OD 2024

If the roots of equation $ax^2+bx+c=0,\ a\neq 0$ are real and equal then discriminant D must be zero.

Discriminant $D = b^2-4ac$

$0 = b^2-4ac$

$b^2 = 4ac$

Thus (c) is correct option.

2. If the quadratic equation $ax^2+bx+c=0$ has two real and equal roots, then c is equal to

(a) $\frac{-b}{2a}$ (b) $\frac{b}{2a}$

(c) $\frac{-b^2}{4a}$ (d) $\frac{b^2}{4a}$

Sol : OD 2023

For equation having real and equal roots,

$D = b^2-4ac$

$0 = b^2-4ac$

$b^2 = 4ac$

$c = \frac{b^2}{4a}$

Thus (d) is correct option.

3. The sum and product of the zeroes of a quadratic polynomial are 3 and -10 respectively. The quadratic polynomial is

(a) $x^2-3x+10$ (b) $x^2+3x-10$

(c) $x^2-3x-10$ (d) $x^2+3x+10$

Sol : [Board 2020 Delhi Basic]

Sum of zeroes, $\alpha+\beta = 3$

and product of zeroes, $\alpha\beta = -10$

Quadratic polynomial,

$p(x) = x^2-(\alpha+\beta)+\alpha\beta$

$= x^2-3x-10$

Thus (c) is correct option.

4. If the sum of the zeroes of the quadratic polynomial $kx^2+2x+3k$ is equal to their product, then k equals

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

Sol : [Board 2020 OD Basic]

We have $p(x) = kx^2+2x+3k$

Comparing it by ax^2+bx+c, we get $a=k,\ b=2$ and $c=3k$.

Sum of zeroes, $\alpha+\beta = -\frac{b}{a} = -\frac{2}{k}$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{3k}{k} = 3$

According to question, we have

$\alpha+\beta = \alpha\beta$

$-\frac{2}{k} = 3 \Rightarrow k = -\frac{2}{3}$

Thus (d) is correct option.

5. If α and β are the zeroes of the polynomial x^2+2x+1, then $\frac{1}{\alpha}+\frac{1}{\beta}$ is equal to

(a) -2 (b) 2

(c) 0 (d) 3

Sol : [Board 2020 Delhi Basic]

Since α and β are the zeros of polynomial x^2+2x+1,

Sum of zeroes, $\alpha+\beta = -\frac{2}{1} = -2$

and product of zeroes, $\alpha\beta = \frac{1}{1} = 1$

Now, $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= -\frac{2}{1} = -2$

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Thus (a) is correct option.

6. If α and β are the zeroes of the polynomial $2x^2-13x+6$, then $\alpha+\beta$ is equal to

(a) -3 (b) 3

(c) $\frac{13}{2}$ (d) $-\frac{13}{2}$

Sol : [Board 2020 Delhi Basic]

We have $p(x) = 2x^2-13x+6$

Comparing it with ax^2+bx+c we get $a = 2$, $b = -13$ and $c = 6$

Sum of zeroes $\alpha+\beta = -\frac{b}{a}$

$$= -\frac{(-13)}{2} = \frac{13}{2}$$

Thus (c) is correct option.

7. The roots of the quadratic equation $x^2-0.04=0$ are

(a) ± 0.2 (b) ± 0.02

(c) 0.4 (d) 2

Sol : [Board 2020 OD Standard]

We have $x^2-0.04 = 0$

$$x^2 = 0.04$$

$$x = \pm\sqrt{0.04}$$

$$x = \pm 0.2.$$

Thus (a) is correct option.

8. If ½ is a root of the equation $x^2+kx-\frac{5}{4}=0$, then the value of k is

(a) 2 (b) -2

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Sol :

We have $x^2+kx-\frac{5}{4} = 0$

Since, ½ is a root of the given quadratic equation, it must satisfy it.

Thus $\left(\frac{1}{2}\right)^2+k\left(\frac{1}{2}\right)-\frac{5}{4} = 0$

$$\frac{1}{4}+\frac{k}{2}-\frac{5}{4} = 0$$

$$\frac{1+2k-5}{4} = 0$$

$$2k-4 = 0 \Rightarrow k = 2$$

Thus (a) is correct option.

9. Each root of $x^2-bx+c=0$ is decreased by 2. The resulting equation is $x^2-2x+1=0$, then

(a) $b=6, c=9$ (b) $b=3, c=5$

(c) $b=2, c=-1$ (d) $b=-4, c=3$

Sol : [Board Term - 2 SQP 2011]

For $x^2-bx+c=0$ we have

$$\alpha+\beta = b \text{ and } \alpha\beta = c$$

Now $\alpha-2+\beta-2 = \alpha+\beta-4 = b-4$

$$(\alpha-2)(\beta-2) = \alpha\beta-2(\alpha+\beta)+4$$

$$= c-2b+4$$

For $x^2-2x+1=0$ we have

$$2 = b-4 \Rightarrow b = 6$$

and $1 = c-2b+4$

$$= c-2\times 6+4$$

$$= c-8$$

$$c = 1+8 = 9$$

Thus (a) is correct option.

10. Value(s) of k for which the quadratic equation $2x^2-kx+k=0$ has equal roots is/are

(a) 0 (b) 4

(c) 8 (d) 0, 8

Sol : [Board Term - 2 Comt. 2011]

We have $2x^2-kx+k = 0$

Comparing with $ax^2+bx+c = 0$ we $a = 2$, $b = -k$ and $c = k$.

For equal roots, the discriminant must be zero.

Thus $b^2-4ac = 0$

$$(-k)^2-4(2)k = 0$$

$$k^2-8k = 0$$

$$k(k-8) = 0 \Rightarrow k = 0, 8$$

Hence, the required values of k are 0 and 8.

Thus (d) is correct option.

11. If the equation $(m^2+n^2)x^2-2(mp+nq)x+p^2+q^2=0$ has equal roots, then

(a) $mp=nq$ (b) $mq=np$

(c) $mn=pq$ (d) $mq=\sqrt{np}$

Sol : [Board Term - 2 Foreign 2017]

For equal roots, $b^2 = 4ac$

$$4(mp+nq)^2 = 4(m^2+n^2)(p^2+q^2)$$

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$$m^2q^2 + n^2p^2 - 2mnpq = 0$$
$$(mq - np)^2 = 0$$
$$mq - np = 0$$
$$mq = np$$

Thus (b) is correct option.

12. The linear factors of the quadratic equation $x^2 + kx + 1 = 0$ are

(a) $k \geq 2$ (b) $k \leq 2$

(c) $k \geq -2$ (d) $2 \leq k \leq -2$

Sol : [Board Term - 2 SQP 2012]

We have, $x^2 + kx + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we get $a = 1$, $b = k$ and $c = 1$.

For linear factors, $b^2 - 4ac \geq 0$

$$k^2 - 4 \times 1 \times 1 \geq 0$$
$$(k^2 - 2^2) \geq 0$$
$$(k-2)(k+2) \geq 0$$
$$k \geq 2 \text{ and } k \leq -2$$

Thus (d) is correct option.

13. If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then

(a) $b = c$ (b) $a = b$

(c) $ac = 1$ (d) $a = c$

Sol : [Board Term - 2 Comt. 2016]

If one root is α, then the other $\frac{1}{\alpha}$.

Product of roots, $\alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$

$$1 = \frac{c}{a} \Rightarrow a = c$$

Thus (d) is correct option.

14. The quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Sol :

We have $2x^2 - \sqrt{5x} + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we get $a = 2$, $b = -\sqrt{5}$ and $c = 1$,

Now $b^2 - 4ac = (-\sqrt{5})^2 - 4 \times (2) \times (1)$

$$= 5 - 8 = -3 < 0$$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has no real roots i.e., imaginary roots.

Thus (c) is correct option.

15. The real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are

(a) $1, 8$ (b) $-1, -8$

(c) $-1, 8$ (d) $1, -8$

Sol : [Board Term - 2 Comt. 2012]

We have $x^{2/3} + x^{1/3} - 2 = 0$

Substituting $x^{1/3} = y$ we obtain,

$$y^2 + y - 2 = 0$$
$$(y-1)(y+2) = 0 \Rightarrow y = 1 \text{ or } y = -2$$

Thus $x^{1/3} = 1 \Rightarrow x = (1)^3 = 1$

or $x^{1/3} = -2 \Rightarrow x = (-2)^3 = -8$

Hence, the real roots of the given equations are $1, -8$.

Thus (d) is correct option.

16. $(x^2 + 1)^2 - x^2 = 0$ has

(a) four real roots (b) two real roots

(c) no real roots (d) one real root

Sol : [Board Term - 2 Comt. 2015]

We have $(x^2 + 1)^2 - x^2 = 0$

$$x^4 + 1 + 2x^2 - x^2 = 0$$
$$x^4 + x^2 + 1 = 0$$
$$(x^2)^2 + x^2 + 1 = 0$$

Let $x^2 = y$ then we have

$$y^2 + y + 1 = 0$$

Comparing with $ay^2 + by + c = 0$ we get $a = 1$, $b = 1$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$$= (1)^2 - 4(1)(1) = -3$$

Since, $D < 0$, $y^2 + y + 1 = 0$ has no real roots.

i.e. $x^4 + x^2 + 1 = 0$ or $(x^2 + 1)^2 - x^2 = 0$ has no real roots.

Thus (c) is correct option.

17. The equation $2x^2 + 2(p+1)x + p = 0$, where p is real, always has roots that are

(a) Equal

(b) Equal in magnitude but opposite in sign

(c) Irrational

(d) Real

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Sol : [Board Term - 2 Foreign 2011]

We have $2x^2+2(p+1)x+p=0$,

Comparing with $ax^2+bx+c=0$ we get $a=2$, $b=2(p+1)$ and $c=p$.

Now $b^2-4ac = [2(p+1)]^2-4(2p)$

$= 4(p+1)^2-8p$

$= 4p^2+8p+4-8p$

$= 4(p^2+1)$

For any real value of p, $4(p^2+1)$ will always be positive as p^2 cannot be negative for real p. Hence, the discriminant b^2-4ac will always be positive.

When the discriminant is greater than 0 or is positive, then the roots of a quadratic equation are real.

Thus (d) is correct option.

18. The condition for one root of the quadratic equation $ax^2+bx+c=0$ to be twice the other, is

(a) $b^2=4ac$ (b) $2b^2=9ac$

(c) $c^2=4a+b^2$ (d) $c^2=9a-b^2$

Sol : [Board Term - 2 SQP 2017]

Sum of zeroes $\alpha+2\alpha = -\frac{b}{a}$

$3\alpha = -\frac{b}{a} \Rightarrow \alpha = -\frac{b}{3a}$

Product of zeroes $\alpha \times 2\alpha = \frac{c}{a}$

$2\alpha^2 = \frac{c}{a}$

$2\left(-\frac{b}{3a}\right)^2 = \frac{c}{a}$

$\frac{2b^2}{9a^2} = \frac{c}{a}$

$2ab^2-9a^2c = 0$

$a(2b^2-9ac) = 0$

Since, $a \neq 0$, $2b^2 = 9ac$

Hence, the required condition is $2b^2=9ac$.

Thus (b) is correct option.

19. If $x^2+y^2=25$, $xy=12$, then x is

(a) $(3,\ 4)$ (b) $(3,-3)$

(c) $(3,4,-3,-4)$ (d) $(3,-3)$

Sol : [Board Term - 2 Foreign 2015]

We have $x^2+y^2 = 25$

and $xy = 12$

$x^2+\left(\frac{12}{x}\right)^2 = 25$

$x^4+144-25x^2 = 0$

$(x^2-16)(x^2-9) = 0$

Hence, $x^2 = 16$

$x = \pm 4$

and $x^2 = 9$

$x = \pm 3$

Thus (c) is correct option.

20. The quadratic equation $2x^2-3\sqrt{2}\,x+\frac{9}{4}=0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Sol : [Board Term - 2 SQP 2016]

We have $2x^2-3\sqrt{2}\,x+\frac{9}{4} = 0$

Here $a = 2,\ b = -3\sqrt{2},\ c = \frac{9}{4}$

Discriminant $D = b^2-4ac$

$= (-3\sqrt{2})^2-4\times 2\times \frac{9}{4}$

$= 18-18 = 0$

Thus, $2x^2-3\sqrt{2}\,x+\frac{9}{4}=0$ has real and equal roots.

Thus (b) is correct option.

21. The quadratic equation $x^2+x-5=0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Sol : [Board Term - 2 Foreign 2012]

We have $x^2+x-5 = 0$

Here, $a = 1,\ b = 1,\ c = -5$

Now, $D = b^2-4ac$

$= (1)^2-4\times 1\times(-5)$

$= 21 > 0$

So $x^2+x-5=0$ has two distinct real roots.

Thus (a) is correct option.

22. The quadratic equation $x^2+3x+2\sqrt{2}=0$ has

(a) two distinct real roots

(b) two equal real roots

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(c) no real roots

(d) more than 2 real roots

Sol : [Board 2019 OD]

We have $x^2+3x+2\sqrt{2}=0$

Here, $a=1,\ b=3$ and $c=2\sqrt{2}$

Now, $D=b^2-4ac$

$=(3)^2-4(1)(2\sqrt{2})$

$=9-8\sqrt{2}<0$

Hence, roots of the equation are not real.

Thus (c) is correct option.

23. The quadratic equation $5x^2-3x+1=0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Sol : [Board 2019 Delhi]

We have $5x^2-3x+1=0$

Here $a=5,\ b=-3,\ c=1$

Now, $D=b^2-4ac$

$=(-3)^2-4(5)(1)$

$=9-20<0$

Hence, roots of the equation are not real.

Thus (c) is correct option.

24. The quadratic equation $x^2-4x+3\sqrt{2}=0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Sol : [Board SQP 2018]

We have $x^2-4x+3\sqrt{2}=0$

Here $a=1,\ b=-4$ and $c=3\sqrt{2}$

Now $D=b^2-4ac$

$=(-4)^2-4(1)(3\sqrt{2})$

$=16-12\sqrt{2}$

$=16-12\times(1.41)$

$=16-16.92=-0.92$

$b^2-4ac<0$

Hence, the given equation has no real roots.

Thus (c) is correct option.

25. The quadratic equation $x^2+4x-3\sqrt{2}=0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Sol : [Board Term - 2 SQP 2015]

We have $x^2+4x-3\sqrt{2}=0$

Here $a=1,\ b=4$ and $c=-3\sqrt{2}$

Now $D=b^2-4ac=(4)^2-4(1)(-3\sqrt{2})$

$=16+12\sqrt{2}>0$

Hence, the given equation has two distinct real roots,

Thus (a) is correct option.

26. The quadratic equation $x^2-4x-3\sqrt{2}=0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Sol : [Board 2018]

We have $x^2-4x-3\sqrt{2}=0$

Here $a=1,\ b=-4$ and $c=-3\sqrt{2}$

Now $D=b^2-4ac$

$=(-4)^2-4(1)(-3\sqrt{2})$

$=16+12\sqrt{2}>0$

Hence, the given equation has two distinct real roots.

Thus (a) is correct option.

27. The quadratic equation $3x^2+4\sqrt{3}x+4$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Sol :

We have $3x^2+4\sqrt{3}x+4=0$

Here, $a=3,\ b=4\sqrt{3}$ and $c=4$

Now $D=b^2-4ac=(4\sqrt{3})^2-4(3)(4)$

$=48-48=0$

Hence, the equation has real and equal roots.

Thus (b) is correct option.

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28. Which of the following equations has 2 as a root?

(a) $x^2-4x+5=0$

(b) $x^2+3x-12=0$

(c) $2x^2-7x+6=0$

(d) $3x^2-6x-2=0$

Sol : [Board Term - 2 SQP 2014]

(a) Substituting, $x=2$ in x^2-4x+5, we get

$(2)^2-4(2)+5 = 4-8+5 = 1 \neq 0$

So, $x=2$ is not a root of $x^2-4x+5=0$

(b) Substituting, $x=2$ in $x^2+3x-12$, we get

$(2)^2+3(2)-12 = 4+6-12 = -2 \neq 0$

So, $x=2$ is not a root of $x^2+3x-12=0$.

(c) Substituting, $x=2$ in $2x^2-7x+6$, we get

$2(2)^2-7(2)+6 = 2(4)-14+6$

$= 8-14+6$

$= 14-14 = 0.$

So, $x=2$ is a root of the equation $2x^2-7x+6=0$.

(d) Substituting, $x=2$ in $3x^2-6x-2$, we get

$3(2)^2-6(2)-2 = 12-12-2 = -2 \neq 0$

So, $x=2$ is not a root of $3x^2-6x-2=0$.

Thus (c) is correct option.

29. Which of the following equations has the sum of its roots as 3 ?

(a) $2x^2-3x+6=0$ (b) $-x^2+3x-3=0$

(c) $\sqrt{2}\,x^2-\frac{3}{\sqrt{2}}x+1=0$ (d) $3x^2-3x+3=0$

Sol : [Board Term - 2 Foreign 2016]

Sum of the roots,

$$\alpha+\beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$$

Option a : $\alpha+\beta = -\left(\frac{-3}{2}\right) = \frac{3}{2} \neq 3$

Option b : $\alpha+\beta = -\left(\frac{3}{-1}\right) = 3$

Option c : $\alpha+\beta = -\left(\frac{\frac{3}{\sqrt{2}}}{\sqrt{2}}\right) = \frac{3}{2} \neq 3$

Option d $\alpha+\beta = -\left(\frac{-3}{3}\right) = 1 \neq 3$

Thus (b) is correct option.

30. **Assertion (A):** If $5+\sqrt{7}$ is a root of a quadratic equation with rational coefficients, then its other root is $5-\sqrt{7}$.

Reason (R): Surd roots of quadratic equation with rational coefficients occur in conjugate pairs.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Sol : OD 2023

In quadratic equation with rational coefficient, irrational roots occur in conjugate pairs. If one root is $5+\sqrt{7}$ then second root will be $5-\sqrt{7}$.

Hence, assertion is true and reason is also true and correct explanation.

Thus (a) is correct option.

31. **Assertion :** The values of x are $-\frac{a}{2}, a$ for a quadratic equation $2x^2+ax-a^2=0$.

Reason : For quadratic equation $ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $2x^2+ax-a^2 = 0$

$$x = \frac{-a\pm\sqrt{a^2+8a^2}}{4}$$

$$= \frac{-a+3a}{4}$$

$$= \frac{2a}{4}, \frac{-4a}{4}$$

$$x = \frac{a}{2}, -a$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

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32. **Assertion :** $4x^2-12x+9=0$ has repeated roots.

Reason : The quadratic equation $ax^2+bx+c=0$ have repeated roots if discriminant $D>0$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Reason is false because if $D=0$, equation has repeated roots.

Assertion $4x^2-12x+9=0$

$$D=b^2-4ac$$
$$=(-12)^2-4(4)(9)$$
$$=144-144=0$$

Roots are repeated.

Assertion (A) is true but reason (R) is false.

Thus (c) is correct option.

33. **Assertion :** The roots of the quadratic equation $x^2+2x+2=0$ are imaginary.

Reason : If discriminant $D=b^2-4ac<0$ then the roots of quadratic equation $ax^2+bx+c=0$ are imaginary.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $x^2+2x+2=0$

Discriminant, $D=b^2-4ac$

$$=(2)^2-4\times1\times2$$
$$=4-8=-4<0$$

Roots are imaginary.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

34. **Assertion :** The equation $x^2+3x+1=(x-2)^2$ is a quadratic equation.

Reason : Any equation of the form $ax^2+bx+c=0$ where $a\neq0$, is called a quadratic equation.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have, $x^2+3x+1=(x-2)^2=x^2-4x+4$

$$x^2+3x+1=x^2-4x+4$$
$$7x-3=0$$

It is not of the form $ax^2+bx+c=0$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

35. **Assertion :** The equation $8x^2+3kx+2=0$ has equal roots then the value of k is $\pm\frac{8}{3}$.

Reason : The equation $ax^2+bx+c=0$ has equal roots if $D=b^2-4ac=0$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $8x^2+3kx+2=0$

Discriminant, $D=b^2-4ac$

$$=(3k)^2-4\times8\times2$$
$$=9k^2-64$$

For equal roots, $D=0$

$$9k^2-64=0$$
$$9k^2=64$$
$$k^2=\frac{64}{9}\Rightarrow k=\pm\frac{8}{3}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

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ONE MARK QUESTIONS

36. If α and β are the roots of $ax^2 - bx + c = 0 (a \neq 0)$, then calculate $\alpha + \beta$.

Sol : [Board Term-1 2014]

We know that

$$\text{Sum of the roots} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Thus $\alpha + \beta = -\left(\frac{-b}{a}\right) = \frac{b}{a}$

37. If one root of the equation $(k-1)x^2 - 10x + 3 = 0$ is the reciprocal of the other then the value of k is .....

Sol : [Board 2020 SQP Standard]

We have $(k-1)x^2 - 10x + 3 = 0$

Let one root be α, then another root will be $\frac{1}{\alpha}$

Now $\alpha \cdot \frac{1}{\alpha} = \frac{c}{a} = \frac{3}{(k-1)}$

$$1 = \frac{3}{(k-1)}$$

$$k - 1 = 3 \Rightarrow k = 4$$

38. Find the roots of the quadratic equation $x^2 - 0.04 = 0$

Sol : [Board 2020 OD Standard]

We have $x^2 - 0.04 = 0$

$$x^2 = 0.04$$

$$x = \pm\sqrt{0.04}$$

$$x = \pm 0.2.$$

39. Find the nature of roots of the quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$.

Sol : [Board Term - 2 2017]

We have $2x^2 - \sqrt{5x} + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we get $a = 2$, $b = -\sqrt{5}$ and $c = 1$,

Now $b^2 - 4ac = \left(-\sqrt{5}\right)^2 - 4 \times (2) \times (1)$

$$= 5 - 8 = -3 < 0$$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has no real roots i.e., imaginary roots.

40. If ½ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then what is the value of k?

Sol : [Board 2009]

We have $x^2 + kx - \frac{5}{4} = 0$

Since, ½ is a root of the given quadratic equation, it must satisfy it.

Thus $\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$

$$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\frac{1 + 2k - 5}{4} = 0$$

$$2k - 4 = 0 \Rightarrow k = 2$$

PRACTICE

✎ If $x = -\frac{1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.

[Board Term-2 Delhi 2015]

Ans : $-\frac{9}{4}$

✎ If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.

[Board 2018]

Ans : ½

41. If quadratic equation $3x^2 - 4x + k = 0$ has equal roots, then the value of k is ..... .

Sol : [Board 2020 Delhi Basic]

Given, quadratic equation is $3x^2 - 4x + k = 0$

Comparing with $ax^2 + bx + c = 0$, we get $a = 3$, $b = -4$ and $c = k$

For equal roots, $b^2 - 4ac = 0$

$$(-4)^2 - 4(3)(k) = 0$$

$$16 - 12k = 0$$

$$k = \frac{16}{12} = \frac{4}{3}$$

PRACTICE

✎ What are the values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots?

[Board 2010]

Ans : 0 and 8.

42. Find the nature of roots of the quadratic equation $x^2 + x - 5 = 0$.

Sol : [Board Term - 2 2015]

We have $x^2 + x - 5 = 0$

Here, $a = 1, b = 1, c = -5$

Now, $D = b^2 - 4ac$

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$$= (1)^2 - 4 \times 1 \times (-5)$$
$$= 21 > 0$$

So $x^2 + x - 5 = 0$ has two distinct real roots.

43. Find the value of k for which the roots of the equations $3x^2 - 10x + k = 0$ are reciprocal of each other.

Sol : [Board 2019 Delhi]

We have $3x^2 - 10x + k = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 3,\ b = -10, c = k$

Let one root be α so other root is $\frac{1}{\alpha}$.

Product of roots $\alpha \times \frac{1}{\alpha} = \frac{c}{a}$

$$1 = \frac{k}{3}$$
$$k = 3$$

Hence, value of k is 3.

PRACTICE

✎ If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then show that $a = c$.

[Board Term - 2 2011]

Ans : Proof

44. What are the real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$?

Sol : [Board Term - 2 2012]

We have $x^{2/3} + x^{1/3} - 2 = 0$

Substituting $x^{1/3} = y$ we obtain,

$$y^2 + y - 2 = 0$$
$$(y-1)(y+2) = 0 \Rightarrow y = 1 \text{ or } y = -2$$

Thus $x^{1/3} = 1 \Rightarrow x = (1)^3 = 1$

or $x^{1/3} = -2 \Rightarrow x = (-2)^3 = -8$

Hence, the real roots of the given equations are $1, -8$.

45. If $x^2 + y^2 = 25,\ xy = 12$, then what is the value of x?

Sol : [Board Term - 2 2016]

We have $x^2 + y^2 = 25$

and $xy = 12$

$$x^2 + \left(\frac{12}{x}\right)^2 = 25$$
$$x^4 + 144 - 25x^2 = 0$$
$$(x^2 - 16)(x^2 - 9) = 0$$

Hence, $x^2 = 16 \Rightarrow x = \pm 4$

and $x^2 = 9 \Rightarrow x = \pm 3$

46. Find the nature of the roots of the quadratic equation $4x^2 + 4\sqrt{3x} + 3 = 0$.

Sol : [Board 2019 OD]

We have $4x^2 + 4\sqrt{3x} + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 4,\ b = 4\sqrt{3}$ and $c = 3$.

Now, $D = b^2 - 4ac$

$$= (4\sqrt{3})^2 - 4 \times 4 \times 3$$
$$= 48 - 48 = 0$$

Since, $b^2 - 4ac = 0$, then roots of the given equation are real and equal.

PRACTICE

✎ Find the nature of roots of the quadratic equation $2x^2 - 3\sqrt{2}\,x + \frac{9}{4} = 0$

[Board Term - 2 2013]

Ans : Real and equal roots

47. Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.

Sol : [Board 2019 OD]

We have $2x^2 - 4x + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 2\ b = -4,\ c = 3$

Now $D = b^2 - 4ac$

$$= (-4)^2 - 4(2) \times (3)$$
$$= -8 < 0 \text{ or } (-\text{ve})$$

Hence, the given equation has no real roots.

PRACTICE

✎ Find the nature of roots of the quadratic equation $5x^2 - 3x + 1 = 0$.

[Board Term - 2 2012]

Ans : No real roots.

48. Find the nature of roots of the quadratic equation $x^2 - 4x + 3\sqrt{2} = 0$.

Sol : [Board Term-2 2011]

We have $x^2 - 4x + 3\sqrt{2} = 0$

NODIA

Here $a = 1,\ b = -4$ and $c = 3\sqrt{2}$

Now $D = b^2 - 4ac$

$= (-4)^2 - 4(1)(3\sqrt{2})$

$= 16 - 12\sqrt{2} = 16 - 12 \times (1.41)$

$= 16 - 16.92 = -0.92$

$b^2 - 4ac < 0$

Hence, the given equation has no real roots.

PRACTICE

Find the nature of roots of the quadratic equation $x^2 + 3x + 2\sqrt{2} = 0$.

[Board Term - 2 2014]

Ans : Not Real

49. Find the nature of roots of the quadratic equation $x^2 + 4x - 3\sqrt{2} = 0$.

Sol : [Board Term - 2 2016]

We have $x^2 + 4x - 3\sqrt{2} = 0$

Here $a = 1,\ b = 4$ and $c = -3\sqrt{2}$

Now $D = b^2 - 4ac$

$= (4)^2 - 4(1)(-3\sqrt{2})$

$= 16 + 12\sqrt{2} > 0$

Hence, the given equation has two distinct real roots.

PRACTICE

Find the nature of roots of the quadratic equation $x^2 - 4x - 3\sqrt{2} = 0$

[Board Term - 2 2015]

Ans : Two distinct real roots

50. Find the nature of roots of the quadratic equation $3x^2 + 4\sqrt{3}\,x + 4$.

Sol : [Board Term - 2 Delhi 2014]

We have $3x^2 + 4\sqrt{3}\,x + 4 = 0$

Here, $a = 3,\ b = 4\sqrt{3}$ and $c = 4$

Now $D = b^2 - 4ac = (4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48 = 0$

Hence, the equation has real and equal roots.

51. Value of the roots of the quadratic equation, $x^2 - x - 6 = 0$ are ..... .

Sol : [Board 2020 OD Basic]

$x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$(x-3)(x+2) = 0 \Rightarrow x = 3$ and $x = -2$

52. Find the positive root of $\sqrt{3x^2 + 6} = 9$.

Sol : [Board Term-2 2015]

We have $\sqrt{3x^2 + 6} = 9$

$3x^2 + 6 = 81$

$3x^2 = 81 - 6 = 75$

$x^2 = \frac{75}{3} = 25$

Thus $x = \pm 5$

Hence 5 is positive root.

53. If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then find the value of k.

Sol : [Board Term-2 Foreign 2017]

We have $6x^2 - x - k = 0$

Substituting $x = \frac{2}{3}$, we get

$6\left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$

$6 \times \frac{4}{9} - \frac{2}{3} - k = 0$

$\frac{8}{3} - \frac{2}{3} - k = 0$

$\frac{8-2}{3} - k = 0$

$2 - k = 0$

Thus $k = 2$.

54. Find the value(s) of k if the quadratic equation $3x^2 - k\sqrt{3}\,x + 4 = 0$ has real roots.

Sol : [Board SQP 2017]

If discriminant D
$= b^2 - 4ac$ of quadratic equation is equal to zero, or more than zero, then roots are real.

We have $3x^2 - k\sqrt{3}\,x + 4 = 0$

Comparing with $ax^2 + bx + c = 0 = 0$ we get

$a = 3, b = -k\sqrt{3}$ and $c = 4$

For real roots $b^2 - 4ac \geq 0$

$(-k\sqrt{3})^2 - 4 \times 3 \times 4 \geq 0$

$3k^2 - 48 \geq 0$

$k^2 - 16 \geq 0$

$(k-4)(k+4) \geq 0$

Thus $k \leq -4$ and $k \geq 4$

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TWO MARKS QUESTIONS

55. For what values of k, the roots of the equation $x^2+4x+k=0$ are real?

Sol : [Board 2019 Delhi]

We have $x^2+4x+k=0$.

Comparing the given equation with $ax^2+bx+c=0$ we get $a=1,\ b=4, c=k$.

Since, given the equation has real roots,

$$\begin{aligned} D &\geqslant 0 \\ b^2-4ac &\geqslant 0 \\ 4^2-4\times 1\times k &\geqslant 0 \\ 4k &\leqslant 16 \\ k &\leqslant 4 \end{aligned}$$

56. Find the roots of the quadratic equation $6x^2-x-2=0$.

Sol : [Board Term-2 Delhi 2012]

We have
$$\begin{aligned} 6x^2-x-2 &= 0 \\ 6x^2+3x-4x-2 &= 0 \quad (3\times 4 = 2\times 6) \\ 3x(2x+1)-2(2x+1) &= 0 \\ (2x+1)(3x-2) &= 0 \\ 3x-2=0 \text{ or } 2x+1 &= 0 \\ x = \frac{2}{3} \text{or } x &= -\frac{1}{2} \end{aligned}$$

Hence roots of equation are $\frac{2}{3}$ and $-\frac{1}{2}$.

57. Solve for x : $2x^2+6\sqrt{3}\,x-60=0$

Sol : [Board Term-2 OD 2015]

We have
$$\begin{aligned} 2x^2+6\sqrt{3}\,x-60 &= 0 \\ x^2+3\sqrt{3}\,x-30 &= 0 \\ x^2+5\sqrt{3}\,x-2\sqrt{3}\,x-30 &= 0 \\ x(x+5\sqrt{3})-2\sqrt{3}(x+5\sqrt{3}) &= 0 \\ (x+5\sqrt{3})(x-2\sqrt{3}) &= 0 \end{aligned}$$

Thus $x = -5\sqrt{3}, 2\sqrt{3}$

PRACTICE

✍ Find the roots of the quadratic equation : $15x^2-10\sqrt{6}\,x+10=0$

[Board Term-2 OD 2012]

Ans : $\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

58. Solve the following quadratic equation for x : $4\sqrt{3}\,x^2+5x-2\sqrt{3}=0$

Sol : [Board Term-2 2013, 2012]

We have
$$\begin{aligned} 4\sqrt{3}\,x^2+5x-2\sqrt{3} &= 0 \\ 4\sqrt{3}\,x^2+8x-3x-2\sqrt{3} &= 0 \\ 4x(\sqrt{3}\,x+2)-\sqrt{3}(\sqrt{3}\,x+2) &= 0 \\ (\sqrt{3}\,x+2)(4x-\sqrt{3}) &= 0 \end{aligned}$$

Thus $x = -\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

PRACTICE

✍ Find the roots of the quadratic equation $\sqrt{3}\,x^2-2x-\sqrt{3}=0$

[Board Term-2 2012, 2011]

Ans : $\sqrt{3}, -\frac{1}{\sqrt{3}}$

59. Find the roots of the quadratic equation $\sqrt{2}\,x^2+7x+5\sqrt{2}=0$

Sol : [Board Term-2 OD 2017]

We have
$$\begin{aligned} \sqrt{2}\,x^2+7x+5\sqrt{2} &= 0 \\ \sqrt{2}\,x^2+2x+5x+5\sqrt{2} &= 0 \\ \sqrt{2}\,x(x+\sqrt{2})+5(x+\sqrt{2}) &= 0 \\ (x+\sqrt{2})(\sqrt{2}\,x+5) &= 0 \end{aligned}$$

Thus $x = -\sqrt{2}$ and $= -\frac{5}{\sqrt{2}}$

PRACTICE

✍ Solve for x : $\sqrt{3}\,x^2+10x+7\sqrt{3}=0$

[Board Term-2 Foreign 2017]

Ans : $-\sqrt{3}$ and $-\frac{7}{\sqrt{3}}$

60. Solve for x : $\sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3}=0$

Sol : [Board Term-2 OD 2015, Foreign 2014]

We have
$$\begin{aligned} \sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3} &= 0 \\ \sqrt{3}\,x^2-[3\sqrt{2}-\sqrt{2}]x-2\sqrt{3} &= 0 \\ \sqrt{3}\,x^2-3\sqrt{2}\,x+\sqrt{2}\,x-2\sqrt{3} &= 0 \\ \sqrt{3}\,x^2-\sqrt{3}\sqrt{3}\sqrt{2}\,x+\sqrt{2}\,x-\sqrt{2}\sqrt{2}\sqrt{3} &= 0 \\ \sqrt{3}\,x(x-\sqrt{3}\sqrt{2})+\sqrt{2}\,(x-\sqrt{2}\sqrt{3}) &= 0 \\ \sqrt{3}\,x[x-\sqrt{6}]+\sqrt{2}[x-\sqrt{6}] &= 0 \\ (x-\sqrt{6})(\sqrt{3}\,x+\sqrt{2}) &= 0 \end{aligned}$$

Thus $x = \sqrt{6} = -\sqrt{\frac{2}{3}}$

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61. Solve for x : $x^2-(\sqrt{3}+1)x+\sqrt{3}=0$

Sol : [Board Term-2 Foreign 2015]

We have

$$\begin{aligned} x^2-(\sqrt{3}+1)x+\sqrt{3} &= 0 \\ x^2-\sqrt{3}\,x-1x+\sqrt{3} &= 0 \\ x(x-\sqrt{3})-1(x-\sqrt{3}) &= 0 \\ (x-\sqrt{3})(x-1) &= 0 \end{aligned}$$

Thus $x=\sqrt{3}, x=1$

62. Find the roots of the following quadratic equation:

$(x+3)(x-1)=3\left(x-\frac{1}{3}\right)$

Sol : [Board Term-2 2012]

We have

$$\begin{aligned} (x+3)(x-1) &= 3\left(x-\frac{1}{3}\right) \\ x^2+3x-x-3 &= 3x-1 \\ x^2-x-2 &= 0 \\ x^2-2x+x-2 &= 0 \\ x(x-2)+1(x-2) &= 0 \\ (x-2)(x+1) &= 0 \end{aligned}$$

Thus $x=2,-1$

63. Find the roots of the following quadratic equation:

$\frac{2}{5}x^2-x-\frac{3}{5}=0$

Sol : [Board Term-2 2012]

We have

$$\begin{aligned} \frac{2}{5}x^2-x-\frac{3}{5} &= 0 \\ \frac{2x^2-5x-3}{5} &= 0 \\ 2x^2-5x-3 &= 0 \\ 2x^2-6x+x-3 &= 0 \\ 2x(x-3)+1(x-3) &= 0 \\ (2x+1)(x-3) &= 0 \end{aligned}$$

Thus $x=-\frac{1}{2},\ 3$

64. Solve the following quadratic equation for x :

$4x^2-4a^2x+(a^4-b^4)=0$

Sol : [Board Term-2 Delhi 2015]

We have $4x^2-4a^2x+(a^4-b^4)=0$

Comparing with $Ax^2+Bx+C=0$ we have

$A=4, B=-4a^2, C=(a^4-b^4)$

$$\begin{aligned} x &= \frac{-B\pm\sqrt{B^2-4AC}}{2A} \\ &= \frac{4a^2\pm\sqrt{(-4a^2)^2-4\times 4(a^4-b^4)}}{2\times 4} \\ &= \frac{4a^2\pm\sqrt{16a^2-16a^4+16b^4}}{8} \\ &= \frac{4a^2\pm\sqrt{16b^4}}{8} \end{aligned}$$

or, $x=\frac{4a^2\pm 4b^2}{8}=\frac{a^2\pm b^2}{2}$

Thus either $x=\frac{a^2+b^2}{2}$ or $x=\frac{a^2-b^2}{2}$

PRACTICE

Solve the following quadratic equation for x :

$9x^2-6b^2x-(a^4-b^4)=0$

[Board Term-2 Delhi 2015]

Ans : $\frac{1}{3}(a^2+b^2),\ \frac{1}{3}(b^2-a^2)$

65. Solve the following quadratic equation for x :

$x^2-2ax-(4b^2-a^2)=0$

Sol : [Board Term-2 2015]

We have

$$\begin{aligned} x^2-2ax-(4b^2-a^2) &= 0 \\ x^2-2ax+a^2-4b^2 &= 0 \\ (x-a)^2-(2b)^2 &= 0 \\ (x-a+2b)(x-a-2b) &= 0 \end{aligned}$$

Thus $x=a-2b, x=a+2b$

66. Solve the quadratic equation, $2x^2+ax-a^2=0$ for x.

Sol : [Board Term-2 Delhi 2014]

We have $2x^2+ax-a^2=0$

Comparing with $Ax^2+Bx+C=0$ we have

$A=2, B=a, C=-a^2$

Now

$$\begin{aligned} x &= \frac{-B\pm\sqrt{B^2-4AC}}{2A} \\ &= \frac{-a\pm\sqrt{a^2-4\times 2\times(-a^2)}}{2\times 2} \\ &= \frac{-a\pm\sqrt{a^2+8a^2}}{4} \\ &= \frac{-a\pm\sqrt{9a^2}}{4} \\ &= \frac{-a\pm 3a}{4} \\ x &= \frac{-a+3a}{4}, \frac{-a-3a}{4} \end{aligned}$$

Thus $x=\frac{a}{2}, -a$

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67. Find the roots of the quadratic equation $4x^2-4px+(p^2-q^2)=0$

Sol : [Board Term-2 2014]

We have $4x^2-4px+(p^2-q^2)=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=4, b=-4p, c=(p^2-q^2)$$

The roots are given by the quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{4p\pm\sqrt{16p^2-4\times4\times(p^2-q^2)}}{2\times4}$$

$$=\frac{4p\pm\sqrt{16p^2-16p^2+16q^2}}{8}$$

$$=\frac{4p\pm4q}{8}$$

Thus roots are $\frac{p+q}{2}, \frac{p-q}{2}$.

Alternative :

We have $4x^2-4px+p^2-q^2=0$

$$(2x-p)^2-q^2=0$$

$$(2x-p+q)(2x-p-q)=0$$

Thus $x=\frac{(p-q)}{2}$ and $x=\frac{p+q}{2}$

PRACTICE

- Solve the following equation for x : $4x^2+4bx-(a^2-b^2)=0$

  [Board Term-2 OD 2012]

  Ans : $-\frac{1}{2}(a+b)$ and $\frac{1}{2}(a-b)$

- Solve the following quadratic equation for x : $x^2-2ax-(4b^2-a^2)=0$

  [Board Term-2 2015]

  Ans : $a-2b,\ a+2b$

68. Solve for x (in terms of a and b) : $\frac{a}{x-b}+\frac{b}{x-a}=2, x\neq a, b$

Sol : [Board Term-2 Foreign 2016]

We have $\frac{a(x-a)+b(x-b)}{(x-b)(x-a)}=2$

$$a(x-a)+b(x-b)=2\left[x^2-(a+b)x+ab\right]$$

$$ax-a^2+bx-b^2=2x^2-2(a+b)x+2ab$$

$$2x^2-3(a+b)x+(a+b)^2=0$$

$$2x^2-2(a+b)x-(a-b)x+(a+b)^2=0$$

$$\left[2x-(a+b)\right]\left[x-(a+b)\right]=0$$

Thus $x=a+b, \frac{a+b}{2}$

69. If $x=\frac{2}{3}$ and $x=-3$ are roots of the quadratic equation $ax^2+7x+b=0$, find the values of a and b.

Sol : [Board Term-2 Delhi 2016]

We have $ax^2+7x+b=0$ (1)

Substituting $x=\frac{2}{3}$ in above equation we obtain

$$\frac{4}{9}a+\frac{14}{3}+b=0$$

$$4a+42+9b=0$$

$$4a+9b=-42 \quad (2)$$

and substituting $x=-3$ in (1) we obtain

$$9a-21+b=0$$

$$9a+b=21 \quad (3)$$

Solving (2) and (3), we get $a=3$ and $b=-6$

70. Solve for x : $\sqrt{6x+7}-(2x-7)=0$

Sol : [Board Term-2 OD 2016]

We have $\sqrt{6x+7}-(2x-7)=0$

or, $\sqrt{6x+7}=(2x-7)$

Squaring both sides we get

$$6x+7=(2x-7)^2$$

$$6x+7=4x^2-28x+49$$

$$4x^2-34x+42=0$$

$$2x^2-17x+21=0$$

$$2x^2-14x-3x+21=0$$

$$2x(x-7)-3(x-7)=0$$

$$(x-7)(2x-3)=0$$

Thus $x=7$ and $x=\frac{2}{3}$.

PRACTICE

- Solve for x : $\sqrt{2x+9}+x=13$

  [Board Term-2 OD 2016]

  Ans : 8 and 20

71. Find the value of k for which the roots of the quadratic equation $2x^2+kx+8=0$ will have the equal roots ?

Sol : [Board Term-2 OD Compt., 2017]

We have $2x^2+kx+8=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=2, b=k, \text{ and } c=8$$

For equal roots, $D=0$

$$b^2-4ac=0$$
$$k^2-4\times 2\times 8=0$$
$$k^2=64$$
$$k=\pm\sqrt{64}$$

Thus $k=\pm 8$

72. Find k so that the quadratic equation $(k+1)x^2-2(k+1)x+1=0$ has equal roots.

Sol : [Board Term-2 2016]

We have $(k+1)x^2-2(k+1)x+1=0$

Comparing with $Ax^2+Bx+C=0$ we get

$A=(k+1), B=-2(k+1), C=1$

If roots are equal, then $D=0$, i.e.

$$B^2=4AC$$
$$4(k+1)^2=4(k+1)$$
$$k^2+2k+1=k+1$$
$$k^2+k=0$$
$$k(k+1)=0$$
$$k=0,-1$$

$k=-1$ does not satisfy the equation, thus $k=0$

73. If 2 is a root of the equation $x^2+kx+12=0$ and the equation $x^2+kx+q=0$ has equal roots, find the value of q.

Sol : [Board Term 2 SQP 2016]

We have $x^2+kx+12=0$

If 2 is the root of above equation, it must satisfy it.

$$(2)^2+2k+12=0$$
,
$$2k+16=0$$
$$k=-8$$

Substituting $k=-8$ in $x^2+kx+q=0$ we have

$$x^2-8x+q=0$$

For equal roots,

$$(-8)^2-4(1)q=0$$
$$64-4q=0$$
$$4q=64 \Rightarrow q=16$$

PRACTICE

✍ If 2 is a root of the quadratic equation $3x^2+px-8=0$ and the quadratic equation $4x^2-2px+k=0$ has equal roots, find k.

[Board Term-2 Foreign 2014]

Ans : 1

74. Find the values of k for which the quadratic equation $x^2+2\sqrt{2k}\,x+18=0$ has equal roots.

Sol : [Board 2020 SQP Standard]

We have $x^2+2\sqrt{2k}\,x+18=0$

Comparing it by ax^2+bx+c, we get $a=1$, $b=2\sqrt{2k}$ and $c=18$.

Given that, equation $x^2+2\sqrt{2}\,kx+18=0$ has equal roots.

$$b^2-4ac=0$$
$$(2\sqrt{2}\,k)^2-4\times 1\times 18=0$$
$$8k^2-72=0$$
$$8k^2=72$$
$$k^2=\frac{72}{8}=9$$
$$k=\pm 3$$

PRACTICE

✍ Find the values of p for which the quadratic equation $4x^2+px+3=0$ has equal roots.

[Board Term-2 2014]

Ans : $\pm 4\sqrt{3}$

✍ Find the values of k for which the quadratic equation $9x^2-3kx+k=0$ has equal roots.

[Board Term-2 Delhi, OD 2014]

Ans : 4

✍ If the equation $kx^2-2kx+6=0$ has equal roots, then find the value of k.

[Board Term-2 2012]

Ans : 6

75. Find the positive value of k for which $x^2-8x+k=0$, will have real roots.

Sol : [Board Term-2 2014]

We have $x^2-8x+k=0$

Comparing with $Ax^2+Bx+C=0$ we get

$A=1, B=-8, C=k$

Since the given equation has real roots, $B^2-4AC>0$

$$(-8)^2-4(1)(k)\geq 0$$
$$64-4k\geq 0$$
$$16-k\geq 0$$

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$$16 \geq k$$

Thus $k \leq 16$

76. Find the nature of the roots of the quadratic equation :
$13\sqrt{3}\,x^2 + 10x + \sqrt{3} = 0$

Sol : [Board Term-2 2012]

We have $13\sqrt{3}\,x^2 + 10x + \sqrt{3} = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 13\sqrt{3}, b = 10, c = \sqrt{3}$$
$$b^2 - 4ac = (10)^2 - 4(13\sqrt{3})(\sqrt{3})$$
$$= 100 - 156$$
$$= -56$$

As $D < 0$, the equation has not real roots.

77. For what value of k, the roots of the quadratic equation $kx(x - 2\sqrt{5}) + 10 = 0$ are equal ?

Sol : [Board Term-2 Delhi 2014, 2013]

We have $kx(x - 2\sqrt{5}) + 10 = 0$

or, $kx^2 - 2\sqrt{5}\,kx + 10 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$a = k, b = -2\sqrt{5}\,k$ and $c = 10$

Since, roots are equal, $D = b^2 - 4ac = 0$

$$(-2\sqrt{5}\,k)^2 - 4 \times k \times 10 = 0$$
$$20k^2 - 40k = 0$$
$$20k(k - 2) = 0$$
$$k(k - 2) = 0$$

Since $k \neq 0$, we get $k = 2$

78. Find the nature of the roots of the following quadratic equation. If the real roots exist, find them : $3x^2 - 4\sqrt{3}\,x + 4 = 0$

Sol : [Board Term-2 2012]

We have $3x^2 - 4\sqrt{3}\,x + 4 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 3,\ b = -4\sqrt{3},\ c = 4$$
$$b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4)$$
$$= 48 - 48 = 0$$

Thus roots are real and equal.

Roots are $\left(-\frac{b}{2a}\right), \left(-\frac{b}{2a}\right)$ or $\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

79. Find the value of k, for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is six times the other.

Sol : [Board Term-2 2016]

We have $kx^2 - 14x + 8 = 0$

Let one root be α and other root be 6α.

Sum of roots, $\alpha + 6\alpha = \frac{14}{k}$

$$7\alpha = \frac{14}{k} \text{ or } \alpha = \frac{2}{k} \quad ...(1)$$

Product of roots , $\alpha(6\alpha) = \frac{8}{k}$

or $6\alpha^2 = \frac{8}{k}$ ...(2)

Solving (1) and (2), we obtain

$$6\left(\frac{2}{k}\right)^2 = \frac{8}{k}$$
$$6 \times \frac{4}{k^2} = \frac{8}{k}$$
$$\frac{3}{k^2} = \frac{1}{k}$$
$$3k = k^2$$
$$3k - k^2 = 0$$
$$k[3 - k] = 0$$
$$k = 0 \text{ or } k = 3$$

Since $k = 0$ is not possible, therefore $k = 3$.

80. Solve the quadratic equation $(x-1)^2 - 5(x-1) - 6 = 0$

Sol : [Board Term-2 2015]

We have $(x-1)^2 - 5(x-1) - 6 = 0$

$$x^2 - 2x + 1 - 5x + 5 - 6 = 0$$
$$x^2 - 7x + 6 - 6 = 0$$
$$x^2 - 7x = 0$$
$$x(x - 7) = 0$$

Thus $x = 0, 7$.

THREE MARKS QUESTIONS

81. The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, find the two numbers.

Sol : [Board 2023 OD Standard]

Let x be first number, then other number will be $15 - x$.

Since the sum of their reciprocals is $\frac{3}{10}$, we have

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Hence, roots are $\frac{a+2b}{3}$ and $\frac{2a+b}{3}$.

100. If the roots of the equation $(a^2+b^2)x^2-2(ac+bd)x+(c^2+d^2)=0$ are equal, prove that $\frac{a}{b}=\frac{c}{d}$.

Sol : [Board Term-2 2016]

We have $(a^2+b^2)x^2-2(ac+bd)x+(c^2+d^2)=0$

Comparing with $Ax^2+Bx+C=0$ we get

$A=(a^2+b^2), B=-2(ac+bd), C=(c^2+d^2)$

If roots are equal, $D = B^2-4AC=0$

or $B^2 = 4AC$

Now $\left[-2(ac+bd)\right]^2 = 4(a^2+b^2)(c^2+d^2)$

$4(a^2c^2+2abcd+b^2d^2)=4(a^2c^2+a^2d^2+b^2c^2+b^2d^2)$

$a^2c^2+2abcd+b^2d^2 = a^2c^2+a^2d^2+b^2c^2+b^2d^2$

$2abcd = a^2d^2+b^2c^2$

$0 = a^2d^2-2abcd+b^2c^2$

$0 = (ad-bc)^2$

$0 = ad-bc$

Thus $ad = bc$

$\frac{a}{b} = \frac{c}{d}$ Hence Proved

PRACTICE

Show that if the roots of the following equation are equal then $ad=bc$ or $\frac{a}{b}=\frac{c}{d}$.

$x^2(a^2+b^2)+2(ac+bd)x+c^2+d^2 = 0$

[Board Term-2 OD Compt. 2017]

Ans : Proof

101. Determine the positive value of k for which the equation $x^2+kx+64=0$ and $x^2-8x+k=0$ will both have real and equal roots.

Sol : [Board Term-2 2012, 2014]

We have $x^2+kx+64 = 0$

Comparing with $ax^2+bx+c=0$ we get

$a=1, b=k, c=64$

For real and equal roots, $b^2-4ac=0$

Thus $k^2-4\times1\times64 = 0$

$k^2-256 = 0 \Rightarrow k = \pm 16$ (1)

Now for equation $x^2-8x+k=0$ we have

$b^2-4ac = 0$

$(-8)^2-4\times1\times k = 0$

$64 = 4k$

$k = \frac{64}{4}=16$ (2)

From (1) and (2), we get $k=16$. Thus for $k=16$, given equations have equal roots.

102. Find that non-zero value of k, for which the quadratic equation $kx^2+1-2(k-1)x+x^2=0$ has equal roots. Hence find the roots of the equation.

Sol : [Board Term-2 Delhi 2015]

We have $kx^2+1-2(k-1)x+x^2 = 0$

$(k+1)x^2-2(k-1)x+1 = 0$

Comparing with $ax^2+bx+c=0$ we get

$a=k+1, b=-2(k-1), c=1$

For real and equal roots, $b^2-4ac=0$

$4(k-1)^2-4(k+1)\times1 = 0$

$4k^2-8k+4-4k-4 = 0$

$4k^2-12k = 0$

$4k(k-3) = 0$

As k can't be zero, thus $k=3$.

103. Find the value of k for which the quadratic equation $(k-2)x^2+2(2k-3)x+(5k-6)=0$ has equal roots.

Sol : [Board Term-2 2015]

We have $(k-2)x^2+2(2k-3)x+(5k-6)=0$

Comparing with $ax^2+bx+c=0$ we get

$a=k-2, b=2(2k-3), c=(5k-6)$

For real and equal roots, $b^2-4ac=0$

$\{2(2k-3)\}^2-4(k-2)(5k-6) = 0$

$4(4k^2-12k+9)-4(k-2)(5k-6) = 0$

$4k^2-12k+9-5k^2+6k+10k-12 = 0$

$k^2-4k+3 = 0$

$k^2-3k-k+3 = 0$

$k(k-3)-1(k-3) = 0$

$(k-3)(k-1) = 0$

Thus $k = 1,3$

104. If the roots of the quadratic equation $(a-b)x^2+(b-c)x+(c-a)=0$ are equal, prove that $2a=b+c$.

Sol : [Board Term-2 Delhi 2016]

We have $(a-b)x^2+(b-c)x+(c-a) = 0$

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$$x = \frac{-121 \pm \sqrt{5049}}{22}$$
$$= \frac{-121 \pm 71.06}{22}$$
$$x = \frac{-49.94}{22}, \frac{-192.06}{22}$$
$$x = -2.27, -8.73.$$

86. Solve for x :

$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; x \neq 1, -2, 2$

Sol : [Board Term-2 OD 2016]

We have $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$

$$\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$$
$$\frac{2x^2+4}{x^2+x-2} = \frac{2x-11}{x-2}$$
$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$
$$5x^2+19x-30 = 0$$
$$(5x-6)(x+5) = 0$$
$$x = -5, \frac{6}{5}$$

87. Solve for x :

$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -\frac{3}{2}$

Sol : [Board Term-2 Delhi 2016]

We have $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$

$$2x(2x+3)+(x-3)+(3x+9) = 0$$
$$4x^2+6x+x-3+3x+9 = 0$$
$$4x^2+10x+6 = 0$$
$$2x^2+5x+3 = 0$$
$$(x+1)(2x+3) = 0$$

Thus $x = -1, x = -\frac{3}{2}$

88. Solve the following quadratic equation for x :

$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

Sol : [Board Term-2 OD 2016]

We have $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

$$x^2 + \frac{a}{a+b}x + \frac{a+b}{a}x + 1 = 0$$
$$x\left(x + \frac{a}{a+b}\right) + \frac{a+b}{a}\left(x + \frac{a}{a+b}\right) = 0$$
$$\left(x + \frac{a}{a+b}\right)\left(x + \frac{a+b}{a}\right) = 0$$

Thus $x = \frac{-a}{a+b}, \frac{-(a+b)}{a}$

89. Solve for x :

$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}; x \neq 1, 2, 3$

Sol : [Board Term-2 OD 2016]

We have $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$

$$\frac{x-3+x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$
$$\frac{2x-4}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$
$$\frac{2(x-2)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$
$$\frac{2}{(x-1)(x-3)} = \frac{2}{3}$$
$$3 = (x-1)(x-3)$$
$$x^2-4x+3 = 3$$
$$x^2-4x = 0$$
$$x(x-4) = 0$$

Thus $x = 0$ or $x = 4$

90. Solve for x : $x^2 + 5x - (a^2+a-6) = 0$

Sol : [Board 2019 OD, Foreign 2015]

We have $x^3 + 5x - (a^2+a-6) = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Thus $x = \frac{-5 \pm \sqrt{25+4(a^2+a-6)}}{2}$

$$= \frac{-5 \pm \sqrt{25+4a^2+4a-24}}{2}$$
$$= \frac{-5 \pm \sqrt{4a^2+4a+1}}{2}$$
$$= \frac{-5 \pm (2a+1)}{2}$$
$$= \frac{2a-4}{2}, \frac{-2a-6}{2}$$

Thus $x = a-2$, $x = -(a+3)$

Alternative :

We have $x^2 + 5x - (a^2+a-6) = 0$

$$x^2 + 5x - [a^2+3a-2a-6] = 0$$
$$x^2 + 5x - [a(a+3) - 2(a+3)] = 0$$
$$x^2 + 5x - (a+3)(a-2) = 0$$
$$x^2 + [a+3-(a-2)]x - (a+3)(a-2) = 0$$
$$x^2 + (a+3)x - (a-2)x - (a+3)(a-2) = 0$$
$$x[x+(a+3)] - (a-2)[x+(a+3)] = 0$$

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$2x^2+px-15=0$, while the quadratic equation $x^2-4px+k=0$ has equal roots. Find the value of k.

[Board Term-2 OD Compt. 2017]

Ans : 4

FIVE MARKS QUESTIONS

109. Solve for x : $\left(\frac{2x}{x-5}\right)^2+\left(\frac{2x}{x-5}\right)-24=0, x\neq 5$

Sol : [Board Term-2 2016]

We have $\left(\frac{2x}{x-5}\right)^2+5\left(\frac{2x}{x-5}\right)-24=0$

Let $\frac{2x}{x-5}=y$ then we have

$$y^2+5y-24=0$$
$$(y+8)(y-3)=0$$
$$y=3,-8$$

Taking $y=3$ we have

$$\frac{2x}{x-5}=3$$
$$2x=3x-15 \Rightarrow x=15$$

Taking $y=-8$ we have

$$\frac{2x}{x-5}=-8$$
$$2x=-8x+40$$
$$10x=40 \Rightarrow x=4$$

Hence, $x=15,4$

110. Solve for x : $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}$ $x\neq -1,-2,-4$

Sol : [Board Term-2 OD 2016]

We have $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}$

$$\frac{x+2+2(x+1)}{(x+1)(x+2)}=\frac{4}{x+4}$$
$$\frac{3x+4}{x^2+3x+2}=\frac{4}{x+4}$$
$$(3x+4)(x+4)=4(x^2+3x+2)$$
$$3x^2+16x+16=4x^2+12x+8$$
$$x^2-4x-8=0$$

Now $x=\frac{-b\sqrt{b^2+4ac}}{2a}$

$$=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-8)}}{2\times 1}$$
$$=\frac{4\pm\sqrt{16+32}}{2}$$
$$=\frac{4\pm\sqrt{48}}{2}=\frac{4\pm 4\sqrt{3}}{2}$$
$$=2\pm 2\sqrt{3}$$

Hence, $x=2+2\sqrt{3}$ and $2-2\sqrt{3}$

PRACTICE

Solve for x : $\frac{3}{x+1}+\frac{4}{x-1}=\frac{29}{4x-1}$; $x\neq -1,\ 1,\ \frac{1}{4}$

[Board Term-2 Delhi 2015]

Ans : 4 and -7

111. Find x in terms of a, b and c :

$$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}, x\neq a,b,c$$

Sol : [Board Term-2 Delhi 2016]

We have $\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$

$$a(x-b)(x-c)+b(x-a)(x-c)$$
$$=2c(x-a)(x-b)$$
$$ax^2-abx-acx+abc+bx^2-bax-bcx+abc$$
$$=2cx^2-2cxb-2cxa+2abc$$
$$ax^2+bx^2-2cx^2-abx-acx-bax-bcx+2cbx+2acx$$
$$=0$$
$$x^2(a+b-2c)-2abx+acx+bcx=0$$
$$x^2(a+b-2c)+x(-2ab+ac+bc)=0$$

Thus $x=-\left(\frac{ac+bc-2ab}{a+b-2c}\right)$

112. Solve for x : $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2$ where $x\neq -\frac{1}{2}, 1$

Sol : [Board Term-2 OD 2015]

We have $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1}=\frac{1}{y}$

Substituting this value we obtain

$$y+\frac{1}{y}=2$$
$$y^2+1=2y$$
$$y^2-2y+1=0$$
$$(y-1)^2=0$$
$$y=1$$

Substituting $y=\frac{x-1}{2x+1}$ we have

$$\frac{x-1}{2x+1}=1 \text{ or } x-1=2x+1$$

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$$= \frac{a \pm b}{3}$$

$$x = \frac{(a+b)}{3}, \frac{(a-b)}{3}$$

Thus $x = \frac{a+b}{3}, x = \frac{a-b}{3}$

95. Solve the equation $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, $x \neq -4, 7$ for x.

Sol : [Board Term-2 2012]

We have, $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$$\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-1}{(x+4)(x-7)} = \frac{1}{30}$$

$$(x+4)(x-7) = -30$$

$$x^2 - 3x - 28 = -30$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$(x-1)(x-2) = 0$$

Thus $x = 1$ and 2.

96. Find the roots of the quadratic equation :

$$a^2b^2x^2 + b^2x - a^2x - 1 = 0$$

Sol : [Board Term-2 2012]

We have $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$$b^2x(a^2x+1) - 1(a^2x+1) = 0$$

$$(b^2x-1)(a^2x+1) = 0$$

$$x = \frac{1}{b^2} \text{ or } x = -\frac{1}{a^2}$$

Hence, roots are $\frac{1}{b^2}$ and $-\frac{1}{a^2}$.

97. If $(x^2+y^2)(a^2+b^2) = (ax+by)^2$, prove that $\frac{x}{a} = \frac{y}{b}$

Sol : [Board Term-2 2014]

We have $(x^2+y^2)(a^2+b^2) = (ax+by)^2$

$$x^2a^2 + x^2b^2 + y^2a^2 + y^2b^2 = a^2x^2 + b^2y^2 + 2abxy$$

$$x^2b^2 + y^2a^2 - 2abxy = 0$$

$$(xb - ya)^2 = 0$$

$$xb = ya$$

Thus $\frac{x}{a} = \frac{y}{b}$ Hence Proved.

98. Solve the following quadratic equation for x :

$p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Sol : [Board Term-2 2012]

We have $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = p^2, b = p^2 - q^2, c = -q^2$$

The roots are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(p^2-q^2) - \sqrt{(p^2-q^2)^2 - 4(p^2)(-q^2)}}{2p^2}$$

$$= \frac{-(p^2-q^2) - \sqrt{p^4 + q^4 - 2p^2q^2 + 4p^2q^2}}{2p^2}$$

$$= \frac{-(p^2-q^2) - \sqrt{p^4 + q^4 + 2p^2q^2}}{2p^2}$$

$$= \frac{-(p^2-q^2) - \sqrt{(p^2+q^2)^2}}{2p^2}$$

$$= \frac{-(p^2-q^2) \pm (p^2+q^2)}{2p^2}$$

Thus $x = \frac{-(p^2-q^2) + (p^2+q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$

and $x = \frac{-(p^2-q^2) - (p^2+q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$

Hence, roots are $\frac{q^2}{p^2}$ and -1.

99. Solve the following quadratic equation for x :

$$9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$$

Sol : [Board Term-2 Foreign 2016]

We have $9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$

Now $2a^2 + 5ab + 2b^2 = 2a^2 + 4ab + ab + 2b^2$

$$= 2a[a+2b] + b[a+2b]$$

$$= (a+2b)(2a+b)$$

Hence the equation becomes

$$9x^2 - 9(a+b)x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3[3a+3b]x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3[(a+2b)+(2a+b)]x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3(a+2b)x - 3(2a+b)x + (a+2b)(2a+b) = 0$$

$$3x[3x-(a+2b)] - (2a+b)[3x-(a+2b)] = 0$$

$$[3x-(a+2b)][3x-(2a+b)] = 0$$

$$3x-(2a+b) = 0$$

$$x = \frac{a+2b}{3}$$

$$3x-(2a+b) = 0$$

$$x = \frac{2a+b}{3}$$

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$$[x+(a+3)][x-(a-2)] = 0$$

Thus $x = -(a+3)$ and $x = (a-2)$

Hence, roots of given equations are $x = -(a+3)$ and $x = a-2$.

PRACTICE

Solve for x : $x^2+6x-(a^2+2a-8)$

[Board Term-2 Foreign 2015]

Ans : $a-2, -a-4$

91. Solve for x : $x^2-(2b-1)x+(b^2-b-20) = 0$

Sol : [Board Term-2 Foreign 2015]

We have $x^2-(2b-1)x+(b^2-b-20) = 0$

Comparing with $Ax^2+Bx+C = 0$ we have

$$A = 1, B = -(2b-1), C = (b^2-b-20)$$

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$x = \frac{(2b-1) \pm \sqrt{(2b-1)^2-4(b^2-b-20)}}{2}$$

$$= \frac{(2b-1) \pm \sqrt{4b^2-4b+1-4b^2+4b+80}}{2}$$

$$= \frac{(2b-1) \pm \sqrt{81}}{2} = \frac{(2b-1) \pm 9}{2}$$

$$= \frac{2b+8}{2}, \frac{2b-10}{2}$$

$$= b+4,\ b-5$$

Thus $x = b+4$ and $x = b-5$

92. Solve the equation for x : $\frac{4}{x}-3 = \frac{5}{2x+3}$; $x \neq 0, \frac{-3}{2}$

Sol : [Board Term-2 Delhi 2014]

We have $\frac{4}{x}-3 = \frac{5}{2x+3}$

$$\frac{4}{x}-\frac{5}{2x+3} = 3$$

$$\frac{4(2x+3)-5x}{x(2x+3)} = 3$$

$$8x+12-5x = 3x(2x+3)$$

$$3x+12 = 6x^2+9x$$

$$6x^2+6x-12 = 0$$

$$x^2+x-2 = 0$$

$$x^2+2x-x-2 = 0$$

$$x(x+2)-(x+2) = 0$$

$$(x+2)(x-1) = 0$$

Thus $x = 1, -2$

PRACTICE

Solve for x : $\frac{16}{x}-1 = \frac{15}{x+1}$; $x \neq 0, -1$

[Board Term-2 OD 2014]

Ans : -4 and 4

93. Find the roots of the equation $2x^2+x-4 = 0$, by the method of completing the squares.

Sol : [Board Term-2 OD 2014]

We have $2x^2+x-4 = 0$

$$x^2+\frac{x}{2}-2 = 0$$

$$x^2+2x\left(\frac{1}{4}\right)-2 = 0$$

Adding and subtracting $\left(\frac{1}{4}\right)^2$, we get

$$x^2+2x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2 = 0$$

$$\left(x+\frac{1}{4}\right)^2-\left(\frac{1}{16}+2\right) = 0$$

$$\left(x+\frac{1}{4}\right)^2-\left(\frac{1+32}{16}\right) = 0$$

$$\left(x+\frac{1}{4}\right)^2-\frac{33}{16} = 0$$

$$\left(x+\frac{1}{4}\right)^2 = \frac{33}{16}$$

$$\left(x+\frac{1}{4}\right) = \pm\frac{\sqrt{33}}{4}$$

Thus roots are $x = \frac{-1+\sqrt{33}}{4}, \frac{-1-\sqrt{33}}{4}$

PRACTICE

Find the roots of $x^2-4x-8 = 0$ by the method of completing square.

[Board Term-2 2015]

Ans : $2+2\sqrt{3},\ 2-2\sqrt{3}$

94. Solve for x : $9x^2-6ax+(a^2-b^2) = 0$

Sol : [Board Term-2 2012]

We have $9x^2-6ax+a^2-b^2 = 0$

Comparing with $Ax^2+Bx+C = 0$ we have

$$A = 9, B = -6a, C = (a^2-b^2)$$

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$x = \frac{6a \pm \sqrt{(-6a)^2-4\times 9k(a^2-b^2)}}{2\times 9}$$

$$= \frac{6a \pm \sqrt{36a^2-36a^2+36b^2}}{18}$$

$$= \frac{6a \pm \sqrt{36b^2}}{18} = \frac{6a \pm 6b}{18}$$

Comparing with $ax^2+bx+c=0$ we get

$a=(a-b,),\ b=(b-c),\ c=c-a$

For real and equal roots, $b^2-4ac=0$

$$(b-c)^2-4(a-b)(c-a)=0$$
$$b^2+c^2-2bc-4(ac-a^2-bc+ab)=0$$
$$b^2+c^2-2bc-4ac+4a^2+4bc-4ab=0$$
$$4a^2+b^2+c^2+2bc-4ab-4ac=0$$

Using $a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2$

$$(-2a+b+c)^2=0$$
$$-2a+b+c=0$$

Hence, $b+c=2a$

105. If $ad\neq bc$, then prove that the equation $(a^2+b^2)x^2+2(ac+bd)x+(c^2+d^2)=0$ has no real roots.

Sol : [Board Term-2 OD 2017]

We have $(a^2+b^2)x^2+2(ac+bd)x+(c^2+d^2)=0$

Comparing with $Ax^2+Bx+C=0$ we get

$A=(a^2+b^2), B=2(ac+bd)$ and $C=(c^2+d^2)$

For no real roots, $D=B^2-4AC<0$

$$\begin{aligned}D&=B^2-4AC\\&=\left[2(ac+bd)\right]^2-4(a^2+b^2)(c^2+d^2)\\&=4[a^2c^2+2abcd+b^2d^2]-4[a^2c^2+a^2d^2+b^2c^2+b^2d^2]\\&=4[a^2c^2+2abcd+b^2d^2-a^2c^2-a^2d^2-b^2c^2-b^2d^2]\\&=-4[a^2d^2+b^2c^2-2abcd]\\&=-4(ad-bc)^2\end{aligned}$$

Since $ad\neq bc$, therefore $D\neq 0$ and always negative.

Hence the equation has no real roots.

106. Find the value of c for which the quadratic equation $4x^2-2(c+1)x+(c+1)=0$ has equal roots.

Sol : [Board Term-2 Delhi 2017]

We have $4x^2-2(c+1)x+(x+1)=0$

Comparing with $Ax^2+Bx+C=0$ we get

$A=4, B=2(c+1), C=(c+1)$

If roots are equal, $B^2-4AC=0$

$$[2(c+1)]^2-4\times 4(c+1)=0$$
$$4(c^2+2c+1)-4(4c+4)=0$$
$$4(c^2+2c+1-4c-4)=0$$
$$c^2-2c-3=0$$
$$c^2-3c+c-3=0$$
$$c(c-3)+1(c-3)=0$$
$$(c-3)(c+1)=0$$
$$c=3,-1$$

Hence for equal roots $c=3,-1$.

107. Solve $\dfrac{1}{(a+b+x)}=\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{x},\ a+b\neq 0.$

Sol : [Board Term-2 SQP 2016]

We have $\dfrac{1}{a+b+x}=\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{x}$

$$\frac{1}{a+b+x}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$$
$$\frac{x-(a+b+x)}{x(a+b+x)}=\frac{a+b}{ab}$$
$$\frac{x-a-b-x}{x(a+b+x)}=\frac{a+b}{ab}$$
$$\frac{-(a+b)}{x(a+b+x)}=\frac{a+b}{ab}$$
$$x(a+b+x)=-ab$$
$$x^2+(a+b)x+ab=0$$
$$(x+a)(x+b)=0$$
$$x=-a \text{ or } x=-b$$

108. If (-5) is a root of the quadratic equation $2x^2+px+15=0$ and the quadratic equation $p(x^2+x)+k=0$ has equal roots, then find the values of p and k.

Sol : [Board Term-2 Delhi 2015]

We have $2x^2+px-15=0$

Since $x=-5$ is the root of above equation. It must satisfy it.

$$2(-5)^2+p(-5)-15=0$$
$$50-5p-15=0$$
$$5p=35\Rightarrow p=7$$

Now $p(x^2+x)+k=0$ has equal roots

or $7x^2+7x+k=0$

Taking $b^2-4ac=0$ we have

$$7^2-4\times 7\times k=0$$
$$7-4k=0$$
$$k=\frac{7}{4}$$

Hence $p=7$ and $k=\dfrac{7}{4}$.

PRACTICE

If -3 is a root of quadratic equation

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$$\frac{1}{x}+\frac{1}{15-x} = \frac{3}{10}$$
$$\frac{15-x+x}{x(15-x)} = \frac{3}{10}$$
$$\frac{15}{x(15-x)} = \frac{3}{10}$$
$$\frac{5}{x(15-x)} = \frac{1}{10}$$
$$50 = x(15-x)$$
$$50 = 15x-x^2$$
$$x^2-15x+50 = 0$$
$$x^2-10x-5x+50 = 0$$
$$x(x-10)-5(x-10) = 0$$
$$(x-10)(x-5) = 0$$
$$x = 10,\ x=5$$

If first number is 10, then other number will $15-10=5$. If first number is 5, then other number will $15-5=10$. Thus required two numbers are 5 and 10.

82. If α and β are roots of the quadratic equation $x^2-7x+10=0$, find the quadratic equation whose roots are α^2 and β^2.

Sol : [Board 2023 OD Standard]

We have
$$x^2-7x+10 = 0$$
$$x^2-5x-2x+10 = 0$$
$$x(x-5)-2(x-5) = 0$$
$$(x-5)(x-2) = 0$$

Thus $\alpha = 5$ and $\beta = 2$

Thus $\alpha^2 = 25$ and $\beta^2 = 4$

Quadratic equation having roots α^2 and β^2 will be
$$x^2-(\alpha^2+\beta^2)x+\alpha^2\beta^2 = 0$$
$$x^2-(25+4)x+25\times 4 = 0$$
$$x^2-29x+100 = 0$$

83. Solve the following equation: $\frac{1}{x}-\frac{1}{x-2} = 3$, $x\neq 0,\ 2$

Sol : [Board 2020 SQP Standard]

We have $\frac{1}{x}-\frac{1}{x-2} = 3 \qquad (x\neq 0,2)$
$$\frac{x-2-x}{x(x-2)} = 3$$
$$\frac{-2}{x(x-2)} = 3$$
$$3x(x-2) = -2$$
$$3x^2-6x+2 = 0$$

Comparing it by ax^2+bx+c, we get $a=3$, $b=-6$ and $c=2$.

Now, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$$= \frac{-(-6)\pm\sqrt{(-6)^2-4(3)(2)}}{2(3)}$$
$$= \frac{6\pm\sqrt{36-24}}{6} = \frac{6\pm\sqrt{12}}{6}$$
$$= \frac{6\pm 2\sqrt{3}}{6}$$
$$= \frac{3+\sqrt{3}}{3},\ \frac{3-\sqrt{3}}{3}$$

84. Solve for x : $\frac{1}{x}+\frac{2}{2x-3} = \frac{1}{x-2}, x\neq 0,\ \frac{2}{3},\ 2$.

Sol : [Board Term-2 Foreign 2016]

We have
$$\frac{1}{x}+\frac{2}{2x-3} = \frac{1}{x-2}$$
$$\frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2}$$
$$\frac{4x-3}{x(2x-3)} = \frac{1}{x-2}$$
$$(x-2)(4x-3) = 2x^2-3x$$
$$4x^2-11x+6 = 2x^2-3x$$
$$2x^2-8x+6 = 0$$
$$x^2-4x+3 = 0$$
$$(x-1)(x-3) = 0$$

Thus $x = 1,3$

85. Solve for x : $\frac{1}{x+4}-\frac{1}{x+7} = \frac{11}{30}$ $x\neq -4,\ -7$.

Sol : [Board 2020 OD Standard]

We have
$$\frac{1}{x+4}-\frac{1}{x+7} = \frac{11}{30}$$
$$\frac{x+7-x-4}{(x+4)(x+7)} = \frac{11}{30}$$
$$\frac{3}{x^2+4x+7x+28} = \frac{11}{30}$$
$$\frac{3}{x^2+11x+28} = \frac{11}{30}$$
$$11x^2+121x+308 = 90$$
$$11x^2+121x+218 = 0$$

Comparing with $ax^2+bx+c=0$, we get $a=11$, $b=121$ and $c=218$ we obtain
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-121\pm\sqrt{14641-9592}}{22}$$

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or $x = -2$

113. Find for x : $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$; $x \neq 0, 1, 2$

Sol : [Board Term-2 OD 2017]

We have $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$

$\frac{x-1+2x-4}{(x-2)(x-1)} = \frac{6}{x}$

$3x^2 - 5x = 6x^2 - 18x + 12$

$3x^2 - 13x + 12 = 0$

$3x^2 - 4x - 9x + 12 = 0$

$x(3x-4) - 3(3x-4) = 0$

$(3x-4)(x-3) = 0$

$x = \frac{4}{3}$ and 3

Hence, $x = 3, \frac{4}{3}$

114. Solve, for x : $\sqrt{3}\,x^2 + 10x + 7\sqrt{3} = 0$

Sol : [Board Term-2 Foreign 2017]

We have $\sqrt{3}\,x^2 + 10x + 7\sqrt{3} = 0$

$\sqrt{3}\,x^2 + 3x + 7x + 7\sqrt{3} = 0$

$(x+\sqrt{3})(\sqrt{3}\,x+7) = 0$

$(x+\sqrt{3})(\sqrt{3}\,x+7) = 0$

$x = -\sqrt{3}$ and $x = \frac{-7}{\sqrt{3}}$

Hence roots $x = -\sqrt{3}$ and $x = \frac{-7}{\sqrt{3}}$

115. Solve for x : $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$; $x \neq 0, 2$

Sol : [Board Term -2 Delhi Compt. 2017]

We have $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$

$\frac{x(x+3) - (1-x)(x-2)}{x(x-2)} = \frac{17}{4}$

$\frac{(x^2+3x) - (-x^2+3x-2)}{x^2-2x} = \frac{17}{4}$

$\frac{2x^2+2}{x^2-2x} = \frac{17}{4}$

$8x^2 + 8 = 17x^2 - 34x$

$9x^2 - 34x - 8 = 0$

$9x^2 - 36x + 2x - 8 = 0$

$9x(x-4) + 2(x-4) = 0$

$(x-4)(9x+2) = 0$

$x = 4$ or $x = -\frac{2}{9}$

Hence, $x = 4, -\frac{2}{9}$

116. Solve for x : $4x^2 + 4bx - (a^2 - b^2) = 0$

Sol : [Board Term-2 Foreign 2017]

We have $4x^2 + 4bx - (a^2 - b^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$A = 4, B = 4b$ and $C = b^2 - a^2$

$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$= \frac{-4b \pm \sqrt{(4b)^2 - 4.4(b^2 - a^2)}}{2.4}$

$= \frac{-4b \pm \sqrt{16b^2 - 16b^2 + 16a^2}}{8}$

$= \frac{-4b \pm 4a}{8}$

$= -\frac{(a+b)}{2}, \frac{(a-b)}{2}$

Hence the roots are $-\frac{(a+b)}{2}$ and $\frac{(a-b)}{2}$

117. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

Sol : [Board 2019 OD]

We have $x^2 + px + 16 = 0$ ...(1)

If this equation has equal roots, then discriminant $b^2 - 4ac$ must be zero.

i.e., $b^2 - 4ac = 0$ ...(2)

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 1$, $b = p$ and $c = 16$

Substituting above in equation (2) we have

$p^2 - 4 \times 1 \times 16 = 0$

$p^2 = 64 \Rightarrow p = \pm 8$

When $p = 8$, from equation (1) we have

$x^2 + 8x + 16 = 0$

$x^2 + 2 \times 4x + 4^2 = 0$

$(x+4)^2 = 0 \Rightarrow x = -4, -4$

Hence, roots are -4 and -4.

When $p = -8$ from equation (1) we have

$x^2 - 8x + 16 = 0$

$x^2 - 2 \times 4x + 4^2 = 0$

$(x-4)^2 = 0 \Rightarrow x = 4, 4$

Hence, the required roots are either $-4, -4$ or $4, 4$

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$$x^2+(a+b)x+ab = 0$$
$$(x+a)(x+b) = 0$$
$$x = -a, x = -b$$

Hence $x = -a, -b$

126. Check whether the equation $5x^2-6x-2=0$ has real roots if it has, find them by the method of completing the square. Also verify that roots obtained satisfy the given equation.

Sol : [Board Term-2 SQP 2017]

We have $5x^2-6x-2=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=5, b=(-6) \text{ and } c=(-2)$$
$$b^2-4ac = (-6)^2-4\times 5\times -2$$
$$= 36+40 = 76 > 0$$

So the equation has real and two distinct roots.

$$5x^2-6x = 2$$

Dividing both the sides by 5 we get

$$\frac{x^2}{5}-\frac{6}{5}x = \frac{2}{5}$$
$$x^2-2x\left(\frac{3}{5}\right) = \frac{2}{5}$$

Adding square of the half of coefficient of x

$$x^2-2x\left(\frac{3}{5}\right)+\frac{9}{25} = \frac{2}{5}+\frac{9}{25}$$
$$\left(x-\frac{3}{5}\right)^2 = \frac{19}{25}$$
$$x-\frac{3}{5} = \pm\frac{\sqrt{19}}{5}$$
$$x = \frac{3+\sqrt{19}}{5} \text{ or } \frac{3-\sqrt{19}}{5}$$

Verification :

$$5\left[\frac{3+\sqrt{19}}{5}\right]^2-6\left[\frac{3+\sqrt{19}}{5}\right]-2$$
$$= \frac{9+6\sqrt{19}+19}{5}-\left(\frac{18+6\sqrt{19}}{5}\right)-2$$
$$= \frac{28+6\sqrt{19}}{5}-\frac{18+6\sqrt{19}}{5}-2$$
$$= \frac{28+6\sqrt{19}-18-6\sqrt{19}-10}{5}$$
$$= 0$$

Similarly

$$5\left[\frac{3-\sqrt{19}}{5}\right]^2-6\left[\frac{3-\sqrt{19}}{5}\right]-2 = 0$$

Hence verified.

WORD PROBLEMS

127. Sum of the areas of two squares is 468 m2. If the difference of their perimeter is 24 m, find the sides of the squares.

Sol : [Board Term-2 2012, 2011]

Let the side of the smaller square be y and the side of the longer square be x. As per question the difference of their perimeter is 24. Thus we have

$$4x-4y = 24$$
$$x-y = 6 \quad (1)$$

According to the question we get

$$x^2+y^2 = 468 \quad (2)$$

Substituting $x=y+6$ from equation (1) in (2) we have

$$(y+6)^2+y^2 = 468$$
$$2y^2+12y+36 = 468$$
$$2y^2+12y-432 = 0$$
$$y^2+6y-216 = 0$$
$$(y+18)(y-12) = 0$$

Thus $y = -18, 12$

As side can not be negative, $y=12$ and $x=12+6=18$

Hence, the side of larger square 18 m and that of smaller square 12 m.

PRACTICE

Sum of the areas of two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares.

[Board Term-2 2013]

Ans : 16 cm

128. A two digit number is four times the sum of the digits. It also equal to 3 times the product of digits. Find the number.

Sol : [Board Term-2 Foreign 2016]

Let units digit and tens digit of the two digit number be x and y respectively.

Thus number is $10y+x$

According to question, we have

$$10y+x = 4(y+x)$$
$$10y+x = 4y+4x$$
$$10y-4y = 4x-x$$
$$6y = 3x \Rightarrow 2y = x$$

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$$4x^2-24x+36 = 0$$
or $$x^2-6x+9 = 0$$
or, $$(x-3)(x-3) = 0$$
$$x = 3,3$$

Thus roots are 3 and 3.

PRACTICE

✎ Find the values of k for which the quadratic equations $(k+4)x^2+(k+1)x+1=0$ has equal roots. Also, find the roots.

[Board Term-2 Delhi 2014]

Ans : For $k=5, x=-\frac{1}{3}$; $k=-3, x=1$

122. If the equation $(1+m^2)x^2+2mcx+(c^2-a^2)=0$ has equal roots, prove that $c^2=a^2(1+m^2)$

Sol : [Board Term-2 Delhi 2015]

We have $(1+m^2)x^2+2mcx+(c^2-a^2)=0$

Comparing with $Ax^2+Bx+C=0$ we get

$$A=1+m^2, B=2mc, C=(c^2-a^2)$$

If roots are equal, $B^2-4AC=0$

$$(2mc)^2-4(1+m^2)(c^2-a^2) = 0$$
$$4m^2c^2-4(1+m^2)(c^2-a^2) = 0$$
$$m^2c^2-(c^2-a^2+m^2c^2-m^2a^2) = 0$$
$$m^2c^2-c^2+a^2-m^2c^2+m^2a^2 = 0$$
$$-c^2+a^2+m^2a^2 = 0$$
$$c^2 = a^2(1+m^2)$$

Hence Proved.

PRACTICE

✎ If the quadratic equation, $(1+a^2)b^2x^2+2abcx+(c^2-m^2)=0$ in x has equal roots, prove that $c^2=m^2(1+a^2)$

[Board Term-2 2014]

Ans : Proof

123. If the roots of the quadratic equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are equal. Then show that $a=b=c$.

Sol : [Board Term-2 Delhi 2015]

We have

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a) = 0$$
$$x^2-ax-bx+ab++x^2-bx-cx+bc+$$
$$+x^2-cx-ax+ac = 0$$
$$3x^2-2ac-2bx-2cx+ab+bc+ca = 0$$

For equal roots $B^2-4AC=0$

$$\{-2(a+b+c)\}^2-4\times3(ab+bc+ca) = 0$$
$$4(a+b+c)^2-12(ab+bc+ca) = 0$$
$$a^2+b^2+c^2-3(ab+bc+ca) = 0$$
$$a^2+b^2+c^2+2ab+2bc+2ac-3ab-3bc-3ac = 0$$
$$a^2+b^2+c^2-ab-ac-bc = 0$$
$$\tfrac{1}{2}[2a^2+2b^2+2c^2-2ab-2ac-2bc] = 0$$
$$\tfrac{1}{2}[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)]=0$$
$$\tfrac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2] = 0$$
or, $$(a-b)^2+(b-c)^2+(c-a)^2 = 0$$

If $a\neq b\neq c$

$(a-b)^2>0$, $(b-c)^2>0$, $(c-a)^2>0$

If $(a-b)^2=0 \Rightarrow a=b$

$(a-c)^2=0 \Rightarrow b=c$

$(c-a)^2=0 \Rightarrow c=a$

Thus $a=b=c$ Hence Proved

124. If the roots of the quadratic equation $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$ in x are equal then show that either $a=0$ or $a^3+b^3+c^3=3abc$

Sol : [Board Term-2 OD 2017]

We have

$(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$

Comparing with $Ax^2+Bx+C=0$ we get

$A=(c^2-ab), B=(a^2-bc), C=(b^2-ac)$

If roots are equal, $B^2-4AC=0$

$$[2(a^2-bc)]^2-4(c^2-ab)(b^2-ac) = 0$$
$$4[a^4+b^2c^2-2a^2bc]-4(b^2c^2-c^3a-ab^3-a^2bc) = 0$$
$$4[a^4+b^2c^2-2a^2bc-b^2c^2+c^3a+ab^3-a^2bc] = 0$$
$$4[a^4+ac^3+ab^3-3a^2bc] = 0$$
$$a(a^3+c^3+b^3-3abc) = 0$$
$$a=0 \text{ or } a^3+b^3+c^3 = 3abc$$

125. Solve for x : $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$

where $a+b+x\neq0$ and $a,b,x\neq0$

Sol : [Board Term-2 Foreign 2017]

We have $$\frac{1}{a+b+x}-\frac{1}{x} = \frac{1}{a}+\frac{1}{b}$$
$$\frac{-(a+b)}{x^2+(a+b)x} = \frac{b+a}{ab}$$

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$$x^2-14+5x = 0$$
$$x^2+5x-14 = 0$$
$$x^2+7x+-2x-14 = 0$$
$$x(x+7)-2(x+7) = 0$$
$$(x+7)(x-2) = 0$$

Thus $x=-7$ and $x=2$

Since, the digits cannot be negative, $x=2$

Thus, $y=\frac{14}{x}=\frac{14}{2}=7$

Therefore, number is $(10x+y)=27$

133. The sum of the squares of two consecutive naturals is 421. Find the numbers.

Sol : [Board Term-2 2012]

Let the first natural number be x. The second consecutive natural will be $x+1$.

According to the question we have,

$$x^2+(x+1)^2 = 421$$
$$x^2+x^2+2x+1 = 421$$
$$x^2+x-210 = 0$$
$$x^2+15x-14x-210 = 0$$
$$x(x+15)-14(x+15) = 0$$
$$(x+15)(x-14) = 0$$
$$x+15 = 0 \text{ or } x-14=0$$
$$x = -15 \text{ or } x=14$$

Rejecting negative value we get $x=14$.

Therefore first number is 14 and consecutive number is 15.

PRACTICE

- The sum of squares of two consecutive even numbers is 340. Find the numbers.
  [Board Term-2 Foreign 2014]
  Ans : 12 and 14

- The sum of squares of two consecutive multiples of 7 is 637. Find the multiples.
  [Board Term-2 Foreign 2014]
  Ans : 14 and 21

- The sum of the squares of two consecutive odd numbers is 394. Find the numbers.
  [Board Term-2 Foreign 2012, 2014]
  Ans : 13 and 15

134. The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

Sol : [Board Term-2 Foreign 2016]

Let numerator be x then fraction will be $\frac{x}{2x+1}$

As per the question we have

$$\frac{x}{2x+1}+\frac{2x+1}{x} = 2\frac{16}{21}=\frac{58}{21}$$
$$21\left[x^2+(2x+1)^2\right] = 58(2x^2+x)$$

or, $$11x^2-26x-21 = 0$$
$$11x^2-33x+7x-20 = 0$$
$$x = 3, -\frac{7}{11} \text{ (rejected)}$$

We reject $x=-\frac{7}{3}$, thus $x=3$ and fraction will be $\frac{3}{6+1}=\frac{3}{7}$.

PRACTICE

- The denominator of a fraction is two more than its numerator. If the sum of the fraction and its reciprocal is $\frac{34}{15}$, find the fraction.
  [Board Term-2 OD 2012]
  Ans : $\frac{3}{5}$.

135. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction.

Sol : [Board Term-2 Delhi 2015]

Let the denominator be x, then numerator will be $x-3$. So the fraction will be $\frac{x-3}{x}$

By the given condition, new fraction will

$$\frac{x-3+2}{x+2} = \frac{x-1}{x+2}$$

Now $$\frac{x-3}{x}+\frac{x-1}{x+2} = \frac{29}{20}$$
$$20\left[(x-3)(x+2)+x(x-1)\right] = 29(x^2+2x)$$
$$20(x^2-x-6+x^2-x) = 29x^2+58x$$
$$20(2x^2-2x-6) = 29x^2+58x$$
$$40x^2-40x-120 = 29x^2+58x$$
$$11x^2-98x-120 = 0$$
$$11x^2-110x+12x-120 = 0$$
$$(11x+20)(x-10) = 0,$$

We take $x=10$ and fraction will be is $\frac{10-3}{10}=\frac{7}{10}$

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Also, $10y + x = 3xy$

Now substituting $x = 2y$ we have

$$\begin{aligned} 10y + 2y &= 3(2y)y \\ 12y &= 6y^2 \\ 6y^2 - 12y &= 0 \\ 6y(y-2) &= 0 \\ y &= 0 \text{ or } y = 2 \end{aligned}$$

As the number can't be zero, $x = 4$ and $x = 2y = 4$.

Thus required number is 24.

129. The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

Sol : [Board 2020 Delhi Basic]

Since difference of two natural numbers is 5, let the x and $(x+5)$ be two natural numbers.

Reciprocals of the numbers are $\frac{1}{x}$ and $\frac{1}{x+5}$.

According to question, we have

$$\begin{aligned} \frac{1}{x} - \frac{1}{x+5} &= \frac{1}{10} \\ \frac{x+5-x}{x(x+5)} &= \frac{1}{10} \\ \frac{5}{x^2+5x} &= \frac{1}{10} \\ x^2 + 5x - 50 &= 0 \\ x^2 + 10x - 5x - 50 &= 0 \\ x(x+10) - 5(x-10) &= 0 \\ (x+10)(x-5) &= 0 \\ \Rightarrow \quad x &= 5,\ x = -10 \end{aligned}$$

But given two numbers are natural numbers. Therefore, $x = 5$.

Here, the required natural numbers are $x = 5$ and $x + 5 = 10$.

130. Three consecutive positive integers are such that the sum of the square of the first and product of the other two is 46. Find the integers.

Sol : [Board 2020 OD Basic]

Let the numbers be x, $x+1$, $x+2$.

According to question, we obtain

$$\begin{aligned} x^2 + (x+1)(x+2) &= 46 \\ x^2 + x^2 + 3x + 2 &= 46 \\ 2x^2 + 3x - 44 &= 0 \\ 2x^2 + 11x - 8x - 44 &= 0 \\ x(2x+11) - 4(2x+11) &= 0 \\ (2x+11)(x-4) &= 0 \\ 2x + 11 = 0 \text{ and } x - 4 &= 0 \end{aligned}$$

or, $x = -\frac{11}{2}$ and $x = 4$

But x can't be negative, thus $x = 4$.

So, numbers are 4, 5 and 6.

PRACTICE

Three consecutive natural number are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the number.

[Board Term-2 OD 2016]

Ans : 9, 10 and 11

131. The product of two consecutive positive integers is 306. Find the integers.

Sol :

Let x and $x+1$ be two consecutive positive integers.

According to question, product of x and $(x+1)$ is 306.

Thus

$$\begin{aligned} x(x+1) &= 306 \\ x^2 + x - 306 &= 0 \\ x^2 + 18x - 17x - 306 &= 0 \\ x(x+18) - 17(x+18) &= 0 \\ (x+18)(x-17) &= 0 \\ x &= 17 \text{ and } -18 \end{aligned}$$

Neglecting negative value we get $x = 17$. Thus numbers are 17 and 18.

132. A two digit number is such that product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number.

Sol : [Board 2020 Delhi Basic]

Let x be the tens digit and y be the ones digit, then number will $10x + y$.

Given, $xy = 14$ ...(1)

If 45 is added to this number the digits get interchanged i.e. new number will be $10y + x$

Hence, $10x + y + 45 = 10y + x$

$$\begin{aligned} 9x - 9y + 45 &= 0 \\ x - y + 5 &= 0 \quad \text{...(2)} \end{aligned}$$

From equation (1) and (2), we get

$$x - \frac{14}{x} + 5 = 0$$

NODIA

Let x be the original price of the book.

Number of books bought at original price for Rs 600,

$$n_1 = \frac{600}{x}$$

If the price of a book is reduced by Rs 5, then the new price of books is $(x-5)$.

Number of book bought at reduced price for Rs 600,

$$n_2 = \frac{600}{x-5}$$

According to equation,

$$\begin{aligned} n_2 &= n_1 + 4 \\ n_2 - n_1 &= 4 \\ \frac{600}{x-5} - \frac{600}{x} &= 4 \\ 600\left[\frac{x-x+5}{x(x-5)}\right] &= 4 \\ \frac{3000}{x^2-5x} &= 4 \\ x^2-5x &= 750 \\ x^2-5x-750 &= 0 \\ x^2-30x+25x-750 &= 0 \\ x(x-30)+25(x-30) &= 0 \Rightarrow x = 30, -25 \end{aligned}$$

But price cannot be negative, thus $x = 30$.

Hence, the original price of the book is Rs 30.

PRACTICE

If the price of a book is reduced by Rs. 5, a person can buy 5 more book for Rs. 300. Find the original list price of the book.

[Board Term-2 2012]

Ans : Rs 20

140. A person on tour has Rs 4200 for his expenses. If he extends his tour for 3 days, he has to cut down his daily expenses by Rs 70. Find the original duration of the tour.

Sol : [Board 2020 Delhi Basic]

Let x be the original duration of tour in days.

Amount with the person is Rs 4200.

Daily expenses, $R_1 = \frac{4200}{x}$

If tour extended for 3 days, total number of days will be $(x+3)$.

Now daily expenses, $R_2 = \frac{4200}{(x+3)}$

According to question,

$$\begin{aligned} R_1 - R_2 &= 70 \\ \frac{4200}{x} - \frac{4200}{x+3} &= 70 \\ 4200\left(\frac{1}{x} - \frac{1}{x+3}\right) &= 70 \\ 60\left[\frac{x+3-x}{x(x+3)}\right] &= 1 \\ 60\left[\frac{3}{x(x+3)}\right] &= 1 \\ 180 &= x^2+3x \\ x^2+3x-180 &= 0 \\ x^2+15x-12x-180 &= 0 \\ x(x+15)-12(x+15) &= 0 \\ (x+15)(x-12) &= 0 \end{aligned}$$

Thus $x = -15$ and $x = 12$

Since, x cannot be negative, $x = 12$

Thus, the original duration of the tour is 12 days.

141. The sum of ages (in years) of a son and his father is 35 years and product of their ages is 150 years, find their ages.

Sol : [Board Term-2 2014, 2012]

Let the age of father be x years and age of son be y years

Now $x + y = 35$ (1)

and $xy = 150$ (2)

Substituting the value of y, in (1) we have

$$\begin{aligned} x(35-x) &= 150 \\ x^2-35x+150 &= 0 \\ (x-30)(x-5) &= 0 \\ x &= 30, x = 5 \text{ (Rejected)} \end{aligned}$$

Age of father can't be 5 years, so we reject $x = 5$ and take $x = 30$.

Now $y = 5$

Hence the age of father is 30 years and the age of son is 5 years.

142. One fourth of a herd of camels was seen in forest. Twice of square root of the herd had gone to mountain and remaining 15 camels were seen on the bank of a river, find the total number of camels.

Sol : [Board Term-2 2012]

Let the total number of camels be x.

According to the condition given we have

$$\begin{aligned} \frac{x}{4} + 2\sqrt{x} + 15 &= x \\ 3x - 8\sqrt{x} - 60 &= 0 \end{aligned}$$

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136. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

Sol : [Board Term-2 2014, Delhi]

Let the first number be x, then second number will be $x+5$.

Now according to the question we have

$$\begin{aligned}\frac{1}{x}-\frac{1}{x+5} &= \frac{1}{10}\\ \frac{x+5-x}{x(x+5)} &= \frac{1}{10}\\ 50 &= x^2+5x\\ x^2+5x-50 &= 0\\ x^2+10x-5x-50 &= 0\\ x(x+10)-5(x+10) &= 0\\ (x+10)(x-5) &= 0\\ x &= 5,-10\end{aligned}$$

Rejecting the negative value, numbers are 5 and 10.

Let the odd number be $2x+1$, then consecutive odd number will be $2x+1+2=2x+3$.

Now, according to question we have

$$\begin{aligned}(2x+1)^2+(2x+3)^2 &= 394\\ 4x^2+4x+1+4x^2+12x+9 &= 394\\ 8x^2+16x-384 &= 0\\ x^2+2x-48 &= 0\\ x^2+8x-6x-48 &= 0\\ x(x+8)-6(x+8) &= 0\\ x &= -8,6\end{aligned}$$

Rejecting the negative value,

$$\begin{aligned}\text{Ist number} &= 2\times 6+1 = 13\\ \text{and second odd number} &= 15\end{aligned}$$

137. In a cricket match, Harbhajan took three wickets less than twice the number of wickets taken by Zahir. The Product of the number of wickets taken by these two is 20. Represent the above situation in the form of quadratic equation.

Sol : [Board Term-2 2015]

Let the number of wickets is taken by Zahir be x , then number of wickets taken by Harbhajan will be $2x-3$.

According to question, we have

$$\begin{aligned}x(2x-3) &= 20\\ 2x^2-3x &= 20\end{aligned}$$

Thus required quadratic equation,

$$2x^2-3x-20 = 0$$

138. A train covers a distance of 360 km at a uniform speed. Had the speed been 5 km/h more, it would have taken 48 min less for the journey. Find the original speed of the train.

Sol : [Board 2020 SQP Standard]

Let x be the original speed of the train.

Time taken to covers 360 km by original speed x ,

$$t_1 = \frac{360}{x}$$

Time taken to cover 360 km when the speed is increased by 5 km/h,

$$t_2 = \frac{360}{x+5}$$

It is given that the time to cover 300 km is reduced by 48 min i.e. $\frac{48}{60}$ hour.

Thus $t_1-t_2 = \frac{48}{60}$

$$\begin{aligned}\frac{360}{x}-\frac{360}{(x+5)} &= \frac{48}{60}\\ \frac{360}{x}-\frac{360}{x+5} &= \frac{4}{5}\\ 360\left[\frac{x+5-x}{x(x+5)}\right] &= \frac{4}{5}\\ 90\left[\frac{5}{x(x+5)}\right] &= \frac{1}{5}\\ 450\times 5 &= x^2+5x\\ x^2+5x-2250 &= 0\\ x^2+50x-45x-2250 &= 0\\ x(x+50)-45(x+50) &= 0\\ (x+50)(x-45) = 0 \Rightarrow x &= 45 \quad [x\neq -50]\end{aligned}$$

Hence, original speed of the train is 45 km/h.

PRACTICE

A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

[Board 2020 Delhi Standard]

Ans : 40 km/hr

139. If the price of a book is reduced by Rs 5, a person can by 4 more books for Rs 600. Find the original price of the book.

Sol : [Board 2020 OD Basic]

NODIA

Time taken by slow train, $t_2 = \frac{60}{x-10}$

According to questions, we have

$$\begin{aligned} t_2 - t_1 &= 3 \\ \frac{600}{x-10} - \frac{600}{x} &= 3 \qquad \left[t = \frac{d}{s}\right] \\ 600\left[\frac{x-x+10}{(x-10)x}\right] &= 3 \\ \frac{6000}{x^2-10x} &= 3 \\ x^2 - 10x - 2000 &= 0 \\ x^2 - 50x + 40x - 2000 &= 0 \\ x(x-50) + 40(x-50) &= 0 \\ (x-50)(x+40) &= 0 \quad x = 50, -40 \end{aligned}$$

But negative speed can not be possible. Thus, the speed of fast train is 50 km/hr, and the speed of slow train is $50 - 10 = 40$ km.

PRACTICE

A journey of 192 km from a town A to town B takes 2 hours more by an ordinary passenger train than a super fast train. If the speed of the faster train is 16 km/h more, find the speed of the faster and the passenger train.

[Board Term-2 2012]

Ans : 32 km/h

146. A car covers a distance of 2592 km with a uniform speed. The number of hours taken for journey is one half the number representing the speed in km/hour.

Find the time taken to cover the distance.

Sol : [Board Term-2 Delhi Compt 2017]

Let the speed of the car be x km/hr.

Therefore time taken $= \frac{x}{2}$ hour

Now $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$$\begin{aligned} x &= \frac{2592}{\frac{x}{2}} \\ x^2 &= 2592 \times 2 = 5184 \\ x &= \sqrt{5184} = 72 \end{aligned}$$

Hence the time taken $\frac{72}{2} = 36$ hours.

147. Speed of a boat in still water is 15 km/hour. It goes 30 km up stream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream.

Sol : [Board Term-2 Delhi 2017]

Let the speed of the Stream be x km/hr.

Speed of boat up stream $= 15 - x$

and speed of boat down stream $= 15 + x$

According to the question we get

$$\begin{aligned} \frac{30}{15-x} + \frac{30}{15+x} &= 4\frac{1}{2} \\ \frac{30(15+x)+30(15-x)}{15^2-x^2} &= \frac{9}{2} \\ 900 \times 2 &= 9(15^2 - x^2) \\ 9x^2 &= 2025 - 1800 = 225 \\ x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$

Hence, the speed of the stream $= 5$ km/hr

148. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

Sol : [Board 2019 Delhi]

Let t be the time taken by the smaller diameter top. Time for larger tap diameter will be $t-2$.

$$\text{Total time taken} = 1\frac{7}{8} = \frac{15}{8} h.$$

Portion filled in one hour by smaller diameter tap will $\frac{1}{t}$ and by lager diameter tap will be $\frac{1}{t-2}$

According to the problem,

$$\begin{aligned} \frac{1}{t} + \frac{1}{t-2} &= \frac{8}{15} \\ \frac{t-2+t}{t(t-2)} &= \frac{8}{15} \\ 15(2t-2) &= 8t(t-2) \\ 30t - 30 &= 8t^2 - 16t \\ 8t^2 - 46t + 30 &= 0 \\ 4t^2 - 23t + 15 &= 0 \\ 4t^2 - 20t - 3t + 30 &= 0 \\ (4t-3)(t-5) &= 0 \\ \Rightarrow \quad t &= \frac{3}{4} \text{ or } t = 5 \end{aligned}$$

If $t = \frac{3}{4}$ then $t - 2 = \frac{3}{4} - 2 = \frac{-5}{4}$

Since, time cannot be negative, we neglect $t = \frac{3}{4}$

Therefore, $t = 5$

and $t - 2 = 5 - 2 = 3$

Hence, time taken by larger tap is 3 hours and time taken by smaller is 5 hours.

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Let $\sqrt{x} = y$, then we have

$$\begin{aligned} 3y^2 - 8y - 60 &= 0 \\ 3y^2 - 18y + 10y - 60 &= 0 \\ 3y(y-6) + 10(y-6) &= 0 \\ (3y+10)(y-6) &= 0 \\ y &= 6 \text{ or } y = -\frac{10}{3} \end{aligned}$$

Here $y = -\frac{10}{3}$ is not possible.

Thus $y = 6$ or $y^2 = 36$, $\quad x = y^2 = 36$

Hence the number of camels is 36.

143. In a class test, the sum of Arun's marks in Hindi and English is 30. Had he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

Sol : [Board 2019 OD]

Let x be the marks in Hindi y be the marks in English.

According to question,

$$x + y = 30 \qquad \ldots(1)$$

If he had got 2 marks more in Hindi, then his marks would be $x+2$ and if he had 3 marks less in English, then his marks would be $y-3$.

According to question

$$\begin{aligned} (x+2)(y-3) &= 210 \\ (x+2)(30-x-3) &= 210 \quad (y = 30-x) \\ (x+2)(27-x) &= 210 \\ 27x - x^2 + 54 - 2x &= 210 \\ -x^2 + 25x - 156 &= 0 \\ x^2 - 25x + 156 &= 0 \\ x^2 - 13x - 12x + 156 &= 0 \\ x(x-13) - 12(x-13) &= 0 \\ (x-12)(x-13) &= 0 \Rightarrow x = 12, 13 \end{aligned}$$

When $x = 12$ then $y = 30 - 12 = 18$

When $x = 13$ then $y = 30 - 13 = 17$

Hence, the marks in Hindi is 12 and marks in English is 18.

In other case the marks in Hindi is 13 and marks in English is 17.

PRACTICE

✎ In a class test Raveena got a total of 30 mark in English and Mathematics. Had she got 2 more marks in Mathematics and 3 marks less in English then the product of her marks obtained would have be 210. Find the individual marks obtained in two subjects.

[Board Term-2 OD Compt. 2017]

Ans : 17 and 13

✎ In a class test, the sum of the marks obtained by a student in mathematics and science is 28. Had he got 3 marks more in mathematics and 4 marks less in science, the product of the marks would have been 180. Find his marks in two subjects.

[Board Term-2 2012]

Ans : 12 and 16

✎ A student scored a total of 32 marks in class tests in mathematics ans science. Had he scored 2 marks less in science and 4 more in mathematics, the product of his marks would have been 253. Find his marks in two subjects.

[Board Term-2 2012]

Ans : 7 and 25, 19 and 13

144. A shopkeeper buys a number of books for Rs. 1200. If he had bought 10 more books for the same amount, each book would have cost him Rs. 20 less. How many books did he buy?

Sol : [Board Term-2 2012]

Let the number of books bought be x.

As per question we have

$$\begin{aligned} \frac{1200}{x} - \frac{1200}{x+10} &= 20 \\ x^2 + 10x - 600 &= 0 \\ (x+30)(x-20) &= 0 \\ x &= -30 \text{ or } x = 20 \end{aligned}$$

Since number of books cannot be negative, $x = 20$

Thus number of books bought is 20.

145. A fast train takes 3 hours less than a slow train for a journey of 60 km. If the speed of the slow train is 10 km/h less than that of the fast train, find the speed of each train.

Sol : [Board 2020 OD Standard]

Total distance of a journey = 600 km

Let x be the speed of fast train then speed of slow will be $(x-10)$.

Time taken by fast train, $t_1 = \frac{60}{x}$

NODIA

118. Find the positive values of k for which quadratic equations $x^2+kx+64=0$ and $x^2-8x+k=0$ both will have the real roots.

Sol : [Board Term-2 Foreign 2016]

(1) For $x^2+kx+64=0$ to have real roots

$$\begin{aligned} k^2-256 &\geq 0 \\ k^2 &\geq 256 \\ k &\geq 16 \text{ or } k<-16 \end{aligned}$$

(2) For $x^2-8x+k=0$ to have real roots

$$\begin{aligned} 64-4k &\geq 0 \\ 16-k &\geq 0 \\ 16 &\geq k \end{aligned}$$

For (1) and (2) to hold simultaneously, $k=16$

119. Find the values of k for which the equation $(3k+1)x^2+2(k+1)x+1$ has equal roots. Also find the roots.

Sol : [Board Term-2 2014]

We have $(3k+1)x^2+2(k+1)x+1$

Comparing with $Ax^2+Bx+C=0$ we get

$$A=(3k+1), B=2(k+1), C=1$$

If roots are equal, $B^2-4AC=0$

$$\begin{aligned} \left[2(k+1)\right]^2-4(3k+1)(1) &= 0 \\ 4(k^2+2k+1)-(12k+4) &= 0 \\ 4k^2+8k+4-12k-4 &= 0 \\ 4k^2-4k &= 0 \\ 4k(k-1) &= 0 \\ k &= 0,1. \end{aligned}$$

Substituting $k=0$, in the given equation,

$$\begin{aligned} x^2+2x+1 &= 0 \\ (x+1)^2 &= 0 \\ x &= -1 \end{aligned}$$

Again substituting $k=1$, in the given equation,

$$\begin{aligned} 4x^2+4x+1 &= 0 \\ (2x+1)^2 &= 0 \end{aligned}$$

or, $$x = -\frac{1}{2}$$

Hence, roots are $-1, -\frac{1}{2}$.

120. If $x=-2$ is a root of the equation $3x^2+7x+p=0$, find the value of k so that the roots of the equation $x^2+k(4x+k-1)+p=0$ are equal.

Sol : [Board Term-2 Foreign 2015]

We have $3x^2+7x+p=0$

Since $x=-2$ is the root of above equation, it must satisfy it.

Thus $$\begin{aligned} 3(-2)+7(-2)+p &= 0 \\ p &= 2 \end{aligned}$$

Since roots of the equation $x^2+4kx+k^2-k+2=0$ are equal,

$$\begin{aligned} 16k^2-4(k^2-k+2) &= 0 \\ 16k^2-4k^2+4k-8 &= 0 \\ 12k^2+4k-8 &= 0 \\ 3k^2+k-2 &= 0 \\ (3k-2)(k+1) &= 0 \\ k &= \frac{2}{3}, -1 \end{aligned}$$

Hence, roots $=\frac{2}{3}, -1$

PRACTICE

If $x=-4$ is a root of the equation $x^2+2x+4p=0$, find the values of k for which the equation $x^2+px(1+3k)+7(3+2k)=0$ has equal roots.

[Board Term-2 Foreign 2015]

Ans : 2 and $-\frac{10}{9}$

121. Find the value of p for which the quadratic equation $(p+1)x^2-6(p+1)x+3(p+9)=0$, $p\neq -1$ has equal roots. Hence find the roots of the equation.

Sol : [Board Term-2 2015]

We have $(p+1)x^2-6(p+1)x+3(p+9)=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=p+1, b=-6(p+1), c=3(p+9)$$

For real and equal roots, $b^2-4ac=0$

$$\begin{aligned} 36(p+1)^2-4(p+1)\times 3(p+9) &= 0 \\ 3(p^2+2p+1)-(p+1)(p+9) &= 0 \\ 3p^2+6p+3-(p^2+9p+p+9) &= 0 \\ 2p^2-4p-6 &= 0 \\ p^2-2p-3 &= 0 \\ p^2-3p+p-3 &= 0 \\ p(p-3)+1(p-3) &= 0 \\ (p-3)(p+1) &= 0 \\ p &= -1,3 \end{aligned}$$

Neglecting $p\neq -1$ we get $p=3$

Now the equation becomes

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PRACTICE

✍ Two water taps together can fill a tank in 9 hours 36 minutes. The tap of larger diameter takes 8 hours less than the smaller one to fill the tank. Find the time in which each tap can separately fill the tank.

[Board Term-2 Foreign 2016]

Ans : 24 hours and 16 hours

149. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank?

Sol : [Board Term-2 OD 2017]

Tow tap running together fill the tank in $3\frac{1}{13}$ hr.

$$= \frac{40}{13} \text{ hours}$$

Thus it will fill in 1 hour $= \frac{13}{40}$ tank.

If first tap alone fills the tank in x hrs, then second tap alone fills it in $(x+3)$ hr.

Now,
$$\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$$
$$\frac{x+3+x}{x(x+3)} = \frac{13}{40}$$
$$80x + 120 = 13x^2 + 39x$$

or,
$$13x^2 - 41x - 120 = 0$$
$$13x^2 - (65-24)x + 120 = 0$$
$$(x-5)(13x+24) = 0$$
$$x = 5,\ x = -\frac{24}{13}$$

Here $x = -\frac{24}{13}$ is not possible. Hence, 1st tap takes 5 hours and 2nd tap takes $= 5+3 = 8$ hours

PRACTICE

✍ Two taps running together can fill a cistern in $2\frac{8}{11}$ minutes. If one tap takes 1 minute more than the other to fill the cistern, find the time in which each tap separately can fill the cistern.

[Board Term-2 OD 2017]

Ans : 5 minute and 6 minute

✍ Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank separately.

[Board Term-2 OD 2016]

Ans : 20 minute and 25 minute

150. Ram takes 6 days less than Bhagat to finish a place of work. If both of them together can finish the work in 4 days, in how many days Bhagat alone can finish the work ?

Sol : [Board Term-2 Delhi Compt. 2017, OD 2017]

Ram and Bhagat together do the work in 4 days

Ram and Bhagat will do in one days $= \frac{1}{4}$ work

Let Bhagat alone does the same work in x days.

Ram will take $= (x-6)$ days

Now
$$\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$$
$$\frac{x-6+x}{x(x-6)} = \frac{1}{4}$$
$$8x - 24 = x^2 - 6x$$
$$x^2 - 14x + 24 = 0$$
$$x^2 - (12+2)x + 24 = 0$$
$$x^2 - 12x - 2x + 24 = 0$$
$$x(x-12) - 2(x-12) = 0$$
$$(x-12)(x-12) = 0$$
$$x = 12, x = 2$$

If Bhagat complete the work in 2 days, Ram will take $= 2-6 = -4$ days that is impossible. Hence, Bhagat can finish in 12 days.

PRACTICE

✍ A and B working together can do a work in 6 days. If A takes 5 days less than B to finish the work, in how many days B alone can do it alone?

[Board Term-2 OD 2017]

Ans : 15 days

✍ A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.

[Board Term-2 2012]

Ans : 12 days

151. In a flight of 600 km, an aircraft was slowed down due to bad weather. The average speed of the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight.

Sol : [Board 2020 Delhi, OD STD]

Let x be the original speed of flight.

Flight distance $d = 600$ km

Original duration, $t = \frac{600}{x}$ $\left(t = \frac{d}{s}\right)$

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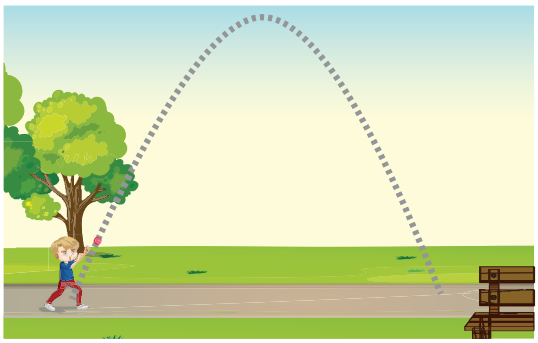
(iii) What is the meaning of two value of t in (ii)?



Sol :

(i) At $t = 2$ sec we have

$$h(2) = -16 \times 2^2 + 176 \times 2$$
$$= 288 \text{ feet}$$

(ii) At ground, $h(t) = 0$, then

$$0 = -16t^2 + 176t$$
$$0 = -16t(t-11)$$
$$t = 0, 11 \text{ sec}$$

173. Fencing a Backyard : Tina and Shriya have just purchased a purebred German Shepherd, and need to fence in their backyard so the dog can run.

(i) What is the maximum rectangular area they can enclose with 200 ft of fencing, if they use fencing material along all four sides? What are the dimensions of the rectangle?

(ii) What is the maximum area if they use the house as one of the sides? What are the dimensions of this rectangle?

Sol :

(i) Let x represent the width, then $100 - x$ represent the length.

Now area, $A(x) = x(100 - x)$

$$= 100x - x^2$$
$$= -x^2 + 100x - 50^2 + 50^2$$
$$= -(x^2 - 100x + 50^2) + 2500$$
$$= -(x-50)^2 + 2500$$

From above equation it is clear that $A(x)$ is maximum at $x = 50$ and this maximum value is 2500.

$$\text{Length} = 100 - x = 100 - 50 = 50$$

Thus dimensions is $50 \text{ ft} \times 50 \text{ ft}$.

(ii) Let x represent the width if they use the house as one of the sides. In this cases length will be $200 - 2x$.

Now are $A(x) = x(200 - 2x)$

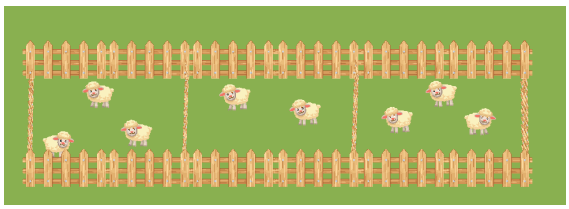
$$= 200x - 2x^2$$
$$= -2(x^2 - 100x)$$
$$= -2(x^2 - 100x + 50^2 - 50^2)$$
$$= -2(x^2 - 100x + 50^2 - 2500)$$
$$= -2(x^2 - 100x + 50^2) + 5000$$
$$= -2(x-50) + 5000$$

From above equation it is clear that $A(x)$ is maximum at $x = 50$ and this maximum value is 5000.

174. Building Sheep Pens : Darsh and Darpan are fencing off a large rectangular area to build some temporary holding pens. To prep the males, females, and kids, they are separated into three smaller and equal-size pens partitioned within the large rectangle.

(i) If 384 ft of fencing is available and the maximum area is desired, what will be the dimensions of the larger, outer rectangle?

(ii) What will be the dimensions of the smaller holding pens?



Sol :

(i) Let x represent the width, then length

$$L = \frac{384 - 4x}{2} = 192 - 2x$$

Now area, $A(x) = LW$

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$$x^2-36x+3x-108 = 0$$
$$x(x-36)+3(x-36) = 0$$
$$(x-36)(x+3) = 0$$
$$x = -3,\ 36$$

Negative value is rejected, thus first speed of train is 36 km/h.

PRACTICE

- A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck.

  [Board Term-2 OD 2015]

  Ans : 60 km/h

- A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete the total journey, what is the original average speed?

  [Board 2018]

  Ans : 42 km/h

155. A train travelling at a uniform speed for 360 km have taken 48 minutes less to travel the same distance if its speed were 5 km/hour more. Find the original speed of the train.

Sol : [Board Term-2 SQP 2017]

Let the original speed of the train be x km/hr.

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x} \text{ hours}$$

Time taken at increased speed$= \frac{360}{x+5}$ hours.

According to the question

$$\frac{360}{x}-\frac{360}{x+5} = \frac{48}{60}$$
$$360\left[\frac{1}{x}-\frac{1}{x+5}\right] = \frac{4}{5}$$
$$\frac{1800}{x^2+5x} = \frac{4}{5}$$
$$x^2+5x-2250 = 0$$
$$x^2+(50-45)x-2250 = 0$$
$$x^2-50x-45x-2250 = 0$$
$$(x+50)(x-45) = 0$$
$$x = -50 \text{ or } x = 45$$

As speed can't be negative, original speed of train is 45 km/hr.

PRACTICE

- A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

  [Board Term-2 2012]

  Ans : 25 km/h

156. A motor boat whose speed is 18 km/hr in still water takes 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol : [Board 2018]

Let x be the speed of stream.

Speed of motor boat in still water = 18 km/hr

The speed of the boat upstream $= (18-x)$ km/hr and speed of the boat downstream $= (18+x)$ km/hr

Time taken to cover 24 km in up stream

$$t_1 = \frac{\text{Distance}}{\text{Speed}} = \frac{24}{18-x}\text{hr}$$

Time taken to cover 24 km in upstream,

$$t_2 = \frac{24}{18+x}\text{hr}$$

As given in the question,

$$t_1-t_2 = 1 \text{ hr}$$
$$\frac{24}{18-x}-\frac{24}{18+x} = 1$$
$$24\left[\frac{1}{18-x}-\frac{1}{18+x}\right] = 1$$
$$24\left[\frac{18+x-18+x}{(18-x)(18+x)}\right] = 1$$
$$\frac{24\times 2x}{324-x^2} = 1$$
$$48x = 324-x^2$$
$$x^2+48x-324 = 0$$
$$x^2+54x-6x-324 = 0$$
$$x(x+54)-6(x+54) = 0$$
$$(x+54)(x-6) = 0 \Rightarrow x = 6,\ -54$$

Thus speed of the stream is 6 km/hr.

PRACTICE

- A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

  [Board Term-2 OD 2014]

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$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{32 \pm \sqrt{1024-4(9)(-16)}}{2(9)}$$

$$= \frac{32 \pm \sqrt{1024+576}}{18}$$

$$= \frac{32 \pm 40}{18}$$

$$= \frac{16 \pm 20}{9}$$

$$x = 4 \text{ and } x = \frac{-4}{9}$$

169. Maximum Profit : An automobile manufacturer can produce up to 300 cars per day. The profit made from the sale of these vehicles can be modelled by the function $P(x) = -x^2 + 350x - 6600$ where $P(x)$ is the profit in thousand Rupees and x is the number of automobiles made and sold. Answer the following questions based on this model:

(i) When no cars are produce what is a profit/ loss?

(ii) What is the break even point ? (Zero profit point is called break even) ?

(iii) What is the profit/loss if 175 cars are produced ?

(iv) What is the profit if 400 cars are produced ?

Sol :

(i) When no cars are produced, $x = 0$

$$P(0) = 0 + 0 - 6600$$

$$= -6600$$

Thus when no cars are produced, there is loss of Rs 6600 Thousand.

(ii) At break-even point $P(x) = 0$, thus

$$0 = -x^2 + 350x - 6600$$

$$x^2 + 350x + 6600 = 0$$

$$x^2 - 330x - 20x + 6600 = 0$$

$$x(x-330) - 20(x+330) = 0$$

$$(x-330)(x-20) = 0$$

$$x = 20, 330$$

Thus break even occur if 20 or 330 cars are produced.

(iii) When 175 cars are produce,

$$P(175) = -(175)^2 + 350(175) - 6600$$

$$= 24025$$

There is profit of Rs 24025 thousand if 175 car are produced.

(iv) When 400 cars are produce,

$$P(400) = -(400)^2 + 350(400) - 6600$$

$$= -26600$$

There is loss of Rs 26600 thousand if 400 cars are produced.

170. Model Rocketry : A model rocket is a small rocket designed to reach low altitudes and be recovered by a variety of means. Flying model rockets is a relatively safe and inexpensive way for person to learn the basics of forces and the response of a vehicle to external forces. Like an airplane, a model rocket is subjected to the forces of weight, thrust, and aerodynamics during its flight.

Shalvi is a member of first rocket club of India named STAR Club. She launches her latest rocket from a large field. At the moment its fuel is exhausted, the rocket has a velocity of 240 ft/sec and an altitude of 544 ft. After t sec, its height $h(t)$ above the ground is given by the function $h(t) = -16t^2 + 240t + 544$.

(i) How high is the rocket 5 sec after the fuel is exhausted?

(ii) How high is the rocket 10 sec after the fuel is exhausted?

(iii) How could the rocket be at the same height at $t = 5$ and at $t = 10$?

(iv) What is the maximum height attained by the

160. The difference between the radii of the smaller circle and the larger circle is 7 cm and the difference between their areas of the two circles is 1078 sq. cm. Find the radius of the smaller circle.

Sol : [Board Term-2 OD Compt. 2017]

We have $r_2 - r_1 = 7$ cm, $r_2 > r_1$ (1)

and $\pi(r_2^2 - r_1^2) = 1078$ cm^2

$\pi(r_2 - r_1)(r_2 + r_1) = 1078$

$\frac{22}{7} \times 7(r_2 + r_1) = 1078$

$r_2 + r_1 = \frac{1078}{22} = 49$ (2)

Adding (1) and (2) we get

$2r_2 = 56$

$r_2 = 28$ cm

and $r_1 = 49 - 28 = 21$

Hence radii of two circles are 28 cm and 21 cm.

161. The perimeter of a rectangular field is 82 m and its area is 400 square metre. Find the length and breadth of the rectangle.

Sol : [Board Term-2 2012]

We have Perimeter $= 2(l + b) = 82$ m

or, $l + b = 41$ m

Let length be x m, then breadth $= (41 - x)$ m.

Area $= l \times b = 400$ m^2

$x(41 - x) = 400$

$41x - x^2 = 400$

$x^2 - 41x + 400 = 0$

$(x - 16)(x - 25) = 0$

$x = 16$ or $x = 25$

If length is 16 m, then breadth will be 25 m.

If length is 25 m, then breadth will be 16 m.

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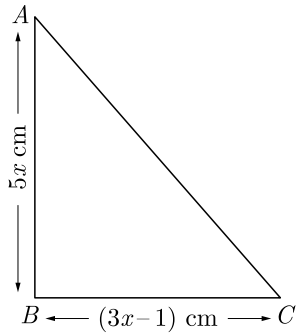
162. The length of the sides forming right angle of a right triangle are $5x$ cm and $(3x - 1)$ cm. If the area of the triangle is 60 cm^2, find its hypotenuse.

Sol : [Board Term-2 2012]

According to the question we have drawn figure below.



Now Area of triangle $= \frac{1}{2} \times$ base $\times$ height

$60 = \frac{1}{2} \times 5x \times (3x - 1)$

$15x^2 - 5x = 120$

$3x^2 - x - 24 = 0$

$3x^2 - 9x + 8x - 24 = 0$

$3x(x - 3) + 8(x - 3) = 0$

$(x - 3)(3x + 8) = 0$

$x = 3,\ x = -\frac{8}{3}$

Length can't be negative, so $x = 3$

Now $AB = 5 \times 3 = 15$ cm, $BC = 3x - 1 = 9 - 1 = 8$ cm

Now $AC = \sqrt{15^2 + 8^2}$

$= \sqrt{225 + 64}$

$= \sqrt{289} = 17$ cm.

Hence hypotenuse is 17 cm.

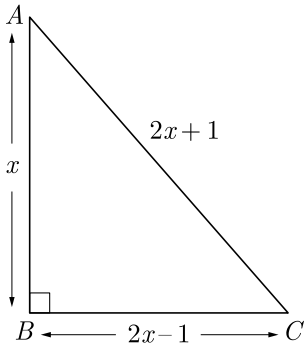
163. The length of the hypotenuse of a right triangle exceeds the length of its base by 2 cm and exceeds twice the length of altitude by 1 cm. Find the length of each side of the triangle.

Sol : [Board Term-2 2012]

Let altitude of triangle be x, then according to question we have

Hypotenuse of triangle $= 2x + 1$

and base of triangle $= 2x - 1$



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Duration with reduced speed $x-200$,

$$t_1 = \frac{600}{x-200}$$

Flight time increased by 30 minute i.e. $\frac{1}{2}$ hour with reduced speed, thus

$$t_1 - t = \frac{1}{2}$$

$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2} \qquad \left(t = \tfrac{d}{s}\right)$$

$$600\left[\frac{1}{x-200} - \frac{1}{x}\right] = \frac{1}{2}$$

$$\frac{x-x+200}{x(x-200)} = \frac{1}{2\times 600}$$

$$\frac{200}{x^2-200x} = \frac{1}{1200}$$

$$x^2-200x = 240000$$

$$x^2-200x-240000 = 0$$

$$x^2-600x+400x-240000 = 0$$

$$x(x-600)+400(x-600) = 0$$

$$(x-600)(x+400) = 0 \Rightarrow x = 400,\ 600$$

Since, speed cannot be negative, therefore original speed is 600 km/hr.

Original distance $d = 600\,\text{km}$

Time $t = \frac{600\,\text{km}}{600\,\text{km/hr}} = 1\ \text{hr}$

Hence, the original duration of flight is 1 hr.

152. A plane left 30 minutes late than its scheduled time and in order to reach destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

Sol : [Board 2018]

Let x be the usual speed of plane.

Time taken to cover 1500 km with speed x,

$$t_1 = \frac{\text{Distance}}{\text{Speed}} = \frac{1500}{x}\text{hr}$$

Time taken to cover 1500 km with increased speed,

$$t_2 = \frac{1500}{x+100}\text{hr}$$

Now we have,

$$t_1 - t_2 = 30\ \text{Minutes} = \frac{1}{2}\text{hr}$$

Substituting values of of t_1 and t_2 in above we obtain

$$\frac{1500}{x} - \frac{1500}{x+100} = \frac{1}{2}$$

$$\frac{1}{x} - \frac{1}{x+100} = \frac{1}{2\times 1500} = \frac{1}{3000}$$

$$\frac{x+100-x}{x(x+100)} = \frac{1}{3000}$$

$$\frac{100}{x^2+100x} = \frac{1}{3000}$$

$$x^2+100x = 300000$$

$$x^2+100x-300000 = 0$$

$$x^2+600x-500x-300000 = 0$$

$$x(x+600)-500(x+600) = 0$$

$$(x+600)(x-500) = 0$$

Thus $x = -600,\ 500$

Speed can not be negative. So we consider only positive value $x = 500$ km/hr

153. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in the return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction.

Sol : [Board Term-2 Delhi 2016]

Let the speed while going be x km/h.

Speed while returning $= (x+10)$ km/h.

According to question we have

$$\frac{150}{x} - \frac{150}{x+10} = \frac{5}{2}$$

$$\frac{150(x+10)-150x}{x(x+10)} = \frac{5}{2}$$

$$\frac{1500}{x(x+10)} = \frac{5}{2}$$

$$x^2+10x-600 = 0$$

$$(x+30)(x-20) = 0$$

Rejecting negative value we get $x = 20$

Speed while going is 20 km/h and speed while returning will be $= 20+10 = 30$ km/h

154. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed ?

Sol : [Board Term-2 OD 2015]

Let the speed of the train be x km/hr for first 54 km. and for next 63 km, speed be $(x+6)$ km/hr.

According to the question we have

$$\frac{54}{x} + \frac{63}{x+6} = 3$$

$$\frac{54(x+6)+63x}{x(x+6)} = 3$$

$$54x+324+63x = 3x(x+6)$$

$$117x+324 = 3x^2+18x$$

$$3x^2-99x-324 = 0$$

$$x^2-33x-108 = 0$$

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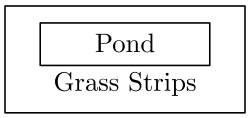
$$x^2+17x-6x-102 = 0$$
$$x(x+17)-6(x+17) = 0$$
$$(x-6)(x+17) = 0$$
$$x = 6 \text{ or } x = -17$$

As x can't be negative, $x = 6$.

167. In a rectangular part of dimension $50\text{ m} \times 40\text{ m}$ a rectangular pond is constructed so that the area of grass strip of uniform breadth surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.



Sol : [Board Term-2 Foreign 2017]

Let width of grass strip be x.

Length of pond $= (50-2x)$ m.

and breadth of pond $= (40-2x)$ m.

and area of park – area of pond = area of grass strip

$$(50 \times 40)-(50-2x)(40-2x) = 1184$$
$$2000-2000+180x-4x^2 = 1184$$
$$x^2-45x+296 = 0$$
$$x^2-37x-8x+296 = 0$$
$$x(x-37)-8(x-37) = 0$$
$$x = 8, 37$$

Here 37 is not possible, thus $x = 8$

$$\text{Length of pond} = 50-2\times 8$$
$$= 34 \text{ m}$$
$$\text{and breadth of pond} = 40-2\times 8$$
$$= 24 \text{ m}.$$

COMPETENCEY BASED QUESTIONS

168. A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

(i) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information.

(ii) Write the corresponding quadratic equation in standard form.

(iii) Find the value of x, the length of side of a tile by factorisation.

(iv) Solve the quadratic equation for x, using quadratic formula.

Sol : [Board 2024 OD Standard]

(i) Quadratic Equation

Area of rectangular floor

= Number of tiles × area of each tile

Total area of rectangular floor $= 200x^2$

$$200x^2 = 128(x+1)^2$$

(ii) Quadratic equation in standard form

$$200x^2 = 128(x+1)^2$$
$$200x^2 = 128(x^2+1+2x)$$
$$72x^2-256x-128 = 0$$
$$9x^2-32x-16 = 0$$

(iii) Value of x,

$$9x^2-32x-16 = 0$$
$$9x^2-36x+4x-16 = 0$$
$$9x(x-4)+4(x-4) = 0$$
$$x = 4,\ x = = \frac{-4}{9}$$

Since $x = -\frac{4}{9}$ is not possible, length of side of tile is 4 units.

(iv) Solution of quadratic equation

By using quadratic formula :

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Ans : 6 km/h

A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

[Board Term-2 OD 2016]

Ans : 8 km/h

157. The total cost of a certain length of a piece of cloth is ₹200. If the piece was 5 m longer and each metre of cloth costs ₹2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre?

Sol : [Board 2019 OD]

Let x be the length of the cloth and y be the cost of cloth per meter.

Now $x \times y = 200$

$$y = \frac{200}{x} \quad ...(1)$$

According to given conditions,

1. If the piece were 5 m longer
2. Each meter of cloth costed ₹ 2 less

i.e., $(x+5)(y-2) = 200$

$$xy - 2x + 5y - 10 = 200$$

$$xy - 2x + 5y = 210$$

$$x\left(\frac{200}{x}\right) - 2x + 5\left(\frac{200}{x}\right) = 210$$

$$200 - 2x + \frac{1000}{x} = 210$$

$$\frac{1000}{x} - 2x = 10$$

$$1000 - 2x^2 = 10x$$

$$x^2 + 25x - 20x - 500 = 0$$

$$x(x+25) - 20(x+25) = 0$$

$$(x+25)(x-20) = 0$$

$$x = -25, 20$$

Neglecting $x = -25$ we get $x = 20$.

Now from equation (1), we have

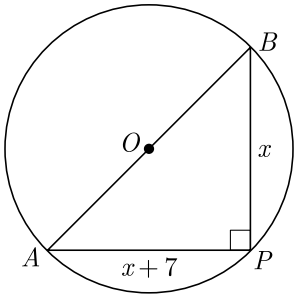
$$y = \frac{200}{x} = \frac{200}{20} = 10$$

Hence, length of the piece of cloths is 20 m and rate per meter is ₹ 10.

158. A pole has to erected at a point on the boundary of a circular park of diameter 17 m in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 meters. Find the distances from the two gates where the pole is to be erected.

Sol : [Board Term-2 Foreign 2016]

As per question the figure is shown below.



Let p be the location of the pole such that its distance from gate B is x.

Thus $AP = x + 7$

Here $AB = 17$ m is diameter and $\angle APB = 90^\circ$

$$x^2 + (x+7)^2 = (17)^2$$

$$x^2 + x^2 + 14x + 49 = 289$$

$$x^2 + x^2 + 14x - 240 = 0$$

$$x^2 + 7x - 120 = 0$$

$$x = \frac{-7 \pm \sqrt{49 + 480}}{2}$$

$$= \frac{-7 \pm 23}{2} = 8, -15$$

Thus $x = 8$ m and $x + 7 = 15$ m.

Hence distance between two gates are 8 m and 15 m.

159. The product of Tanay's age (in years) five years ago and his age ten years later is 16. Determine Tanay's present age.

Sol : [Board Term-2 2012]

Let the present age of Tanay's be x years.

According to question we have

$$(x-5)(x+10) = 16$$

$$x^2 + 5x - 50 = 16$$

$$x^2 + 5x - 66 = 0$$

$$x^2 + 11x - 6x - 66 = 0$$

$$x(x+11) - 6(x+11) = 0$$

$$(x+11)(x-6) = 0 \Rightarrow$$

$$x = -11, 6.$$

As age cannot be negative, we reject $x = -11$. Thus present age of Tanay is 6 years.

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rocket?

(v) How many seconds was the rocket airborne after its fuel was exhausted?

Sol :

We have $h(t) = -16t^2 + 240t + 544$

(i) 5 sec after the fuel is exhausted,

$$h(5) = -16(5)^2 + 240(5) + 544$$
$$= 1344 \text{ feet}$$

(ii) 10 sec after the fuel is exhausted,

$$h(10) = -16(10)^2 + 240(10) + 544$$
$$= 1344 \text{ feet}$$

(iii) The particle is at initial position at $t = 0$. After 11 second it is again at ground because it is coming back down.

(iv) $$h(t) = -16t^2 + 240t + 544$$
$$= -16(t^2 - 15t) + 544$$
$$= -16(t^2 - 15t + 7.5^2 - 7.5^2) + 544$$
$$= -16(t - 7.5)^2 + 16 \times 7.5^2 + 544$$
$$= -16(t - 7.5)^2 + 900 + 544$$
$$= -16(t - 7.5)^2 + 1444$$

From above equation it is clear that $h(t)$ is maximum at $t = 7.5$ second and this maximum value is 1444 feet.

(v) For airbonre time, h will be zero.

$$0 = -16t^2 + 240t + 544$$
$$0 = t^2 - 15t - 34$$
$$0 = (t - 17)(t + 2)$$
$$t = 17 \text{ sec}$$

171. Maximum Profit : A kitchen utensils manufacturer can produce up to 200 utensils per day. The profit made from the sale of these utensils can be modelled by the function $P(x) = -0.5x + 175x - 330$, where $P(x)$ is the profit in Rupees, and x is the number of utensils made and sold. Based on this model,

(i) Find the y-intercept and explain what it means in this context.

(ii) Find the x-intercepts and explain what they mean in this context.

(iii) How many utensils should be sold to maximize profit?

(iv) What is the maximum profit?

Sol :

We have $P(x) = -0.5x^2 + 175x - 3300$

(i) If no utensils are sold, $x = 0$

$$P(0) = -0.5x^2 + 175x - 3300$$
$$= -3300$$

If no utensils are sold, the loss will be Rs 3300 . Here y-intercept $(0, -3300)$ is loss because at $x = 0$ $P(0)$ is plotted on y axis.

(ii) If there is no profit, i.e. $P(x) = 0$

$$-0.5x^2 + 175x - 3300 = 0$$
$$-0.5(x^2 - 350x + 6600) = 0$$
$$-0.5(x - 20)(x - 330) = 0$$
$$x = 20, 330; \; x = 330$$

Thus (20, 0) and (330, 0). If less than 20 or more than 330 utensils are made sold, there will be no profit. Thus x intercept is break even point because $P(x) = 0$ it is plotted on x.

(iii) For maximum profit,

$$P(x) = -0.5x^2 + 175x - 3300$$
$$= -0.5(x^2 - 350x) - 3300$$
$$= -0.5(x^2 - 350x + 175^2 - 175^2) - 3300$$
$$= -0.5[(x - 175)^2 - 30625] - 3300$$
$$= -0.5(x - 175)^2 + 15312.5 - 3300$$

or $$P(x) = -0.5(x - 175)^2 + 12012.5$$

From above equation it is clear that $P(x)$ is maximum at $x = 175$ and this maximum value is 12012.5.

(iv) The maximum profit is Rs 12012.5.

172. Height of a Projectile : A projectile is thrown upward with an initial velocity of 176 ft/sec. After t sec, its height $h(t)$ above the ground is given by the function $h(t) = -16t^2 + 176t$.

(i) What is the projectile's height above the ground after 2 sec.

(ii) How many seconds after it is thrown will the projectile strike the ground?

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Using Pythagoras theorem,

$$\begin{aligned}(2x+1)^2 &= x^2+(2x-1)^2\\ 4x^2+1+4x &= x^2+4x^2+1-4x\\ x^2-8x &= 0\\ x(x-8) &= 0\end{aligned}$$

Rejecting $x=0$, we get $x=8$.

Thus altitude of triangle is 8 cm

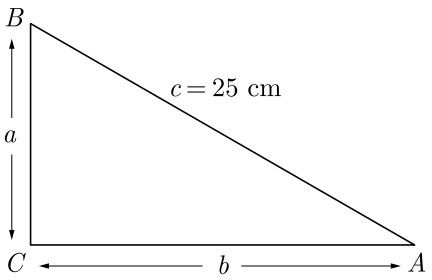
Hypotenuse of triangle is $2\times 8+1=17$ cm

and base of triangle is $2\times 8-1=15$ cm

164. The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle.

Sol : [Board Term-2 Delhi 2016]

As per question statement figure is given below.



Here $a+b+c = 60, c=25$

$$a+b = 60-c = 60-25 = 35$$

Using Pythagoras theorem

$$a^2+b^2 = 25^2 = 625$$

Substituting the values in

$$\begin{aligned}(a+b)^2 &= a^2+b^2+2ab\\ 35^2 &= 625+2ab\\ 1225-625 &= 2ab\end{aligned}$$

or, $ab = 300$

Hence, Area of ΔABC, $\frac{1}{2}ab = 150\,\text{cm}^2$.

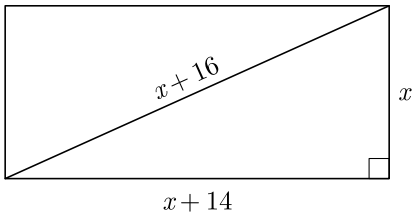
165. The diagonal of a rectangular field is 16 metre more than the shorter side. If the longer side is 14 metre more than shorter side, then find the length of the of the field.

Sol : [Board Term-2 OD 2015]

Let the length of shorter side be x.

Length of diagonal $=(x+16)$ m

and, Length of longer side $=(x+14)$ m



Now as per question we have

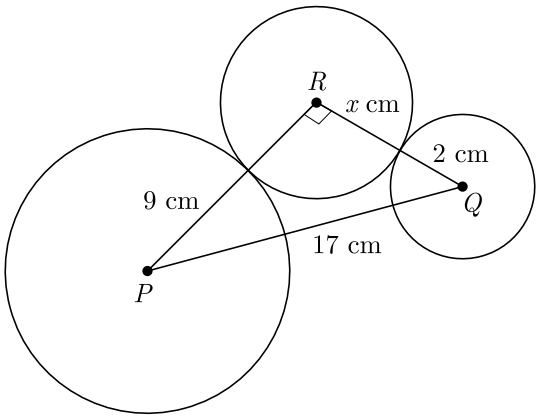
$$\begin{aligned}x^2+(x+14)^2 &= (x+16)^2\\ x^2+x^2+28x+196 &= x^2+32x+256\\ x^2-4x-60 &= 0\\ x^2+6x-10x-60 &= 0\\ x(x+6)-10(x+6) &= 0\\ x &= -6, x=10\end{aligned}$$

As length can't be negative $x = 10$ m. Therefore length of sides are 10 m and 24 m.

166. P and Q are centres of circles of radii 9 cm and 2 cm respectively. $PQ=17$ cm. R is the centre of the circle of radius x cm which touch given circles externally. Given that angle PRQ is 90°. Write an equation in x and solve it.

Sol : [Board Term-2 SQP 2016]

As per question statement figure is given below.



In right ΔPQR,

$$\begin{aligned}PQ^2 &= PR^2+RQ^2\\ 17^2 &= (x+9)^2+(x+2)^2\\ 289 &= x^2+18x+81+x^2+4x+4\\ 289 &= 2x^2+22x+85\\ 0 &= 2x^2+22x+85-289\\ 0 &= 2x^2+22x-204\\ x^2+11x-102 &= 0\end{aligned}$$

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$$= x(192-2x)$$
$$= 192x-2x^2$$
$$= -2(x^2-96x)$$
$$= -2(x^2-96x+48^2-48^2)$$
$$= -2(x^2-96x+48^2)+2\times 48^2$$
$$= -2(x-48)^2+4608$$

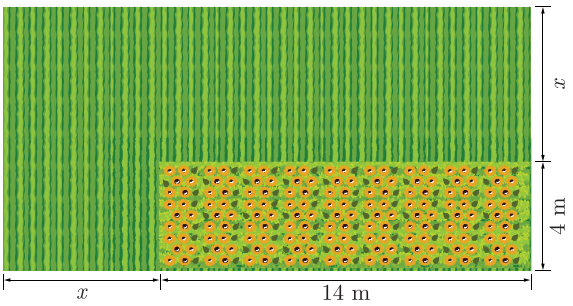
From above equation it is clear that $A(x)$ is maximum at $x=48$ and this maximum value is 4608.

$$\text{Length} = (192-2x)$$
$$= (192-2\times 48) = 96$$

(i) The dimension of larger outer rectangle is 48 ft by 96 ft.

(ii)The one dimension of smaller holding pan is 48 feet. Other dimension is $\frac{96}{3}=32$ feet.

175. Changing Area : Advik's flower bed is rectangular in shape with a length of 14 meter and a width of 4 meter. He wants to increase the length and width by the same amount to obtain a flower bed with an area of 200 square meter. What should the amount of increase be?



Sol :

Let x be the amount of increase. The length and width of the new flower bed are $x+14$ meter and $x+4$ meter as shown in figure. Because the area is to be 200 m^2, we have

$$(x+14)(x+4) = 200$$
$$x^2+18x+56 = 200$$
$$x^2+18x-144 = 0$$
$$x = \frac{-18\pm\sqrt{18^2-4(1)(-144)}}{2(1)}$$
$$= \frac{-18\pm\sqrt{324+576}}{2}$$
$$= \frac{-18\pm\sqrt{900}}{2} = \frac{-18\pm 30}{2}$$
$$= -9\pm 15 = 6,-24$$

Because the value of x must be positive, the exact increase is 6 feet.

176. Cycling : It is a healthy, low-impact exercise that can be enjoyed by people of all ages, from young children to older adults. It is also fun, cheap and good for the environment. Cycling increases stamina, strength and aerobic fitness. Unlike some other sports, cycling does not require high levels of physical skill. Most people know how to ride a bike and, once you learn, you don't forget.

Sanya was traveling across the desert on her bicycle. Before lunch she travelled 60 km; after lunch she travelled 48 km. She put in 1 hour more after lunch than before lunch, but her speed was 4 kmph slower than before. What was her speed before lunch and after lunch?

Sol :

Let x represent her speed before lunch and $x-4$ her speed after lunch. Since $T=\frac{D}{R}$, her time before lunch was $\frac{60}{x}$ and her time after lunch was $\frac{48}{x-4}$.

Since he put in one hour more after lunch, we can write the following equation.

$$\frac{48}{x-4}-1 = \frac{60}{x}$$
$$\frac{48-x+4}{x-4} = \frac{60}{x}$$
$$\frac{52-x}{x-4} = \frac{60}{x}$$
$$52x-x^2 = 60x-240$$
$$-x^2-8x+240 = 0$$
$$x^2+8x-240 = 0$$
$$x = \frac{-8\pm\sqrt{64-4(1)(-240)}}{2(1)}$$
$$= \frac{-8\pm\sqrt{1024}}{2} = \frac{-8\pm 32}{2}$$
$$= -4\pm 16 = 12,-20$$

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We disregard negative number, thus his speed before lunch was 12 kmh and after lunch was 4 kmph slower i.e. 8 kmh.

177. Extreme Hardship : Chaitanya starts to walk 4 km to school at 7:30 AM. with a temperature of 0°F. Her brother Ekansh starts at 8:15 AM. on his bicycle, traveling 12 kmph faster than Chaitanya. If they get to school at the same time, then how fast is each one traveling?

Sol :

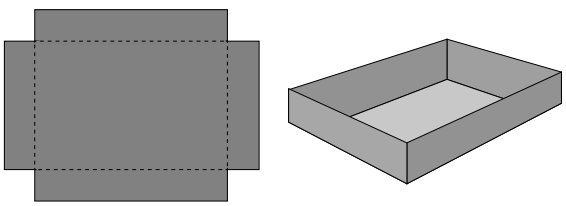
Let x represent Chaitanya's speed and $x+12$ represent Ekansh's speed. Chaitanya's time is $\frac{4}{x}$ hours and Ekansh's time is $\frac{4}{x+12}$ hours.

Since Chaitanya's time is $\frac{3}{4}$ hour longer than Ekansh's, we can write the following equation.

$$\begin{aligned}
\frac{4}{x} &= \frac{4}{x+12}+\frac{3}{4} \\
\frac{4}{x} &= \frac{16+3x+36}{4(x+12)} \\
\frac{4}{x} &= \frac{52+3x}{4(x+12)} \\
16x+192 &= 52x+3x^2 \\
-3x^2-36x+192 &= 0 \\
x^2+12x-64 &= 0 \\
x^2+16x-4x-64 &= 0 \\
(x+16)(x-4) &= 0 \;\Rightarrow\; x=4,-16
\end{aligned}$$

We disregard negative number, thus Chaitanya's speed was 4 kmh and Ekansh's speed was 16 kmh.

178. Open-top Box : Jihan is going to make an open-top box by cutting equal squares from the four corners of an 20 cm by 30 cm sheet of cardboard and folding up the sides. If the area of the base of box is to be 336 square cm, then what size square should be cut from each corner?



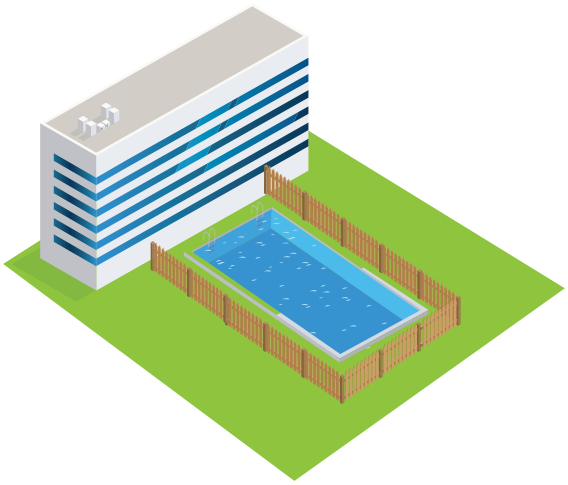
Sol :

Let x represent the length of the side of the square. After squares are cut from each of the corners, the dimensions of the base will be $30-2x$ and $20-2x$. Since the base is 336 square cm, we can write the following equation.

$$\begin{aligned}
(30-2x)(20-2x) &= 336 \\
600-60x-40x+4x^2 &= 336 \\
4x^2-100x+264 &= 0 \\
x^2-25x+66 &= 0 \\
x^2-22x-3x+66 &= 0 \\
(x-3)(x-22) &= 0 \;\Rightarrow\; x=3,22
\end{aligned}$$

Since $x=22$ is too large, the value of x is 3 cm.

179. Swimming Pool Design : Nihal has designed a motel pool within a rectangular area that is fenced on three sides as shown in the figure. If he uses 60 metres of fencing to enclose an area of 352 square metres, then what are the dimensions marked L and W in the figure? Assume L is greater than W.



Sol :

Here $2L+W=60$, $W=60-2L$.

Since the area is 352 square metres,

$$\begin{aligned}
LW &= 352 \\
L(60-2L) &= 352 \\
-2L^2+60L-352 &= 0 \\
L^2-30L+176 &= 0 \\
(L-22)(L-8) &= 0 \;\Rightarrow\; L=22 \text{ or } L=8 \\
W &= 60-2L = 16 \\
\text{or} \qquad W &= 60-2L = 44
\end{aligned}$$

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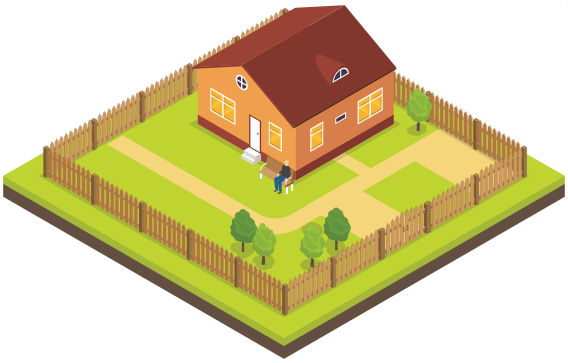
Since L is greater than W, $L = 22$ metre and $W = 16$ metre

180. Reyansh used 70 meters of fencing to enclose a rectangular region. To be sure that the region was a rectangle, he measured the diagonals and found that they were 25 meters each. (If the opposite sides of a quadrilateral are equal and the diagonals are equal, then the quadrilateral is a rectangle.) What are the length and width of the rectangle?



Sol :

The perimeter of a rectangle is twice the length plus twice the width, $P = 2L + 2W$. Because the perimeter is 70 meters, the sum of one length and one width is 35 meters i.e. $W + L = 35$. If we let x represent the width, then $35 - x$ is the length. Now we use the Pythagorean theorem to get a relationship among the length, width, and diagonal.

$$\begin{aligned} x^2 + (35 - x)^2 &= 25^2 \\ x^2 + 1225 - 70x + x^2 &= 625 \\ 2x^2 - 70x + 600 &= 0 \\ x^2 - 35x + 300 &= 0 \\ (x - 20)(x - 15) &= 0 \Rightarrow x = 20 \text{ or } x = 15 \end{aligned}$$

Now length $35 - 15 = 20$ or $35 - 20 = 15$ m.

Solving the equation gives two possible rectangles: a 20 by 15 rectangle or a 15 by 20 rectangle. However, those are identical rectangles. The rectangle is 15 meters by 20 meters.

181. Supreme Court : The "Conference handshake" has been a tradition since the late 19th century. When the Justices of the Supreme Court assemble to go on the Bench each day, each Justice shakes hands with each of the other Justices for a total of 36 handshakes. The total number of handshakes h possible for n people is given by $h = \frac{1}{2}(n^2 - n)$.

(i) Write an equation which represent above situation

(ii) Solve above equation to determine the number of Justices on the Supreme Court.

Sol :

(i) Since, $h = 32$ we have

$$\begin{aligned} 36 &= \frac{1}{2}(n^2 - n) \\ 72 &= n^2 - n \\ n^2 - n - 72 &= 0 \end{aligned}$$

Thus $n^2 - n - 72 = 0$ represent the given situation.

(ii) $$\begin{aligned} n^2 - 9n + 8n - 72 &= 0 \\ n(n-9) + 8(n-9) &= 0 \\ (n-9)(n+8) &= 0 \Rightarrow n = 9, -8 \end{aligned}$$

There are 9 Justices on the Supreme Court.

182. A tour bus in Jaipur serves 400 customers a day. The charge is Rs 50 per person. The owner of the bus service estimates that the company would lose 10 passengers a day for each Rs 5 fare increase.

(i) How much should the fare be in order to maximize the income for the company?

(ii) What is the maximum income the company can expect to make?

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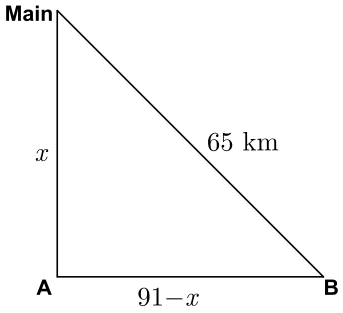
below.



Applying Pythagoras theorem we have

$$\begin{aligned} x^2+(91-x)^2 &= 65^2 \\ x^2+8281+x^2-182x &= 4225 \\ 2x^2-182x+4056 &= 0 \\ x^2-91x+2028 &= 0 \\ x^2-52x-39x+2028 &= 0 \\ x(x-52)-39(x-52) &= 0 \\ (x-52)(x-39) &= 0 \\ x &= 52, 39 \end{aligned}$$

If warehouse A is at 52 km from main warehouse, then, distance between warehouse A and warehouse B is $91-52=39$ km.

If warehouse A is at 39 km from main warehouse, then, distance between warehouse A and warehouse B is $91-39=52$ km.

Since it was farther from the main Delhi warehouse to warehouse A than it was from warehouse A to warehouse B, 52 km is correct answer.

187. Braking Distance : The distance that a car travels between the time the driver makes the decision to hit the brakes and the time the car actually stops is called the braking distance. For a certain car traveling v, the braking distance d is given by $d=v+\frac{1}{20}v^2$.

(i) Find the braking distance when v is 50 km/h.

(ii) If a driver decides to brake 120 metre from a stop sign, how fast can the car be going and still stop by the time it reaches the sign?

Sol :

We have $d = v+\frac{1}{20}v^2$

Substituting $v=50$ we have

$$\begin{aligned} d &= 50+\tfrac{1}{20}\times 50^2 \\ &= 50+125=175 \text{ m} \end{aligned}$$

If a driver decides to brake 120 metre from a stop sign, substituting $v=120$ in $d=v+\frac{1}{20}v^2$ we have

$$\begin{aligned} 120 &= v+\tfrac{1}{20}v^2 \\ 2400 &= 20v+v^2 \\ v^2+20v-2400 &= 0 \\ v^2+60v-40v-2400 &= 0 \\ v(v+60)-40(v-60) &= 0 \\ (v+60)(v-40) &= 0 \\ v &= 40, -60 \end{aligned}$$

(i) Braking distance is 175 m when v is 50 km/h.

(ii) If a driver decides to brake 120 metre from a stop sign, the car be going by 40 km/h and still stop by the time it reaches the sign.

188. Quantity Discount : Peter Paris Co. sells running trousers to dealers for Rs 400 per trouser if less than 50 trousers are ordered. If 50 or more trousers are ordered (up to 400), the price per trouser is reduced at a rate of Rs 0.4 times the number ordered. How many trousers can a dealer purchase for Rs 84000?

Sol :

Let the number of trousers that dealer purchases is x. Now the price of single unit will be $(400-0.4x)$ and total price of x trousers will be $x(400-0.4x)$

Then, according to the question, we have

$$\begin{aligned} x(400-0.4x) &= 84000 \\ 400x-0.4x^2 &= 84000 \\ 0.4x^2-400x+84000 &= 0 \\ x^2-1000x+210000 &= 0 \end{aligned}$$

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For 1999, $n = 0$

Thus $P(0) = 0 - 0 + 570 = 570$

Now $P(n) = 8n^2 - 112n + 570$

$= 8(n^2 - 14n) + 570$

$= 8(n^2 - 14n + 49 - 49) + 570$

$= 8(n^2 - 14n + 49) - 8 \times 49 + 570$

$= 8(n-7)^2 - 392 + 570$

$= 8(n-7)^2 + 178$

From above equation it is clear that $P(n)$ is minimum at $n = 7$ and this minimum value is 178.

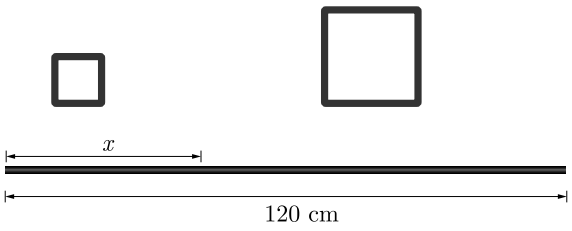
(i) In 1999, there were 570 deer.

(ii) The deer population was minimum in $1999 + 7 = 2006$.

(iii) Minimum population was 178.

(iv) Above equation wont be zero for any value of n. Thus the deer population will never reach zero, according to this model.

185. A piece of wire is 120 cm long. The wire is cut into two pieces and then each piece is bent into a square. Find the length of each piece if the sum of the areas of these squares is to be 500 square cm.



Sol :

We have $\left(\frac{x}{4}\right)^2 + \left(\frac{120-x}{4}\right)^2 = 500$

$\frac{x^2}{16} + \frac{(120-x)^2}{16} = 500$

$x^2 + (120-x)^2 = 8000$

$x^2 + 14400 + x^2 - 240x = 8000$

$2x^2 - 240x + 6400 = 0$

$x^2 - 120x + 3200 = 0$

$x^2 - 80x - 40x + 3200 = 0$

$x(x-80) - 40(x-80) = 0$

$(x-40)(x-80) = 0$

$x = 40, 80$

When $x = 40$ cm, length of other piece $120 - 40 = 80$ cm. When $x = 80$ cm, length of other piece $120 - 80 = 40$ cm. Thus both values of x gives same set of length of wire.

186. TRANSPORTATION : Amazon delivery truck leaves main Delhi Amazon warehouse and travels south to warehouse A. From warehouse A the truck travels west to warehouse B and then returns directly to the main Delhi warehouse (It makes right triangle.).

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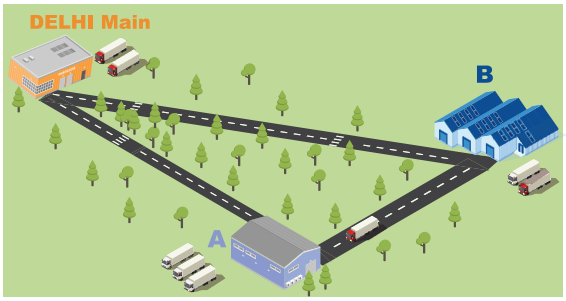

The driver recorded the truck's odometer reading at the warehouse at both the beginning and the end of the trip and also at warehouse B, but forgot to record it at warehouse A (odometer reading given).

The driver does recall that it was farther from the main Delhi warehouse to warehouse A than it was from warehouse A to warehouse B. Because delivery charges are based on distance from the warehouse, the driver needs to know how far warehouse A is from the main Delhi warehouse. Find this distance.

Sol :

Let x be the distance of main warehouse form warehouse A.

Distance travelled from main warehouse to warehouse B via warehouse A is known and can be computed from odometer readings. That is $= 652937 - 652846 = 91$ km.

Now, distance between warehouse A and warehouse B is $(91 - x)$. Again distance between warehouse B and main warehouse is $653002 - 652937 = 65$ km.

Now, we make the digram of situation as given

The expanse of different item was as follows.

S. No.	Article	Cost per student
1	Entry ticket	Rs 5
2	Coffee	Rs 10
3	Food	Rs 25
4	Travelling cost	Rs 50
5	Ice-cream	Rs 15

(i) If x is the number of students planned for picnic, find the quadratic equation that describe the situation.

(ii) What is the number of students planned for picnic ?

(iii) What is the number of students who attended the picnic?

(iv) What is the total expanse for this picnic ?

(v) How much money they spent for travelling ?

Sol :

(i) We have $\frac{2000}{x}+20 = \frac{2000}{x-5}$

$$2000(x-5)+20x(x-5) = 2000x$$
$$-10000+20x^2-100x = 0$$
$$x^2-5x-500 = 0$$

(ii) We have $x^2-5x-500 = 0$

$$x^2-25x+20x-500 = 0$$
$$x(x-25)+20(x-25) = 0$$
$$(x-25)(x+20) = 0$$
$$x = 25, -20$$

(iii) $x = 25-5 = 20$ Students attended picnic

(iv) Expanse per student $= 5+10+25+50+15$
$= 105$

Total expanse, $105 \times 20 = 2100$

(v) Expanse on travelling $50 \times 20 = 1000$

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$$x^2-300x-700x+210000 = 0$$
$$x(x-300)-700(x-300) = 0$$
$$(x-300)(x-700) = 0$$
$$x = 300, 700$$

Since $x < 400$, $x = 300$

So, dealer purchases 300 trousers.

189. Riya has a lawn with a flowerbed and grass land. The grass land is in the shape of rectangle while flowerbed is in the shape of square. The length of the grassland is found to be 3 m more than twice the length of the flowerbed. Total area of the whole lawn is 1260 m^2.

(i) If the length of the flowerbed is x m then what is the total length of the lawn ?

(ii) What will be the perimeter of the whole field?

(iii) What is the value of x if the area of total lawn is 1260 m^2 ?

(iv) What is the area of grassland ?

(v) What is the ratio of area of flowerbed to area of grassland ?

Sol :

(i) The length of the grassland is 3 m more than twice the length of the flowerbed. Thus it will be $2x+3$. Now the total length of field is $2x+3+x = 3x+3$.

(ii) Perimeter $= 2(3x+3+x)$
$= 2(4x+3) = (8x+6)$

(iii) We have $A = (3x+3)x$
$1260 = 3x^2+3x$
$420 = x^2+x$
$x^2+x-420 = 0$
$(x+21)(x-20) = 0$

Thus, $x = 20$ is only possible value.

(iv) Area of grassland, $A_g = (2x+3)x$
$= (2\times 20+3)20$
$= 860\ m^2$

(v) Area of flowerbed,
$A_f = x^2 = 20^2 = 400\ m^2$
Ratio $= \frac{400}{860} = \frac{20}{43}$

190. John and Priya went for a small picnic. After having their lunch Priya insisted to travel in a motor boat. The speed of the motor boat was 20 km/hr. Priya being a Mathematics student wanted to know the speed of the current. So she noted the time for upstream and downstream.

She found that for covering the distance of 15 km the boat took 1 hour more for upstream than downstream.

(i) Let speed of the current be x km/hr. then speed of the motorboat in upstream will be

(ii) What is the relation between speed distance and time?

(iii) Write the correct quadratic equation for the speed of the current ?

(iv) What is the speed of current ?

(v) How much time boat took in downstream ?

Sol :

(i) In this case speed of the motorboat in upstream will be $(20-x)$ km/hr.

(ii) distance = (speed)/time

(iii) As per question,

$$\frac{15}{20-x} = \frac{15}{20+x}+1$$
$$15(20+x) = 15(20-x)+(20-x)(20+x)$$
$$15x = -15x+(20^2-x^2)$$
$$30x = -x^2+400$$
$$x^2+30x-400 = 0$$

(iv) We have $x^2+30x-400 = 0$
$x^2+40x-10x-400 = 0$
$x(x+40)-10x(x+40) = 0$

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Sol :

Let x represent the number of Rs 5 fare increases. Then $50+5x$ is the price per passenger and $400-10x$ is the number of passengers.

The income is the number of passengers multiplied by the price per ticket. Let $I(x)$ represent income as a function of x.

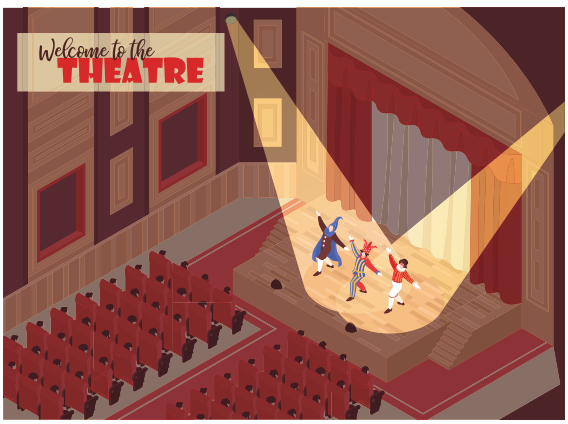
$$\begin{aligned}\text{Now}\quad I(x) &= (400-10x)(50+5x)\\ &= 10(40-x)(5)(10+x)\\ &= 50(40-x)(10+x)\\ &= 50(400+40x-10x-x^2)\\ &= 50(400+30x-x^2)\\ &= -50(x^2-30x-400)\\ &= -50(x^2-30x+15^2-15^2-400)\\ &= -50(x^2-30x+15^2-625)\\ &= -50(x^2-30x+15^2)+50\times 625\\ &= -50(x-15)^2+31250\end{aligned}$$

From above equation it is clear that $I(x)$ is maximum at $x=15$ and this maximum value is 31250. This means the company should make 15 fare increases of Rs 5 to maximize its income. Thus, the ticket price should be $50+5\times 15=125$ Rs.

183. Drama Ticket : Last year, 300 people attended the Ravindra Manch Drama Club's winter play. The ticket price was Rs 70. The advisor estimates that 20 fewer people would attend for each Rs 10 increase in ticket price.

(i) What ticket price would give the most income for the Drama Club?

(ii) If the Drama Club raised its tickets to this price, how much income should it expect to bring in?



Sol :

Let x represent the number of Rs 10 fare increases. Then $70+10x$ is the price per passenger and $300-20x$ is the number of passengers.

The income is the number of passengers multiplied by the price per ticket. Let $I(x)$ represent income as a function of x.

$$\begin{aligned}\text{Now}\quad I(x) &= (300-20x)(70+10x)\\ &= 20(15-x)(10)(7+x)\\ &= 200(105+8x-x^2)\\ &= -200(x^2-8x-105)\\ &= -200(x^2-8x+16-16-105)\\ &= -200(x^2-8x+16-121)\\ &= -200(x^2-8x+16)+24200\\ &= -200(x-4)^2+24200\end{aligned}$$

From above equation it is clear that $I(x)$ is maximum at $x=4$ and this maximum value is 24200. This means the company should make 4 fare increases of Rs 10 to maximize its income. Thus, the ticket price should be $70+4\times 10=110$

184. A biologist predicts that the deer population, $P(n)$, in a certain national park can be modelled by $P(n)=8n^2-112n+570$, where n is the number of years since 1999.

(i) According to this model, how many deer were in the park in 1999?

(ii) In which year was the deer population a minimum?

(iii) How many deer were in the park when their population was a minimum?

(iv) Will the deer population ever reach zero, according to this model?

Sol :

We have $P(n)=8n^2-112n+570$

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(ii) Theatre can charge Rs 20 hundred also. This is also break even point.

(iii) At, $x = 4$, we have

$$P(2) = -(4)^2 + 22 \times 4 - 40 = 32$$

(iv) At, $x = 25$, we have

$$P(5) = -(25)^2 + 22 \times 25 - 40 = -115$$

(v) We have $P(x) = -x^2 + 22x - 40$

Rearranging the profit equation we have

$$P(x) = -(x^2 - 22x + 121 - 81)$$
$$= -(x-11)^2 + 81$$

From above equation it is clear that maximum value of above equation is 81.

193. Auditorium, the part of a public building where an audience sits, as distinct from the stage, the area on which the performance or other object of the audience's attention is presented. In a large theatre an auditorium includes a number of floor levels frequently designed as stalls, private boxes, dress circle, balcony or upper circle, and gallery. A sloping floor allows the seats to be arranged to give a clear view of the stage. The walls and ceiling usually contain concealed light and sound equipment and air extracts or inlets and may be highly decorated.

In an auditorium, seats are arranged in rows and columns. The number of rows are equal to the number of seats in each row. When the number of rows are doubled and the number of seats in each row is reduced by 10, the total number of seats increases by 300.

(i) If x is taken as number of row in original arrangement, write the quadratic equation that describes the situation ?

(ii) How many number of rows are there in the original arrangement?

(iii) How many number of seats are there in the auditorium in original arrangement ?

(iv) How many number of seats are there in the auditorium after re-arrangement.

(v) How many number of columns are there in the auditorium after re-arrangement?

Sol :

(i) Since number of rows are equal to the number of seats in each row in original arrangement, total seats are x^2.

In new arrangement row are $2x$ and seats in each row are $x-10$. Hence total $2x(x-10)$ seats are there.

Total seats are 300 more than previous seats so total number of seats are $x^2 + 300$.

Thus $2x(x-10) = x^2 + 300$

$$2x^2 - 20x = x^2 + 300$$
$$x^2 - 20x - 300 = 0$$

(ii) We have $x^2 - 20x - 300 = 0$

$$x^2 - 30x + 10x - 300 = 0$$
$$x(x-30) + 10(x-30) = 0$$
$$(x-30)(x+10) = 0 \Rightarrow x = 30, -10$$

(iii) Number of seats in original arrangement,

$$x^2 = 30^2 = 900$$

(iv) Total seats in rearrangement $= 30^2 + 300$

$$= 900 + 300 = 1200$$

(v) Number of row are 30 in original arrangement. In rearrangement number of rows are $2 \times 30 = 60$

Number of Column after rearrangement,

$$= \frac{\text{Total seats}}{\text{Row}} = \frac{1200}{60} = 20 \text{ Column}$$

194. Raju and his classmates planned a picnic in zoo. The total budget for picnic was Rs 2000 but 5 students failed to attend the picnic and thus the contribution for each student was increased by Rs 20.

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$$(x+40)(x-10) = 0$$
$$x = 10, -40$$

Here $x = 10$ is only possible.

(v) In downstream speed of boat $= 20 + 10 = 30$ km/hr

Time take to cover distance 15 km will be 30 minutes.

191. Nidhi and Ria are very close friends. Nidhi's parents own a Maruti Alto. Ria's parents own a Toyota Liva. Both the families decide to go for a picnic to Somnath temple in Gujrat by their own cars.

Nidhi's car travels x km/h while Ria's car travels 5 km/h more than Nidhi's car. Nidhi's car took 4 hrs more than Ria's car in covering 400 km.

(i) What will be the distance covered by Ria's car in two hour?

(ii) Write the quadratic equation that describe the speed of Nidhi's car?

(iii) What is the the speed of Nidhi's car?

(iv) How much time did Ria take to travel 400 km?

(iv) How much time did Nidhi take to travel 400 km?

Sol :

(i) Nidhi's car travels x km/h while Ria's car travels 5 km/h more than Nidhi's car. Thus Ria's car speed is $x+5$ km/hour. Distance covered in two hour is $2(x+5)$.

(ii) As per question,

$$\frac{400}{x} = \frac{400}{x+5} + 4$$
$$400(x+5) = 400x + 4x(x+5)$$
$$2000 = 4x^2 + 20x$$
$$500 = x^2 + 5x$$
$$x^2 + 5x - 500 = 0$$

(iii) We have $x^2 + 5x - 500 = 0$

$$x^2 + 25x - 20x - 500 = 0$$
$$x(x+25) - 20(x+25) = 0$$
$$(x+25)(x-20) = 0$$
$$x = 20, -25$$

Since $x = -25$ is not possible, we get $x = 20$

(iv) Rias car speed $= 20 + 5 = 25$ km/hour

$$\text{Time taken} = \frac{400}{25} = 16 \text{ hour}$$

(v) Time Taken by Nidhi $16 + 4 = 20$ Hours

192. Optimal Pricing Strategy : The director of the National School of Drama must decide what to charge for a ticket to the comedy drama. If the price is set too low, the theatre will lose money; and if the price is too high, people won't come. From past experience she estimates that the profit P from sales (in hundreds) can be approximated by $P(x) = -x^2 + 22x - 40$ where x is the cost of a ticket and $0 \le x \le 25$ hundred rupees.

(i) What is the lowest cost of a ticket that would allow the theatre to break even?

(ii) What is the highest cost that the theatre can charge to break even?

(iii) If theatre charge Rs 4 hundred for each ticket, what is the profit/loss ?

(iv) If theatre charge Rs 25 hundred for each ticket, what is the profit/loss ?

(v) What is the maximum profit which can be earned by theatre ?

Sol :

(i) At break even $P(x) = 0$, thus

$$-x^2 + 22x - 40 = 0$$
$$x^2 - 22x + 40 = 0$$
$$(x-2)(x-20) = 0 \Rightarrow x = 2, 20$$

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CHAPTER 5

Arithmetic Progression

OBJECTIVE QUESTIONS

1. In an A.P., if the first term $a = -16$ and the common difference $d = -2$, then the sum of first 10 terms is

(a) -200 (b) -70

(c) -250 (d) 250

Sol : [Board 2024 OD Standard]

We have $a = -16$

and $d = -2$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_{10} = \frac{10}{2}[2(-16) + (10-1)(-2)]$$

$$= 5[-32 + 9(-2)]$$

$$= 5[-32 - 18]$$

$$= 5[-50]$$

$$= -250$$

Thus (c) is correct option.

2. If the sum of the first n terms of an A.P. be $3n^2 + n$ and its common difference is 6, then its first term is

(a) 2 (b) 3

(c) 1 (d) 4

Sol : [Board 2023 OD Standard]

We have $S_n = 3n^2 + n$

and $d = 6$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$3n^2 + n = \frac{n}{2}[2a + 6n - 6]$$

$$6n^2 + 2n = 2an + 6n^2 - 6n$$

$$\frac{8n}{2n} = a$$

$$a = 4$$

Thus, first term is 4.

Thus (d) is correct option.

3. The n^{th} term of the AP $a, 3a, 5a, \ldots$ is

(a) na (b) $(2n-1)a$

(c) $(2n+1)a$ (d) $2na$

Sol : [Board 2020 OD Standard]

Given AP is $a, 3a, 5a, \ldots$

First term is a and $d = 3a - a = 2a$

n^{th} term $a_n = a + (n-1)d$

$$= a + (n-1)2a$$

$$= a + 2na - 2a$$

$$= 2na - a = (2n-1)a$$

Thus (b) is correct option.

4. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ is

(a) 1 (b) $\frac{1}{p}$

(c) -1 (d) $-\frac{1}{p}$

Sol : [Board 2020 OD Standard]

Given AP is $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p} \ldots$

Common difference

$$d = \frac{1-p}{p} - \frac{1}{p}$$

$$= \frac{1-p-1}{p} = \frac{-p}{p} = -1$$

Thus (c) is correct option.

5. The value of x for which $2x, (x+10)$ and $(3x+2)$ are the three consecutive terms of an AP, is

(a) 6 (b) -6

(c) 18 (d) -18

Sol : [Board 2020 Delhi Standard]

Since $2x$, $(x+10)$ and $(3x+2)$ are in AP we obtain,

$$(x+10) - 2x = (3x+2) - (x+10)$$

$$-x + 10 = 2x - 8$$

$$-x - 2x = -8 - 10$$

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$$= 7 - 4n - 4 - 7 + 4n$$
$$= -4$$

Both are correct. Reason is the correct explanation.

Thus (a) is correct option.

41. **Assertion :** Common difference of the AP -5, -1, 3, 7, ... is 4.

Reason : Common difference of the AP $a, a+d, a+2d, ...$ is given by $d = a_2 - a_1$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Common difference, $d = -1-(-5) = 4$

So, both A and R are correct and R explains A.

Thus (c) is correct option.

42. **Assertion :** If sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Then its n^{th} term is $a_n = 6n - 7$.

Reason : n^{th} term of an AP, whose sum to n terms is S_n, is given by $a_n = S_n - S_{n-1}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

nth term of an AP,

$$a_n = S_n - S_{n-1}$$
$$= 3n^2 - 4n - 3(n-1)^2 + 4(n-1)$$
$$= 6n - 7$$

So, both A and R are correct and R explains A.

Thus (a) is correct option.

43. **Assertion :** Sum of first 10 terms of the arithmetic progression -0.5, -1.0, -1.5, ... is 31.

Reason : Sum of n terms of an AP is given as $S_n = \frac{n}{2}[2a + (n-1)d]$ where a is first term and d common difference.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Assertion, $S_{10} = \frac{10}{2}[2(-0.5) + (10-1)(-0.5)]$

$$= 5[-1 - 4.5]$$
$$= 5(-5.5) = 27.5$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

44. **Assertion :** $a_n - a_{n-1}$ is not independent of n then the given sequence is an AP.

Reason : Common difference $d = a_n - a_{n-1}$ is constant or independent of n.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Common difference of an AP $d = a_n - a_{n-1}$ is independent of n or constant.

So, A is correct but R is incorrect.

Thus (d) is correct option.

ONE MARK QUESTIONS

45. What is the n^{th} term of the AP a, $3a$, $5a$, ... ?

Sol : [Board 2020 OD Standard]

Given AP is a, $3a$, $5a$, ...

First term is a and $d = 3a - a = 2a$

n^{th} term $a_n = a + (n-1)d$

$$= a + (n-1)2a$$
$$= a + 2na - 2a$$

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Thus (c) is correct option.

13. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(a) 8 (b) -8

(c) -4 (d) 4

Sol : (a) 8

We have $a_{18} - a_{14} = 32$

In an AP, $a_n = a + (n-1)\,d$

$$a + (18-1)\,d - [a + (14-1)\,d] = 32$$

$$a + 17d - a - 13d = 32$$

$$4d = 32 \Rightarrow d = 8$$

Hence, the required common difference of the given AP is 8.

Thus (a) is correct option.

14. The 4th term from the end of an AP -11, -8, -5, ..., 49 is

(a) 37 (b) 40

(c) 43 (d) 58

Sol : (b) 40

Common difference,

$$d = -8 - (-11)$$

$$= -8 + 11 = 3$$

Last term, $l = 49$

nth term of an AP from the end is

$$a_n = l - (n-1)\,d$$

$$a_4 = 49 - (4-1) \times 3$$

$$= 49 - 9 = 40$$

15. The famous mathematician associated with finding the sum of the first 100 natural numbers is

(a) Pythagoras (b) Newton

(c) Gauss (d) Euclid

Sol :

Gauss is the famous mathematician associated with finding the sum of the first 100 natural numbers, i.e., 1, 2, 3, ..., 100 = 5050.

Thus (c) is correct option.

16. If the first term of an AP is -5 and the common difference is 2, then the sum of the first 6 terms is

(a) 0 (b) 5

(c) 6 (d) 15

Sol : (a) 0

We have $a = -5$ and $d = 2$

$$S_n = \frac{n}{2}\{2a + (n-1)\,d\}$$

$$S_6 = \frac{6}{2}[2a + (6-1)\,d]$$

$$= 3[2(-5) + 5(2)]$$

$$= 3(-10 + 10) = 0$$

Thus (a) is correct option.

17. The sum of first 16 terms of the AP 10, 6, 2, ... is

(a) -320 (b) 320

(c) -352 (d) -400

Sol : (a) -320

Given, AP, is 10, 6, 2 .....

We have $a = 10$ and $d = (6-10) = -4$

$$S_n = \frac{n}{2}\{2a + (n-1)\,d\}$$

$$S_{16} = \frac{16}{2}[2a + (16-1)\,d]$$

$$= 8[2 \times 10 + 15(-4)]$$

$$= 8(20 - 60)$$

$$= 8(-40) = -320$$

Thus (a) is correct option.

18. In an AP, if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is equal to

(a) 19 (b) 21

(c) 38 (d) 42

Sol : (c) 38

We have $a = 1$, $a_n = 20$ and $S_n = 399$

Now, $S_n = \frac{n}{2}(a + a_n)$

$$399 = \frac{n}{2}(1 + 20)$$

$$n = \frac{399 \times 2}{21} = 38.$$

19. The sum of first five multiples of 3 is

(a) 45 (b) 55

(c) 65 (d) 75

Sol : (a) 45

The first five multiples of 3 are 3, 6, 9, 12 and 15.

Here, first term, $a = 3$, $d = 6 - 3 = 3$ and $n = 5$

$$S_n = \frac{n}{2}\{2a + (n-1)\}\,d$$

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$$4(a+3d) = 18(a+17d)$$
$$2a+6d = 9a+153d$$
$$7a = -147d$$
$$a = -21d$$
$$a+21d = 0$$
$$a+(22-1)d = 0 \Rightarrow a_{22} = 0$$

Hence, the 22^{nd} term of the AP is 0.

54. If the first three terms of an AP are b, c and $2b$, then find the ratio of b and c.

Sol : [Board 2020 SQP Standard]

Given, b, c and $2b$ are in AP.

Thus
$$c-b = 2b-c$$
$$2c = 3b$$
$$\frac{2}{3} = \frac{b}{c}$$
$$\frac{b}{c} = \frac{2}{3} \Rightarrow b:c = 2:3$$

55. The n^{th} term of an AP is $(7-4n)$, then what is its common difference?

Sol : [Board 2020 Delhi Basic]

We have $a_n = 7-4n$

Putting $n = 1$, $a_1 = 7-4 = 3$

Putting $n = 2$, $a_2 = 7-8 = -1$

Common difference $d = a_2 - a_1$
$= -1-3 = -4$

56. In an AP, if the common difference $d = -4$, and the seventh term a_7 is 4, then find the first term.

Sol : [Board 2018]

We have $d = -4$

and $a_7 = 4$

Now
$$a_n = a+(n-1)d$$
$$a_7 = a+(7-1)d$$
$$4 = a+(7-1)(-4)$$
$$4 = a-24 \Rightarrow a = 4+24$$
$$= 28$$

First term of the AP is 28.

57. Find the sum of first 8 multiples of 3.

Sol : [Board 2018]

First 8 multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 which are in AP where $a = 3$, $d = 3$ and $n = 8$.

Now
$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_8 = \frac{8}{2}[2\times 3+(8-1)3]$$
$$= 4[6+21]$$
$$S_8 = 4\times 27 = 108$$

Thus, sum of first 8 multiples of 3 is 108.

58. Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both.

Sol : [Board 2020 SQP Standard]

If any number is divisible by 2 and 5, it must be divisible by LCM of 2 and 5, i.e. 10.

Numbers between 102 ..... 998 which are divisible by 2 and 5 are 110, 120, 130, ...990

Here, $a = 110$, $d = 120-110 = 10$ and $a_n = 990$

$$a_n = a+(n-1)d$$
$$990 = 110+(n-1)10$$
$$880 = 10(n-1)$$
$$88 = n-1$$
$$n = 88+1 = 89$$

59. Is -150 a term of the AP 11, 8, 5, 2, ...?

Sol : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

We have $a = 11$, $d = -3$, $a_n = -150$

Now
$$a_n = a+(n-1)d$$
$$-150 = 11+(n-1)(-3)$$
$$-150 = 11-3n+3$$
$$3n = 164$$

or,
$$n = \frac{164}{3} = 54.66$$

Since, 54.66 is not a whole number, -150 is not a term of the given AP.

60. Which of the term of AP 5, 2, -1,..... is -49?

Sol : [Board Term-2 2012]

Let the first term of an AP be a and common difference d.

We have $a = 5$, $d = -3$

Now
$$a_n = a+(n-1)d$$

Substituting all values we have
$$-49 = 5+(n-1)(-3)$$
$$-49 = 5-3n+3$$

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(a) -1 (b) -8

(c) 7 (d) -9

Sol : (c) 7

4th term of first AP,

$$a_4 = -1 + (4-1)\,d = -1 + 3d$$

and 4th term of second AP,

$$a'_4 = -8 + (4-1)\,d = -8 + 3d$$

Now, the difference between their 4th terms,

$$a'_4 - a_4 = (-1 + 3d) - (-8 + 3d)$$
$$= -1 + 3d + 8 - 3d = 7$$

Hence, the required difference is 7.

Thus (c) is correct option.

26. An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then the fourth term is

(a) 2 (b) 3

(c) 5 (d) 6

Sol : (a) 2

We have $S_{11} = 33$

$$\frac{11}{2}[2a + 10d] = 33$$

$$a + 5d = 3$$

i.e. $a_6 = 3 \Rightarrow a_4 = 2$

Since, alternate terms are integers and the given sum is possible, $a_4 = 2$.

Thus (a) is correct option.

27. If the sum of the first $2n$ terms of 2, 5, 8, ... is equal to the sum of the first n terms of 57, 59, 61, ..., then n is equal to

(a) 10 (b) 12

(c) 11 (d) 13

Sol : (c) 11

$$\frac{2n}{2}\{2 \times 2 + (2n-1)3\} = \frac{n}{2}\{2 \times 57 + (n-1)2\}$$

$$2(6n+1) = 112 + 2n$$

$$10n = 110 \Rightarrow n = 11$$

Thus (c) is correct option.

28. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then a is equal to

(a) 6 (b) 7

(c) 20 (d) 28

Sol : (d) 28

In an AP, $a_n = a + (n-1)d$

$$4 = a + (7-1)(-4)$$

$$4 = a + 6(-4)$$

$$4 + 24 = a \Rightarrow a = 28$$

Thus (d) is correct option.

29. The first four terms of an AP whose first term is -2 and the common difference is -2 are

(a) $-2, 0, 2, 4$

(b) $-2, 4, -8, 16$

(c) $-2, -4, -6, -8$

(d) $-2, -4, -8, -16$

Sol : (c) $-2, -4, -6, -8$

Let the first four terms of an AP are $a, a+d, a+2d$ and $a+3d$.

Given, that first term, $a = -2$ and common difference, $d = -2$, then we have an AP as follows

$$-2, \quad -2-2, \quad -2+2(-2), \quad -2+3(-2)$$
$$= -2, -4, -6, -8$$

Thus (c) is correct option.

30. The 21^{th} term of an AP whose first two terms are -3 and 4, is

(a) 17 (b) 137

(c) 143 (d) -143

Sol : (b) 137

Given, first two terms of an AP are

$$a = -3$$

and $a + d = 4$

$$-3 + d = 4 \Rightarrow d = 7$$

For an AP, $a_n = a + (n-1)d$

Thus $a_{21} = a + (21-1)d$

$$= -3 + (20)7$$
$$= -3 + 140 = 137$$

Thus (b) is correct option.

31. The number of two digit numbers which are divisible by 3 is

(a) 33 (b) 31

(c) 30 (d) 29

Sol : (c) 30

Two digit numbers which are divisible by 3 are 12, 15, 18, ..., 99;

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$$S_5 = \frac{5}{2}[2a + (5-1)d]$$
$$= \frac{5}{2}[2 \times 3 + 4 \times 3]$$
$$= \frac{5}{2}(6+12) = \frac{5}{2} \times 18 = 45$$

Thus (a) is correct option.

20. If the sum of the series $2 + 5 + 8 + 11$ ..... is 60100, then the number of terms are

(a) 100 (b) 200

(c) 150 (d) 250

Sol : (b) 200

We have $a = 2$. $d = 5 - 2 = 3$ and $S_n = 60100$

$$\frac{n}{2}[2a + (n-1)d] = S_n$$
$$\frac{n}{2}[4 + (n-1)3] = 60100$$
$$n(3n+1) = 120200$$
$$3n^2 + n - 120200 = 0$$
$$(n-200)(3n+601) = 0$$
$$n = 200, \frac{601}{3}$$

Thus $n = 200$ because n can not be fraction.

Thus (b) is correct option.

21. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(a) 5 (b) 20

(c) 25 (d) 30

Sol : (c) 25

Given, the common difference of AP i.e., $d = 5$

Now $a_n = a + (n-1)d$

Now, $a_{18} - a_{13} = a + (18-1)d - [a + (13-1)d]$
$= a + 17 \times 5 - a - 12 \times 5$
$= 85 - 60 = 25$

Thus (c) is correct option.

22. There are 60 terms is an AP of which the first term is 8 and the last term is 185. The 31st term is

(a) 56 (b) 94

(c) 85 (d) 98

Sol : (d) 98

Let d be the common difference;

Now $a_n = a + (n-1)d$

Then 60th term, $a_{60} = 8 + (60-1)d$

$$185 = 8 + 59d$$
$$59d = 177$$
$$d = 3$$

31th term $a_{31} = 8 + 30 \times 3 = 98$

Thus (d) is correct option.

23. The first and last term of an AP are a and ℓ respectively. If S is the sum of all the terms of the AP and the common difference is $\frac{\ell^2 - a^2}{k - (\ell + a)}$, then k is equal to

(a) S (b) $2S$

(c) $3S$ (d) None of these

Sol : (b) $2S$

We have, $S = \frac{n}{2}(a + \ell)$

$$\frac{2S}{a+\ell} = n \quad \ldots(1)$$

Also, $\ell = a + (n-1)d$

$$d = \frac{\ell - a}{n-1} = \frac{\ell - a}{\frac{2S}{a+\ell} - 1}$$
$$= \frac{\ell^2 - a^2}{2S - (\ell + a)}$$

Thus $k = 2S$

Thus (b) is correct option.

24. If the nth term of an AP is given by $a_n = 5n - 3$, then the sum of first 10 terms if

(a) 225 (b) 245

(c) 255 (d) 270

Sol : (b) 245

We have $a_n = 5n - 3$

Substituting $n = 1$ and 10 we have

$$a = 2$$
$$a_{10} = 47$$

Thus $S_n = \frac{n}{2}(a + a_n)$

$$S_{10} = \frac{10}{2}(2 + 47)$$
$$= 5 \times 49 = 245$$

Thus (b) is correct option.

25. Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4th terms is

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and $a+3x$.

Sum $S_4 = 4a = 20$

Thus $a = 5$

Also, $\frac{(a-3x)(a+3x)}{(a-x)(a+x)} = \frac{2}{3}$

$3(a^2-(3x)^2) = 2(a^2-x^2)$

$3a^2-27x^2 = 2a^2-2x^2$

$a^2 = 25x^2$

$a = 5x \Rightarrow x = \frac{a}{5} = \frac{5}{5} = 1$

Thus $x = 1$. Common difference, $d = 2x = 2$

Thus (d) is correct option.

37. The sum of 11 terms of an AP whose middle term is 30, is

(a) 320 (b) 330

(c) 340 (d) 350

Sol : (b) 330

Middle term is $\frac{11+1}{2} = 6$ th term.

Now $a_n = a+(n-1)d$

$a_6 = a+5d$

$30 = a+5d$

$a = 30-5d$

Now $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{11} = \frac{11}{2}(2a+10d)$

Substituting value of a we have

$S_{11} = \frac{11}{2}[2(30-5d)+10d]$

$= \frac{11}{2}[60-10d+10d]$

$= 11 \times 30$

$S_{11} = 330$

Thus (b) is correct option.

38. Five distinct positive integers are in a arithmetic progression with a positive common difference. If their sum is 10020, then the smallest possible value of the last term is

(a) 2002 (b) 2004

(c) 2006 (d) 2007

Sol : (c) 2006

Let the five integers be $a-2d$, $a-d$, a, $a+d$, $a+2d$.

Then, we have,

$(a-2d)+(a-d)+a+(a+d)+(a+2d) = 10020$

$5a = 10020 \Rightarrow a = 2004$

Now, as smallest possible value of d is 1.

Hence, the smallest possible value of $a+2d$ is 2004+2 = 2006

Thus (c) is correct option.

39. If the 2nd term of an AP is 13 and 5th term is 25, what is its 7th term?

(a) 30 (b) 33

(c) 37 (d) 38

Sol : (b) 33

We have $a_2 = 13$, and $a_5 = 25$

In an AP, $a_n = a+(n-1)d$

$a_2 = a+(2-1)d = 13$

$a+d = 13$ ...(1)

and $a_5 = a+(5-1)d = 25$

$a+4d = 25$ ...(2)

Subtracting equation (1) from equation (2), we get

$3d = 25-13 = 12 \Rightarrow d = 4$

From equation (1), $a = 13-4 = 9$

Now, 7th term, $a_7 = a+(7-1)d$

$= 9+6\times 4 = 33$

Thus (b) is correct option.

40. **Assertion :** If n^{th} term of an AP is $7-4n$, then its common differences is -4.

Reason : Common difference of an AP is given by $d = a_{n+1}-a_n$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Assertion, $a_n = 7-4n$

$d = a_{n+1}-a_n$

$= 7-4(n+1)-(7-4n)$

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Here $a = 12$, $d = 3$ and $a_n = 99$

For an AP, $a_n = a + (n-1)d$

So, $99 = 12 + (n-1) \times 3$

$99 - 12 = 3n - 3$

$99 - 12 + 3 = 3n$

$90 = 3n \Rightarrow n = 30$

Thus (c) is correct option.

32. The list of numbers $-10, -6, -2, 2, ...$ is

(a) an AP with $d = -16$

(b) an AP with $d = 4$

(c) an AP with $d = -4$

(d) not an AP

Sol : (b) an AP with $d = 4$

The given numbers are $-10, -6, -2, 2, ...$

Here, $a_1 = 10$, $a_2 = -6$, $a_3 = -2$ and $a_4 = 2$, ...

Since, $d_1 = a_2 - a_1 = -6 - (-10)$

$= -6 + 10 = 4$

$d_2 = a_3 - a_2 = -2 - (-6)$

$= -2 + 6 = 4$

$d_3 = a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

Since, $d_1 = d_2 = d_3 = ... = 4$

i.e., each successive term of given list has same difference. So, the given list forms an AP with common difference, $d = 4$.

Thus (b) is correct option.

33. If the nth term of an AP is $4n + 1$, then the common difference is

(a) 3 (b) 4

(c) 5 (d) 6

Sol : (b) 4

Given that the n^{th} term of an AP is $4n + 1$.

$a_n = 4n + 1$

Substituting $n = 1, 2, 3, ...$ we have

$a_1 = 4(1) + 1 = 5$

$a_2 = 4(2) + 1 = 9$

Common difference,

$d = a_2 - a_1$

$= 9 - 5 = 4$

Thus (b) is correct option.

34. If a, b, c, d, e, f are in AP, then $e - c$ is equal to

(a) $2(c - a)$ (b) $2(d - c)$

(c) $2(f - d)$ (d) $(d - c)$

Sol : (b) $2(d - c)$

Let x be the common difference of the AP a, b, c, d, e, f.

For an AP, $a_n = a + (n-1)d$

$e = a + (5-1)x$

$e = a + 4x$ ...(1)

and $c = a + (3-1)x$

$c = a + 2x$ ...(2)

Using equation (1) and (2), we get

$e - c = a + 4x - a - 2x$

$= 2x = 2(d - c)$

Thus (b) is correct option.

35. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its term will be

(a) 7 (b) 11

(c) 18 (d) 0

Sol : (d) 0

In an AP, $a_n = a + (n-1)d$

Now, according to the question,

$7a_7 = 11a_{11}$

$7[a + (7-1)d] = 11[a + (11-1)d]$

$7(a + 6d) = 11(a + 10d)$

$7a + 42d = 11a + 110d$

$4a + 68d = 0$

$4(a + 17d) = 0$

$a + 17d = 0$ ...(1)

18th term of an AP,

$a_{18} = a + (18-1)d = a + 17d$

But from equation (1) this is zero.

36. What is the common difference of four terms in AP such that the ratio of the product of the first and fourth term to that of the second and third term is 2 : 3 and the sum of all four terms is 20?

(a) 3 (b) 1

(c) 4 (d) 2

Sol : (d) 2

We take the four terms as $a - 3x$, $a - x$, $a + x$

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$$= 2na - a$$
$$= (2n-1)\,a$$

46. What is the common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$?

Sol : [Board 2020 OD Standard]

Given AP is $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}$ ...

Common difference

$$d = \frac{1-p}{p} - \frac{1}{p}$$
$$= \frac{1-p-1}{p}$$
$$= \frac{-p}{p} = -1$$

47. What is the value of x for which $2x, (x+10)$ and $(3x+2)$ are the three consecutive terms of an AP?

Sol : [Board 2020 Delhi Standard]

Since $2x$, $(x+10)$ and $(3x+2)$ are in AP we obtain,

$$(x+10) - 2x = (3x+2) - (x+10)$$
$$-x+10 = 2x-8$$
$$-x-2x = -8-10$$
$$-3x = -18 \Rightarrow x = 6$$

48. If the first term of AP is p and the common difference is q, then what is its 10th term?

Sol : [Board 2020 Delhi Standard]

We have $a = p$ and $d = q$

$$a_{10} = a + (10-1)\,d$$
$$= p + 9q$$

49. Fill the two blanks in the sequence 2 ....., 26, ... so that the sequence forms an AP.

Sol : [Board 2020 SQP Standard]

Let a and b be the two numbers. AP will be 2, a, 26, b.

Now, $26 - a = a - 2$

$$2a = 28 \Rightarrow a = \frac{28}{2} = 14$$

and $b - 26 = 26 - a$

$$a + b = 52$$
$$14 + b = 52 \Rightarrow b = 38$$

Thus $a = 14$ and $b = 38$.

50. The sum of first 20 terms of the AP 1, 4, 7, 10 .... is

Sol : [Board 2020 Delhi Standard]

Given AP is 1, 4, 7, 10 ...

Here, $a = 1$, $d = 4-1 = 3$ and $n = 20$

$$S_{20} = \frac{n}{2}[2a + (n-1)\,d]$$
$$= \frac{20}{2}[2 \times 1 + (20-1)\,3]$$
$$= 10\,(2+57)$$
$$= 10 \times 59 = 590$$

51. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

Sol : [Board 2020 Delhi Standard]

Given, $(a-b)^2, (a^2+b^2)$ and $(a+b)^2$.

Common difference,

$$d_1 = (a^2+b^2) - (a-b)^2$$
$$= (a^2+b^2) - (a^2+b^2-2ab)$$
$$= a^2+b^2-a^2-b^2+2ab$$
$$= 2ab$$

and $d_2 = (a+b)^2 - (a^2+b^2)$

$$= a^2+b^2+2ab-a^2-b^2 = 2ab$$

Since, $d_1 = d_2$, thus, $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

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52. Find the sum of all 11 terms of an AP whose middle term is 30.

Sol : [Board 2020 OD Standard]

In an AP with 11 terms, the middle term is $\frac{11+1}{2} = 6^{\text{th}}$ term.

Now, $a_6 = a + 5d = 30$

Thus, $S_{11} = \frac{11}{2}[2a + 10d]$

$$= 11\,(a+5d)$$
$$= 11 \times 30 = 330$$

53. If 4 times the 4^{th} term of an AP is equal to 18 times the 18^{th} term, then find the 22^{nd} term.

Sol : [Board 2020 Delhi Basic]

Let a be the first term and d be the common difference of the AP.

Now $a_n = a + (n-1)\,d$

As per the information given in question

$$4 \times a_4 = 18 \times a_{18}$$

$$-3x = -18 \Rightarrow x = 6$$

Thus (a) is correct option.

6. The first term of AP is p and the common difference is q, then its 10th term is

(a) $q+9p$ (b) $p-9q$

(c) $p+9q$ (d) $2p+9q$

Sol : [Board 2020 Delhi Standard]

We have $a = p$ and $d = q$

$$a_{10} = a+(10-1)\,d$$
$$= p+9q$$

Thus (c) is correct option.

7. In an AP, if $d=-4$, $n=7$ and $a_n=4$, then a is equal to

(a) 6 (b) 7

(c) 20 (d) 28

Sol : (d) 28

In an AP, $a_n = a+(n-1)\,d$

$$4 = a+(7-1)(-4)$$
$$4 = a+6(-4)$$
$$4+24 = a \Rightarrow a = 28$$

Thus (d) is correct option.

8. In an AP, if $a=3.5$, $d=0$ and $n=101$, then a_n will be

(a) 0 (b) 3.5

(c) 103.5 (d) 104.5

Sol : (b) 3.5

As, $d = 0$ all the terms are same whatever the value of n. So, $a_n = 3.5$.

Alternate Method :

In an AP, $a_n = a+(n-1)\,d$

$$a_n = 3.5+(101-1)\times 0 = 3.5$$

Thus (b) is correct option.

9. The 11th term of an AP $-5, \frac{-5}{2}, 0, \frac{5}{2}, ...$, is

(a) -20 (b) 20

(c) -30 (d) 30

Sol : (b) 20

Here, $a = -5$, $d = \frac{-5}{2}-(-5) = \frac{5}{2}$

nth term, $a_n = a+(n-1)\,d$

$$a_{11} = -5+(11-1)\times\left(\frac{5}{2}\right)$$
$$a_{11} = -5+25 = 20$$

Thus (b) is correct option.

10. In an AP, if $a=3.5$, $d=0$ and $n=101$, then a_n will be

(a) 0 (b) 3.5

(c) 103.5 (d) 104.5

Sol : (b) 3.5

For an AP, $a_n = a+(n-1)d$

$$= 3.5+(101-1)\times 0$$
$$= 3.5$$

Thus (b) is correct option.

11. Which term of an AP, 21, 42, 63, 84, ... is 210?

(a) 9th (b) 10th

(c) 11th (d) 12th

Sol : (b) 10th

Let nth term of given AP be 210,

First term, $a = 21$

Common difference, $d = 42-21 = 21$

and $a_n = 210$

In an AP, $a_n = a+(n-1)\,d$

$$210 = 21+(n-1)21$$
$$210 = 21+21n-21$$
$$210 = 21n \Rightarrow n = 10$$

Hence, the 10th term of the given AP is 210.
Thus (b) is correct option.

12. If the common difference of an AP is 5, then what is $a_{18}-a_{13}$?

(a) 5 (b) 20

(c) 25 (d) 30

Sol : (c) 25

Given, the common difference of AP i.e., $d=5$

Using, $a_n = a+(n-1)\,d$

We have, $a_{18} = a+(18-1)\,d$

and $a_{13} = a+(13-1)\,d$

Now, $a_{18}-a_{13} = a+(18-1)\,d-[a+(13-1)\,d]$

$$= a+17\times 5-a-12\times 5$$
$$= 85-60 = 25$$

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$$3n = 49+5+3$$
$$n = \frac{57}{3} = 19^{th} \text{ term.}$$

61. Find the first four terms of an AP, whose first term is -2 and common difference is -2.

Sol : [Board Term-2 2012]

We have $a_1 = -2,$

$$a_2 = a_1 + d = -2+(-2) = -4$$
$$a_3 = a_2 + d = -4+(-2) = -6$$
$$a_4 = a_3 + d = -6+(-2) = -8$$

Hence first four terms are $-2, -4, -6, -8$

62. Find the tenth term of the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}$, ...

Sol : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

Given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}$ or $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}$...

where, $a = \sqrt{2}, d = \sqrt{2}, n = 10$

Now $a_n = a+(n-1)d$

$$a_{10} = \sqrt{2}+(10-1)\sqrt{2}$$
$$= \sqrt{2}+9\sqrt{2} = 10\sqrt{2}$$

Therefore tenth term of the given sequence $\sqrt{200}$.

63. Find the next term of the series $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ ....

Sol : [Board Term-2 2012]

Let the first term of an AP be a and common difference d.

Here, $a = \sqrt{2},\ a+d = \sqrt{8} = 2\sqrt{2}$

$$d = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$
$$\text{Next term} = \sqrt{32} + \sqrt{2}$$
$$= 4\sqrt{2} + \sqrt{2}$$
$$= 5\sqrt{2} = \sqrt{50}$$

PRACTICE

✍ What is the next term of an AP $\sqrt{7}, \sqrt{28}, \sqrt{63}, ...?$

[Board Term-2 Foreign 2014]

Ans : $\sqrt{112}$

64. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$ an AP? Give reason.

Sol : [Board Term-2 2015]

Let common difference be d then we have

$$d = a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2}-1)$$
$$d = a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$$
$$d = a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3$$

As common difference are not equal, the given series is not in AP

65. If the common difference of an AP is -6, find $a_{16} - a_{12}$.

Sol : [Board Term-2 2014]

Let the first term of an AP be a and common difference be d.

Now $d = -6$

$$a_{16} = a+(16-1)(-6) = a-90$$
$$a_{12} = a+(12-1)(-6) = a-66$$
$$a_{16} - a_{12} = (a-90)-(a-66)$$
$$= a-90-n+66 = -24$$

66. The first three terms of an AP are $3y-1$, $3y+5$ and $5y+1$ respectively then find y.

Sol : [Board Term-2 Delhi 2015]

If x, y and z are in AP then we have

$$y - x = z - y$$

Therefore if $3y-1, 3y+5$ and $5y+1$ in AP

$$(3y+5)-(3y-1) = (5y+1)-(3y+5)$$
$$3y+5-3y+1 = 5y+1-3y-5$$
$$6 = 2y-4$$
$$2y = 6+4$$
$$y = \frac{10}{2} = 5$$

67. If $3k-2$, $4k-6$ and $k+2$ are three consecutive terms of AP, then find the value of k.

Sol : [Board 2020 OD Basic]

To be term of an AP the difference between two consecutive terms must be the same.

If $3k-2$, $4k-6$ and $k+2$ are terms of an AP, then

$$4k-6-(3k-2) = k+2-(4k-6)$$
$$4k-6-3k+2 = k+2-4k+6$$
$$k-4 = 8-3k$$
$$4k = 12 \Rightarrow k = 3$$

Hence, the value of k is 3.

NODIA

115. If the n^{th} term of a sequence is $3-2n$. Find the sum of fifteen terms.

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a_n = 3-2n$

Taking $n=1$, $a_1 = 3-2=1$

15th term, $a_{15} = 3-2\times 15$

$= 3-30 = -27$

Now $S_n = \frac{n}{2}(a_1+a_n)$

$S_{15} = \frac{15}{2}(a_1+a_{15})$

$= \frac{15}{2}[1+(-27)]$

$= \frac{15}{2}[-26]$

$= 15\times(-13) = -195$

PRACTICE

✍ If the n^{th} term of an AP is $7-3n$, find the sum of twenty five terms.

[Board Term-2 2012]

Ans : -800

116. The n^{th} term of an AP is given by $(-4n+15)$. Find the sum of first 20 terms of this AP.

Sol : [Board Term-2 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a_n = -4n+15$

$a_1 = -4\times 1+15 = 11$

$a_2 = -4\times 2+15 = 7$

$a_3 = -4\times 3+15 = 3$

$d = a_2-a_1 = 7-11 = -4$

Now, we have $a = 11$, $d=-4$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{20} = \frac{20}{2}[2\times 11+(20-1)\times(-4)]$

$= 10[22-76] = 10\times(-54) = -540$

Thus $S_{20} = -540$.

PRACTICE

✍ Find the sum of first 24 terms of an AP whose n^{th} term is given by $a_n = 3+2n$.

[Board Term-2 OD Comptt. 2017]

Ans : 672

117. The sum of first n terms of an AP is $5n-n^2$. Find the n^{th} term of the AP.

Sol : [Board Term-2 Foreign 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have, $S_n = 5n-n^2$

Now, n^{th} term of AP,

$a_n = S_n - S_{n-1}$

$= (5n-n^2)-[5(n-1)-(n-1)^2]$

$= 5n-n^2-[5n-5-(n^2+1-2n)]$

$= 5n-n^2-(5n-5-n^2-1+2n)$

$= 5n-n^2-7n+6+n^2$

$= -2n+6$

$a_n = -2(n-3)$

Thus n^{th} term is $= -2(n-3)$

118. The first and last term of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a=5, a_n = 45$

Now $45 = 5+(n-1)d$

$(n-1)d = 40$ ...(1)

Given, $S_n = 400$

Now $S_n = \frac{n}{2}(a+a_n)$

$400 = \frac{n}{2}(5+45)$

$800 = 50n$

$n = 16$

Substituting this value of n in (1) we have

$(n-1)d = 40$

$15d = 40$

$d = \frac{40}{15} = \frac{8}{3}$

119. If the sum of the first 7 terms of an AP is 49 and that of the first 17 terms is 289, find the sum of its first n terms.

Sol : [Board Term-2 Foreign 2012]

Let the first term be a, common difference be d,

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$$a_{25} - a_{20} = \{a+(25-1)d\} - \{a+(20-1)d\}$$
$$45 = a + 24d - a - 19d$$
$$45 = 5d$$
$$d \quad \frac{45}{5} = 9$$

76. Find the sum of first ten multiple of 5.

Sol : [Board Term-2 Delhi, 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a = 5, n = 10, d = 5$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_{10} = \frac{10}{2}[2\times 5+(10-1)5]$$
$$= 5[10+9\times 5]$$
$$= 5[10+45]$$
$$= 5\times 55 = 275$$

Hence the sum of first ten multiple of 5 is 275.

PRACTICE

✎ Find the sum of first five multiples of 2.

[Board Term-2 2012]

Ans : 30

77. Find the sum of first 16 terms of the AP 10, 6, 2, ...

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a = 10, d = 6-1 = -4, n = 16$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_{16} = \frac{16}{2}[2\times 10+(16-1)(-4)]$$
$$= 8[20+15\times(-4)]$$
$$= 8[20-60] = 8\times(-40) = -320$$

78. What is the sum of five positive integer divisible by 6.

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of $n$the term be S_n

Here, $a = 6, d = 6, n = 5$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_5 = \frac{5}{2}[2\times 6+(5-1)(6)]$$
$$= \frac{5}{2}[12+4\times 6]$$
$$= \frac{5}{2}[12+24] = \frac{5}{2}[36]$$
$$= 5\times 18 = 90$$

79. If the sum of n terms of an AP is $2n^2+5n$, then find the 4^{th} term.

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now, $S_n = 2n^2+5n$

n^{th} term of AP,

$$a_n = S_n - S_{n-1}$$
$$a_n = (2n^2+5n) - [2(n-1)^2+5(n-1)]$$
$$= 2n^2+5n-[2n^2-4n+2+5n-5]$$
$$= 2n^2+5n-2n^2-n+3$$
$$= 4n+3$$

Thus 4^{th} term $a_4 = 4\times 4+3 = 19$

PRACTICE

✎ The sum of first n terms of an AP is given by $S_n = 2n^2+8n$. Find the sixteenth term of the AP.

[Board 2010]

Ans : 65

80. If the sum of first k terms of an AP is $3k^2-k$ and its common difference is 6. What is the first term?

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d , nth term be a_n. Let the sum of k terms of AP is S_k.

We have $S_k = 3k^2-k$

Now k^{th} term of AP,

$$a_k = S_k - S_{k-1}$$
$$a_k = (3k^2-k) - [3(k-1)^2-(k-1)]$$
$$= 3k^2-k-[3k^2-6k+3-k+1]$$
$$= 3k^2-k-3k^2+7k-4 = 6k-4$$

First term $a = 6\times 1-4 = 2$

81. Which term of the AP $8, 14, 20, 26, ...$ will be 72 more than its 41^{st} term.

Sol : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

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Solving equations (1) and (2) we have

$$d = 3$$

105. Find the middle term of the AP 213, 205, 197, ... 37.

Sol : [Board Term-2 Delhi 2015]

Let the first term of an AP be a, common difference be d and number of terms be m.

Here, $a = 213, d = 205 - 213 = -8, a_m = 37$

$$\begin{aligned} a_m &= a + (m-1)d \\ 37 &= 213 + (m-1)(-8) \\ 37 - 213 &= -8(m-1) \\ m - 1 &= \frac{-176}{-8} = 22 \\ m &= 22 + 1 = 23 \end{aligned}$$

The middle term will be $= \frac{23+1}{2} = 12^{th}$

$$\begin{aligned} a_{12} &= a + (12-1)d \\ &= 213 + (12-1)(-8) \\ &= 213 - 88 = 125 \end{aligned}$$

Middle term will be 125.

PRACTICE

Find the middle term of the AP 6, 13, 20, ... 216.

[Board Term-2 Delhi 2015]

Ans : 111

106. If the number $x+3, 2x+1$ and $x-7$ are in AP find the value of x.

Sol : [Board Term-2 2012]

If x, y and z are three consecutive terms of an AP then we have

$$\begin{aligned} y - x &= z - y \\ (2x+1) - (x+3) &= (x-7) - (2x+1) \\ 2x + 1 - x - 3 &= x - 7 - 2x - 1 \\ x - 2 &= -x - 8 \\ 2x &= -6 \quad \Rightarrow \quad x = -3 \end{aligned}$$

107. Find the values of a, b and c, such that the numbers a, 10, b, c, 31 are in AP

Sol : [Board Term-2 2012]

Let the first term be a and common difference be d.

Since $a, 10, b, c, 31$ are in AP, then

$$\begin{aligned} a + d &= 10 \qquad (1) \\ a + 4d &= a_5 \\ a + 4d &= 31 \qquad (2) \end{aligned}$$

Solving (1) and (2) we have

$$d = 7 \text{ and } a = 3$$

Now $a = 3, b = 3 + 14 = 17, c = 3 + 21 = 24$

Thus $a = 3, b = 17, c = 24$.

108. For AP show that $a_p + a_{p+2q} = 2a_{p+q}$.

Sol : [Board Term-2 2012]

Let the first term be a and the common difference be d. Let a_n be the nth term.

$$\begin{aligned} a_p &= a + (p-1)d \\ a_{p+2q} &= a + (p+2q-1)d \\ a_p + a_{p+2q} &= a + (p-1)d + a + (p+2q-1)d \\ &= a + pd - d + a + pd + 2qd - d \\ &= 2a + 2pd + 2qd - 2d \end{aligned}$$

or $a_p + a_{p+2q} = 2\left[a + (p+q-1)d\right]$ ...(1)

But $2a_{p+q} = 2\left[a + (p+q-1)d\right]$ ...(2)

From (1) and (2), we get $a_p + a_{p+2q} = 2a_{p+q}$

109. The 4^{th} term of an AP is zero. Prove that the 25^{th} term of the AP is three times its 11^{th} term.

Sol : [Board Term-2 OD 2016]

Let the first term be a, common difference be d and nth term be a_n.

We have, $a_4 = 0$

$$\begin{aligned} a + (n-1)d &= 0 \\ a + 3d &= 0 \\ 3d &= -a \\ -3d &= a \qquad ...(1) \end{aligned}$$

Now, $a_{25} = a + 24d = -3d + 24d = 21d$ ...(2)

$a_{11} = a + 10d = -3d + 10d = 7d$ ...(3)

From equation (2) and (3) we have

$a_{25} = 3a_{11}$ Hence Proved.

110. How many terms of the AP $65, 60, 55, ...$ be taken so that their sum is zero?

Sol : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 65, d = -5, S_n = 0$

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difference of the given AP. Then,

$$S_m = S_n$$

$$\frac{m}{2}\{2a+(m-1)d\} = \frac{n}{2}\{2a+(n-1)d\}$$

$$2a(m-n)+\{m(m-1)-n(n-1)d\} = 0$$

$$2a(m-n)+[(m^2-n^2)-(m-n)d] = 0$$

$$(m-n)[2a+(m+n-1)d] = 0$$

$$2a+(m+n-1)d = 0$$

Now, $S_{m+n} = \frac{m+n}{2}\{2a+(m+n-1)d\}$

$$= \frac{m+n}{2} \times 0 = 0$$

89. How many terms of AP 3, 5, 7, 9, ... must be taken to get the sum 120?

Sol : [Board 2020 OD Basic]

Given AP : 3, 5, 7, 9, ...

We have $a = 3$, $d = 2$ and $S_n = 120$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$120 = \frac{n}{2}[2\times 3+(n-1)2]$$

$$120 = n(3+n-1)$$

$$120 = n(n+2)$$

$$n^2+2n-120 = 0$$

$$n^2+12n-10n-120 = 0$$

$$(n+12)(n-10) = 0 \Rightarrow n = 10 \text{ or } n = -12$$

Neglecting $n = -12$ because n can't be negative we get $n = 10$. Hence, 10 terms must be taken to get the sum 120.

90. How many two digits numbers are divisible by 3?

Sol : [Board 2019 Delhi]

Numbers divisible by 3 are 3, 6, 9, 12, 15, ..., 96 and 99. Lowest two digit number divisible by 3 is 12 and highest two digit number divisible by 3 is 99.

Hence, the sequence start with 12, ends with 99 and common difference is 3.

So, the AP is 12, 15, 18, ..., 96, 99.

Here, $a = 12$, $d = 3$ and $a_n = 99$

$$a_n = a+(n-1)d$$

$$99 = 12+(n-1)3$$

$$99-12 = 3(n-1)$$

$$n-1 = \frac{87}{3} = 29 \Rightarrow n = 30$$

Therefore, there are 30, two digit numbers divisible by 3.

PRACTICE

✎ How many two digit numbers are divisible by 7?

[Board 2019, SQP 2016]

Ans : 13

91. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

Sol : [Board 2019 Delhi]

Given AP is 3, 15, 27, 39.....

Here, first term, $a = 3$ and common difference, $d = 12$

Now, 21^{st} term of AP is

$$a_n = a+(n-1)d$$

$$a_{21} = 3+(21-1)\times 12$$

$$= 3+20\times 12 = 243$$

Therefore, 21^{st} term is 243.

Now we need to calculate term which is 120 more than 21^{st} term i.e it should be $243+120 = 363$

Therefore, $a_n = a+(n-1)d$

$$363 = 3+(n-1)12$$

$$360 = 12(n-1)$$

$$n-1 = 30 \Rightarrow n = 31$$

So, 31^{st} term is 120 more than 21^{st} term.

92. If S_n the sum of first n terms of an AP is given by $S_n = 3n^2-4n$, find the n^{th} term.

Sol : [Board 2019 Delhi]

We have $S_n = 3n^2-4n$

Substituting n by $n-1$ we get

$$S_{n-1} = 3(n-1)^2-4(n-1)$$

Now n^{th} term

$$a_n = S_n - S_{n-1}$$

$$= 3n^2-4n-3(n-1)^2+4(n-1)$$

$$= 3n^2-4n-3(n^2-2n+1)+4(n-1)$$

$$= -3(-2n+1)+4(-1)$$

$$= 6n-7$$

Therefore, n^{th} term is $6n-7$.

93. Find the 21^{st} term of the AP $-4\frac{1}{2}, -3, -1\frac{1}{2}, \ldots$

Sol : [Board 2019 OD]

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We have $a = 8, d = 6$.

Since n^{th} term is 72 more than 41^{st} term. we get

$$\begin{aligned} a_n &= a_{41} + 72 \\ 8 + (n-1)6 &= 8 + 40 \times 6 + 72 \\ 6n - 6 &= 240 + 72 \\ 6n &= 312 + 6 = 318 \\ n &= 53 \end{aligned}$$

82. If the n^{th} term of an AP $-1, 4, 9, 14, \ldots\ldots$ is 129. Find the value of n.

Sol : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = -1$ and $d = 4 - (-1) = 5$

$$\begin{aligned} -1 + (n-1) \times 5 &= a_n \\ -1 + 5n - 5 &= 129 \\ 5n &= 135 \\ n &= 27 \end{aligned}$$

Hence 27^{th} term is 129.

83. Write the n^{th} term of the AP $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, \ldots\ldots$

Sol : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = \frac{1}{m}$

$$d = \frac{1+m}{m} - \frac{1}{m} = 1$$

$$a_n = \frac{1}{m} + (n-1)1$$

Hence , $a_n = \frac{1}{m} + n - 1$

84. What is the common difference of an AP which $a_{21} - a_7 = 84$.

Sol : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_{21} - a_7 = 84$

$$\begin{aligned} a + 20d - a - 6d &= 84 \\ 14d &= 84 \\ d &= \frac{84}{14} = 6 \end{aligned}$$

Hence common difference is 6.

85. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$ .... is the first negative.

Sol : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 20$ and $d = -\frac{3}{4}$

Let the n^{th} term be first negative term, then

$$\begin{aligned} a + (n-1)d &< 0 \\ 20 + (n-1)\left(-\frac{3}{4}\right) &< 0 \\ 20 - \frac{3}{4}n + \frac{3}{4} &< 0 \\ 3n &> 83 \\ n &> \frac{83}{3} = 27\frac{2}{3} \end{aligned}$$

Hence 28^{th} term is first negative.

86. If S_n denotes the sum of n terms of an AP whose common difference is d and first term is a, find $S_n - 2S_{n-1} + S_{n-2}$.

Sol : [Board Term-2 2011]

We have $a_n = S_n - S_{n-1}$

$$a_{n-1} = S_{n-1} - S_{n-2}$$

Subtracting above equation we have

$$\begin{aligned} a_n - a_{n-1} &= S_n - S_{n-1} - S_{n-1} + S_{n-2} \\ d &= S_n - 2S_{n-1} + S_{n-2}. \end{aligned}$$

87. Find the sum of first 15 multiples of 8.

Sol : [Board Term-2 Delhi Compt 2017]

Let the first term be $a = 8$, common difference be $d = 8$, nth term be a_n and sum of n term be S_n.

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ S_{15} &= \frac{15}{2}[2 \times 8 + (15-1)8] \\ &= \frac{15}{2}[16 + 112] \\ &= \frac{15}{2} \times 128 = 996 \end{aligned}$$

Hence, the sum of 15 terms is 960.

TWO MARKS QUESTIONS

88. If the sum of first m terms of an AP is the same as the sum of its first n terms, show that the sum of its first $(m+n)$ terms is zero.

Sol : [Board 2020 SQP Standard]

Let a be the first term and d be the common

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$$a_{70} = a + 69d$$
$$= 9d + 69d = 78d$$
$$a_{31} = a + 30d$$
$$= 9d + 30d = 39d$$
$$a_{70} = 2a_{31} \quad \text{Hence Proved.}$$

99. The 8^{th} term of an AP is zero. Prove that its 38^{th} term is triple of its 18^{th} term.

Sol : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

We have, $a_8 = 0$ or, $a + 7d = 0$ or, $a = -7d$

Now $a_{38} = a + 37d$

$a_{38} = -7d + 37d = 30d$

$a_{18} = a + 17d = -7d + 17d = 10d$

$a_{38} = 30d = 3 \times 10d = 3 \times a_{18}$

$a_{38} = 3a_{18}$ Hence Proved

100. If five times the fifth term of an AP is equal to eight times its eighth term, show that its 13^{th} term is zero.

Sol : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

Now $5a_5 = 8a_8$

$5(a + 4d) = 8(a + 7d)$

$5a + 20d = 8a + 56d$

$3a + 36d = 0$

$3(a + 12d) = 0$

$a + 12d = 0$

$a_{13} = 0$ Hence Proved

101. The fifth term of an AP is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference.

Sol : [Board Term-2 Foreign 2015]

Let the first term be a and common difference be d.

$a + 4d = 20$ ...(1)

$a + 6d + a + 10d = 64$

$a + 8d = 32$ ...(2)

Solving equations (1) and (2), we have

$d = 3$

102. The ninth term of an AP is -32 and the sum of its eleventh and thirteenth term is -94. Find the common difference of the AP

Sol : [Board Term-2 Foreign 2015]

Let the first term be a and common difference be d.

Now $a + 8d = a_9$

$a + 8d = -32$ ...(1)

and $a_{11} + a_{13} = -94$

$a + 10d + a + 12d = -94$

$a + 11d = -47$ ...(2)

Solving equation (1) and (2), we have

$d = -5$

PRACTICE

The fifth term of an AP is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference.

[Board Term-2 Foreign 2015]

Ans : 3

103. The seventeenth term of an AP exceeds its 10^{th} term by 7. Find the common difference.

Sol : [Board Term-2 2015, 2014]

Let the first term be a and common difference be d.

Now $a_{17} = a_{10} + 7$

$a + 16d = a + 9d + 7$

$16d - 9d = 7$

$7d = 7 \Rightarrow d = 1$

Thus common difference is 1.

104. The fourth term of an AP is 11. The sum of the fifth and seventh terms of the AP is 34. Find the common difference.

Sol : [Board Term-2 Foreign 2015]

Let the first term be a and common difference be d.

Now $a_4 = 11$

$a + 3d = 11$ ...(1)

and $a_5 + a_7 = 34$

$a + 4d + a + 6d = 34$

$2a + 10d = 34$

$a + 5d = 17$ ...(2)

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PRACTICE

- For what value of k will the consecutive terms $2k+1$, $3k+3$ and $5k-1$ form an AP?
  [Board Term-2 Foreign 2016]
  Ans : 6

- For what value of k, $k+9$, $2k-1$ and $2k+7$ are the consecutive terms of an AP.
  [Board Term-2 OD 2016]
  Ans : 18

- For what value of k; $k+2$, $4k-6$, $3k-2$ are three consecutive terms of an AP.
  [Board Term-2 Delhi 2014, 2012]
  Ans : 3

68. Find the 25^{th} term of the AP $-5, \frac{-5}{2}, \frac{5}{2}, ...$

Sol : [Board Term-2 Foreign 2015]

Let the first term of an AP be a and common difference be d.

Here, $a = -5,\ d = -\frac{5}{2} - (-5) = \frac{5}{2}$

$$a_n = a + (n-1)d$$

$$a_{25} = 5 + (25-1) \times \left(\frac{5}{2}\right)$$

$$= -5 + 60 = 55$$

69. What is the common difference of an AP in which $a_{21} - a_7 = 84$?

Sol : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

$$a_{21} - a_7 = 84$$

$$a + (21-1)d - \left[a + (7-1)d\right] = 84$$

$$a + 20d - a - 6d = 84$$

$$14d = 84$$

$$d = 6$$

70. In the AP $2, x, 26$ find the value of x.

Sol : [Board Term-2 2012]

If x, y and z are in AP then we have

$$y - x = z - y$$

Since $2, x$ and 26 are in AP we have

$$x - 2 = 26 - x$$

$$2x = 26 + 2$$

$$x = \frac{28}{2} = 14$$

71. If $18, a, b, -3$ are in AP, then find $a+b$.

Sol : [Board Term-2 2012]

If $18, a, b, -3$ are in AP, then,

$$a - 18 = -3 - b$$

$$a + b = -3 + 18$$

$$a + b = 15$$

72. Find the common difference of the AP $\frac{1}{3q}, \frac{1-6q}{3q}, \frac{1-12q}{3q}, ...$

Sol : [Board Term-2 Delhi 2011]

Let common difference be d then we have

$$d = \frac{1-6q}{3q} - \frac{1}{3q}$$

$$= \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$$

73. Find the first four terms of an AP whose first term is $3x+y$ and common difference is $x-y$.

Sol : [Board Term-2 2012]

Let the first term of an AP be a and common difference be d.

Now $a_1 = 3x + y$

$$a_2 = a_1 + d = 3x + y + x - y = 4x$$

$$a_3 = a_2 + d = 4x + x - y = 5x - y$$

$$a_4 = a_3 + d = 5x - y + x - y = 6x - 2y$$

So, the four terms are $3x+y$, $4x, 5x-y$ and $6x-2y$.

74. Find the 37^{th} term of the AP $\sqrt{x}, 3\sqrt{x}, 5\sqrt{x}$.

Sol : [Board Term-2 2012]

Let the nth term of an AP be a_n and common difference be d.

Here, $a_1 = \sqrt{x}$

$$a_2 = 3\sqrt{x}$$

$$d = a_2 - a_1 = 3\sqrt{x} - \sqrt{x} = 2\sqrt{x}$$

$$a_n = a + (n-1)d$$

$$a_{37} = \sqrt{x} + (37-1)2\sqrt{x}$$

$$= \sqrt{x} + 36 \times 2\sqrt{x} = 73\sqrt{x}$$

75. For an AP, if $a_{25} - a_{20} = 45$, then find the value of d.

Sol : [Board Term-2 2011]

Let the first term of an AP be a and common difference be d.

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Now $S_n = \frac{n}{2}[2a+(n-1)d]$

Let sum of n term be zero, then we have

$$\frac{n}{2}[130+(n-1)(-5)] = 0$$
$$\frac{n}{2}[130+5n+5] = 0$$
$$135n-5n^2 = 0$$
$$n(135-5n) = 0$$
$$5n = 135 \Rightarrow n = 27$$

PRACTICE

- How many terms of the AP $18, 16, 14$ ..... be taken so that their sum is zero?
  [Board Term-2 Delhi 2016]
  Ans : 19

- How many terms of the AP $27, 24, 21$.... should be taken so that their sum is zero?
  [Board Term-2 Delhi 2016]
  Ans : 19

111. How many terms of the AP $-6, -\frac{11}{2}, -5, -\frac{9}{2}$.... are needed to give their sum zero.

Sol : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = -6, d = -\frac{11}{2} - (-6) = \frac{1}{2}$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Let sum of n term be zero, then we have

$$\frac{n}{2}\left[2\times(-6)+(n-1)\frac{1}{2}\right] = 0$$
$$\frac{n}{2}\left[-12+\frac{n}{2}-\frac{1}{2}\right] = 0$$
$$\frac{n}{2}\left[\frac{n}{2}-\frac{25}{2}\right] = 0$$
$$n^2-25n = 0$$
$$n(n-25) = 0$$
$$n = 25$$

Hence 25 terms are needed.

112. In an AP, if $S_5+S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of first n terms.

Sol : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_5+S_7 = 167$$
$$\frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d) = 167$$
$$5a+10d+7a+21d = 167$$
$$12a+31d = 167 \quad ...(1)$$

Now we have $S_{10} = 235$, thus

$$\frac{10}{2}[2a+(10-1)d] = 235$$
$$5(2a+9d) = 235$$
$$2a+9d = 47 \quad (2)$$

Solving (1) and (2), we get

$$a = 1, d = 5$$

Thus AP is 1, 6, 11....

113. Find the sum of sixteen terms of an AP $-1, -5, -9, \ldots\ldots$

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a_1 = -1, a_2 = -5$ and $d = -4$

Now $S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_{16} = \frac{16}{2}[2\times(-1)+(16-1)(-4)]$$
$$= 8[-2-60] = 8(-62)$$
$$= -496$$

114. If the 1^{st} term of a series is 7 and 13^{th} term is 35. Find the sum of 13 terms of the sequence.

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a = 7, a_{13} = 35$

$$a_n = a+(n-1)d$$
$$a_{13} = a+12d$$
$$35 = 7+12d \Rightarrow d = \frac{7}{3}$$

Now $S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_{13} = \frac{13}{2}\left[2\times 7+12\times\left(\frac{7}{3}\right)\right]$$
$$= \frac{13}{2}[14+28]$$
$$= \frac{13}{2}\times 42 = 273$$

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Given AP is $-4\frac{1}{2}, -3, -1\frac{1}{2}, \ldots$ or $-\frac{9}{2}, -3, -\frac{3}{2}, \ldots$

First term, $a = \frac{-9}{2}$

Common difference,

$$d = -3 - \left(-\frac{9}{2}\right) = -3 + \frac{9}{2}$$
$$= \frac{-6+9}{2} = \frac{3}{2}$$

Now $a_n = a + (n-1)d$

$$a_{21} = \left(-\frac{9}{2}\right) + (21-1)\left(\frac{3}{2}\right)$$
$$= -\frac{9}{2} + 20 \times \frac{3}{2} = -\frac{9}{2} + 30$$
$$= \frac{-9+30}{2}$$
$$= \frac{51}{2} = 25\frac{1}{2}$$

Hence, 21^{st} term of given AP is $25\frac{1}{2}$.

94. If the sum of first n terms of an AP is n^2, then find its 10th term.

Sol : [Board 2019 Delhi]

We have $S_n = n^2$ ...(1)

Substituting n by $n-1$ we get

$$S_{n-1} = (n-1)^2$$

Now n^{th} term, $a_n = S_n - S_{n-1}$

$$= n^2 - (n-1)^2$$
$$= n^2 - (n^2 - 2n + 1)$$
$$= 2n - 1$$

Now, 10^{th} term of AP,

$$a_{10} = 2 \times 10 - 1 = 19$$

Hence, the 10^{th} term of AP is 19.

95. Is 184 a term of the sequence $3, 7, 11, \ldots\ldots$?

Sol : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and number of terms be n.

Let $a_n = 184$

Here, $a = 3, d = 7 - 3 = 11 - 7 = 4$

Now $a_n = a + (n-1)d$,

$$184 = 3 + (n-1)4$$
$$\frac{181}{4} = n - 1$$
$$45.25 = n - 1$$
$$46.25 = n$$

Since 46.25 is not an whole number, thus 184 is not a term of given AP

96. Find, 100 is a term of the AP $25, 28, 31, \ldots\ldots$ or not.

Sol : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and number of terms be n.

Let $a_n = 100$

Here $a = 25, d = 28 - 25 = 31 - 28 = 3$

Now $a_n = a + (n-1)d$,

$$100 = 25 + (n-1) \times 3$$
$$100 - 25 = 75 = (n-1) \times 3$$
$$25 = n - 1$$
$$n = 26$$

Since 26 is an whole number, thus 100 is a term of given AP.

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97. Find the 7^{th} term from the end of AP $7, 10, 13, \ldots. 184$.

Sol : [Board Term-2 2012]

Let us write AP in reverse order i.e., $184, \ldots\ldots 13, 10, 7$

Let the first term of an AP be a and common difference be d.

Now $d = 7 - 10 = -3$

$a = 184,\ n = 7$

7^{th} term from the original end,

$$a_7 = a + 6d$$
$$a_7 = 184 + 6(-3)$$
$$= 184 - 18 = 166.$$

Hence, 166 is the 7^{th} term from the end.

98. In a certain AP 32^{th} term is twice the 12^{th} term. Prove that 70^{th} term is twice the 31^{st} term.

Sol : [Board Term-2 2015, 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

Now we have $a_{32} = 2a_{12}$

$$a + 31d = 2(a + 11d)$$
$$a + 31d = 2a + 22d$$
$$a = 9d$$

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nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Now $$S_7 = \frac{7}{2}(2a+6d) = 49$$

$$a+3d = 7 \quad ...(1)$$

and $$S_{17} = \frac{17}{2}(2a+16d) = 289$$

$$a+8d = 17$$

Subtracting (1) from (2), we get

$$5d = 10 \Rightarrow d = 2$$

Substituting this value of d in (1) we have

$$a = 1$$

Now $$S_n = \frac{n}{2}[2\times 1+(n-1)2]$$

$$= \frac{n}{2}[2+2n-2] = n^2$$

Hence, sum of n terms is n^2.

120. Which term of the AP $3, 12, 21, 30, ...$ will be 90 more than its 50^{th} term.

Sol : [Board Term-2 Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 3, d = 9$

Now $$a_n = a+(n-1)d$$

$$a_{50} = 3+49\times 9 = 444$$

Now, $$a_n - a_{50} = 90$$

$$3+(n-1)9-444 = 90$$

$$(n-1)9 = 90+441$$

$$(n-1) = \frac{531}{9} = 49$$

$$n = 49+1 = 50$$

121. Find how many integers between 200 and 500 are divisible by 8.

Sol : [Board Term-2 Delhi Compt. 2017]

Number divisible by 8 are 208, 2016, 224, ... 496. It is an AP

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 208, d = 8$ and $a_n = 496$

Now $$a+(n-1)d = a_n$$

$$208+(n-1)d = 496$$

$$(n-1)8 = 496-208$$

$$n-1 = \frac{288}{8} = 36$$

$$n = 36+1 = 37$$

Hence, required numbers divisible by 8 is 37.

122. The fifth term of an AP is 26 and its 10^{th} term is 51. Find the AP

Sol : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$a_5 = a+4d = 26 \quad ...(1)$$

$$a_{10} = a+9d = 51 \quad ...(2)$$

Subtracting (1) from (2) we have

$$5d = 25 \Rightarrow d = 5$$

Substituting this value of d in equation (1) we get $a = 6$.

Hence, the AP is 6, 11, 16, ...

123. Find the AP whose third term is 5 and seventh term is 9.

Sol : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

Now $$a_3 = a+2d = 5 \quad ...(1)$$

and $$a_7 = a+6d = 9 \quad ...(2)$$

Subtracting (2) from (1) we have

$$4d = 4 \Rightarrow d = 1$$

Substituting this value of d in (1) we get

$$a = 3$$

Hence AP is 3, 4, 5, 6, ...

124. Find whether -150 is a term of the AP $11, 8, 5, 2, ...$

Sol : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

Let the n^{th} term of given AP $11, 8, 5, 2, ...$ be -150

Hence $a = 11$, $d = 8-11 = -3$ and $a_n = -150$

$$a+(n-1)d = a_n$$

$$11+(n-1)(-3) = -150$$

$$(n-1)(-3) = -161$$

$$(n-1) = \frac{-161}{-3} = 53\frac{2}{3}$$

which is not a whole number. Hence -150 is not a term of given AP.

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Sol : [Board Term-2 Foreign 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$a_{12} = a + 11d = -13 \quad ...(1)$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now $$S_4 = 2[2a + 3d] = 24$$

$$2a + 3d = 12 \quad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it we get

$$(2a + 22d) - (2a + 3d) = -26 - 12$$

$$19d = -38$$

$$d = -2$$

Substituting the value of d in (1) we get

$$a + 11 \times -2 = -13$$

$$a = -13 + 22 = 9$$

Now, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}(2 \times 9 + 9 \times -2)$$

$$= 5 \times (18 - 18) = 0$$

Hence, $S_{10} = 0$

PRACTICE

✎ The tenth term of an AP, is -37 and the sum of its first six terms is -27. Find the sum of its first eight terms.

[Board Term-2 Foreign 2015]

Ans : -76

150. Find the sum of first seventeen terms of AP whose 4^{th} and 9^{th} terms are -15 and -30 respectively.

Sol : [Board Term-2 2014]

Let the first term be a, common difference be d and nth term be a_n.

Now $$a_4 = a + 3d = -15 \quad ...(1)$$

$$a_9 = a + 8d = -30 \quad ...(2)$$

Subtracting eqn (1) from eqn (2), we obtain

$$(a + 8d) - (a + 3d) = -30 - (-15)$$

$$5d = -15 \Rightarrow d = \frac{-15}{5} = -3$$

Substituting the value of d in (1) we get

$$a + 3d = -15$$

$$a + 3(-3) = -15$$

$$a = -15 + 9 = -6$$

Now $$S_{17} = \frac{17}{2}[2 \times (-6) + (17-1)(-3)]$$

$$= \frac{17}{2}[-12 + 16 \times (-3)]$$

$$= \frac{17}{2}[-12 - 48]$$

$$= \frac{17}{2}[-60] = 17 \times (-30)$$

$$= -510$$

Thus $S_{17} = -510$.

151. The common difference of an AP is -2. Find its sum, if first term is 100 and last term is -10.

Sol : [Board Term-2 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $$a = 100, d = -2, t_n = -10$$

Now $$a_n = a + (n-1)d$$

$$-10 = 100 + (n-1)(-2)$$

$$-10 = 100 - 2n + 2$$

$$2n = 112$$

$$n = 56$$

Thus 56^{th} term is -10 and number of terms in AP are 56.

Now $$S_n = \frac{n}{2}(a + 1)$$

$$S_{56} = \frac{56}{2}(100 - 10)$$

$$= \frac{56}{2}(90) = 56 \times 45 = 2520$$

Thus $S_n = 2520$

152. The 13^{th} term of an AP is four times its 3^{rd} term. If the fifth term is 16, then find the sum of its first ten terms.

Sol : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a_{13} = 4a_3$

$$a + 12d = 4(a + 2d)$$

$$3a = 4d \quad ...(1)$$

and $$a_5 = 16$$

$$a + 4d = 16 \quad ...(2)$$

Substituting the value of $a = \frac{4}{3}d$ in (2) we have

$$\frac{4}{3}d + 4d = 16$$

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Now, sum of next 7 terms,

$$S_{8-14} = 161$$

$$S_{8-14} = \frac{7}{2}(a_8 + a_{14})$$

$$161 = \frac{7}{2}(a + 7d + a + 13d)$$

$$161 = \frac{7}{2}(2a + 20d)$$

$$23 = a + 10d \qquad ...(2)$$

Subtracting equation (1) from (2) we have

$$14 = 7d \Rightarrow d = 2$$

Substituting the value of d in (1), we get

$$a = 3$$

Hence, the AP is 3, 5, 7, 9, ...

130. Which term of the AP 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term.

Sol : [Board 2020 OD Standard]

Here, $a = 20$

and $d = \frac{77}{4} - 20 = -\frac{3}{4}$

Let a_n is the first negative term, thus $a_n < 0$.

Now $a_n = a + (n-1)\,d$

$$20 + (n-1)\left(-\frac{3}{4}\right) < 0$$

$$80 - 3n + 3 < 0$$

$$83 - 3n < 0$$

$$n > \frac{83}{3}$$

$$n > 27.6$$

$$n = 28$$

Hence, the first negative term is 28th term.

PRACTICE

✎ Which term of an AP 150, 147, 144, ... is its first negative term?

[Board 2010]

Ans : 52^{nd} term

131. Find the middle term of the AP 7, 13, 19, ..., 247.

Sol : [Board 2020 OD Standard]

In this AP, $a = 7$

$$d = 13 - 7 = 6$$

$$a_n = a + (n-1)\,d$$

$$247 = 7 + (n-1)6$$

$$6(n-1) = 240$$

$$n - 1 = 40 \Rightarrow n = 41$$

Hence, the middle term

$$\frac{n+1}{2} = \frac{41+1}{2} = \frac{42}{2} = 21.$$

$$a_{21} = 7 + (21-1)6 = 127$$

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132. Show that the sum of all terms of an AP whose first term is a, the second term is b and last term is c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

Sol : [Board 2020 OD Standard]

Given, first term, $A = a$

and second term $A_2 = b$

Common difference, $D = b - a$

Last term, $A_n = c$

$$A + (n-1)\,d = c$$

$$a + (n-1)(b-a) = c$$

$$(b-a)(n-1) = c - a$$

$$n - 1 = \frac{c-a}{b-a}$$

$$n = \frac{c-a}{b-a} + 1$$

$$= \frac{c - a + b - a}{b-a}$$

$$n = \frac{b + c - 2a}{b-a}$$

Now sum of all terms

$$S_n = \frac{n}{2}[A + A_n] = \frac{(b+c-2a)}{2(b-a)}[a+c]$$

$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$ Hence Proved

133. If in an AP, the sum of first m terms is n and the sum of its first n terms is m, then prove that the sum of its first $(m+n)$ terms is $-(m+n)$.

Sol : [Board 2020 OD Standard]

Let 1^{st} term of series be a and common difference be d, then we have

$$S_m = n$$

and $$S_n = m$$

$$\frac{m}{2}[2a + (m-1)\,d] = n \qquad ...(1)$$

$$\frac{n}{2}[2a + (n-1)\,d] = m \qquad ...(2)$$

Subtracting we have

$$a(m-n) + \frac{d}{2}[m(m-1) - n(n-1)] = n - m$$

$$a = A+(p-1)D$$
$$b = A+(q-1)D$$
$$c = A+(r-1)D$$

Now $a(q-r) = [A+(p-1)D][q-r]$

$b(r-p) = [A+(q-1)D][r-p]$

and $c[p-q] = [A+(r-1)D][p-q]$

$$\begin{aligned}
&a(q-r)+b(r-p)+c(p-q)\\
&=[A+(p-1)D][q-r]+\\
&\quad+[A+(q-1)D][r-p]+\\
&\quad\quad+[A+(r-1)D][p-q]+\\
&=A[p-q+q-p+q-r]+\\
&\quad+D(p-1)(q-r)+\\
&\quad\quad+D(q-1)(r-p)+\\
&\quad\quad\quad+D(r-1)(p-q)\\
&=A[0]+\\
&\quad+D[p(q-r)-(q-r)]\\
&\quad\quad+D[q(r-p)-(r-p)]\\
&\quad\quad\quad+D[r(p-q)-(p-q)]\\
&=D[p(q-r)+q(r-p)+r(p-q)]+\\
&\quad-D[(q-r)+(r-p)+(p-q)]\\
&=D[pq-pr+qr-qp+rp-rq]+0\\
&=D[0]=0
\end{aligned}$$

142. The sum of n terms of an AP is $3n^2+5n$. Find the AP. Hence find its 15^{th} term.

Sol : [Board Term-2 2013, 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now $S_n = 3n^2+5n$

$$\begin{aligned}
S_{n-1} &= 3(n-1)^2+5(n-1)\\
&= 3(n^2+1-2n)+5n-5\\
&= 3n^2+3-6n+5n-5\\
&= 3n^2-n-2
\end{aligned}$$

$$\begin{aligned}
a_n &= S_n-S_{n-1}\\
&= 3n^2+5n-(3n^2-n-2)\\
&= 6n+2
\end{aligned}$$

Thus AP is 8, 14, 20, ...

Now $a_{15} = a+14d = 8+14(6) = 92$

PRACTICE

The sum of first n terms of an AP is given by $S_n = 3n^2-4n$. Determine the AP and the 12^{th} term.

[Board Term-2 Delhi 2014, 2012]

Ans : $-1, 5, 11, ...; 65$

143. For what value of n, are the n^{th} terms of two APs 63, 65, 67, ... and 3, 10, 17, ... equal?

Sol : [Board Term-2 OD 2016]

Let a, d and A, D be the 1^{st} term and common difference of the 2 APs respectively.

n is same

For 1st AP, $a = 63,\ d = 2$

For 2nd AP, $A = 3, D = 7$

Since nth term is same,

$$\begin{aligned}
a_n &= A_n\\
a+(n-1)d &= A+(n-1)D\\
63+(n-1)2 &= 3+(n-1)7\\
63+2n-2 &= 3+7n-7\\
61+2n &= 7n-4\\
65 &= 5n \Rightarrow n = 13
\end{aligned}$$

When n is 13, the n^{th} terms are equal i.e., $a_{13} = A_{13}$

144. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each AP is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1+S_3 = 2S_2$.

Sol : [Board Term-2 OD 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $S_1 = 1+2+3+.....n$

$S_2 = 1+3+5+.....$ up to n terms

$S_3 = 1+4+7+.....$ upto n terms

Now $S_n = \dfrac{n(n+1)}{2}$

$S_2 = \dfrac{n}{2}[2+(n-1)2] = \dfrac{n}{2}[2n] = n^2$

and $S_3 = \dfrac{n}{2}[2+(n-1)3] = \dfrac{n(3n-1)}{2}$

Now, $S_1+S_3 = \dfrac{n(n+1)}{2}+\dfrac{n(3n-1)}{2}$

$= \dfrac{n[n+1+3n-1]}{2} = \dfrac{n[4n]}{2}$

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Subtracting equation (1) from (2) we get

$$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} \Rightarrow d = \frac{1}{63}$$

Substituting the value of d in (2) we get

$$a + 8 \times \frac{1}{63} = \frac{1}{7}$$

$$a = \frac{1}{7} - \frac{8}{63} = \frac{9-8}{63} = \frac{1}{63}$$

Thus $a_{63} = a + (63-1)\,d$

$$= \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$$

$$= \frac{63}{63} = 1$$

Hence, $a_{63} = 1$.

PRACTICE

- If the 2^{nd} term of an AP is 8 and the 5^{th} term is 17, find its 19^{th} term.
  [Board Term-2 2016]
  Ans : 59

- The 10^{th} term of an AP is -4 and its 22^{nd} term is -16. Find its 38^{th} term.
  [Board Term-2 Delhi Compt. 2017]
  Ans : -32

138. The ninth term of an AP is equal to seven times the second term and twelfth term exceeds five times the third term by 2. Find the first term and the common difference.

Sol : [Board 2008, 2010]

Let the first term be a, common difference be d and nth term be a_n.

Now $a_9 = 7a_2$

$$a + 8d = 7(a + d)$$

$$a + 8d = 7a + 7d$$

$$-6a + d = 0 \quad \ldots(1)$$

and $a_{12} = 5a_3 + 2$

$$a + 11d = 5(a + 2d) + 2$$

$$a + 11d = 5a + 10d + 2$$

$$-4a + d = 2 \quad \ldots(2)$$

Subtracting (2) from (1), we get

$$-2a = -2 \Rightarrow a = 1$$

Substituting this value of a in equation (1) we get

$$-6 + d = 0 \Rightarrow d = 6$$

Hence first term is 1 and common difference is 6.

139. Determine an AP whose third term is 9 and when fifth term is subtracted from 8^{th} term, we get 6.

Sol : [Board Term-2 2015]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = 9$

$$a + 2d = 9 \quad \ldots(1)$$

and $a_8 - a_5 = 6$

$$(a + 7d) - (a + 4d) = 6$$

$$3d = 6 \Rightarrow d = 2$$

Substituting this value of d in (1), we get

$$a + 2(2) = 9 \Rightarrow a = 5$$

So, AP is 5, 7, 9, 11, ...

140. Divide 56 in four parts in AP such that the ratio of the product of their extremes (1^{st} and 4^{rd}) to the product of means (2^{nd} and 3^{rd}) is 5:6.

Sol : [Board Term-2 Foreign 2016]

Let the four numbers be $a-3d, a-d, a+d, a+3d$

Now $a - 3d + a - d + a + d + a + 3d = 56$

$$4a = 56 \Rightarrow a = 14$$

Hence numbers are $14-3d, 14-d, 14+d, 14+3d$

Now, according to question, we have

$$\frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6}$$

$$\frac{196 - 9d^2}{196 - d^2} = \frac{5}{6}$$

$$6(196 - 9d^2) = 5(196 - d^2)$$

$$6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$$

$$(6-5) \times 196 = 49d^2$$

$$d^2 = \frac{196}{49} = 4$$

$$d = \pm 2$$

Thus numbers are $a - 3d = 14 - 3 \times 2 = 8$

$$a - d = 14 - 2 = 12$$

$$a + d = 14 + 2 = 16$$

$$a + 3d = 14 + 3 \times 2 = 20$$

Thus required AP is $8, 12, 16, 20$.

141. An AP has p^{th}, q^{th} and r^{th} terms as a, b and c respectively, Show that $a(q-r) + b(r-p) + c(p-q) = 0$.

Sol : [Board Term-2 Foreign 2016]

Let the first term be A and the common difference be D.

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125. If seven times the 7^{th} term of an AP is equal to eleven times the 11^{th} term, then what will be its 18^{th} term.

Sol : [Board Term-2 Foreign 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$7a_7 = 11a_{11}$$

Now $$7(a+6d) = 11(a+10d)$$

$$7a+42d = 11a+110d$$

$$11a-7a = 42d-110d$$

, $$4a = -68d$$

$$4a+68d = 0$$

$$4(a+17d) = 0$$

$$a+17d = 0$$

Hence, $$a_{18} = 0$$

126. In an AP of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP.

Sol : [Board Term-2 Foreign 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_{10} = 210$$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$210 = \frac{10}{2}(2a+9d)$$

$$42 = 2a+9d \qquad (1)$$

Now $$a_{36} = a+35d$$

$$a_{50} = a+49d$$

Sum of last 15 terms,

$$S_{36-50} = \frac{n}{2}(a_{36}+a_{50})$$

$$2565 = \frac{15}{2}(a+35d+a+49d)$$

$$171 = \frac{1}{2}(2a+84d)$$

$$171 = a+42d \qquad (2)$$

Solving (1) and (2) we get $a = 3$ and $d = 4$

Hence, AP is 3, 7, 11, ...

127. How many multiples of 4 lie between 11 and 266?

Sol : [Board Term-2 2012]

First multiple of 4 is 12 and last multiple of 4 is 264.

It forms a AP. Let multiples of 4 be n.

Let the first term be a, common difference be d, nth term be a_n.

Here, $a = 12, a_n = 264, d = 4$

$$a_n = a+(n-1)d$$

$$264 = 12+(n-1)4$$

$$n = \frac{264-12}{4}+1$$

Hence, there are 64 multiples of 4 that lie between 11 and 266.

THREE MARKS QUESTIONS

128. The sum of four consecutive number in AP is 32 and the ratio of the product of the first and last term to the product of two middle terms is 7 : 15. Find the numbers.

Sol : [Board 2020 Delhi Standard, 2018]

Let the four consecutive terms of AP be $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

As per question statement we have

$$a-3d+a-d+a+d+a+3d = 32$$

$$4a = 32 \Rightarrow a = 8$$

and $$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$\frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$$

$$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$960-135d^2 = 448-7d^2$$

$$7d^2-135d^2 = 448-960$$

$$-128d^2 = -512$$

$$d^2 = 4 \Rightarrow d = \pm 2$$

Hence, the number are 2, 6, 10 and 14 or 14, 10, 6 and 2.

129. The sum of the first 7 terms of an AP is 63 and that of its next 7 terms is 161. Find the AP.

Sol : [Board 2020 Delhi Standard]

We have $$S_7 = 63$$

Now $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$63 = \frac{7}{2}[2a+6d]$$

$$9 = a+3d \qquad ...(1)$$

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$= 2n^2 = 2s_2$ Hence Proved

145. If S_n denotes, the sum of the first n terms of an AP prove that $S_{12} = 3(S_8 - S_4)$.

Sol : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = 6[2a + 11d] = 12a + 66d$$

$$S_8 = 4[2a + 7d] = 8a + 28d$$

$$S_4 = 2[2a + 3d] = 4a + 6d$$

$$3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)]$$

$$= 3[4a + 22d] = 12a + 66d$$

$$= 6[2a + 11d] = S_{12}$$ Hence Proved

146. The 14^{th} term of an AP is twice its 8^{th} term. If the 6^{th} term is -8, then find the sum of its first 20 terms.

Sol : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a_{14} = 2a_8$ and $a_6 = -8$

Now $a + 13d = 2(a + 7d)$

$a + 13d = 2a + 14d$

$a = -d$ ...(1)

and $a_6 = -8$

$a + 5d = -8$ ...(2)

Solving (1) and (2), we get

$a = 2, d = -2$

Now $S_{20} = \frac{20}{2}[2 \times 2 + (20-1)(-2)]$

$= 10[4 + 19 \times (-2)]$

$= 10(4 - 38)$

$= 10 \times (-34) = -340$

147. If the ratio of the sums of first n terms of two AP's is $(7n+1):(4n+27)$, find the ratio of their m^{th} terms.

Sol : [Board Term-2 OD 2016]

Let a, and A be the first term and d and D be the common difference of two AP's, then we have

$$\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$$

$$\frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2A + (n-1)D]} = \frac{7n+1}{4n+27}$$

$$\frac{2a + (n-1)d}{2A + (n-1)D} = \frac{7n+1}{4n+27}$$

$$\frac{a + (\frac{n-1}{2})d}{A + (\frac{n-1}{2})D} = \frac{7n+1}{4n+27}$$

Substituting $\frac{n-1}{2} = m-1$ or $n = 2m-1$ we get

$$\frac{a + (m-1)d}{A + (m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$= \frac{14m-6}{8m+23}$$

Hence, $\frac{a_m}{A_m} = \frac{14m-6}{8m+23}$

148. If the sum of the first n terms of an AP is $\frac{1}{2}[3n^2 + 7n]$, then find its n^{th} term. Hence write its 20^{th} term.

Sol : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Sum of n terms, $S_n = \frac{1}{2}[3n^2 + 7n]$

Sum of $n-1$ terms,

$$S_{n-1} = \frac{1}{2}[3(n-1)^2 + 7(n-1)]$$

Now n th term,

$a_n = S_n - S_{n-1}$

$= \frac{1}{2}[3n^2 + 7n] - \frac{1}{2}[3(n-1)^2 + 7(n-1)]$

$= \frac{1}{2}[3n^2 - 3(n^2 - 2n + 1) + 7n - 7(n-1)]$

$= \frac{1}{2}[-3(-2n+1) - 7(-1)]$

$= \frac{1}{2}[6n - 3 + 7] = 3n + 2$

Thus n^{th} term, $a_n = 3n + 2$

Now 20th term $a_{20} = 3 \times 20 + 2 = 62$

PRACTICE

In an AP the sum of first n terms is $\frac{3n^2}{2} + \frac{13n}{2}$. Find the 25^{th} term.

[Board Term-2 SQP 2015]

Ans : 80

149. In an AP, if the 12^{th} term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms.

$$693 = 306 + (n-1)9$$
$$(n-1)9 = 693 - 306 = 387$$
$$n-1 = \frac{387}{9} = 43$$
$$n = 43 + 1 = 44$$

Hence there are 44 terms.

157. If the sum of the first 14 terms of an AP is 1050 and its first term is 10 find it 20^{th} term.

Sol : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 10$, and $S_{14} = 1050$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{14} = \frac{14}{2}\left[2 \times 10 + (14-1)d\right]$$
$$1050 = 7\left[20 + 13d\right]$$
$$20 + 13d = \frac{1050}{7} = 150$$
$$13d = 130 \Rightarrow d = 10$$
$$a_{20} = a + (n-1)d$$
$$= 10 + 19 \times 10$$
$$= 200$$

Hence $a_{20} = 200$

158. If the tenth term of an AP is 52 and the 17^{th} term is 20 more than the 13^{th} term, find AP

Sol : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

Now $a_{10} = 52$

$$a + 9d = 52 \qquad ...(1)$$

Also $a_{17} - a_{13} = 20$

$$a + 16d - (a + 12d) = 20$$
$$4d = 20$$
$$d = 5$$

Substituting this valued d in (1), we get

$$a = 7$$

Hence AP is 7, 12, 17, 22, ...

159. Find the sum of all odd number between 0 and 50.

Sol : [Board Term-2 Delhi Compt 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Given AP is $1 + 3 + 5 + 7 + ... + 49$

Let total number of terms be n. Here $a = 1$, $d = 2$ and $a_n = 49$.

$$a_n = 1 + (n-1) \times 2$$
$$49 = 1 + 2n - 2$$
$$50 = 2n$$
$$n = 25$$

Now $S_{25} = \frac{n}{2}(a + a_n)$

$$= \frac{25}{2}(1 + 49)$$
$$= 25 \times 25 = 625$$

Hence, Sum of odd number is 625.

PRACTICE

- Find the sum of the integers between 100 and 200 that are divisible by 6.
  [Board Term-2 2012]
  Ans : 2550

- Find the sum of all two digit natural numbers which are divisible by 4.
  [Board Term-2 Delhi Compt. 2017]
  Ans : 1188

- Find the sum of the two digits numbers divisible by 6.
  [Board Term-2 2013]
  Ans : 810

- Find the sum of all two digits odd positive numbers.
  [Board Term-2 2014]
  Ans : 2475

160. If m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ find the sum of first mn terms.

Sol : [Board Term-2 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Now $a_m = a + (m-1)d = \frac{1}{n}$ ...(1)

$$a_n = a + (n-1)d = \frac{1}{m} \qquad ...(2)$$

Subtracting (2) from (1) we get

$$(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$$

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$$16d = 48 \Rightarrow d = 3$$

Thus $a = 4$ and $d = 3$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{10} = \frac{10}{2}[2 \times 4 + (10-1)3]$$
$$= 5[8 + 27] = 5 \times 35 = 175$$

Thus $S_{10} = 175$

PRACTICE

The 16^{th} term of an AP is five times its third term. If its 10^{th} term is 41, then find the sum of its first fifteen terms.

[Board Term-2 OD 2015]

Ans : 495

153. The sum of first 7 terms of an AP is 63 and sum of its next 7 terms is 161. Find 28^{th} term of AP.

Sol : [Board Term-2 Foreign 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now, $S_7 = 63$

$$\frac{7}{2}[2a + 6d] = 63$$
$$2a + 6d = 18 \quad ...(1)$$

Also, sum of next 7 terms,

$$S_{14} = S_{first7} + S_{next7} = 63 + 161$$
$$\frac{14}{2}[2a + 13d] = 224$$
$$2a + 13d = 32 \quad (2)$$

Subtracting equation (1) form (2) we get

$$7d = 14 \Rightarrow d = 2$$

Substituting the value of d in (1) we get

$$a = 3$$

Now $a_n = a + (n-1)d$

$$a_{28} = 3 + 2 \times (27)$$
$$= 57$$

Thus 28^{th} term is 57.

154. Find the sum of all two digit natural numbers which are divisible by 4.

Sol : [Board Term-2 Delhi Compt. 2017]

First two digit multiple of 4 is 12 and last is 96

So, $a = 12, d = 4$. Let n^{th} term be last term $a_n = 96$

Now $a + (n-1)d = a_n$

$$12 + (n-1)4 = 96$$
$$(n-1)4 = 96 - 12 = 84$$
$$n - 1 = 21$$
$$n = 21 + 1 = 22$$

Now, $S_{22} = \frac{22}{2}[12 + 96]$

$$= 11 \times 108$$
$$= 1188$$

155. Find the sum of the following series.

$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)$

Sol : [Board Term-2 Foreign 2017]

The given series can be written as sum of two series

$(5 + 9 + 13 + ... + 81) +$
$+(-41) + (-39) + (-37) + (-35)$
$...(-5) + (-3)$

For the series $(5 + 9 + 13 .....81)$

$a = 5$, $d = 4$ and $a_n = 81$

Now $a_n = a + (n-1)d$

$$81 = 5 + (n-1)4$$
$$81 = 5 + (n-1)4$$
$$(n-1)4 = 76 \Rightarrow n = 20$$
$$S_n = \frac{20}{2}(5 + 81) = 860$$

For series $(-41) + (-39) + (-37) + ... + (-5) + (-3)$

$a_n = -3$, $a = -41$ and $d = 2$

$$a_n = -41 + (n-1)(2)$$
$$-3 = -41 + 2n - 2 \Rightarrow n = 20$$

Now $S_n = \frac{20}{2}[-41 + -3] = -440$

Sum of the series $= 860 - 440 = 420$

156. Find the number of multiple of 9 lying between 300 and 700.

Sol : [Board Term-2 OD Compt. 2017]

The numbers, multiple of 9 between 300 and 700 are 306, 315, 324, ... 693.

Let the first term be a, common difference be d and nth term be $a_n = 693$

$$a_n = 306 + (n-1)9$$

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$$2a(m-n)+d[m^2-n^2-(m-n)] = 2(n-m)$$
$$2a(m-n)+d(m-n)[(m+n)-1] = 2(n-m)$$
$$2a+d[(m+n)-1] = -2$$

Now, $S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d]$

$$= \frac{m+n}{2}(-2)$$
$$= -(m+n)$$

134. The 17^{th} term of an AP is 5 more than twice its 8^{th} term. If 11^{th} term of AP is 43, then find its n^{th} term.

Sol : [Board 2020 OD Basic]

Let a be the first term and d be the common difference.

n^{th} term of an AP,

$$a_n = a+(n-1)d$$

Since 17^{th} term of an AP is 5 more than twice of its 8^{th} term, thus

$$a+(17-1)d = 5+2[a+(8-1)d]$$
$$a+16d = 5+2(a+7d)$$
$$a+16d = 5+2a+14d$$
$$2d-a = 5 \quad ...(1)$$

Since 11^{th} term of AP is 43,

$$a+(11-1)d = 43$$
$$a+10d = 43 \quad ...(2)$$

Solving equation (1) and (2), we have

$$a = 3 \text{ and } d = 4$$

Hence, n^{th} term would be

$$a_n = 3+(n-1)4 = 4n-1$$

135. How many terms of the AP 24, 21, 18, ... must be taken so that their sum is 78?

Sol : [Board 2020 Delhi Basic]

Given : 24, 21, 18, ... are in AP.

Here, $a = 24,\ d = 21-24 = -3$

Sum of n term, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$78 = \frac{n}{2}[2\times 24+(n-1)(-3)]$$
$$156 = n(48-3n+3)$$
$$156 = n(51-3n)$$
$$3n^2-51n+156 = 0$$
$$n^2-17n+52 = 0$$
$$n^2-13n-4n+52 = 0$$
$$(n-4)(n-13) = 0 \Rightarrow n = 4, 13$$

When $n = 4$,

$$S_4 = \frac{4}{2}[2\times 24+(4-1)(-3)]$$
$$= 2(48-9) = 2\times 39 = 78$$

When $n = 13$,

$$S_{13} = \frac{13}{2}[2\times 24+(13-1)(-3)]$$
$$= \frac{13}{2}[48+(-36)] = 78$$

Hence, the number of terms $n = 4$ or $n = 13$.

PRACTICE

✎ How many terms of an AP $9, 17, 25, ...$ must be taken to give a sum of 636?

[Board Term-2 Delhi Compt 2015]

Ans : 12

136. Find the 20^{th} term of an AP whose 3^{rd} term is 7 and the seventh term exceeds three times the 3^{rd} term by 2. Also find its n^{th} term (a_n).

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = a+2d = 7 \quad (1)$

$$a_7 = 3a_3+2$$
$$a+6d = 3\times 7+2 = 23 \quad (2)$$

Solving (1) and (2) we have

$$4d = 16 \Rightarrow d = 4$$
$$a+8 = 7 \Rightarrow a = -1$$
$$a_{20} = a+19d = -1+19\times 4 = 75$$
$$a_n = a+(n-1)d$$
$$= -1+4n-4 = 4n-5.$$

Hence n^{th} term is $4n-5$.

137. If 7^{th} term of an AP is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term.

Sol : [Board Term-2 Delhi 2014]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_7 = \frac{1}{9} \Rightarrow a+6d = \frac{1}{9} \quad (1)$

$$a_9 = \frac{1}{7} \Rightarrow a+8d = \frac{1}{7} \quad (2)$$

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$$d = \frac{1}{mn}$$

Substituting this value of d in equation (1), we get

$$a = \frac{1}{mn}$$

Now, $$S_{mn} = \frac{mn}{2}\left(\frac{2}{mn} + (mn-1)\frac{1}{mn}\right)$$

$$= 1 + \frac{mn}{2} - \frac{1}{2} = \frac{1}{2} + \frac{mn}{2}$$

$$= \frac{1}{2}[mn+1]$$

Hence, the sum of mn term is $\frac{1}{2}[mn+1]$.

PRACTICE

✎ If the p^{th} term of an AP is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$. Prove that the sum of first pq term of the AP is $\frac{1}{2}(pq+1)$.

[Board Term-2 Delhi 2017]

Ans : Proof

161. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Sol : [Board Term-2 OD 2014]

The sequence goes like 110, 120, 130, ... 990

Since they have a common difference of 10, they form an AP. Let the first term be a, common difference be d, nth term be a_n.

Here $a = 110,\ a_n = 990,\ d = 10$

$$a_n = a + (n-1)d$$

$$990 = 110 + (n-1) \times 10$$

$$990 - 110 = 10(n-1)$$

$$880 = 10(n-1)$$

$$88 = n - 1$$

$$n = 88 + 1 = 89$$

Hence, there are 89 terms between 101 and 999 divisible by both 2 and 5.

162. If the ratio of the 11^{th} term of an AP to its 18^{th} term is 2 : 3, find the ratio of the sum of the first five term of the sum of its first 10 terms.

Sol : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now $$\frac{a_{11}}{a_{18}} = \frac{a+10d}{a+17d} = \frac{2}{3}$$

$$2(a+17d) = 3(a+10d)$$

$$a = 4d \quad \ldots(1)$$

Now, $$\frac{S_5}{S_{10}} = \frac{\frac{5}{2}(2a+4d)}{\frac{10}{2}[2a+9d]}$$

$$= \frac{(a+2d)}{[2a+9d]}$$

Substituting the value $a = 4d$ we have

or, $$\frac{S_5}{S_{10}} = \frac{4d+2d}{8d+9d} = \frac{6}{17}$$

Hence $S_5 : S_{10} = 6:17$

163. How many three digit numbers are such that when divided by 7, leave a remainder 3 in each case?

Sol : [Board Term-2 2012]

When a three digit number divided by 7 and leave 3 as remainder are 101, 108, 115, ... 997

These are in AP. Let the first term be a, common difference be d, nth term be a_n.

Here $a = 101, d = 7, a_n = 997$

Now $$a_n = a + (n-1)d$$

$$997 = 101 + (n-1)7$$

$$997 - 101 = 896 = (n-1)7$$

$$\frac{896}{7} = n - 1$$

$$n = 128 + 1 = 129$$

Hence, 129 numbers are divided by 7 which leaves remainder is 3.

164. Prove that the n^{th} term of an AP can not be n^2+1. Justify your answer.

Sol : [Board Term-2 2015]

Let n^{th} term of AP,

$$a_n = n^2 + 1$$

Substituting the value of $n = 1, 2, 3, \ldots..$ we get

$$a_1 = 1^2 + 1 = 2$$

$$a_2 = 2^2 + 1 = 5$$

$$a_3 = 3^2 + 1 = 10$$

The obtained sequence is 2, 5, 10, 17,.....

Its common difference

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3$$

$$5 - 2 \neq 10 - 5 \neq 17 - 10$$

$$3 \neq 5 \neq 7$$

Since the sequence has no. common difference, n^2+1 is not a form of n^{th} term of an AP

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$$S_3 = \frac{3n}{2}\left[2a+(3n-1)d\right]$$

$$3(S_2 - S_1) = 3\left[\frac{2n}{2}\left[2a+(2n-1)d\right] - \frac{n}{2}\left[2a+(n-1)d\right]\right]$$

$$= 3\left[\frac{n}{2}\left[4a+2(2n-1)d\right] - \left[2a+(n-1)d\right]\right]$$

$$= 3\left[\frac{n}{2}(4a+4nd-2d-2a-nd+d)\right]$$

$$= 3\left[\frac{n}{2}(2a+3nd-d)\right]$$

$$= \frac{3n}{2}\left[2a+(3n-1)d\right] = S_3$$

187. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the AP.

Sol : [Board Term-2 SQP 2017]

Let the middle most terms of the AP be $(x-d)$, x and $(x+d)$.

We have $x-d+x+x+d = 225$

$$3x = 225 \Rightarrow x = 75$$

and the middle term $= \frac{37+1}{2} = 19^{th}$ term

Thus AP is

$(x-18d), \ldots (x-2d), (x-d), x, (x+d), (x+2d), \ldots\ldots$

$(x-18d)$

Sum of last three terms,

$$(x+18d)+(x+17d)+(x+16d) = 429$$

$$3x+51d = 429$$

, $$225+51d = 429 \Rightarrow d = 4$$

First term $a_1 = x-18d = 75-18\times 4 = 3$

$$a_2 = 3+4 = 7$$

Hence AP = 3, 7, 11, ..., 147.

WORD PROBLEMS

188. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7:15. Find the numbers.

Sol : [Board 2018]

Let the four consecutive terms of AP be $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$

By given conditions

$$a-3d+a-d+d+a+3d = 32$$

$$4a = 32 \Rightarrow a = 8$$

and $$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$\frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$$

$$d^2 = 4$$

$$d = \pm 2$$

Number are 2, 6, 10 and 14 or 14, 10, 6 and 2

189. The digit of a positive number of three digits are in AP and their sum is 15. The number obtained by reversing the digits is 594 less then the original number. Find the number.

Sol : [Board Term-2 OD 2016]

Let these digit of 3 digit number be $a-d, a, a+d$

Since their sum is 15,

$$a-d+a+a+d = 15$$

$$3a = 15 \Rightarrow a = 5$$

$$\text{Required 3 digit number} = 100(a-d)+10a+a+d$$

$$= 100a-100d+10a+a+d$$

$$= 111a-99d$$

Number obtained by reversing digit,

$$= 100(a+d)+10a+a-d$$

$$= 100a+100d+10a+a-d$$

$$= 111a+99d$$

According the question, we have

$$111a+99d = 111a-99d-594$$

$$2\times 99d = -594 \Rightarrow d = -3$$

Thus number is

$$111a-99d = 111\times 5-99\times -3$$

$$= 555+297$$

$$= 852$$

190. The minimum age of children to be eligible to participate in a painting competition is 8 years. It is observed that the age of youngest boy was 8 years and the ages of rest of participants are having a common difference of 4 months. If the sum of ages of all the participants is 168 years, find the age of eldest participant in the painting competition.

Sol : [Board Term-2 SQP 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 8, d = 4$ month $= \frac{1}{3}$ years, $S_n = 168$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

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$$a^2 - 32a + 60 = 0$$
$$a^2 - 30a - 2a + 60 = 0$$
$$(a-30)(a-2) = 0$$

Thus $a = 2$ and $a = 30$.

Substituting $a = 2$ in (4) we get $d = 4$.

Substituting $a = 30$ in (4) we get $d = -4$.

For $a = 2$ and $d = 4$, sum of first 20 terms

$$S_{20} = \frac{20}{2}[2(2) + 19(4)]$$
$$= 10(4 + 76)$$
$$= 800$$

For $a = 30$ and $d = -4$, sum of first 20 terms

$$S_{20} = \frac{20}{2}[2(30) + 19(-4)]$$
$$= 10(60 - 76)$$
$$= -160$$

168. In an A.P. of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and common difference of A.P. Also, find the sum of all the terms of the A.P.

Sol : [Board 2024 OD Standard]

Let the first term and common difference of A.P. be a and d respectively.

The sum of first 9 terms is 153. Thus

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$153 = \frac{9}{2}[2a + (9-1)d]$$
$$17 = \frac{1}{2}[2a + 8d]$$
$$17 = a + 4d \quad (1)$$

Since the sum of last 6 terms is 687,

$$S_{\text{last 6 term}} = S_{40} - S_{34}$$
$$687 = \frac{40}{2}[2a + 39d] - \frac{34}{2}[2a + 33d]$$
$$687 = 20(2a + 39d) - 17(2a + 33d)$$
$$687 = 6a + 219d \quad \ldots(2)$$

Substituting $a = 17 - 4d$ from (1) we have

$$687 = 6(17 - 4d) + 219d$$
$$687 = 102 - 24d + 219d$$
$$687 = 102 + 195d$$
$$585 = 195d \Rightarrow d = 3$$

Substituting $d = 3$ in (1) have

$$a = 17 - 4(3) = 5$$

Sum of 40 terms,

$$S_{40} = \frac{40}{2}[2(5) + 39(3)]$$
$$= 20(10 + 117)$$
$$= 20(127)$$
$$= 2540$$

169. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

Sol : [Board 2019 Delhi]

Let a be the first term and d be the common difference.

Sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now $S_4 = 40$ and $S_{14} = 280$

$$\frac{4}{2}[2a + (4-1)d] = 40$$
$$2[2a + 3d] = 40$$
$$2a + 3d = 20 \quad (1)$$

and $$\frac{14}{2}[2a + (14-1)d] = 280$$
$$7[2a + 13d] = 280$$
$$2a + 13d = 40 \quad (2)$$

Solving equations (1) and (2), we get

$$a = 7 \text{ and } d = 2$$

Now $$S_n = \frac{n}{2}[2 \times 7 + (n-1)2]$$
$$= \frac{n}{2}[14 + 2n - 2]$$
$$= \frac{n}{2}(12 + 2n) = 6n + n^2$$

Hence, sum of n terms is $6n + n^2$.

170. The first term of an AP is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the AP.

Sol : [Board 2019 Delhi]

First term, $a = 3$

Last term, $a_n = 83$

Sum of n terms, $S_n = 903$

Since, $S_n = \frac{n}{2}(a + a_n)$

$$903 = \frac{n}{2}(3 + 83)$$

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$$= (4+7+10+....+n \text{ terms}) - \frac{1}{n}(1+2+3+...n)$$

$$= \frac{n}{2}[2\times 4+(n-1)(3)] - \frac{1}{n}\times\frac{n}{2}[2\times 1+(n-1)(1)]$$

$$= \frac{n}{2}[8+3n-3] - \frac{1}{2}[2+n-1]$$

$$= \frac{n}{2}(3n+5) - \frac{1}{2}(n+1)$$

$$= \frac{3n^2+5n-n-1}{2}$$

$$= \frac{3n^2+4n-1}{2}$$

PRACTICE

Find the sum of n terms of the series
$\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+.....$
[Board Term-2 Delhi 2017]
Ans : $\frac{1}{2}(7n-1)$

179. Find the 60^{th} term of the AP $8,10,12,....$, if it has a total of 60 terms and hence find the sum of its last 10 terms.

Sol : [Board Term-2 OD 2015]
Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n
We have $a=8, d=10-8=2$

$$a_n = a+(n-1)d$$

Now $a_{60} = 8+(60-1)2 = 8+59\times 2 = 126$

and $a_{51} = 8+50\times 2 = 8+100 = 108$

Sum of last 10 terms,

$$S_{51-60} = \frac{n}{2}(a_{51}+a_{60})$$

$$= \frac{10}{2}(108+126)$$

$$= 5\times 234 = 1170$$

Hence sum of last 10 terms is 1170.

180. An arithmetic progression $5,12,19,.....$ has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

Sol : [Board Term-2 OD 2015]
Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
We have $a=5$, $d=12-5=7$ and $n=50$

$$a_{50} = 5+(50-1)7$$

$$= 5+49\times 7 = 348$$

Also the first term of the AP of last 15 terms be a_{36}

$$a_{36} = 5+35\times 7$$

$$= 5+245 = 250$$

Now, sum of last 15 terms,

$$S_{36-50} = \frac{15}{2}[a_{36}+a_{50}]$$

$$= \frac{15}{2}[250+348]$$

$$= \frac{15}{2}\times 598 = 4485$$

Hence, sum of last 15 terms is 4485.

181. If the sum of first n term of an AP is given by $S_n = 3n^2+4n$. Determine the AP and the n^{th} term.

Sol : [Board Term-2 2014]
Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.
We have $S_n = 3n^2+4n.$

$$a_1 = 3(1)^2+4(1) = 7$$

$$a_1+a_2 = S_2 = 3(2)^2+4(2)$$

$$= 12+8 = 20$$

$$a_2 = S_2-S_1 = 20-7 = 13$$

$$a+d = 13$$

or, $7+d = 13$

Thus $d = 13-7 = 6$

Hence AP is $7,13,19,.....$

Now, $a_n = a+(n-1)d$

$$= 7+(n-1)(6)$$

$$= 7+6n-6 = 6n+1$$

$$a_n = 6n+1$$

182. The sum of the 3^{rd} and 7^{th} terms of an AP is 6 and their product is 8. Find the sum of first 20 terms of the AP.

Sol : [Board Term-2 2012]
Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n
We have $a_3+a_7 = 6$

$$a+2d+a+6d = 6$$

$$a+4d = 3 \quad (1)$$

and $a_3\times a_7 = 8$

$$(a+2d)(a+6d) = 8 \quad (2)$$

Substituting the value $a=(3-4d)$ in (2) we get

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Here it may be easily seen that sum of initial 6 terms is 180. Sum of next 4 terms is zero. Thus sum of 10 terms is also 180.

PRACTICE

How many terms of the AP $-6, \frac{11}{2}, -5, ...$ are needed to given the sum -25? Explain the double answer.

[Board Term-2 2012]

Ans : 5, 20

174. The sum of three numbers in AP is 12 and sum of their cubes is 288. Find the numbers.

Sol : [Board Term-2 Delhi 2016]

Let the three numbers in AP be $a-d,\ a,\ a+d$.

$$a-d+a+a+d = 12$$
$$3a = 12$$
$$a = 4$$

Also, $(4-d)^3+4^3+(4+d)^3 = 288$

$$64-48d+12d^2-d^3+64+64+48d+12d^2+d^3 = 288$$
$$24d^2+192 = 288$$
$$d^2 = 4$$
$$d = \pm 2$$

The numbers are 2, 4, 6 or 6, 4,2

175. Find the value of a, b and c such that the numbers $a, 7, b, 23$ and c are in AP.

Sol : [Board Term-2 2015]

Let the common difference be d.

Since $a, 7, b, 23$ and c are in AP, we have

$$a+d = 7 \quad ..(1)$$
$$a+3d = 23 \quad ...(2)$$

Form equation (1) and (2), we get

$$a = -1, d = 8$$
$$b = a+2d = -1+2\times 8$$
$$= -1+16 = 15$$
$$c = a+4d = -1+4\times 8$$
$$= -1+32 = 31$$

Thus $a = -1, b = 15, c = 31$

176. If S_n denotes the sum of first n terms of an AP, prove that, $S_{30} = 3(S_{20}-S_{10})$

Sol : [Board Term-2 Delhi 2015, Foreign 2014]

Let the first term be a, and common difference be d.

Now $S_{30} = \frac{30}{2}(2a+29d)$ ...(1)

$$= 15(2a+29d)$$

$$3(S_{20}-S_{10}) = 3\left[10(2a+19d)-5(2a+9d)\right]$$
$$= 3[20a+190d-10a-45d]$$
$$= 3[10a+145d]$$
$$= 15[2a+29d] \quad ...(2)$$

Hence $S_{30} = 3(S_{20}-S_{10})$

177. The sum of first 20 terms of an AP is 400 and sum of first 40 terms is 1600. Find the sum of its first 10 terms.

Sol : [Board Term-2 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We know $S_n = \frac{n}{2}[2a+(n-1)d]$

Now $S_{20} = \frac{20}{2}(2a+19d)$

$$400 = \frac{20}{2}(2a+19d)$$
$$400 = 10[2a+19d]$$
$$2a+19d = 40 \quad (1)$$

Also, $S_{40} = \frac{40}{2}(2a+39d)$

$$1600 = 20[2a+39d]$$
$$2a+39d = 80 \quad (2)$$

Solving equation (1) and (2), we get $a = 1$ and $d = 2$.

Now $S_{10} = \frac{10}{2}[2\times 1+(10-1)(2)]$

$$= 5[2+9\times 2]$$
$$= 5[2+18]$$
$$= 5\times 20 = 100$$

178. Find $\left(4-\frac{1}{n}\right)+\left(7-\frac{2}{n}\right)+\left(10-\frac{3}{n}\right)+.....$ upto n terms.

Sol : [Board Term-2 2015]

Let sum of n term be S_n, then we have

$s_n = \left(4-\frac{1}{n}\right)+\left(7-\frac{2}{n}\right)+\left(40-\frac{3}{n}\right)+.....$ upto n terms.

$$= (4+7+10+.....+n \text{ terms})-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}.....+1\right)$$

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FIVE MARKS QUESTIONS

165. The ratio of the 11^{th} term to the 18^{th} term of an A.P. is 2 : 3. Find the ratio of the 5^{th} term to the 21^{st} term. Also, find the ratio of the sum of first 5 terms to the sum of first 21 terms.

Sol : [Board 2023 OD Standard]

Let a be the first term and d be the common difference.

Since the ratio of the 11^{th} term to the 18^{th} term is 2 : 3, we have

$$\frac{a_{11}}{a_{18}} = \frac{2}{3}$$
$$\frac{a+(11-1)d}{a+(18-1)d} = \frac{2}{3}$$
$$\frac{a+10d}{a+17d} = \frac{2}{3}$$
$$3a+30d = 2a+34d$$
$$a = 4d \quad ...(1)$$

Ratio of 5^{th} term to 21^{st} term is

$$\frac{a_5}{a_{21}} = \frac{a+(5-1)d}{a+(21-1)d} = \frac{a+4d}{a+20d}$$

Substitute value of $a = 4d$ from (1) we get

$$\frac{a_5}{a_{21}} = \frac{4d+4d}{4d+20d} = \frac{8d}{24d}$$
$$\frac{a_5}{a_{21}} = \frac{1}{3}$$

Now $S_n = \frac{n}{2}(2a+(n-1)d)$

Ratio of S_5 to S_{21} is

$$\frac{S_5}{S_{21}} = \frac{\frac{5}{2}(2a+4d)}{\frac{21}{2}(2a+20d)} = \frac{5(2a+4d)}{21(2a+20d)} \quad ...(ii)$$

Substitute $a = 4d$ in equation (2) we have

$$\frac{S_5}{S_{21}} = \frac{5(8d+4d)}{21(8d+20d)} = \frac{5\times 12d}{21\times 28d} = \frac{60}{588}$$
$$\frac{S_5}{S_{21}} = \frac{5}{49}$$

Hence $a_5 : a_{21} = 1:3$

$$S_5 : S_{21} = 5:49$$

166. If the sum of first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

Sol : [Board 2023 OD Standard]

We have $S_n = \frac{n}{2}[2a+(n-1)d]$

Now $S_6 = 36$

$$\frac{6}{2}[2a+(6-1)d] = 36$$
$$3[2a+5d] = 36$$
$$2a+5d = 12 \quad ...(1)$$

Now $S_{16} = 256$

$$\frac{16}{2}[2a+(6-1)d] = 256$$
$$8[2a+5d] = 256$$
$$2a+15d = 32 \quad ...(2)$$

Subtract (1) from (2) we have

$$10d = 20 \Rightarrow d = 2$$

Substitute $d = 2$ in equation (1) we have

$$2a+5\times 2 = 12$$
$$2a = 12-10$$
$$2a = 2 \Rightarrow a = 1$$

Thus, the sum of first 10 terms of given AP,

$$S_{10} = \frac{10}{2}[2\times 1+(10-1)2]$$
$$= 5(2+18)$$
$$= 5\times 20 = 100$$

167. The sum of first and eighth terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms.

Sol : [Board 2024 OD Standard]

Let the first term of A.P. be a, common difference be d and eighth term of A.P. be a_8,

$$a_8 = a+7d \quad ...(1)$$

Since the sum of first and eighth terms of an A.P. is 32 and their product is 60. Therefore

$$a+a_8 = 32 \quad ...(2)$$
$$a \cdot a_8 = 60 \quad ...(3)$$

From (1) and (2), we get

$$a+a+7d = 32$$
$$2a+7d = 32 \quad ...(4)$$

From (1) and (3), we get

$$a(a+7d) = 60$$
$$a(32-a) = 60 \quad \text{From (4)}$$
$$32a-a^2 = 60$$

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$$(3-4d+2d)(3-4d+6d) = 8$$
$$(3+2d)(3-2d) = 8$$
$$9-4d^2 = 8$$
$$4d^2 = 1 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$$

CASE 1 : Substituting $d = \frac{1}{2}$ in equation (1), $a = 1$.

$$S_{20} = \frac{n}{2}[2a+(n-1)d]$$
$$= \frac{20}{2}\left[2+\frac{19}{2}\right] = 115$$

Thus $d = \frac{1}{2}, a = 1$ and $S_{20} = 115$

CASE 2 : Substituting $d = -\frac{1}{2}$ in equation (1) $a = 5$

$$S_{20} = \frac{20}{2}\left[2\times 5 + 19\times\left(-\frac{1}{2}\right)\right]$$
$$= 10\left[10-\frac{19}{2}\right] = 15$$

Thus $d = -\frac{1}{2}, a = 5$ and $S_{20} = 15$

183. If the sum of first m terms of an AP is same as the sum of its first n terms $(m \neq n)$, show that the sum of its first $(m+n)$ terms is zero.

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n, and sum of n term be S_n

Now $S_m = S_n$

$$\frac{m}{2}[2a+(m-1)d] = \frac{n}{2}[2a+(n-1)d]$$
$$2ma+m(m-1)d = 2na+n(n-1)d$$
$$2a(m-n)+[(m^2-n^2)-m+n]d = 0$$
$$2a(m-n)+[(m-n)(m+n)-(m-n)]d = 0$$
$$(m-n)[2a+(m+n-1)d] = 0$$
$$2a+(m+n-1)d = 0 \quad [m-n \neq 0]$$
$$S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d]$$
$$= \frac{m+n}{2}\times 0 = 0$$

184. If $1+4+7+10.....+n = 287$, find the value of n.

Sol : [Board 2020 Std, Board Term-2 Foreign 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 1, d = 3$ and $S_n = 287$.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$\frac{n}{2}[2\times 1+(n-1)3] = 287$$
$$\frac{n}{2}[2+(3n-3)] = 287$$
$$3n^2-n = 574$$
$$3n^2-n-574 = 0$$
$$3n^2-42n+41n-574 = 0$$
$$3n(n-14)+41(n-14) = 0$$
$$(n-14)(3n+41) = 0$$

Since negative value is not possible, $n = 14$

$$a_{14} = a+(n-1)d$$
$$= 1+13\times 3 = 40$$

185. Find the number of terms of the AP $-12,-9,-6,.....,21$. If 1 is added to each term of this AP, then find the sum of all the terms of the AP thus obtained.

Sol : [Board Term-2 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = -12, d = -9-(-12) = 3$

$$a_n = a+(n-1)d$$
$$21 = -12+(n-1)\times 3$$
$$21+12 = (n-1)\times 3$$
$$33 = (n-1)\times 3$$
$$n-1 = 11$$
$$n = 11+1 = 12$$

Now, if 1 is added to each term we have a new AP with $-12+1,-a+1,-6+1.....21+1$

Now we have $a = -11, d = 3$ and $a_n = 22$ and $n = 12$

Sum of this obtained AP,

$$S_{12} = \frac{12}{2}[-11+22] = 6\times 11 = 66$$

Hence the sum of new AP is 66.

186. If S_1, S_2, S_3 be the sum of $n, 2n, 3n$ terms respectively of an AP, prove that $S_3 = 3(S_2-S_1)$.

Sol : [Board Term-2 2012]

Let the first term be a, and common difference be d.

Now $S_1 = \frac{n}{2}[2a+(n-1)d]$

$$S_2 = \frac{2n}{2}[2a+(2n-1)d]$$

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Sum of first 14 terms,

$$s_{14} = \frac{14}{2}(a + a_{14})$$
$$= \frac{14}{2}[10 + 49] = 413$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Sum of the last 14 terms,

$$s_{16-29} = \frac{14}{2}(a_{16} + a_{29})$$
$$= \frac{14}{2}[55 + 94]$$
$$= 7 \times 149$$
$$= 1043$$

194. A sum of Rs. 280 is to be used towards four prizes. If each prize after the first is Rs. 20 less than its preceding prize, find the value of each of the prizes.

Sol : [Board Term-2 2012]

Let first prize be Rs. x, then series of prize is

$x,\ x-20,\ x-40,\ x-60, ...$

Here series is AP and $a = x,\ d = -20,\ n = 4$ and $S_n = 280$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$280 = \frac{4}{2}[2x + 3(-20)]$$
$$280 = 2[2x - 60]$$
$$140 = 2x - 60$$
$$x = \frac{140 + 60}{2} = 100$$

Thus prizes are Rs. 100, Rs. 80, Rs. 60, Rs. 40.

195. In a garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row. How many rows are there of rose plants> also find the total number of roes plants in the garden.

Sol : [Board Term-2 2012]

The number of rose plants in the $1^{st}, 2^{nd}, ...$ are $23, 21, 19, ..., 5$.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 23, d = -2, a_n = 5$

$$a_n = a + (n-1)d$$
$$5 = 23 + (n-1)(-2)$$
$$n = 10$$

Total number of roes plants in the flower bed,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{10} = 5(46 - 18) = 140$$

196. A sum of Rs. 1890 is to be used to given seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 50 less than its preceding prize, find the value of each of the prizes.

Sol : [Board Term-2 2012]

Let first prize be Rs. x, then series of prize is

$x,\ x-50,\ x-100,\ x-150, ...$

Here series is AP and $a = x,\ d = -50,\ n = 7$ and $S_n = 1890$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$1890 = \frac{7}{2}[2x + (-50) \times 6]$$
$$270 = x + (-50) \times 3 = x - 150$$
$$x = 270 + 150 = 420$$

The prizes are Rs. 420, Rs. 370, Rs. 320, Rs. 270, Rs. 220, Rs. 170, Rs. 120.

197. A man repays a loan of Rs. 3250 by paying Rs. 20 in the first month and then increases the payment by Rs. 15 every month. How long will it take him to clear the loan?

Sol : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 20, d = 15$ and $S_n = 3250$

Now
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$3250 = \frac{n}{2}[2a + (n-1) \times 15]$$
$$3250 \times 2 = n[40 + 15n - 15]$$
$$6500 = n[25 + 15n]$$
$$1300 = n[5 + 3n]$$
$$3n^2 + 65n - 60n - 1300 = 0$$
$$n(3n + 65) - 20(3n + 65) = 0$$
$$(n - 20)(3n + 65) = 0$$

Since $n = -65/3$, is not possible, $n = 20$

Man will repay loan in 20 months.

198. A spiral is made up of successive semi-circles with centres alternately A and B starting with A, of radii 1 cm, 2 cm, 3 cm, ... as shown in the

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$$168 = \frac{n}{2}\left[2(8)+(n-1)\frac{1}{3}\right]$$
$$n^2+47n-1008 = 0$$
$$n^2+63n-16n-1008 = 0$$
$$(n-16)(n+63) = 0$$

Thus $n = 16$ or $n = -63$

As n cannot be negative, we take $n = 16$

Age of the eldest participant $= a+15d = 13$ years

191. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100/minute in the first minute and increased his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief.

Sol : [Board Term-2 OD 2016]

Let total time to catch the thief be n minutes

Total distance covered by thief will be $100n$

Total distance covered by policeman,

$= 100+110+120+....+(n-1)$ terms

$$100n = \frac{n-1}{2}[200+(n-2)10]$$
$$n^2-3n-18 = 0$$
$$(n-6)(n+3) = 0$$
$$n = 6$$

Policeman takes 5 minutes to catch the thief.

192. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3, when divided by 4. Also find the sum of all numbers on both sides of the middle terms separately.

Sol : [Board Term-2 Foreign 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

The three digit numbers which leaves 3 as remainder when divided by 4 are: $103, 107, 111, ... 999$.

Now, the first number $a = 103$, last number $a_n = 999$ and common difference $d = 4$

Let the number of terms in this sequence be n.

$$a_n = a+(n-1)d$$
$$999 = 103+(n-1)4$$
$$896 = (n-1)4$$
$$(n-1) = \frac{896}{4} = 224$$
$$n = 224+1 = 225$$

$$\text{Middle term} = \frac{225+1}{2}$$
$$= 113^{th} \text{ term}$$
$$a_{113} = 103+112\times 4 = 551$$

and $a_{112} = 551-4 = 547$

Sum of first 112 terms,

$$S_{112} = \frac{112}{2}(a+a_{112})$$
$$= 56(103+547)$$
$$= 56\times 650 = 36400$$

and $a_{114} = 551+4 = 555$

The sum of last 112 terms,

$$S_{114-225} = \frac{112}{2}(a_{114}+a_{225})$$
$$= 56(555+999)$$
$$= 56\times 1554 = 87024$$

PRACTICE

Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 5 when divided by 7. Also find the sum of all numbers on both sides of the middle term separately.

[Board Term-2 Foreign 2014, 2015]

Ans : 49824

193. Find the middle term of the sequence formed all numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately.

Sol : [Board Term-2 Foreign 2015]

The sequence is 10, 13, ... 94

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a = 10,\ d = 3,\ S_n = 94$

$$94 = 10+(n-1)3$$
$$84 = (n-1)3$$
$$n = \frac{84}{3}+1 = 29$$

Therefore $\frac{29+1}{2} = 15^{th}$ term is the middle term.

$$\text{Middle term } a_{15} = a+(15-1)d$$
$$= 10+14\times 3 = 52$$
$$a_{14} = 52-3 = 49$$
$$a_{16} = 52+3 = 55$$

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$$1806 = 86n$$
$$n = \frac{1806}{86} \Rightarrow n = 21$$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$903 = \frac{21}{2}[2 \times 3 + (21-1)d]$$
$$1806 = 21(6 + 20d)$$
$$6 + 20d = 86$$
$$20d = 80 \Rightarrow d = 4$$

Hence, the common difference is 4.

171. Find the common difference of the Arithmetic Progression (AP) $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots (a \neq 0)$

Sol : [Board 2019 OD]

Given AP is $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots \ldots (a \neq 0)$

Here, first term, $a_1 = \frac{1}{a}$

Second term, $a_2 = \frac{3-a}{3a}$

Third term, $a_3 = \frac{3-2a}{3a}$

Common difference,

$$d = a_2 - a_1$$
$$= \frac{3-a}{3a} - \frac{1}{a} = \frac{3-a-3}{3a}$$
$$= \frac{-a}{3a} = \frac{-1}{3}$$

Here, common difference d of given AP is $\frac{-1}{3}$.

172. Which term of the Arithmetic Progression $-7, -12, -17, -22, \ldots$ will be -82? Is -100 any term of the AP ? Given reason for your answer.

Sol : [Board 2019 OD]

Given AP is $-7, -12, -17, -22, \ldots$

Here,

First term, $a_1 = -7$

Second term $a_2 = -12$

Third term, $a_3 = -17$

Common difference,

$$d = a_2 - a_1 = -12 - (-7)$$
$$= -12 + 7 = -5$$
$$d = -5$$

Let a_n be the n^{th} term of AP and it will be -82.

Since, $a_n = a_1 + (n-1)d$

$$-82 = -7 + (n-1)(-5)$$
$$-82 = -7 - 5(n-1)$$
$$82 = 5n + 2$$
$$5n = 80 \Rightarrow n = 16$$

Hence, 16^{th} term of AP is -82. Since, these numbers are not factor of 5, hence -100 will not be a term in the given AP.

173. How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.

Sol : [Board 2019 OD]

Given AP is 45, 39, 33, ...

Here, $a = 45$, $d = 39 - 45 = -6$ and $S_n = 180$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$180 = \frac{n}{2}[2 \times 45 + (n-1)(-6)]$$
$$360 = n(90 - 6n + 6)$$
$$360 = n(96 - 6n)$$
$$60 = n(16 - n)$$
$$n^2 - 16n + 60 = 0$$
$$n^2 - 6n - 10n + 60 = 0$$
$$n(n-6) - 10(n-6) = 0$$
$$(n-10)(n-6) = 0$$
$$n = 10 \text{ or } n = 6$$

Hence, 10 terms or 6 terms can be taken to get the sum of AP as 180.

Now, sum of 6 terms,

$$S_6 = \frac{6}{2}[2 \times 45 + (6-1)(-6)]$$
$$= 3(90 - 30)$$
$$= 3 \times 60 = 180 \quad \text{Hence, verified.}$$

and sum of 10 terms,

$$S_{10} = \frac{10}{2}[2 \times 45 + (10-1)(-6)]$$
$$= 5(90 - 54)$$
$$= 5 \times 36 = 180 \quad \text{Hence, verified.}$$

Here we have two values of n because d is negative. There will be negative terms after some positive terms. Thus first 6 term will give sum 180 and after 10 term it will be again 180 because negative term cancel positive term.

Series will be : 45, 39, 33, 27, 21, 15, 9, 3, -3 , -9 ...

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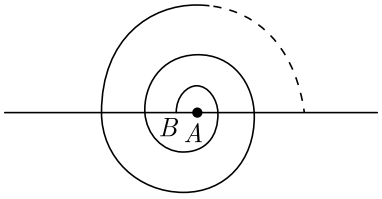
figure. What is the total length of spiral made up of eleven consecutive semi-circles?
(Use $\pi = 3.14$)



Sol : [Board Term-2 2012]

Let r_1, r_2 ..... be the radii of semi-circles and $l_1, l_2,$ ..... be the lengths of circumferences of semi-circles, than

$$\begin{aligned} l_1 &= \pi r_1 = \pi(1) = \pi\, cm \\ l_2 &= \pi r_2 = \pi(2) = 2\pi\, cm \\ l_3 &= 3\pi\, cm \\ &\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots \\ l_{11} &= 11\pi\, cm \end{aligned}$$

Total length of spiral

$$\begin{aligned} L &= l_1 + l_2 + \ldots\ldots\ldots + l_{11} \\ &= \pi + 2\pi + 3\pi + \ldots\ldots\ldots + 11\pi \\ &= \pi(1 + 2 + 3 + \ldots\ldots\ldots + 11) \\ &= \pi \times \frac{11 \times 12}{2} \\ &= 66 \times 3.14 \\ &= 207.24 \text{ cm} \end{aligned}$$

199. The ratio of the sums of first m and first n terms of an AP is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m-1):(2n-1)$.

Sol : [Board Term-2 Delhi 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

$$\begin{aligned} \frac{S_m}{S_n} &= \frac{m^2}{n^2} \\ \frac{\frac{m}{2}\left[2a + (m-1)d\right]}{\frac{n}{2}\left[2a + (n-1)d\right]} &= \frac{m^2}{n^2} \\ \frac{2a + (m-1)d}{2a + (n-1)d} &= \frac{m^2}{n^2} \times \frac{n}{m} = \frac{m}{n} \\ m\left(2a + (n-1)d\right) &= n\left[2a + (m-1)d\right] \\ 2ma + mnd - md &= 2na + nmd - nd \\ 2ma - 2na &= md - nd \\ d &= 2a \end{aligned}$$

Now,

$$\begin{aligned} \frac{a_m}{a_n} &= \frac{a + (m-1)d}{a + (n-1)d} \\ &= \frac{a + (m-1) \times 2a}{a + (n-1) \times 2a} \\ &= \frac{a + 2ma - 2a}{a + 2na - 2a} \\ &= \frac{2ma - a}{2na - a} = \frac{a(2m-1)}{a(2n-1)} \\ &= 2m - 1 : 2n - 1 \end{aligned}$$

200. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the AP.

Sol : [Board Term-2 SQP 2017]

Let the middle most terms of the AP be $(x-d)$, x and $(x+d)$.

We have $x - d + x + x + d = 225$

$$3x = 225 \Rightarrow x = 75$$

and the middle term $= \frac{37+1}{2} = 19^{th}$ term

Thus AP is

$(x-18d), \ldots (x-2d), (x-d), x, (x+d), (x+2d), \ldots\ldots$

$(x-18d)$

Sum of last three terms,

$$\begin{aligned} (x+18d) + (x+17d) + (x+16d) &= 429 \\ 3x + 51d &= 429 \\ 225 + 51d &= 429 \Rightarrow d = 4 \end{aligned}$$

,

First term $a_1 = x - 18d = 75 - 18 \times 4 = 3$

$$a_2 = 3 + 4 = 7$$

Hence AP = 3, 7, 11, ..., 147.

201. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following X.

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Sol : [Board Term-2 OD 2014]

The houses are numbered consecutively from 1 to 49 i.e. 1, 2, 3 ..... $x-1$, x, $x+1$, ... 49.

Sum of numbers of houses proceeding X number house,

$$= \text{sum of no. following } X$$

$$\frac{x-1}{2}\left[1 + (x-1)\right] = \left[(1 + \ldots 49) - (1 + 2 + \ldots\ldots x)\right]$$

(ii) If there is room for 25 rows, how many chairs will be needed to set up the theatre?

Sol :

Arrangement of seats in row makes arithmetic sequence where $a_1 = 80$ and $d = 8$.

(i) For 10th row $n = 10$, thus

$$\begin{aligned} a_n &= a_1 + (n-1)\,d \\ a_{10} &= 80 + (10-1)(8) \\ &= 80 + 9(8) \\ &= 80 + 72 = 152 \text{ seats} \end{aligned}$$

(ii) For 25th row $n = 25$, thus

$$\begin{aligned} a_{25} &= 80 + (25-1)(8) \\ &= 80 + 24(8) \\ &= 80 + 192 = 272 \end{aligned}$$

(iii) Chair needed to setup in theatre

$$\begin{aligned} S_n &= \frac{n(a_1 + a_n)}{2} \\ S_{25} &= \frac{25(80+272)}{2} \\ &= \frac{25(352)}{2} = 4400 \text{ seats} \end{aligned}$$

207. Sales Goals : At the time that I was newly hired, 100 sales per month was what I required. Each following month—the last plus 20 more, as I work for the goal of top sales award. When 2500 sales are thusly made, I get a holiday package.

(i) How many sales were made by this person in the seventh month?

(ii) What were the total sales after the 12th month?

(iii) Was the goal of 2500 total sales met after the 12th month?

Sol :

Sales form a arithmetic sequence with $a_1 = 100$, $d = 20$ and $a_n = 2500$

(i) Sales in the seventh month,

$$\begin{aligned} a_n &= a_1 + (n-1)\,d \\ a_7 &= 100 + (7-1)(20) \\ &= 100 + 6(20) = \$220 \end{aligned}$$

(ii) Total sales after the 12th month,

$$\begin{aligned} a_{12} &= 100 + (12-1)(20) \\ &= 100 + 11(20) = 320 \\ S_n &= \frac{n(a_1 + a_n)}{2} \\ S_{12} &= \frac{12(100+320)}{2} \\ &= \frac{12(420)}{2} = 2520 \text{ Yes} \end{aligned}$$

(iii) Goal of sales 2500 was achieved after 12th month.

208. Temperature Fluctuation: At 5 P.M. in Coldwater, the temperature was a chilly 36° F . If the temperature decreased by 3° F every half-hour for the next 7 hr, at what time did the temperature hit 0° F ?

Sol :

We have an arithmetic sequence, with 36° , 33° , 30° , 27° ....

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$$14{,}000 = 7{,}000 + (11-1)\,d$$
$$7{,}000 = 10d$$
$$d = 700 \text{ Rs}$$

The amount of money received over the 11 years is

$$S_{11} = \frac{n}{2}(a_1 + a_n)$$
$$S_{11} = \frac{11}{2}(7{,}000 + 14{,}000) = 115{,}500 \text{ Rs}$$

204. Arc of a Baby Swing : When Mackenzie's baby swing is started, the first swing (one way) is a 30 inch arc. As the swing slows down, each successive arc is 1.5 inch less than the previous one.

(i) Find the length of the tenth swing.

(ii) How far Mackenzie has travelled during the 10 swings ?

Sol :

Mackenzie's baby swing form a arithmetic sequence.

Here $a_1 = 30,\ d = \frac{-3}{2},\ n = 10$

(i) $a_n = a_1 + (n-1)\,d$

$$a_{10} = 30 + (10-1)\left(\frac{-3}{2}\right)$$
$$= 30 + 9\left(\frac{-3}{2}\right)$$
$$= 30 - \frac{27}{2} = 30 - 13.5$$
$$a_{10} = 16.5 \text{ inches}$$

(ii) $S_n = \frac{n(a_1 + a_n)}{2}$

$$S_{10} = \frac{10(30 + 16.5)}{2}$$
$$= \frac{10(46.5)}{2} = 232.5; \text{ inches}$$

205. Computer Animations : The animation on a new computer game initially allows the hero of the game to jump a (screen) distance of 10 inch over booby traps and obstacles. Each successive jump is limited to $\frac{3}{4}$ inch less than the previous one.

(i) Find the length of the seventh jump

(ii) Find the total distance covered after seven jumps.

Sol :

Successive jump of hero formed arithmetic sequence with $a_1 = 10$, and $d = -\frac{3}{4}$.

(i) Length of the seventh jump

$$a_n = a_1 + (n-1)\,d$$
$$a_7 = 10 + (7-1)\left(\frac{-3}{4}\right)$$
$$= 10 + 6\left(\frac{-3}{4}\right) = 10 - \frac{9}{2}$$
$$a_7 = 5.5 \text{ inches}$$

(ii) The total distance covered after seven jumps,

$$S_n = \frac{n(a_1 + a_n)}{2} = \frac{7(10 + 5.5)}{2}$$
$$= \frac{7(15.5)}{2} = 54.25 \text{ inches}$$

206. Seating Capacity : The Fox Theater creates a "theater in the round" when it shows any of Shakespeare's plays. The first row has 80 seats, the second row has 88, the third row has 96, and so on.

(i) How many seats are in the 10th row?

(ii) How many seats are in the 25th row?

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216. Old Age Home : Your grandmother has assets of Rs 500,000. One option that she is considering involves an adult residential community for a six-year period beginning in 2020. The model $a_n = 1800n + 64130$ describes yearly adult residential community costs n years after 2019. Does your grandmother have enough to pay for the facility?

Sol :

We must find the sum of an arithmetic sequence whose general term is $a_n = 1800n + 64,130$. The first term of the sequence corresponds to the facility's costs in the year 2020. The last term corresponds to costs in the year 2025. Because the model describes costs n years after 2019, $n = 1$ describes the year 2020 and $n = 6$ describes the year 2025.

$$a_n = 1800n + 64,130$$

For year 2020, $n = 1$,

$$a_1 = 1800 \times 1 + 64,130 = 65,930$$

For year 2025, $n = 6$,

$$a_6 = 1800 \times 6 + 64,130$$
$$= 74,930$$

The first year the facility will cost Rs 65,930. By year six, the facility will cost Rs 74,930. Now we must find the sum of the costs for all six years.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_6 = \frac{6}{2}(65,930 + 74,930)$$

$$= 3(140,860) = 422,580 \text{ Rs}$$

Total adult residential community costs for your grandmother are predicted to be Rs 422,580. Because your grandmother's assets are Rs 500,000, she has enough to pay for the facility for the six-year period.

217. TOWER OF PISA : To prove that objects of different weights fall at the same rate, Galileo dropped two objects with different weights from the Leaning Tower of Pisa in Italy. The objects hit the ground at the same time. When an object is dropped from a tall building, it falls about 16 feet in the first second, 48 feet in the second, and 80 feet in the third second, regardless of its weight.

(i) How many feet would an object fall in the sixth second?

(ii) How many feet would an object fall in the six second?

(iii) How many feet would an object fall in the eight second?

Sol :

16, 32, 48.....form a arithmetic sequence with $a_1 = 16$ feet, and $d = 48 - 12 = 32$

Object fall in n^{th} second,

$$a_n = a_1 + (n-1)\,d$$
$$= 16 + (n-1)32$$
$$= 32n - 16$$
$$= 16(2n-1)$$

(i) Object fall in 6th second

$$a_6 = 16(2 \times 6 - 1)$$
$$= 176 \text{ feet}$$

(ii) Total fall in 6 second

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_6 = \frac{6}{2}(16 + 176)$$

$$= 3 \times 192 = 576 \text{ feet}$$

(ii) Total fall in 8 second

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Here $a_1 = 33,\ d = -3,\ a_n = 0$

$$a_n = a_1 + (n-1)\,d$$
$$0 = 33 + (n-1)(-3)$$
$$0 = 33 - 3n + 3$$
$$0 = 36 - 3n$$
$$-36 = -3n \Rightarrow n = 12$$

Thus 12 half-hours after 5 P.M. i.e at 11 P.M.

209. Model Teachers' Earnings : According to the National Teachers Association, teachers in the India earned an average of Rs 30,532 in 2000. This amount has increased by approximately Rs 1472 per year.

(i) Write a formula for the n th term of the arithmetic sequence that models teachers' average earnings n years after 1999.

(ii) How much will India teachers earn, on average, by the year 2030?

Sol :

In this sequence, a_1, the first term, represents the amount teachers earned in 2000. Each subsequent year this amount increases by Rs 1472, so $d = 1472$. We can express teachers' earnings by the arithmetic sequence where $a_1 = 30532$ and $d = 1472$.

(i) Average earnings n years after 1999,

$$a_n = a_1 + (n-1)\,d$$
$$= 30,532 + (n-1)1472$$
$$= 30,532 + 1472n - 1472$$

Thus $a_n = 1472n + 29,060$

(ii) Now we need to find teachers' earnings in 2030. The year 2030 is 31 years after 1999: That is, $2030 - 1999 = 31$. Thus, $n = 31$.

$$a_n = 1472n + 29,060.$$

$$a_{31} = 1472 \times 31 + 29,060 = 74,692$$

The 31st term of the sequence is 74,692. Therefore, Indian teachers are predicted to earn an average of Rs 74,692 by the year 2030.

210. Contest Prizes : A contest offers 15 prizes. The 1st prize is Rs 5000, and each successive prize is Rs 250 less than the preceding prize.

(i) What is the value of the 15th prize?

(ii) What is the total amount of money distributed in prizes?

Sol :

Contest prize form a arithmetic sequence Rs 5000, Rs 4750, Rs 4500, Rs 4525000

Here $a_1 = 5000$ and $d = -250$

For the value of 15th prize, $n = 15$,

$$a_n = a_1 + (n-1)\,d$$
$$= 5000 + (15-1)250$$
$$= 5000 + (15-1)(-250)$$
$$= 5000 - 14 \times 250 = 1500 \text{ Rs}$$

(ii) Total amount of money distributed in prizes,

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$S_{15} = \frac{15}{2}(a_1 + a_{15})$$
$$= \frac{15}{2}(5000 + 1500) = \frac{15 \times 6500}{2}$$
$$= 48750$$

211. Bequests to Charity : At the time our mother left this Earth, she gave Rs 90000 to her children of birth. This we kept and each year added Rs 30000 more, as a lasting memorial from the children she bore. When Rs 4,20,000 is thusly attained, all goes to charity that her memory be maintained.

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(i) What was the balance in the sixth year?

(ii) In what year was the goal of Rs 420,000 met?

Sol :

Here donation forms a arithmetic sequence with $a_0 = 90000$ and $d = 3000$.

Here $a_1 = 120000$ and $a_n = 420000$ also

(i) The balance in the sixth year,

$$\begin{aligned} a_n &= a_1 + (n-1)\,d \\ a_6 &= 120000 + (6-1)\,(30000) \\ &= 120000 + 5\,(30000) \\ &= 270000 \text{ Rs} \end{aligned}$$

(ii) Goal of Rs 4,20,000

$$\begin{aligned} 420000 &= 120000 + (n-1)\,(30000) \\ 14 &= 4 + n - 1 \\ 14 &= 3 + n \;\Rightarrow\; n = 14 - 3 = 11 \end{aligned}$$

The goal was met in 11 years.

212. It takes 5 toothpicks to build the top trapezoid shown at below. You need 9 toothpicks to build 2 adjoined trapezoids and 13 toothpicks for 3 trapezoids.

(i) If 1000 toothpicks are available, how many trapezoids will be in the last complete row?

(ii) How many complete rows will there be?

(iii) How many toothpicks will you use to construct these rows?

(iv) Use the numbers in this problem to carefully describe the difference between a sequence and a series.

Sol :

The number of toothpicks needed to build each row can be expressed as an arithmetic sequence where the first term is 5, the second term is 9, and the third term is 13. The common difference is 4.

(i) If 1000 toothpicks are available, $S_n = 1000$,

$$\begin{aligned} S_n &= \frac{n}{2}[2a_1 + (n-1)\,d] \\ 1000 &= \frac{n}{2}[2 \times 5 + (n-1) \times 4] \\ 1000 &= n[5 + 2\,(n-1)] \\ 1000 &= 5n + 2n^2 - 2n \\ 1000 &= 3n + 2n^2 \\ 2n^2 + 3n - 1000 &= 0 \\ n &= \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-1000)}}{2 \times 2} \\ n &= 21.62 \text{ or } -23.12 \end{aligned}$$

Only a positive integer answer makes sense, so there are 21 trapezoids in the last row.

(ii) The number of trapezoids in each row is the same as the row number, so the 21st row has 21 trapezoids.

(iii)
$$\begin{aligned} S_n &= \frac{n}{2}[2a_1 + (n-1)\,d] \\ S_{21} &= \frac{21}{2}[2 \times 5 + (21-1) \times 4] \\ &= \frac{21}{2}[10 + 20 \times 4] \\ &= \frac{21}{2} \times 90 = 945 \end{aligned}$$

(iv) The numbers of toothpicks in each row form a sequence, whereas the total numbers of toothpicks used form a series.

213. Book Reading : On the first day of October an English teacher suggests to his students that they read five pages of a novel and every day thereafter increase their daily reading by two pages. If his students follow this suggestion, then how many pages will they read during October?

Sol :

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The students read 5 pages the first day, 7 pages the second day, 9 pages the third day, and so on. This form a arithmetic sequence with $d = 2$ and $a = 5$.

For last day of October month, $n = 31$,

$$a_n = a_1 + (n-1)\,d$$
$$a_{31} = 5 + (31-1)\,2 = 65$$

Now $S_n = \frac{n}{2}(a_1 + a_n)$

For October month $n = 31$,

$$S_{31} = \frac{31}{2}(a_1 + a_{31})$$
$$= \frac{31}{2}(5+65) = 1085$$

Thus students read total 1085 pages in October month.

214. Increasing Salary : A lab technician has a salary of Rs 22,000 her first year and is due to get a Rs 500 raise each year,

(i) What will her salary be in her seventh year?

(ii) What is the total salary for 7 years of work for the lab technician ?

Sol :

Here salary forms a arithmetic sequence with $a_1 = 22000$ and $d\ d = 500$.

(i) For salary in seventh year $n = 7$,

$$a_n = a_1 + (n-1)\,d$$
$$a_7 = 22{,}000 + (7-1)\,500$$
$$= 25{,}000 \text{ Rs}$$

(ii) Total salary for 7 years of work,

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$S_7 = \frac{7}{2}(22{,}000 + 25{,}000)$$
$$= 164{,}500 \text{ Rs}$$

215. CONSTRUCTION : The table at the below shows typical costs for a construction company to rent a crane for one, two, three, or four months. If the sequence continues, how much would it cost to rent the crane for twelve months?

Months	Cost (Rs)
1	75,000
2	90,000
3	105,000
4	120,000

Sol :

Since the difference between any two successive costs is Rs 15,000, the costs forms an arithmetic sequence with common difference 15,000. Here $a_1 = 75{,}000$ and $d\ d = 15{,}000$

For the rent of the crane for twelve months $n = 12$,

$$a_n = a_1 + (n-1)\,d$$
$$a_{12} = 75{,}000 + (12-1)\,15{,}000$$
$$= 75{,}000 + 11 \times 15{,}000$$
$$= 15000\,(5+11)$$
$$= 240{,}000$$

If would cost Rs 240,000 to rent the crane for twelve months.

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$$\frac{x-1}{2}[1+x-1] = \left[\frac{49}{2}\times(1+49)-\left[\frac{x}{2}(1+x)\right]\right]$$

$$\frac{(x-1)x}{2} = \frac{49\times 50}{2}-\frac{x}{2}(x+1)$$

$$\frac{x(x-1)}{2}+(x+1)\frac{x}{2} = 49\times 25$$

$$\frac{x^2-x+x^2+x}{2} = 49\times 25$$

$$\frac{2x^2}{2} = 49\times 25$$

$$x^2 \quad x^2 = 7\times 7\times 5\times 5$$

$$x = 7\times 5 = 35$$

COMPETENCEY BASED QUESTIONS

202. Salary : In investigating different job opportunities, you find that firm A will start you at Rs 25,000 per year and guarantee you a raise of Rs 1,200 each year whereas firm B will start you at Rs 28,000 per year but will guarantee you a raise of only Rs 800 each year.

(i) Over a period of 15 years, how much would you receive from firm A?

(ii) Over a period of 15 years, how much would you receive from firm B?

(iii) What would be your annual salary at firm A for the tenth year?

(iv) What would be your annual salary at firm B for the tenth year?

Sol :

With each firm, the salaries form an arithmetic sequence. We are asked for the sum of fifteen terms, or S_{15}, given a_1 and d.

$$S_n = \frac{n}{2}[2a_1+(n-1)d]$$

$$S_{15} = \frac{15}{2}[2a_1+14d] = 15(a_1+7d)$$

Firm A : $a_1 = 25000,\ d = 1200,$

$$S_{15} = 15(25000+7\times 1200)$$

$$= 501{,}000 \text{ Rs}$$

Firm B : $a_1 = 28{,}000,\ d = 800,$

$$S_{15} = 15(28{,}000+7.800)$$

$$= 504{,}000 \text{ Rs}$$

Now $a_n = a_1+(n-1)d,$

For the tenth year $n = 10$, thus

$$a_{10} = a_1+9d,$$

Firm A : $a_1 = 25{,}000,\ d = 1{,}200,$

$$a_{10} = 25{,}000+9(1{,}200)$$

$$= 35{,}800 \text{ Rs}$$

Firm B : $a_1 = 28{,}000,\ d = 800,$

$$a_{10} = 28{,}000+9(800)$$

$$= 35{,}200$$

(i) Rs 501000

(ii) Rs 504000

(iii) Rs 35800

(ii) Rs 35200

203. Investment : Eleven years ago an investment earned Rs 7,000 for the year. Last year the investment earned Rs 14,000. If the earnings from the investment have increased the same amount each year, what is the yearly increase and how much income has accrued from the investment over the past 11 years?

Sol :

This involves an arithmetic sequence. Let d be increase in earnings each year.

Now $a_1 = 7{,}000,\ a_{11} = 14{,}000$

$$a_n = a_1+(n-1)d$$

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$$\begin{aligned} S_n &= \frac{n}{2}[2a_1 + (n-1)\,d] \\ &= \frac{8}{2}[2\times 16 + (8-1)\times 32] \\ &= 4\times 16\,[2 + 7\times 2] \\ &= 64\times 16 = 1024 \text{ feet} \end{aligned}$$

218. Jessica arranges a display of soup cans as shown in figure.

(i) List the number of cans in the top row, the second row, the third row, and so on, down to the tenth row.

(ii) Write a recursive formula for the terms of the sequence.

(iii) If the cans are to be stacked 47 rows high, how many cans will it take to build the display?

(iv) If Jessica uses six cases (273 cans), how tall can she make the display?

Sol :

There are 3 cans in the first row, 6 cans in the second row, and 9 cans in the third row as shown. The number of cans in each row forms an arithmetic sequence where the first term a_1 is 3 and the common difference d is 3.

(i) 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.

(ii) $a_{n+1} = a_n + 3$

(iii) $$\begin{aligned} S_n &= \frac{n}{2}[2a_1 + (n-1)\,d] \\ &= \frac{47}{2}[2\times 3 + (47-1)\times 3] \\ &= \frac{47}{2}[6 + 46\times 3] \\ &= \frac{47}{2}\times 144 = 3384 \text{ can} \end{aligned}$$

(iv) Here $S_n = 273$

$$\begin{aligned} S_n &= \frac{n}{2}[2a_1 + (n-1)\,d] \\ 273 &= \frac{n}{2}[2\times 3 + (n-1)\times 3] \\ 546 &= n[6 + 3(n-1)] \\ 546 &= 6n + 3n^2 - 3n \\ 182 &= n + n^2 \\ n^2 + n - 182 &= 0 \\ n^2 + 14n - 13n - 182 &= 0 \\ (n+14)(n-13) &= 0 \Rightarrow n = 13, -14 \end{aligned}$$

Thus can make 13 row with 273 can.

219. Piono : Suppose you practice the piano 45 min on the first day of the semester and increase your practice time by 5 min each day.

(i) How much total time will you devote to practicing during the first 15 days of the semester?

(ii) How much total time will you devote to practicing during the first 35 days of the semester?

Sol :

Each day represents a term in an arithmetic sequence, so the first term is 45 and the common difference is 5.

(i) For first 15 days of the semester, $n = 15$

Now $$\begin{aligned} S_n &= \frac{n}{2}[2a_1 + (n-1)\,d] \\ &= \frac{15}{2}[2\times 45 + (15-1)\times 5] \\ &= \frac{15}{2}[90 + 14\times 5] \\ &= \frac{15}{2}\times 160 = 1200 \text{ minute or } 20 \text{ hours} \end{aligned}$$

(i) For first 35 days of the semester, $n = 35$

$$S_n = \frac{n}{2}[2a_1 + (n-1)\,d]$$

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$$= \frac{35}{2}[2 \times 45 + (35-1) \times 5]$$
$$= \frac{35}{2}[90 + 34 \times 5]$$
$$= \frac{35}{2} \times 260$$
$$= 4550 \text{ minute or } 75 \text{ hours } 50 \text{ min}$$

220. RADIO : A radio station is giving away a total of Rs 124,000 in August on promotion. If they increase the amount given away each day by Rs 100,

(i) How much should they give away the first day?

(ii) How much have they give away on the 15th day ?

(iii) How much have they give away in 15 day ?

Sol :

Money given away on promotion forms a arithmetic sequence with $d = 100$.

They have given away Rs 124,000 in august i.e in 31 days. Thus $n = 31$ and $S_n = 124000$.

(i) Now $S_n = \frac{n}{2}[2a_1 + (n-1)\,d]$

$$S_{31} = \frac{31}{2}[2a_1 + (31-1)\,100]$$
$$124{,}000 = \frac{31}{2}(2a_1 + 3000)$$
$$8000 = 2a_1 + 3000$$
$$5000 = 2a_1$$
$$2500 = a_1$$

The radio station should give away Rs 2500 the first day.

(ii) Give away on the 15th day,

Now $a = 2500$, $d = 100$, $n = 15$

$$a_n = a_1 + (n-1)\,d$$
$$a_{15} = 2500 + (15-1)\,100$$
$$= 2500 + 14 \times 100$$
$$= 3900$$

(iii) Total give away in 15 days

$$S_{31} = \frac{n}{2}(a_1 + a_n)$$
$$= \frac{15}{2}(2500 + 3900)$$
$$= \frac{15}{2} \times 6400 = 48000$$

221. Road Roller : A road roller (sometimes called a roller-compactor, or just roller) is a compactor-type engineering vehicle used to compact soil, gravel, concrete, or asphalt in the construction of roads and foundations. Similar rollers are used also at landfills or in agriculture. Road rollers are frequently referred to as steamrollers, regardless of their method of propulsion.

RCB Machine Pvt Ltd started making road roller 10 year ago. Company increased its production uniformly by fixed number every year. The company produces 800 roller in the 6th year and 1130 roller in the 9th year.

(i) What was the company's production in first year?

(ii) What was the company's production in the 8th year ?

(iii) What was the company's total production of the first 6 years?

(iv) What was the increase in the company's production every year ?

(v) In which year the company's production was 1350 rollers ?

Sol :

(i) Let a be the production in first year and d be the increase every year in production.

We have $a_6 = 800$

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$$a_9 = 1130$$

Now, $a + (6-1)\,d = 800$

$$a + 5d = 800 \quad ...(1)$$

Similarly $a + 8d = 1130 \quad ...(2)$

Solving (1) and (2), we get

$$d = 110$$

$$a = 800 - 5 \times 110 = 250$$

(ii) Since, $a = 250$ and $d = 110$

$$a_8 = a + (8-1)\,d$$

$$= 250 + 7 \times 110 = 1020$$

(iii) Production in 6^{th} year is 800 rollers.

$$S_n = \frac{n}{2}[a + a_n]$$

$$= \frac{6}{2}[250 + 800]$$

$$= 3 \times 1050 = 3150$$

(iv) In part (i), we have calculated $d = 110$ which is the increase every year in production. Hence, the production increases constantly by 110 every year.

(v) Suppose company produce 1350 rollers in n^{th} year.

Then, $a_n = a + (n-1)\,d$

$$1350 = 250 + (n-1) \times 110$$

$$(n-1)\,110 = 1350 - 250 = 1100$$

$$n - 1 = 10 \Rightarrow n = 11$$

222. Heavy Penalties : If an air-conditioning system is not completed by the agreed upon date, the contractor pays a penalty of Rs 500 for the first day that it is overdue, Rs 600 for the second day, Rs 700 for the third day, and so on. If the system is completed 10 days late, then what is the total amount of the penalties that the contractor must pay?

Sol :

Thus is arithmetic series where $a_1 = 500$, $d = 100$ and $n = 10$.

Now $S_n = \frac{n}{2}[2a_1 + (n-1)\,d]$

$$S_{10} = \frac{10}{2}[2 \times 500 + (10-1)\,100]$$

$$= 5\,[1000 + 900]$$

$$= 9500 \text{ Rs}$$

****

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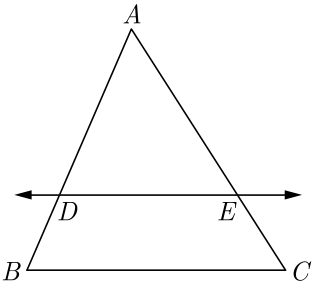
24. In Figure, in ΔABC, $DE \parallel BC$ such that $AD = 2.4$ cm, $AB = 3.2$ cm and $AC = 8$ cm, then what is the length of AE?



Sol : [Board 2020 Delhi Basic]

We have $DE \parallel BC$

By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$

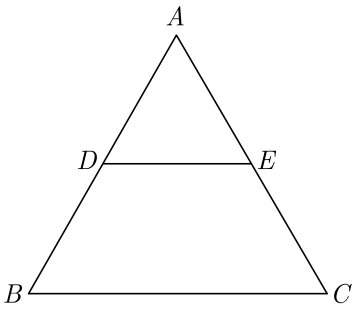
$$\frac{AD}{AD+DB} = \frac{AE}{AE+EC}$$

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\frac{2.4}{3.2} = \frac{AE}{8} \Rightarrow AE = 6 \text{ cm}$$

PRACTICE

In given figure $DE \| BC$. If $AD = 3$ cm $DB = 4$ cm and $AE = 6$ cm then find EC.

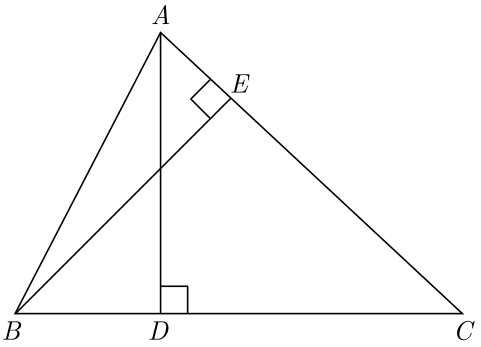


[Board Term-1 2016]

Ans : 6 cm

TWO MARKS QUESTIONS

25. In, ΔABC altitudes AD and BE are drawn. If $AD = 7$ cm, $BE = 9$ cm and $EC = 12$ cm then, find the length of CD.



Sol : [Board 2024 OD Standard]

We have $AD = 7$ cm, $BE = 9$ cm and $EC = 12$ cm

In ΔADC and ΔBEC.

$$\angle ADC = \angle BEC = 90^\circ$$

$$\angle ACD = \angle BCE \quad \text{(Common)}$$

By AA similarity criterion we have

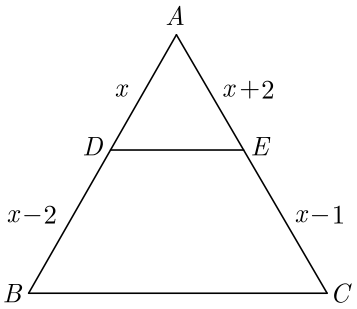
$$\Delta ADC \sim \Delta BEC$$

Thus $\frac{AD}{BE} = \frac{CD}{EC}$

$$\frac{7}{9} = \frac{CD}{12}$$

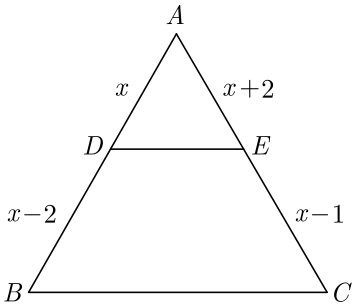
$$CD = \frac{28}{3} \text{ cm}$$

26. In the given figure, ABC is a triangle in which $DE \| BC$. If $AD = x$, $DB = x-2$, $AE = x+2$ and $EC = x-1$, then find the value of x.



Sol : [Board 2023 OD Standard]

We have redrawn the figure below.



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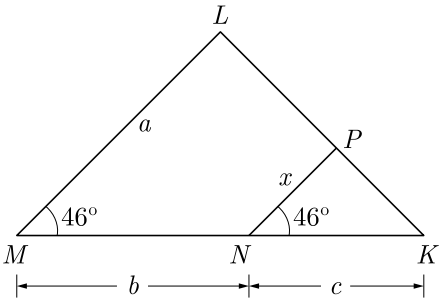
4. In the given figure, x is



(a) $\frac{ab}{a+b}$ (b) $\frac{ac}{b+c}$

(c) $\frac{bc}{b+c}$ (d) $\frac{ac}{a+c}$

Sol :

In ΔKPN and ΔKLM, $\angle K$ is common and we have

$$\angle KNP = \angle KML = 46^\circ$$

Thus by $A-A$ criterion of similarity,

$$\Delta KNP \sim \Delta KML$$

Thus $$\frac{KN}{KM} = \frac{NP}{ML}$$

$$\frac{c}{b+c} = \frac{x}{a} \Rightarrow x = \frac{ac}{b+c}$$

Thus (b) is correct option.

5. ΔABC is an equilateral triangle with each side of length $2p$. If $AD \perp BC$ then the value of AD is

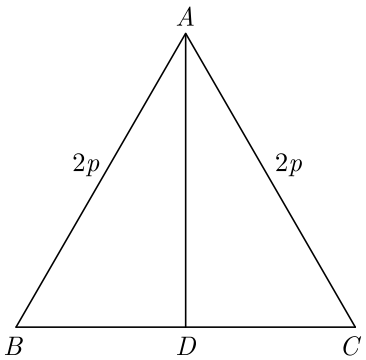
(a) $\sqrt{3}$ (b) $\sqrt{3}\,p$

(c) $2p$ (d) $4p$

Sol :

We have $AB = BC = CA = 2p$

and $AD \perp BC$



In ΔADB, $AB^2 = AD^2 + BD^2$

$$(2p)^2 = AD^2 + p^2$$

$$AD^2 = \sqrt{3}\,p$$

Thus (b) is correct option.

6. Which of the following statement is false?

(a) All isosceles triangles are similar.

(b) All quadrilateral are similar.

(c) All circles are similar.

(d) None of the above

Sol :

Isosceles triangle is a triangle in which two side of equal length. Thus two isosceles triangles may not be similar. Hence statement given in option (a) is false.

Thus (a) is correct option.

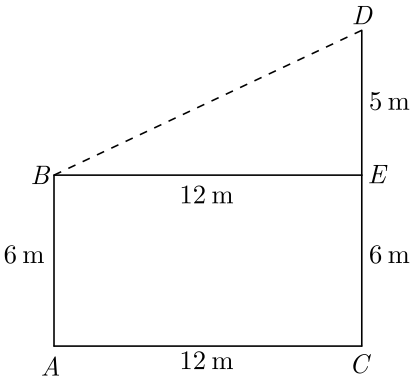
7. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, then distance between their tops is

(a) 12 m (b) 14 m

(c) 13 m (d) 11 m

Sol :

Let AB and CD be the vertical poles as shown below.



We have $AB = 6\,\text{m}$, $CD = 11$ m

and $AC = 12$ m

$DE = CD - CE$

$= (11-6)\,\text{m} = 5$ m

In right angled, ΔBED,

$BD^2 = BE^2 + DE^2$

$= 12^2 + 5^2 = 169$

$BD = \sqrt{169}\,\text{m} = 13$ m

Hence, distance between their tops is 13 m.

Thus (c) is correct option.

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Sol : [Board Term-1 2017]

In ΔKPN and ΔKLM, $\angle K$ is common and we have

$$\angle KNP = \angle KML = 46^\circ$$

Thus by AA criterion of similarity,

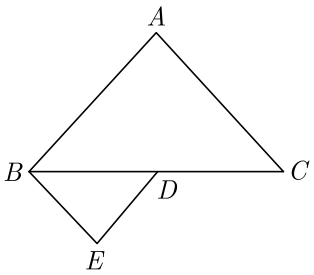
$$\Delta KNP \sim \Delta KML$$

Thus
$$\frac{KN}{KM} = \frac{NP}{ML}$$
$$\frac{c}{b+c} = \frac{x}{a} \Rightarrow x = \frac{ac}{b+c}$$

15. ΔABC and ΔBDE are two equilateral triangle such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is ..... .

Sol : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



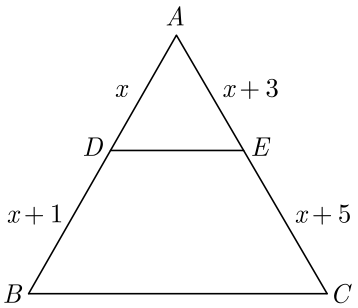
$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta BDE)} = \frac{\frac{\sqrt{3}}{4}(BC)^2}{\frac{\sqrt{3}}{4}(BD)^2} = \frac{(BC)^2}{\left(\frac{1}{2}BC\right)^2}$$
$$= \frac{4BC^2}{BC^2} = \frac{4}{1} = 4:1$$

16. Two triangles are similar if their corresponding sides are ..... .

Sol : [Board 2020 OD Standard]

in the same ratio.

17. In $\Delta ABC, DE \| BC$, find the value of x.

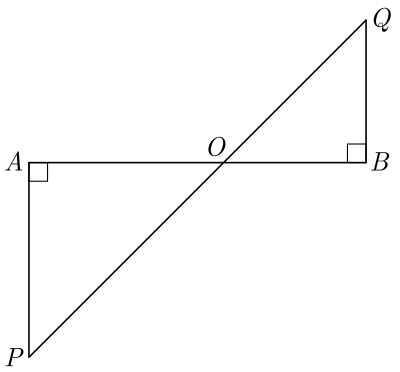


Sol : [Board Term-1 2016]

In the given figure $DE \| BC$, thus

$$\frac{AD}{DB} = \frac{AE}{EC}$$
$$\frac{x}{x+1} = \frac{x+3}{x+5}$$
$$x^2+5x = x^2+4x+3$$
$$x = 3$$

18. In the given figure, if $\angle A = 90^o, \angle B = 90^o$, $OB = 4.5$cm $OA = 6$ cm and $AP = 4$ cm then find QB.



Sol : [Board Term-1, 2015]

In ΔPAO and ΔQBO we have

$$\angle A = \angle B = 90^o$$

Vertically opposite angle,

$$\angle POA = \angle QOB$$

Thus
$$\Delta PAO \sim \Delta QBO$$
$$\frac{OA}{OB} = \frac{PA}{QB}$$
$$\frac{6}{4.5} = \frac{4}{QB}$$
$$QB = \frac{4 \times 4.5}{6} = 3 \text{ cm}$$

Thus $QB = 3$ cm

19. Are two triangles with equal corresponding sides always similar?

Sol : [Board Term-1 2015]

Yes, Two triangles having equal corresponding sides are congruent and all congruent Δs have equal angles, hence they are similar too.

20. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of second triangle is ..... .

Sol : [Board 2020 Delhi Basic]

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We have, $AB^2 + BC^2 = (24)^2 + (10)^2$

$= 576 + 100$

$= 676 = AC^2$

Thus $AB^2 + BC^2 = AC^2$ and ABC is a right angled triangle.

Also, two triangle are similar if their corresponding angles are equal.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

12. **Assertion :** ABC and DEF are two similar triangles such that $BC = 4\,\text{cm}$, $EF = 5\,\text{cm}$ and area of $\Delta ABC = 64\,\text{cm}^2$, then area of $\Delta DEF = 100\,\text{cm}^2$.

Reason : The areas of two similar triangles are in the ratio of the squares of the corresponding altitudes.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Reason is true because this is standard result.

For Assertion, since $\Delta ABC \sim \Delta DEF$

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{(4)^2}{(5)^2} = \frac{16}{25}$$

Ratio of areas of two similar triangle is equal to the ratio of the squares of corresponding sides.

$$\frac{64}{\text{area}(\Delta DEF)} = \frac{16}{25}$$

$$\text{area}(\Delta DEF) = \frac{64 \times 25}{16}$$

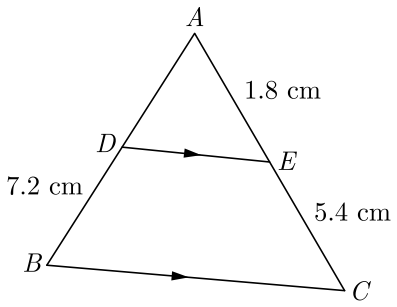
$$= 4 \times 25 = 100\,\text{cm}^2$$

Assertion is true. But Reason is not correct explanation for Assertion.

Thus (b) is correct option.

ONE MARK QUESTIONS

13. In Figure, $DE \| BC$. Find the length of side AD, given that $AE = 1.8\,\text{cm}$, $BD = 7.2\,\text{cm}$ and $CE = 5.4\,\text{cm}$.



Sol : [Board 2019 OD]

Since $DE \| BC$ we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substituting the values, we get

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$AD = \frac{1.8 \times 7.2}{5.4}$$

$$= \frac{12.96}{5.4} = 2.4\ \text{cm}$$

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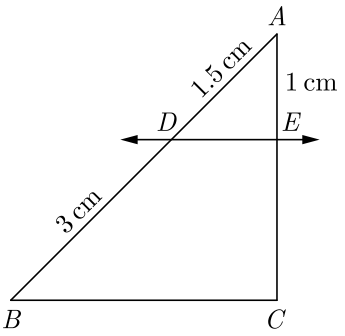
PRACTICE

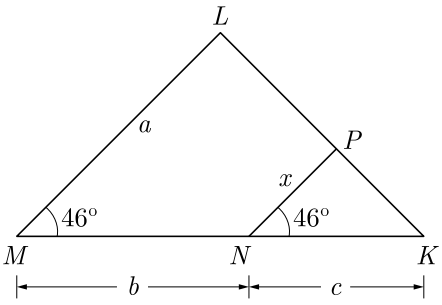
In the given figure, $DE \| BC$. What is the value of EC ?



[Board Term-1 2015]

Ans : 2 cm

14. What is the value of x in given figure?

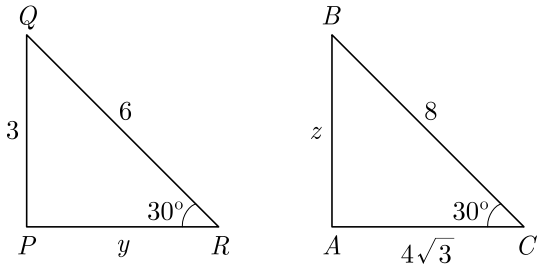


Alternative Method :

In ABC, $DE \parallel AB$, thus

$$\frac{CD}{CA} = \frac{CE}{CB}$$
$$\frac{CD}{CA - CD} = \frac{CE}{CB - CE}$$
$$\frac{CD}{AD} = \frac{CE}{BE}$$
$$\frac{x+3}{2x} = \frac{x}{2x-1}$$
$$5x = 3 \text{ or, } x = \frac{3}{5}$$

30. In the given figure, $\Delta ABC \sim \Delta PQR$. Find the value of $y + z$.



Sol : [Board Term-1 2010]

In the given figure $\Delta ABC \sim \Delta PQR$,

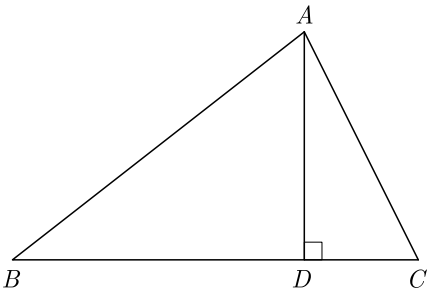
Thus $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$
$$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$$
$$\frac{z}{3} = \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y}$$
$$z = \frac{8 \times 3}{6} \text{ and } y = \frac{4\sqrt{3} \times 6}{8}$$
$$z = 4 \text{ and } y = 3\sqrt{3}$$

Thus $$y + z = 3\sqrt{3} + 4$$

31. In ΔABC, $AD \perp BC$, such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A.

Sol : [Board Term-1 2015]

As per given condition we have drawn the figure below.



We have $$AD^2 = BD \times CD$$
$$\frac{AD}{CD} = \frac{BD}{AD}$$

Since $\angle D = 90^\circ$, by SAS we have

$$\Delta ADC \sim \Delta BDA$$

and $$\angle BAD = \angle ACD;$$

Since corresponding angles of similar triangles are equal

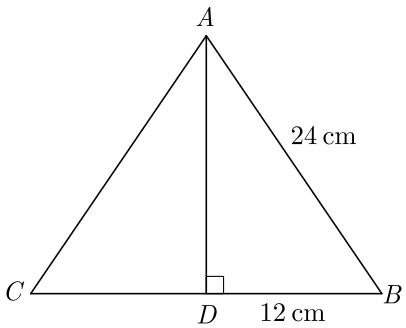
$$\angle DAC = \angle DBA$$
$$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180^\circ$$
$$2\angle BAD + 2\angle DAC = 180^\circ$$
$$\angle BAD + \angle DAC = 90^\circ$$
$$\angle A = 90^\circ$$

Thus ΔABC is right angled at A.

32. In an equilateral triangle of side 24 cm, find the length of the altitude.

Sol : [Board Term-1 2015]

Let ΔABC be an equilateral triangle of side 24 cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now $$BD = \frac{BC}{2} = \frac{24}{2} = 12 \text{ cm}$$
$$AB = 24 \text{ cm}$$
$$AD = \sqrt{AB^2 - BD^2}$$
$$= \sqrt{(24)^2 - (12)^2}$$
$$= \sqrt{576 - 144}$$
$$= \sqrt{432} = 12\sqrt{3}$$

Thus $AD = 12\sqrt{3}$ cm.

33. In the figure, $PQRS$ is a trapezium in which $PQ \parallel RS$. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that $EQ \times GS = GQ \times FS$.

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Ratio of the perimeter of two similar triangles is equal to the ratio of corresponding sides.

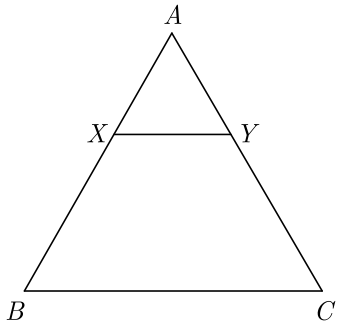
Thus $\frac{25}{15} = \frac{9}{\text{side}}$

$$\text{side} = \frac{9 \times 15}{25} = 5.4 \text{ cm}$$

21. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC parallel or not.

Sol : [Board Term-1 2016, 2015]

As per question we have drawn figure given below.



In this figure we have

$$\frac{AX}{XB} = \frac{3}{4}, AY = 5 \text{ and } YC = 9$$

Now $\frac{AX}{XB} = \frac{3}{4}$ and $\frac{AY}{YC} = \frac{5}{9}$

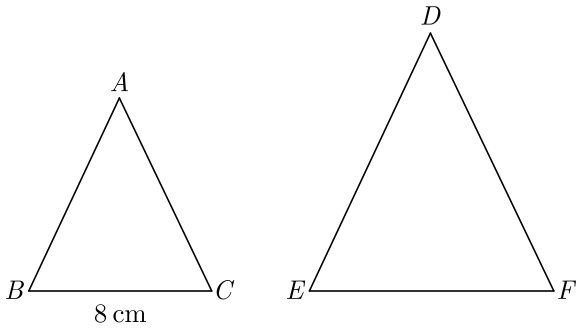
Since $\frac{AX}{XB} \neq \frac{AY}{YC}$

Hence XY is not parallel to BC.

22. If triangle ABC is similar to triangle DEF such that $2AB = DE$ and $BC = 8$ cm then find EF.

Sol :

As per given condition we have drawn the figure below.



Here we have $2AB = DE$ and $BC = 8$ cm

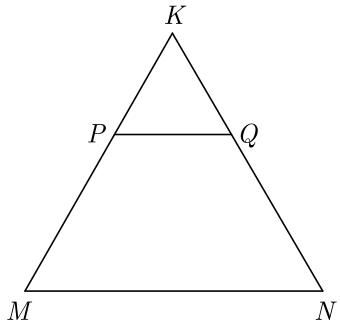
Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{AB}{8} = \frac{2AB}{EF}$$

$$EF = 2 \times 8 = 16 \text{ cm}$$

23. In the figure, PQ is parallel to MN. If $\frac{KP}{PM} = \frac{4}{13}$ and $KN = 20.4$ cm then find KQ.



Sol : [Board Term-1, 2013]

In the given figure $PQ \parallel MN$, thus

$$\frac{KP}{PM} = \frac{KQ}{QN} \quad \text{(By BPT)}$$

$$\frac{KP}{PM} = \frac{KQ}{KN - KQ}$$

$$\frac{4}{13} = \frac{KQ}{20.4 - KQ}$$

$$4 \times 20.4 - 4KQ = 13KQ$$

$$17KQ = 4 \times 20.4$$

$$KQ = \frac{20.4 \times 4}{17} = 4.8 \text{ cm}$$

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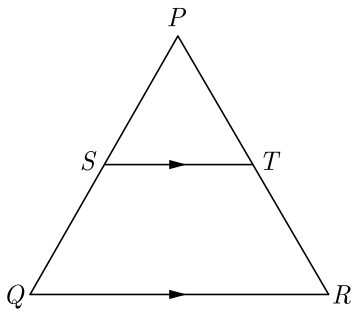
PRACTICE

In the given figure, in a triangle PQR, $ST \parallel QR$ and $\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28$ cm, find PT.



[Board Term-1 2011]

Ans : 10.5 cm

8. In a right angled ΔABC right angled at B, if P and Q are points on the sides AB and BC respectively, then

(a) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$

(b) $2(AQ^2 + CP^2) = AC^2 + PQ^2$

(c) $AQ^2 + CP^2 = AC^2 + PQ^2$

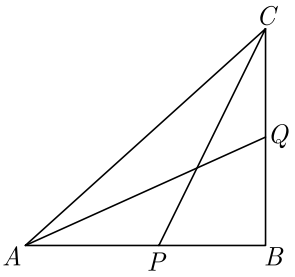
(d) $AQ + CP = \frac{1}{2}(AC + PQ)$

Sol :

In right angled ΔABQ and ΔCPB,

$$CP^2 = CB^2 + BP^2$$

and $$AQ^2 = AB^2 + BQ^2$$



$$\begin{aligned} CP^2 + AQ^2 &= CB^2 + BP^2 + AB^2 + BQ^2 \\ &= CB^2 + AB^2 + BP^2 + BQ^2 \\ &= AC^2 + PQ^2 \end{aligned}$$

Thus (c) is correct option.

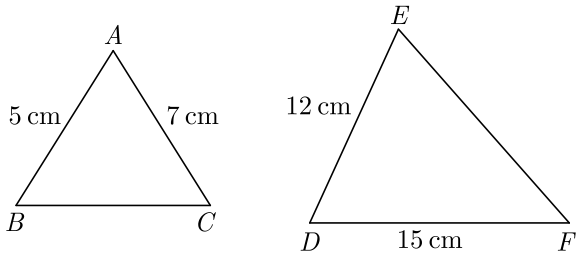
9. It is given that, $\Delta ABC \sim \Delta EDF$ such that $AB = 5\,\text{cm}$, $AC = 7$ cm, $DF = 15$ cm and $DE = 12\,\text{cm}$ then the sum of the remaining sides of the triangles is

(a) 23.05 cm (b) 16.8 cm

(c) 6.25 cm (d) 24 cm

Sol :

We have $\Delta ABC \sim \Delta EDF$



Now $$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

Taking first and second ratios, we get

$$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5}$$

$$= 16.8 \text{ cm}$$

Taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12}$$

$$= 6.25 \text{ cm}$$

Now, sum of the remaining sides of triangle,

$$EF + BC = 16.8 + 6.25 = 23.05 \text{ cm}$$

Thus (a) is correct option.

10. The area of a right angled triangle is 40 sq cm and its perimeter is 40 cm. The length of its hypotenuse is

(a) 16 cm (b) 18 cm

(c) 17 cm (d) data insufficient

Sol : (b) 18 cm

Let c be the hypotenuse of the triangle, a and b be other sides.

Now $$c = \sqrt{a^2 + b^2}$$

We have, $a + b + c = 40$ and $\frac{1}{2}ab = 40$

$\Rightarrow ab = 80$

$$c = 40 - (a + b) \text{ and } ab = 80$$

Squaring $c = 40 - (a + b)$ we have

$$\begin{aligned} c^2 &= [40 - (a + b)]^2 \\ a^2 + b^2 &= 1600 - 2 \times 40(a + b) + (a + b)^2 \\ a^2 + b^2 &= 1600 - 2 \times 40(a + b) + a^2 + 2ab + b^2 \\ 0 &= 1600 - 2 \times 40(a + b) + 2 \times 80 \\ 0 &= 20 - (a + b) + 2 \\ a + b &= 22 \\ c &= 40 - (a + b) = 40 - 22 = 18 \text{ cm} \end{aligned}$$

Thus (b) is correct option.

11. **Assertion :** In the ΔABC, $AB = 24\,\text{cm}$, $BC = 10\,\text{cm}$ and $AC = 26\,\text{cm}$, then ΔABC is a right angle triangle.

Reason : If in two triangles, their corresponding angles are equal, then the triangles are similar.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

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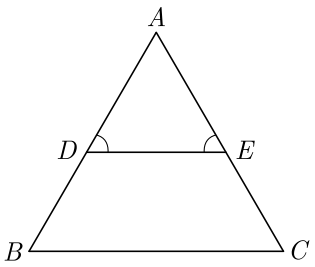
In the given figure $DE || BC$, thus

$$\begin{aligned}\frac{AD}{DB} &= \frac{AE}{EC}\\ \frac{x}{x-2} &= \frac{x+2}{x-1}\\ x(x-1) &= (x+2)(x-2)\\ x^2-x &= x^2-4\\ -x &= -4\\ x &= 4\end{aligned}$$

27. In Figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that ΔBAC is an isosceles triangle.



Sol : [Board 2020 Delhi Standard]

We have, $\angle D = \angle E$

and $\frac{AD}{DB} = \frac{AE}{EC}$

By converse of BPT, $DE \parallel BC$

Due to corresponding angles we have

$\angle ADE = \angle ABC$ and

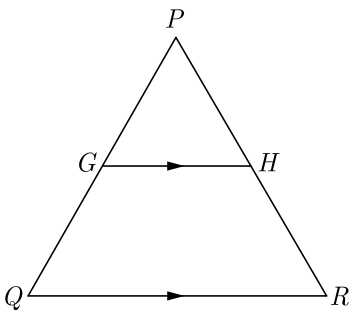
$\angle AED = \angle ACB$

Given $\angle ADE = \angle AED$

Thus $\angle ABC = \angle ACB$

Therefore BAC is an isosceles triangle.

28. In the given figure, G is the mid-point of the side PQ of ΔPQR and $GH || QR$. Prove that H is the mid-point of the side PR or the triangle PQR.



Sol : [Board Term-1 2012]

Since G is the mid-point of PQ we have

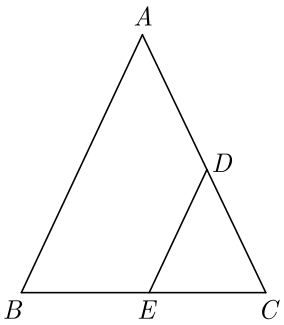
$$\begin{aligned}PG &= GQ\\ \frac{PG}{GQ} &= 1\end{aligned}$$

We also have $GH || QR$, thus by BPT we get

$$\begin{aligned}\frac{PG}{GQ} &= \frac{PH}{HR}\\ 1 &= \frac{PH}{HR}\\ PH &= HR.\end{aligned}$$ Hence proved.

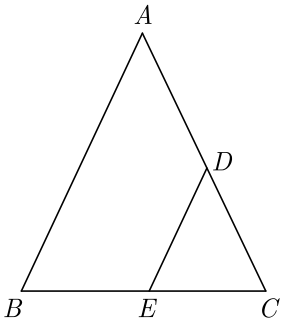
Hence, H is the mid-point of PR.

29. In the figure of ΔABC, the points D and E are on the sides CA, CB respectively such that $DE || AB$, $AD = 2x, DC = x+3, BE = 2x-1$ and $CE = x$. Then, find x.



OR

In the figure of ΔABC, $DE || AB$. If $AD = 2x$, $DC = x+3, BE = 2x-1$ and $CE = x$, then find the value of x.



Sol : [Board Term-1 2015, 2016]

We have $\frac{CD}{AD} = \frac{CE}{BE}$

$$\begin{aligned}\frac{x+3}{2x} &= \frac{x}{2x-1}\\ 5x &= 3 \text{ or, } x = \frac{3}{5}\end{aligned}$$

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CHAPTER 6

Triangles

OBJECTIVE QUESTIONS

1. If $\Delta PQR \sim \Delta ABC$; $PQ = 6\,\text{cm}$, $AB = 8\,\text{cm}$ and the perimeter of ΔABC is 36 cm, then the perimeter of ΔPQR is

(a) 20.25 cm (b) 27 cm

(c) 48 cm (d) 64 cm

Sol : [Board 2023 OD Standard]

We have $\Delta PQR \sim \Delta ABC$

$PQ = 6\,\text{cm}$, $AB = 8\,\text{cm}$

Perimeter of ΔABC is 36 cm.

Since ratio of perimeter of two similar triangles is same as the ratio of their corresponding sides, we have

$$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{8}{6}$$

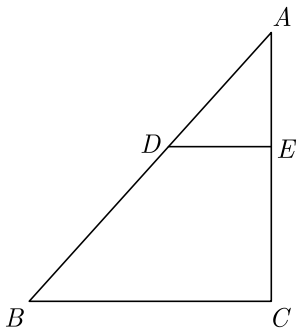
$$\frac{36}{x} = \frac{8}{6}$$

$$x = \frac{36 \times 6}{8} = 27$$

Thus, perimeter of ΔPQR is 27 cm.

Thus (b) is correct option.

2. In the given figure, $DE \| BC$. If $AD = 3\,\text{cm}$, $AB = 7$ cm and $EC = 3$ cm, then the length of AE is



(a) 2 cm (b) 2.25 cm

(c) 3.5 cm (d) 4 cm

Sol : [Board 2023 OD Standard]

In ΔABC $DE \| BC$

Using basic proportionality theorem we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{AD}{AB - AD} = \frac{AE}{EC}$$

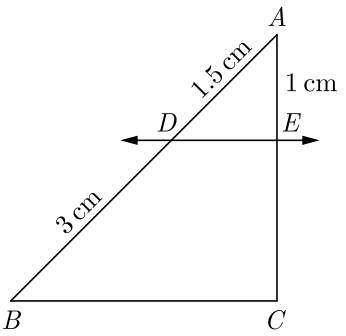
$$\frac{3}{7-3} = \frac{AE}{3}$$

$$\frac{3}{4} = \frac{AE}{3}$$

$$AE = \frac{9}{4} = 2.25\,\text{cm}$$

Thus (b) is correct option.

3. In the given figure, $DE \| BC$. The value of EC is



(a) 1.5 cm

(b) 3 cm

(c) 2 cm

(d) 1 cm

Sol :

Since, $DE \| BC$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC}$$

$$EC = 2\,\text{cm}$$

Thus (c) is correct option.

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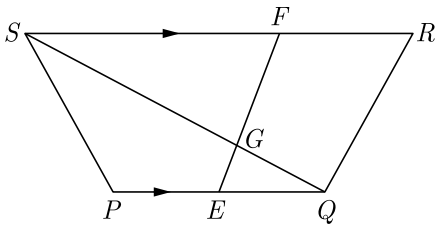


Sol : [Board Term-1 2016]

In ΔGEQ and ΔGFS,

Due to vertical opposite angle,

$$\angle EGQ = \angle FGS$$

Due to alternate angle,

$$\angle EQG = \angle FSG$$

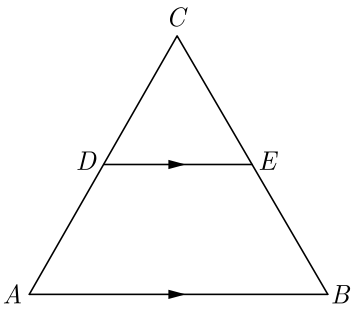
Thus by AA similarity we have

$$\Delta GEQ \sim GFS$$

$$\frac{EQ}{FS} = \frac{GQ}{GS}$$

$$EQ \times GS = GQ \times FS$$

34. In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE || AB$.



Sol : [Board Term-1 2012]

In ΔCAB, we have

$$\angle A = \angle B \quad (1)$$

By isosceles triangle property we have

$$AC = CB$$

But, we have been given

$$AD = BE \quad (2)$$

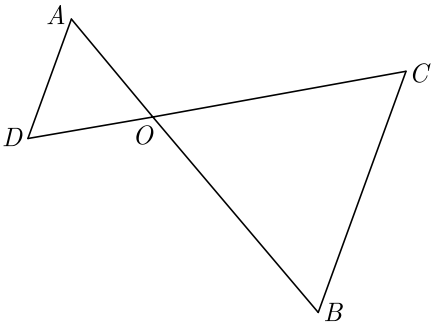
Dividing equation (2) by (1) we get,

$$\frac{CD}{AD} = \frac{CE}{BE}$$

By converse of BPT,

$$DE || AB. \quad \text{Hence Proved}$$

35. In the given figure, $OA \times OB = OC \times OD$, show that $\angle A = \angle C$ and $\angle B = \angle D$.



Sol : [Board Term-1 2012]

We have $OA \times OB = OC \times OD$

$$\frac{OA}{OD} = \frac{OC}{OB}$$

Due to the vertically opposite angles,

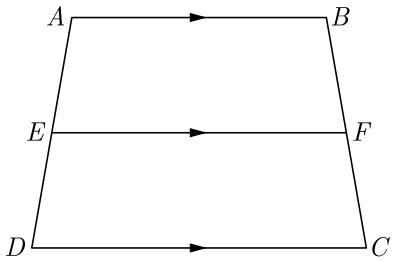
$$\angle AOD = \angle COB$$

Thus by SAS similarity we have

$$\Delta AOD \sim \Delta COB$$

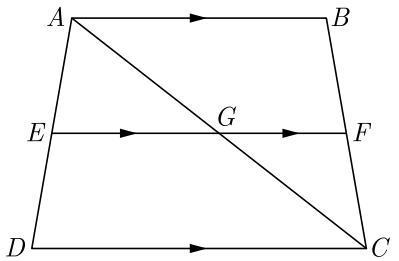
Thus $\angle A = \angle C$ and $\angle B = \angle D$. because of corresponding angles of similar triangles.

36. In the given figure, if $ABCD$ is a trapezium in which $AB || CD || EF$, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$



Sol : [Board Term-1 2012]

We draw, AC intersecting EF at G as shown below.



In ΔCAB, $GF || AB$, thus by BPT we have

$$\frac{AG}{CG} = \frac{BF}{FC} \quad ...(1)$$

In ΔADC, $EG || DC$, thus by BPT we have

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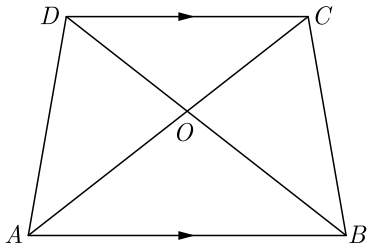
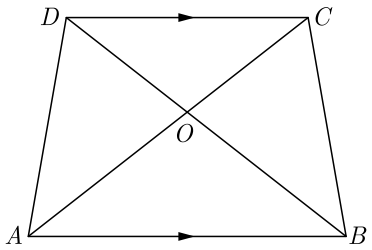
THREE MARKS QUESTIONS

44. Diagonals AC and BD of trapezium $ABCD$with $AB \| DC$ intersect each other at point O. Show that $\frac{OA}{OC} = \frac{OB}{OD}$.



Sol : [Board 2023 OD Standard]

We have redrawn the figure below.



In ΔAOB and ΔCOD, $AB \| CD$,

Thus due to alternate angles

$$\angle OAB = \angle DCO$$

and $$\angle OBA = \angle ODC$$

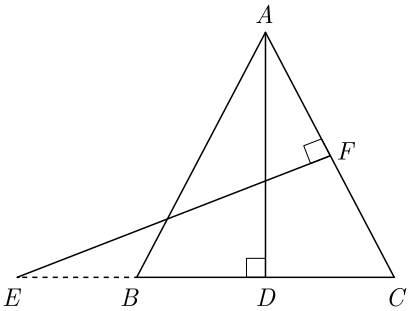
By AA similarity we have

$$\Delta AOB \sim \Delta COD$$

For corresponding sides of similar triangles we have

$$\frac{OA}{OC} = \frac{OB}{OD}$$ Hence Proved

45. In the given figure, E is a point on the side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, them prove that $\Delta ABD - \Delta ECF$.



Sol : [Board 2023 OD Standard]

In ΔABD and ΔCEF, we have

$$AB = AC$$

Thus $$\angle ABC = \angle ACB$$

$$\angle ABD = \angle ECF$$

$$\angle ADB = \angle EFC$$ (Each 90^o)

Due to AA similarity

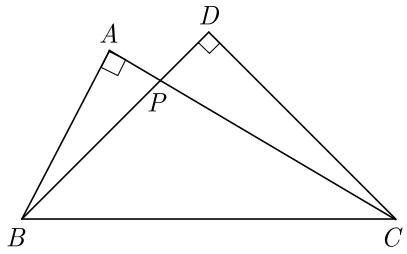
$$\Delta ABD \sim \Delta ECF$$ Hence proved

46. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that $AP \times PC = BP \times DP$.

Sol : [Board 2019 OD]

Let ΔABC, and ΔDBC be right angled at A and D respectively.

As per given information in question we have drawn the figure given below.



In ΔBAP and ΔCDP we have

$$\angle BAP = \angle CDP = 90^\circ$$

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

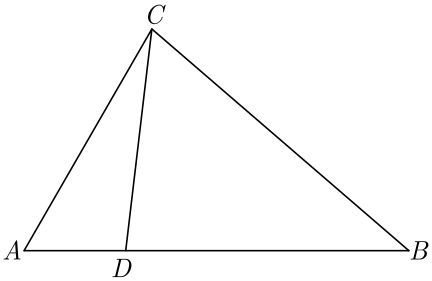
By AA similarity we have

$$\Delta BAP \backsim \Delta CDP$$

Therefore $$\frac{BP}{PC} = \frac{AP}{PD}$$

$$AP \times PC = BP \times PD$$ Hence Proved

47. In the given figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB.



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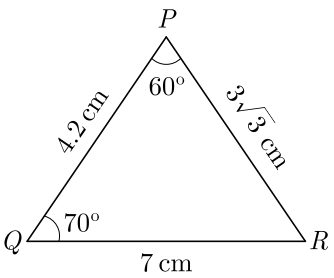
Sol : [Board Term-1 2012]

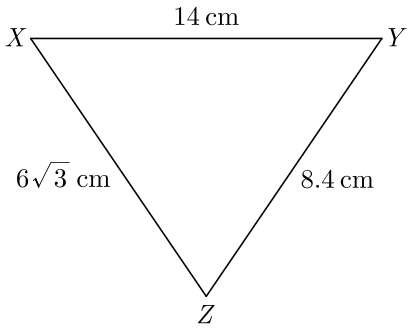
We know that diagonals of a trapezium divide each other proportionally. Therefore

$$\frac{OA}{OC} = \frac{BO}{OD}$$
$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$
$$(x+5)(x-2) = (x-1)(x+3)$$
$$x^2-2x+5x-10 = x^2+3x-x-3$$
$$x^2+3x-10 = x^2+2x-3$$
$$3x-10 = 2x-3$$
$$3x-2x = 10-3 \Rightarrow x = 7$$

Thus $x = 7$.

41. In the given figures, find the measure of $\angle X$.





Sol : [Board Term-1 2012]

From given figures,

$$\frac{PQ}{ZY} = \frac{4.2}{8.4} = \frac{1}{2},$$
$$\frac{PR}{ZX} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

and $$\frac{QR}{YX} = \frac{7}{14} = \frac{1}{2}$$

Thus $$\frac{QP}{ZY} = \frac{PR}{ZX} = \frac{QR}{YX}$$

By SSS criterion we have

$$\Delta PQR \sim \Delta ZYX$$

Thus $\angle X = \angle R$

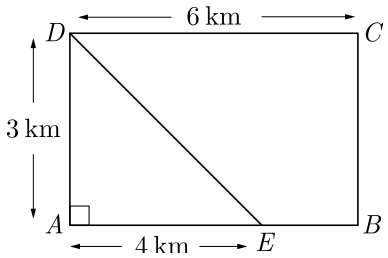
$$= 180^\circ - (60^\circ + 70^\circ) = 50^\circ$$

Thus $\angle X = 50^\circ$

42. In a rectangle $ABCD, E$ is a point on AB such that $AE = \frac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find DE.

Sol : [Board Term-1 2016]

As per given condition we have drawn the figure below.



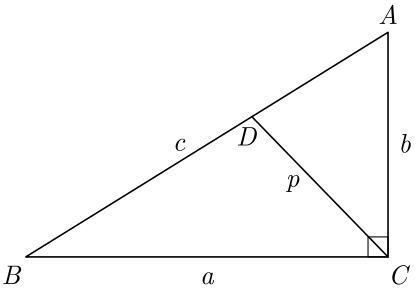
We have $$AE = \frac{2}{3}AB = \frac{2}{3} \times 6 = 4 \text{ km}$$

In right triangle ADE,

$$DE^2 = (3)^2 + (4)^2$$
$$= 25$$

Thus $$DE = 5 \text{ km}$$

43. ABC is a right triangle right angled at C. Let $BC = a$, $CA = b, AB = c$ $PQR, ST \| QR$and p be the length of perpendicular from C to AB. Prove that $cp = ab$.



Sol : [Board Term-1 2012]

In the given figure $CD \perp AB$, and $CD = p$

Area, $$\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times AB \times CD = \frac{1}{2}cp$$

Also,Area of $\Delta ABC = \frac{1}{2} \times BC \times AC = \frac{1}{2}ab$

Thus $$\frac{1}{2}cp = \frac{1}{2}ab$$
$$cp = ab$$ Proved

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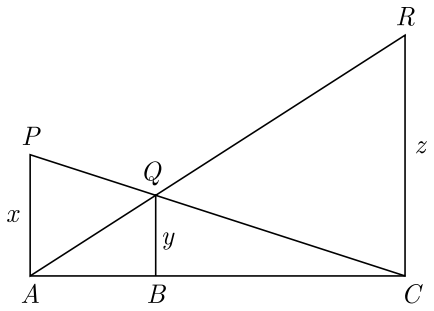
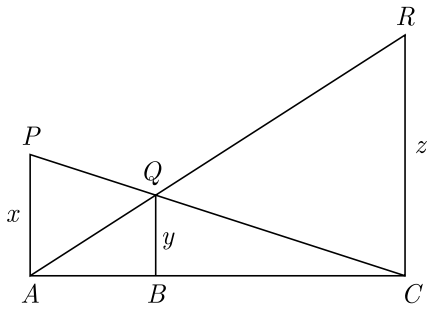


Sol : [Board 2024 OD Standard]

We have redrawn the given figure as below.

Here $AP \perp AC$, $BQ \perp AC$ and $CR \perp AC$

$AP = x$, $BQ = y$, $CR = Z$



In ΔPAC and ΔQBC,

$$\angle PAC = \angle QBC = 90^\circ$$

$$\angle PCA = \angle QCB \quad \text{(Common)}$$

By AA similarity criterion

$$\Delta PAC \sim \Delta QBC$$

$$\frac{BQ}{AP} = \frac{BC}{AC}$$

$$\frac{y}{x} = \frac{BC}{AC} \quad ...(1)$$

Similarly, by AA similarity criterion

$$\Delta ACR \sim \Delta ABQ$$

$$\frac{BQ}{CR} = \frac{AB}{AC}$$

$$\frac{y}{Z} = \frac{AB}{AC} \quad ...(2)$$

Adding (1) and (2), we get

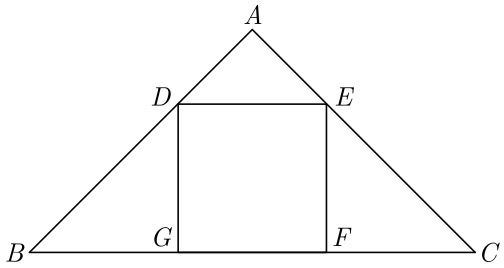
$$\frac{y}{x} + \frac{y}{z} = \frac{BC}{AC} + \frac{AB}{AC}$$

$$y\left(\frac{1}{x} + \frac{1}{z}\right) = \frac{BC + AB}{AC}$$

$$y\left(\frac{1}{x} + \frac{1}{z}\right) = \frac{AC}{AC} = 1$$

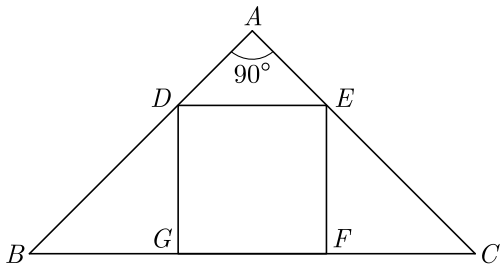
$$\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$$ Hence, proved

62. In the given figure, $DEFG$ is a square and $\angle BAC = 90^\circ$. Show that $FG^2 = BG \times FC$.



Sol : [Board 2020 SQP Standard]

We have redrawn the given figure as shown below.



In ΔADE and ΔGBD, we have

$$\angle DAE = \angle BGD \quad [\text{each } 90^\circ]$$

Due to corresponding angles we have

$$\angle ADE = \angle GDB$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta GBD$$

Now, in ΔADE and ΔFEC,

$$\angle EAD = \angle CFE \quad [\text{each } 90^\circ]$$

Due to corresponding angles we have

$$\angle AED = \angle FCE$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta FEC$$

Since $\Delta ADE \sim \Delta GBD$ and $\Delta ADE \sim \Delta FEC$ we have

$$\Delta GBD \sim \Delta FEC$$

Thus $$\frac{GB}{FE} = \frac{GD}{FC}$$

Since $DEFG$ is square, we obtain,

$$\frac{BG}{FG} = \frac{FG}{FC}$$

Therefore $$FG^2 = BG \times FC$$

Hence Proved

NODIA

Sol : [Board 2020 SQP Standard]

In ΔABC and ΔACD we have

$$\angle ACB = \angle CDA \quad \text{[given]}$$

$$\angle CAB = \angle CAD \quad \text{[common]}$$

By AA similarity criterion we get

$$\Delta ABC \sim \Delta ACD$$

Thus $$\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$$

Now $$\frac{AB}{AC} = \frac{AC}{AD}$$

$$AC^2 = AB \times AD$$

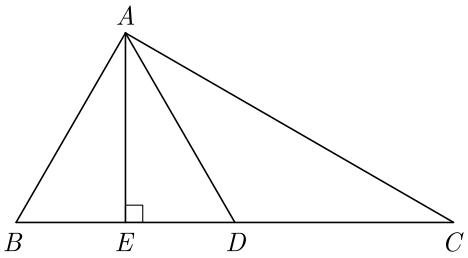
$$6^2 = AB \times 3$$

$$AB = \frac{36}{3} = 12 \text{ cm}$$

48. If in $\Delta ABC, AD$ is median and $AE \perp BC$, then prove that $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.

Sol : [Board Term-1 2015]

As per given condition we have drawn the figure below.



In ΔABE, using Pythagoras theorem we have

$$AB^2 = AE^2 + BE^2$$

$$= AD^2 - DE^2 + (BD - DE)^2$$

$$= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$$

$$= AD^2 + BD^2 - 2BD \times DE \quad ...(1)$$

In ΔAEC, we have

$$AC^2 = AE^2 + EC^2$$

$$= (AD^2 - ED^2) + (ED + DC)^2$$

$$= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC$$

$$= AD^2 + CD^2 + 2ED \times CD$$

$$= AD^2 + DC^2 + 2DC \times DE \quad ...(2)$$

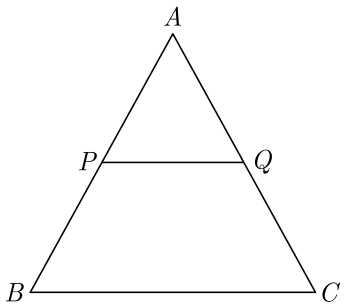
Adding equation (1) and (2) we have

$$AB^2 + AC^2 = 2(AD^2 + BD^2) \quad (BD = DC)$$

$$= 2AD^2 + 2\left(\frac{1}{2}BC\right)^2 \quad \left(BD = \tfrac{1}{2}BC\right)$$

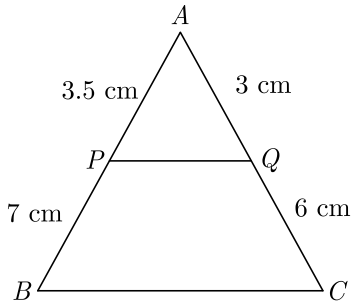
$$= 2AD^2 + \frac{1}{2}BC^2 \quad \text{Hence Proves}$$

49. In the given figure, P and Q are the points on the sides AB and AC respectively of ΔABC, such that $AP = 3.5$cm, $PB = 7$ cm, $AQ = 3$ cm and $QC = 6$ cm. If $PQ = 4.5$ cm, find BC.



Sol : [Board Term-1 2011]

We have redrawn the given figure as below.



We have $$\frac{AP}{AB} = \frac{3.5}{10.5} = \frac{1}{3}$$

and $$\frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$$

In ΔABC, $\frac{AP}{AB} = \frac{AQ}{AC}$ and $\angle A$ is common.

Thus due to SAS we have

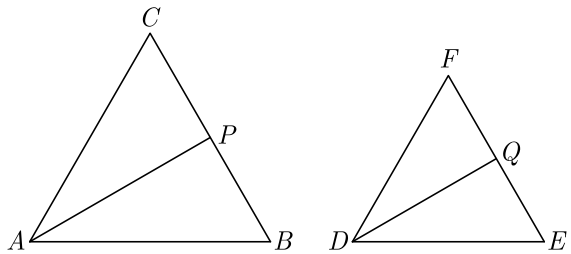
$$\Delta APQ \sim \Delta ABC$$

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{1}{3} = \frac{4.5}{BC}$$

$$BC = 13.5 \text{ cm}.$$

50. In given figure $\Delta ABC \sim \Delta DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.



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Sol : [Board 2020 Delhi STD, 2012]

In ΔABC, $DE \parallel AC$, (Given)

By BPT $\frac{BD}{DA} = \frac{BE}{EC}$ ...(1)

In ΔABE, $DF \parallel AE$, (Given)

By BPT $\frac{BD}{DA} = \frac{BF}{FE}$ ...(2)

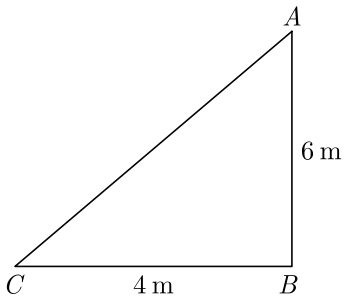
From (1) and (2), we have

$$\frac{BF}{FE} = \frac{BE}{EC}.$$

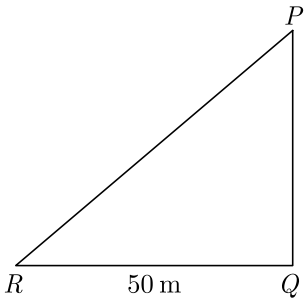
57. A 6 m high tree cast a 4 m long shadow. At the same time, a flag pole cast a shadow 50 m long. How long is the flag pole?

Sol : [Board 2009]

Let AB be height of tree and BC its shadow.



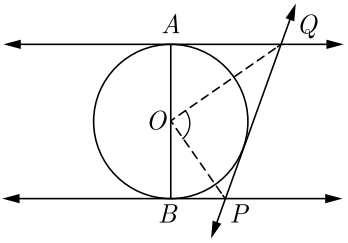
Again, let PQ be height of pole and QR be its shadow. At the same time, the angle of elevation of tree and poles are equal i.e $\Delta ABC \sim PQR$



Thus
$$\frac{AB}{BC} = \frac{PQ}{QR}$$
$$\frac{6}{4} = \frac{PQ}{50}$$
$$PQ = \frac{50 \times 6}{4}$$
$$= 75 \text{ m}$$

FIVE MARKS QUESTIONS

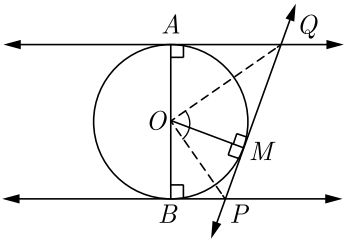
58. In the given figure, AB is a diameter of the circle with centre O. AQ, BP and PQ are tangents to the circle. Prove that $\angle POQ = 90^\circ$.



or

Sol :

We redraw the given figure as shown below.



In ΔOAQ and ΔOMQ, AQ and QM are tangents drawn from common point,

Thus $AQ = QM$

Lengths of tangents from an external point are equal. Thus

$$OQ = OQ$$

Due to radius of circle,

$$OA = OM$$

By SSS congruence criterion we get

$$\Delta OAQ \cong \Delta OMQ$$

By CPCT we obtain

$$\angle AOQ = \angle MOQ \quad (1)$$

Similarly, $\angle BOP = \angle MOP$ (2)

As AB is diameter which is a straight line, from linear pair axiom we have

$$\angle AOQ + \angle MOQ + \angle BOP + \angle MOP = 180^\circ$$

Substituting from (1) and (2) we have

$$\angle MOQ + \angle MOQ + \angle MOP + \angle MOP = 180^\circ$$
$$2\angle MOP + 2\angle MOQ = 180^\circ$$
$$\angle MOP + \angle MOQ = 90^\circ$$

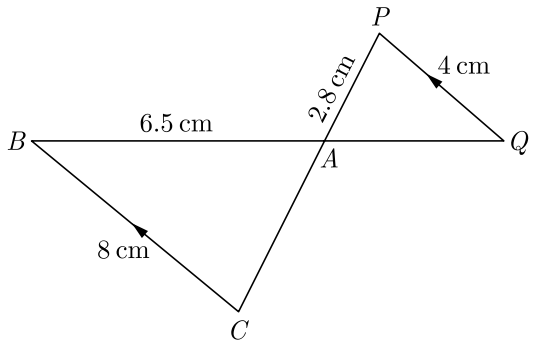
NODIA

$$\frac{AQ}{QC} = \frac{DR}{RC}$$

By converse of BPT,

$$PR \parallel AD$$ Hence proved

53. In the given figure, $BC \parallel PQ$ and $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm $AP = 2.8$ cm Find CA and AQ.



Sol : [Board Term-1 2012]

In ΔABC and ΔAPQ, $AB = 6.5$ cm, $BC = 8$ cm,

$PQ = 4$ cm and $AP = 2.8$ cm.

We have $BC \parallel PQ$

Due to alternate angles

$$\angle CBA = \angle AQP$$

Due to vertically opposite angles,

$$\angle BAC = \angle PAQ$$

Due to AA similarity,

$$\Delta ABC \sim \Delta AQP$$

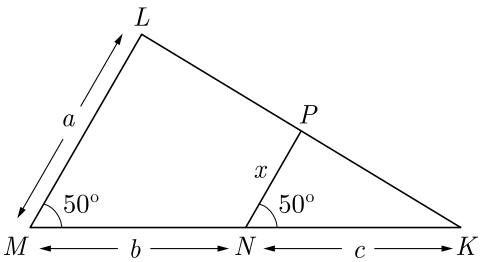
$$\frac{AB}{AQ} = \frac{BC}{QP} = \frac{AC}{AP}$$

$$\frac{6.5}{AQ} = \frac{8}{4} = \frac{AC}{AP}$$

$$AQ = \frac{6.5}{2} = 3.25 \text{ cm}$$

$$AC = 2 \times 2.5 = 5.6 \text{ cm}$$

54. In the given figure, find the value of x in terms of a, b and c.



Sol : [Board Term-1 2012]

In triangles LMK and PNK, $\angle K$ is common and

$$\angle M = \angle N = 50^\circ$$

Due to AA similarity,

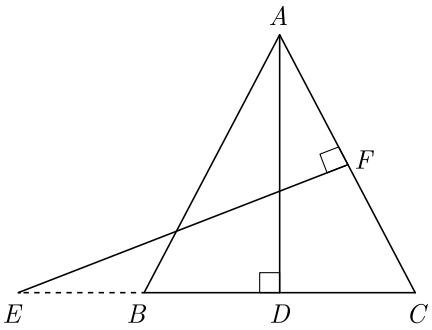
$$\Delta LMK \sim \Delta PNK$$

$$\frac{LM}{PN} = \frac{KM}{KN}$$

$$\frac{a}{x} = \frac{b+c}{c}$$

$$x = \frac{ac}{b+c}$$

55. In the given figure, $AB = AC$. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC, prove that ΔABD is similar to ΔCEF.



Sol : [Board Term-1 2012]

In ΔABD and ΔCEF, we have

$$AB = AC$$

Thus $\angle ABC = \angle ACB$

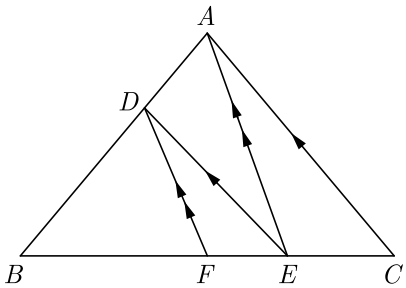
$$\angle ABD = \angle ECF$$

$$\angle ADB = \angle EFC$$ (Each 90°)

Due to AA similarity

$$\Delta ABD \sim \Delta ECF$$ Hence proved

56. In the given figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{BE}{FE} = \frac{BE}{EC}$.



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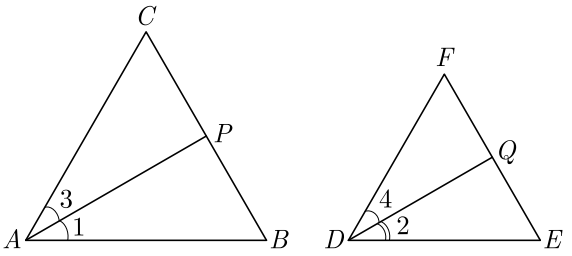
Prove that :

(1) $\frac{AP}{DQ} = \frac{AB}{DE}$

(2) $\Delta CAP \sim \Delta FDQ$.

Sol : [Board Term-1 2016]

As per given condition we have redrawn the figure below.



(1) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D \quad \text{(Corresponding angles)}$$

$$2\angle 1 = 2\angle 2$$

Also $\angle B = \angle E$ (Corresponding angles)

$$\frac{AP}{DQ} = \frac{AB}{DE}$$ Hence Proved

(2) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D$$

and $$\angle C = \angle F$$

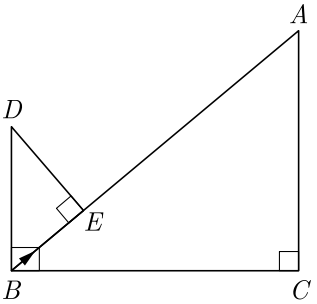
$$2\angle 3 = 2\angle 4$$

$$\angle 3 = \angle 4$$

By AA similarity we have

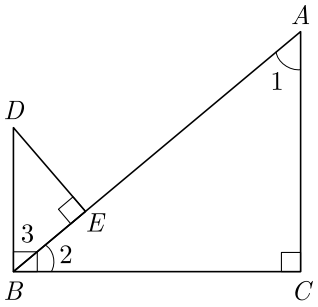
$$\Delta CAP \sim \Delta FDQ$$

51. In the given figure, $DB \perp BC, DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



Sol : [Board Term-1 2011]

As per given condition we have redrawn the figure below.



We have $DB \perp BC, DE \perp AB$ and $AC \perp BC$.

In ΔABC, $\angle C = 90^\circ$, thus

$$\angle 1 + \angle 2 = 90^\circ$$

But we have been given,

$$\angle 2 + \angle 3 = 90^\circ$$

Hence $$\angle 1 = \angle 3$$

In ΔABC and ΔBDE,

$$\angle 1 = \angle 3$$

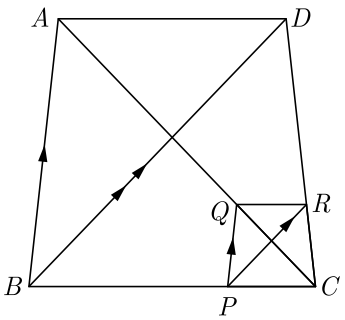
and $$\angle ACB = \angle DEB = 90^\circ$$

Thus by AA similarity we have

$$\Delta ABC \sim \Delta BDE$$

Thus $$\frac{AC}{BC} = \frac{BE}{DE}.$$ Hence Proved

52. In the given figure, two triangles ABC and DBC lie on the same side of BC such that $PQ \| BA$ and $PR \| BD$. Prove that $QR \| AD$.



Sol : [Board Term-1 2011]

In ΔABC, we have $PQ \| AB$ and $PR \| BD$.

By BPT we have

$$\frac{BP}{PC} = \frac{AQ}{QC} \quad \ldots(1)$$

Again in ΔBCD, we have

$$PR \| BD$$

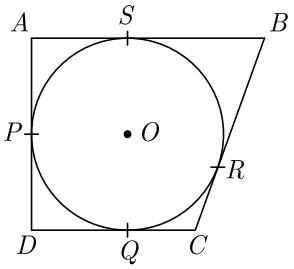
By BPT we have

$$\frac{BP}{PC} = \frac{DR}{RC}$$ (by BPT) ...(2)

NODIA

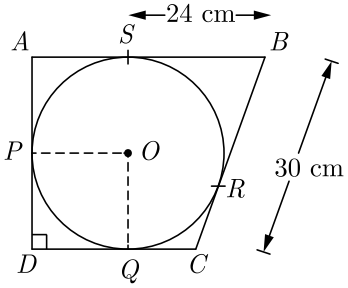
$$\angle POQ = 90^\circ$$

59. A circle with centre O and radius 8 cm is inscribed in a quadrilateral $ABCD$ in which P, Q, R, S are the points of contact as shown. If AD is perpendicular to DC, $BC = 30$ cm and $BS = 24$ cm, then find the length DC.



Sol : [Board 2024 OD Standard]

We redraw the given figure as shown below. Here we have joined OP and OQ.



We have $BS = 24$ cm, $BC = 30$ cm and radius $= 8$ cm

Since length of tangents from an external point are equal, we have

$$BR = BS = 24\text{ cm} \quad (1)$$

Now $BC = 30\text{ cm}$

$$BR + CR = 30\text{ cm}$$

$$24 + CR = 30\text{ cm}$$

$$CR = 6\text{ cm} \quad (2)$$

Since length of tangents from an external point are equal, we have

$$CQ = CR = 6\text{ cm} \quad (3)$$

In $\square POQD$, using angle sum property we have

$$\angle OPD + \angle PDQ + \angle DQO + \angle POQ = 360^\circ$$

$$90^\circ + 90^\circ + 90^\circ + \angle POQ = 360^\circ$$

$$\angle POQ = 90^\circ$$

Thus $\square POQD$ is a square as two adjacent sides are also $[OP = OQ]$ equal.

Here OQ is radius of 8 cm, thus

$$DQ = OQ = 8\text{ cm} \quad ...(4)$$

From (3) and (4)

$$DC = DQ + CQ$$

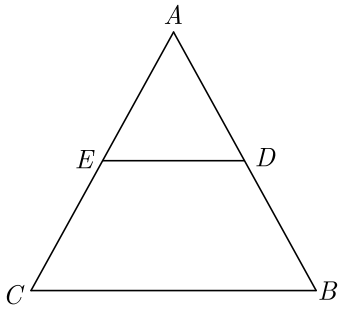
$$= 8 + 6$$

$$= 14\text{ cm}$$

60. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

Sol : [Board 2024 OD Standard]

We have drawn a triangle ABC is given in which $DE \| BC$ as given below. Here ED is parallel to CB.



Here angle EAD is common to both triangle.

Due to corresponding angles

$$\angle AED = \angle ACB$$

$$\angle ADE = \angle ABC$$

From AAA similarity we get

$$\Delta AED \ \Delta ACB$$

Due to similar triangle we have

$$\frac{AC}{AE} = \frac{AB}{AD}$$

$$\frac{AE + AC}{AE} = \frac{AD + AB}{AD}$$

$$\frac{EC}{AE} = \frac{BD}{AD}$$ Hence Proved

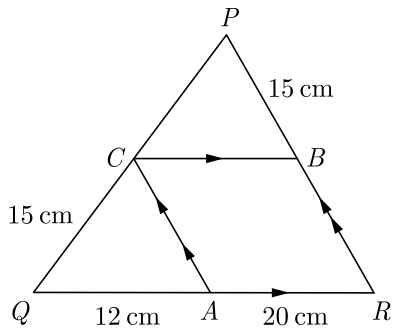
61. In the given figure PA, QB and RC are each perpendicular to AC. If $AP = x$, $BQ = y$ and $CR = z$, then prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$

$$\frac{AE}{ED} = \frac{AG}{CG} \qquad ...(2)$$

From equations (1) and (2),

$$\frac{AE}{ED} = \frac{BF}{FC}. \qquad \text{Hence Proved.}$$

37. In the given figure, $CB \,||\, QR$ and $CA \,||\, PR$. If $AQ = 12$ cm, $AR = 20$ cm, $PB = CQ = 15$ cm, calculate PC and BR.



Sol : [Board Term-1 2012]

In ΔPQR, $CA \,||\, PR$

By BPT similarity we have

$$\frac{PC}{CQ} = \frac{RA}{AQ}$$

$$\frac{PC}{15} = \frac{20}{12}$$

$$PC = \frac{15 \times 20}{12} = 25 \text{ cm}$$

In ΔPQR, $CB \,||\, QR$

Thus

$$\frac{PC}{CQ} = \frac{PR}{BR}$$

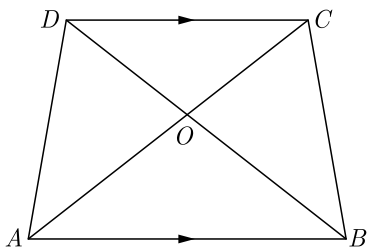
$$\frac{25}{15} = \frac{15}{BR}$$

$$BR = \frac{15 \times 15}{25} = 9 \text{ cm}$$

38. $ABCD$ is a trapezium in which $AB \,||\, CD$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Sol : [Board Term-1 2012]

As per given condition we have drawn the figure below.



In ΔAOB and ΔCOD, $AB \,||\, CD$,

Thus due to alternate angles

$$\angle OAB = \angle DCO$$

and $$\angle OBA = \angle ODC$$

By AA similarity we have

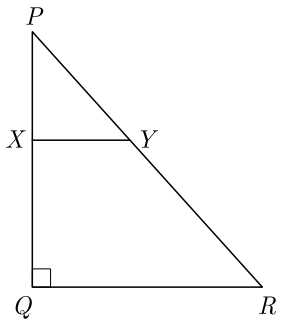
$$\Delta AOB \sim \Delta COD$$

For corresponding sides of similar triangles we have

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\frac{AO}{BO} = \frac{CO}{DO}. \qquad \text{Hence Proved}$$

39. In the given figure, PQR is a triangle right angled at Q and $XY \,||\, QR$. If $PQ = 6$ cm, $PY = 4$ cm and $PX : XQ = 1 : 2$. Calculate the length of PR and QR.



Sol : [Board Term-1 2012]

Since $XY \,||\, OR$, by BPT we have

$$\frac{PX}{XQ} = \frac{PY}{YR}$$

$$\frac{1}{2} = \frac{PY}{PR - PY} = \frac{4}{PR - 4}$$

$$PR - 4 = 8 \Rightarrow PR = 12 \text{ cm}$$

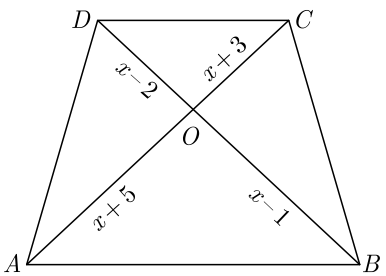
In right ΔPQR we have

$$QR^2 = PR^2 - PQ^2$$

$$= 12^2 - 6^2 = 144 - 36 = 108$$

Thus $QR = 6\sqrt{3}$ cm

40. In the given figure, if $AB \,||\, DC$, find the value of x.



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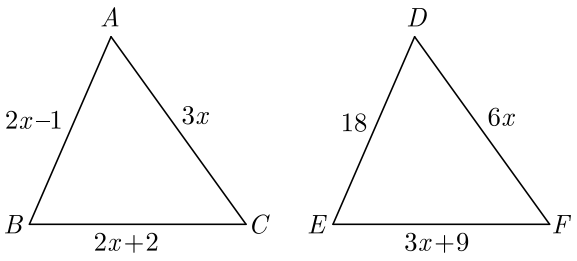
63. In Figure, if $\Delta ABC \sim \Delta DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle.



Sol : [Board 2020 OD Standard]

Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{2x-1}{2x+2} = \frac{18}{3x+9}$$

$$(2x-1)(3x+9) = 18(2x+2)$$

$$(2x-1)(x+3) = 6(2x+2)$$

$$2x^2 - x + 6x - 3 = 12x + 12$$

$$2x^2 + 5x - 12x - 15 = 0$$

$$2x^2 - 7x - 15 = 0$$

$$2x^2 - 10x + 3x - 15 = 0$$

$$2x(x-5) + 3(x-5) = 0$$

$$(x-5)(2x+3) = 0 \Rightarrow x = 5$$

or $$x = \frac{-3}{2}$$

But $x = \frac{-3}{2}$ is not possible, thus $x = 5$.

Now in ΔABC, we get

$$AB = 2x - 1$$
$$= 2 \times 5 - 1 = 9$$
$$BC = 2x + 2$$
$$= 2 \times 5 + 2 = 12$$
$$AC = 3x$$
$$= 3 \times 5 = 15$$

and in ΔDEF, we get

$$DE = 18$$
$$EF = 3x + 9$$
$$= 3 \times 5 + 9 = 24$$
$$DE = 6x$$
$$= 6 \times 5 = 30$$

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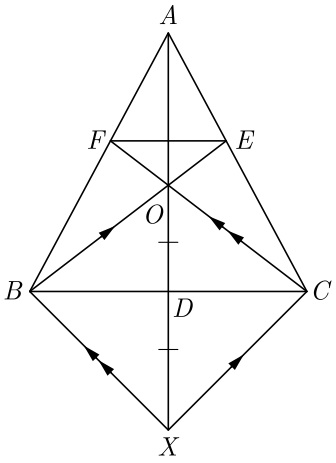
64. In ΔABC, AD is a median and O is any point on AD. BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such that $OD = DX$ as shown in figure.

Prove that :

(1) $EF \,||\, BC$

(2) $AO : AX = AF : AB$



Sol : [Board Term-1 2015]

Since BC and OX bisect each other, $BXCO$ is a parallelogram. Therefore $BE \,||\, XC$ and $BX \,||\, CF$

In ΔABX, by BPT we get,

$$\frac{AF}{FB} = \frac{AO}{OX} \quad ..(1)$$

In ΔAXC, $$\frac{AE}{EC} = \frac{AO}{OX} \quad ...(2)$$

From (1) and (2) we get

$$\frac{AF}{FB} = \frac{AE}{EC}$$

By converse of BPT we have

$$EF \,||\, BC$$

From (1) we get $\frac{OX}{OA} = \frac{FB}{AF}$

$$\frac{OX + OA}{OA} = \frac{FB + AF}{AF}$$

$$\frac{AX}{OA} = \frac{AB}{AF}$$

$$\frac{AO}{AX} = \frac{AF}{AB}$$

Thus $AO : AX = AF : AB$ Hence Proved

65. In the figure, $\angle BED = \angle BDE$ and E is the mid-point of BC. Prove that $\frac{AF}{CF} = \frac{AD}{BE}$.

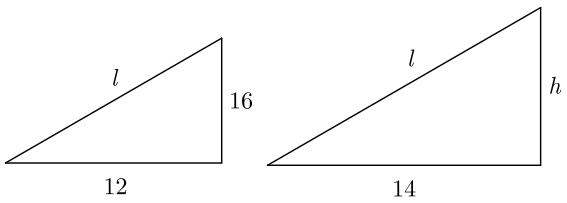
$$x = 6$$
$$y^2 = (21-x)^2 + 8^2$$
$$= (21-6)^2 + 8^2$$
$$= 15^2 + 8^2 = 17^2$$
$$y = 17 \text{ m}$$

Thus Anil should cut 17 m wire.

78. Wall Paint : A painter sets a ladder up to reach the bottom of a second-story window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint, a neighbour's dog bumps the ladder, which moves the base 2 feet farther away from the house. How far up the side of the house does the ladder reach?

Sol :

We draw a diagram of the situation as shown below.



We use the Pythagorean Theorem to find the length of the ladder, represented by y.

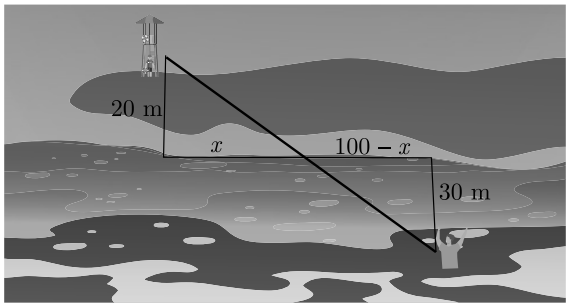
$$12^2 + 16^2 = l^2$$
$$144 + 256 = l^2$$
$$400 = l^2 \Rightarrow l = \sqrt{400} = 20 \text{ feet}$$

The ladder is 20 feet long.

Now
$$(2+12)^2 + h^2 = l^2$$
$$(2+12)^2 + h^2 = 20^2$$
$$14^2 + h^2 = 20^2$$
$$h = \sqrt{20^2 - 14^2} = \sqrt{204}$$
$$= 2\sqrt{51}$$
$$\approx 14.3$$

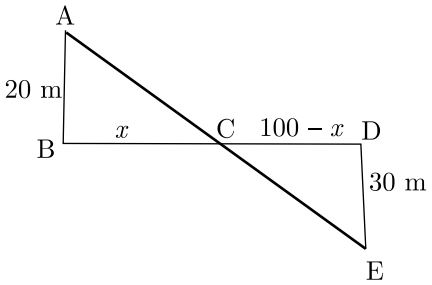
The ladder reaches about 14.3 feet up the side of the house.

79. Swimmer in Distress : A lifeguard located 20 metre from the water spots a swimmer in distress. The swimmer is 30 metre from shore and 100 metre east of the lifeguard. Suppose the lifeguard runs and then swims to the swimmer in a direct line, as shown in the figure. How far east from his original position will he enter the water? (Hint: Find the value of x in the sketch.)



Sol :

Here we will use property of similar triangle. We redraw the diagram of the situation as shown below.



Because of AA similarity we can write,

$$\frac{BC}{CD} = \frac{AB}{DE}$$
$$\frac{x}{100-x} = \frac{20}{30} = \frac{2}{3}$$
$$3x = 2(100-x)$$
$$3x = 200 - 2x$$
$$3x + 2x = 200$$
$$5x = 200 \Rightarrow x = \frac{200}{5} = 40 \text{ m}$$

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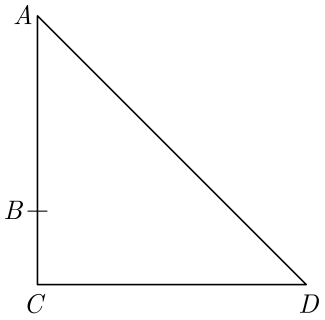
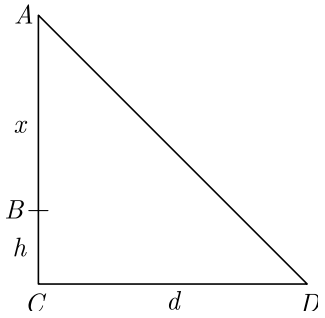


Sol : [Board Term-1 2015]

We have redrawn the given figure as below.



We have $AB + AD = BC + CD$

$$AD = BC + CD - AB$$

$$AD = h + d - x$$

In right ΔACD, we have

$$AD^2 = AC^2 + DC^2$$

$$(h + d - x)^2 = (x + h)^2 + d^2$$

$$(h + d - x)^2 - (x + h)^2 = d^2$$

$$(h + d - x - x - h)(h + d - x + x + h) = d^2$$

$$(d - 2x)(2h + d) = d^2$$

$$2hd + d^2 - 4hx - 2xd = d^2$$

$$2hd = 4hx + 2xd$$

$$= 2(2h + d)x$$

or, $$x = \frac{hd}{2h + d}$$

68. Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides.

[Board 2020 Delhi Basic, 2019 Delhi, 2018]

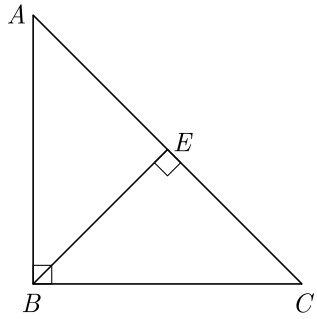
or

Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. Using the above result, prove that, in rhombus $ABCD$, $4AB^2 = AC^2 + BD^2$.

Sol : [Board Term -2 SQP 2017, 2015]

(1) As per given condition we have drawn the figure below. Here $AB \perp BC$.

We have drawn $BE \perp AC$



In ΔAEB and ΔABC $\angle A$ common and

$$\angle E = \angle B \quad \text{(each } 90^\circ\text{)}$$

By AA similarity we have

$$\Delta AEB \sim \Delta ABC$$

$$\frac{AE}{AB} = \frac{AB}{AC}$$

$$AB^2 = AE \times AC$$

Now, in ΔCEB and ΔCBA, $\angle C$ is common and

$$\angle E = \angle B \quad \text{(each } 90^\circ\text{)}$$

By AA similarity we have

$$\Delta AEB \sim \Delta CBA$$

$$\frac{CE}{BC} = \frac{BC}{AC}$$

$$BC^2 = CE \times AC \quad \ldots(2)$$

Adding equation (1) and (2) we have

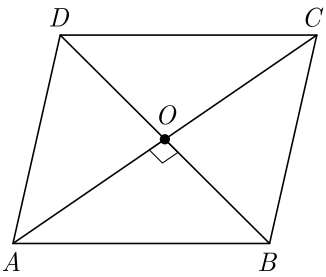
$$AB^2 + BC^2 = AE \times AC + CE \times AC$$

$$= AC(AE + CE)$$

$$= AC \times AC$$

Thus $AB^2 + BC^2 = AC^2$ Hence proved

(2) As per given condition we have drawn the figure below. Here $ABCD$ is a rhombus.



We have drawn diagonal AC and BD.

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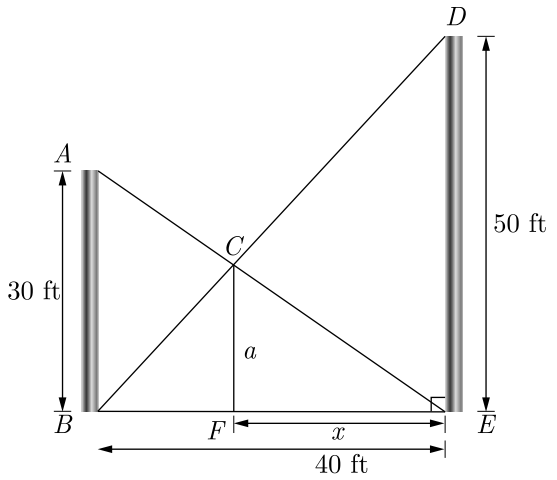


Here $\angle CFE$ is a right angle.

By AA similarity $\triangle AEB \sim \triangle CEF$

Thus
$$\frac{x}{40} = \frac{a}{30}$$
$$30x = 40a$$
$$\frac{30}{40}x = a$$

By AA similarity $\triangle CBF \sim \triangle DBE$
$$\frac{a}{50} = \frac{40-x}{40}$$
$$40a = 50(40-x)$$

Substituting value of a we have
$$40\left(\frac{30}{40}x\right) = 2000 - 50x$$
$$30x = 2000 - 50x$$
$$80x = 2000$$
$$x = 25$$

(i) So, the distance from C to the taller pole is 25 feet.

(ii)
$$\frac{x}{a} = \frac{40}{30}$$
$$30x = 40a$$
$$30(25) = 40a$$
$$750 = 40a$$
$$18.75 = a$$

The coupling is 18.75 feet above the ground.

(iii) Let y represent the length of the wire from the smaller pole.
$$30^2 + 40^2 = y^2 \Rightarrow y = 50$$

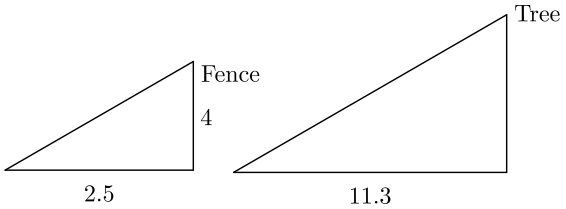
Let z represent the length of the wire from the top of the smaller pole to the coupling.
$$\frac{z}{50-z} = \frac{40-25}{25}$$
$$25z = 15(50-z)$$
$$5z = 3(50-z)$$
$$5z = 150 - 3z$$
$$8z = 150 \Rightarrow z = \frac{150}{8} = 18.75$$

The coupling is 18.75 feet down the wire from the top of the smaller pole.

14. Aakesh wanted to determine the height of a tree on the corner of his block. He knew that a certain fence by the tree was 4 feet tall. At 3 PM, he measured the shadow of the fence to be 2.5 feet tall. Then he measured the tree's shadow to be 11.3 feet. What is the height of the tree?

Sol :

We draw a diagram of the situation as shown below. Let x be the height of the tree.



Since the triangles formed are similar, a proportion can be written.
$$\frac{\text{height of the tree}}{\text{height of the fence}} = \frac{\text{tree shadow length}}{\text{fence shadow length}}$$
$$\frac{x}{4} = \frac{11.3}{2.5}$$
$$x \times 2.5 = 4(11.3)$$
$$2.5x = 45.2 \Rightarrow x = 18.08$$

The height of the tree is 18.08 feet.

15. Helicopter Patrolling : A helicopter is hovering over a crowd of people watching a police standoff in a parking garage across the street. Stewart notices the shadow of the helicopter is lagging

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$$\angle 1 + \angle 3 = \angle 2 + \angle 4$$

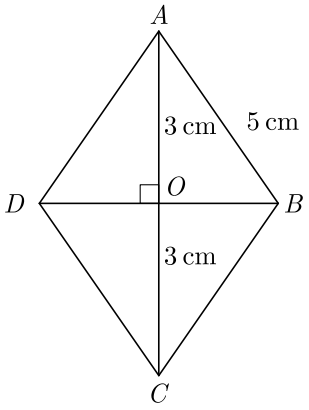
By SAS similarity, we have

$$\Delta ABC \sim \Delta PQR \qquad \text{Hence Proved}$$

71. Find the length of the second diagonal of a rhombus, whose side is 5 cm and one of the diagonals is 6 cm.

Sol : [Board 2010]

As per given condition we have drawn the figure below.



We have $AB = BC = CD = AD = 5$ cm and $AC = 6$ cm

Since $AO = OC$, $\quad AO = 3$ cm

Here ΔAOB is right angled triangle as diagonals of rhombus intersect at right angle.

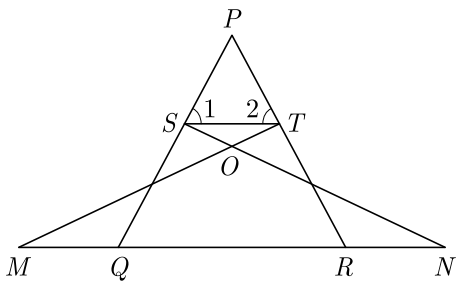
By Pythagoras theorem,

$$OB = 4 \text{ cm.}$$

Since $DO = OB, BD = 8$ cm, length of the other diagonal $= 2(BO)$ where $BO = 4$ cm

Hence $\quad BD = 2 \times BO = 2 \times 4 = 8$ cm

72. In given figure $\angle 1 = \angle 2$ and $\Delta NSQ \sim \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRO$.



Sol : [Board Term-1 SQP 2017]

We have $\quad \Delta NSQ \cong \Delta MTR$

By CPCT we have

$$\angle SQN = \angle TRM$$

From angle sum property we get

$$\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$$

$$\angle 1 + \angle 2 = \angle PQR + \angle PRQ$$

Since $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$ we get

$$2\angle 1 = 2\angle PQR$$

$$\angle 1 = \angle PQR$$

Also $\quad \angle 2 = \angle QPR$ (common)

Thus by AAA similarity,

$$\Delta PTS \sim \Delta PRQ$$

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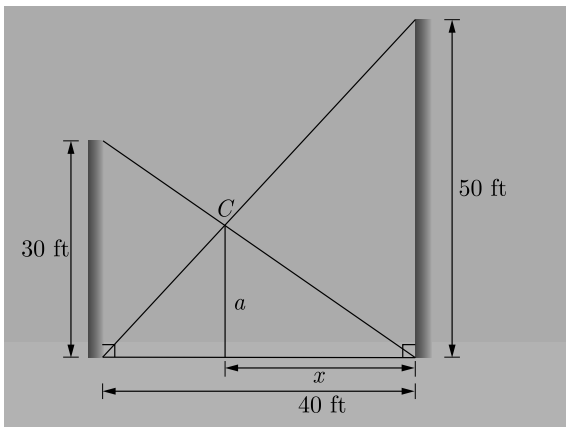
COMPETENCEY BASED QUESTIONS

73. Two poles, 30 feet and 50 feet tall, are 40 feet apart and perpendicular to the ground. The poles are supported by wires attached from the top of each pole to the bottom of the other, as in the figure. A coupling is placed at C where the two wires cross.



(i) What is the horizontal distance from C to the taller pole?

(ii) How high above the ground is the coupling ?

(iii) How far down the wire from the smaller pole is the coupling ?

Sol :

The poles form parallel line segments and the wires are transversals cutting through the ends of the parallel segments.

$$AO = OC = \frac{1}{2}AC$$

and $$BO = OD = \frac{1}{2}BD$$

$$AC \perp BD$$

Since diagonal of rhombus bisect each other at right angle,

$$\angle AOB = 90^\circ$$

$$AB^2 = OA^2 + OB^2$$

$$= \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$= \frac{AC^2}{4} + \frac{BD^2}{4}$$

or $$4AB^2 = AC^2 + BD^2$$ Hence proved

69. In a right triangle ABC, right angled at C. P and Q are points of the sides CA and CB respectively, which divide these sides in the ratio $2:1$.

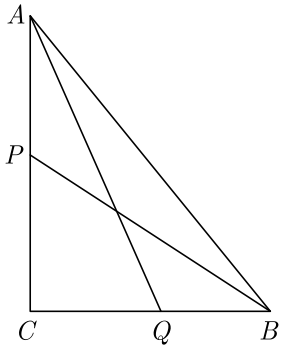
Prove that : $9AQ^2 = 9AC^2 + 4BC^2$

$$9BP^2 = 9BC^2 + 4AC^2$$

$$9(AQ^2 + BP^2) = 13AB^2$$

Sol :

As per given condition we have drawn the figure below.



Since P divides AC in the ratio $2:1$

$$CP = \frac{2}{3}AC$$

and Q divides CB in the ratio $2:1$

$$QC = \frac{2}{3}BC$$

$$AQ^2 = QC^2 + AC^2$$

$$= \frac{4}{9}BC^2 + AC^2$$

or, $$9AQ^2 = 4BC^2 + 9AC^2 \quad ...(1)$$

Similarly, we get

$$9BP^2 = 9BC^2 + 4AC^2 \quad ...(2)$$

Adding equation (1) and (2), we get

$$9(AQ^2 + BP^2) = 13AB^2$$

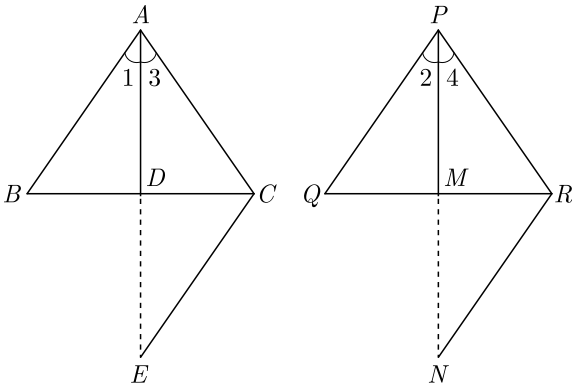
70. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Sol : [Board Term-1 2012]

It is given that in ΔABC and ΔPQR, AD and PM are their medians,

such that $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$$

We have produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. We join CE and RN. As per given condition we have drawn the figure below.



In ΔABD and ΔEDC,

$$AD = DE \quad \text{(By construction)}$$

$$\angle ADB = \angle EDC \quad \text{(VOA)}$$

$$BD = DC \quad (AD \text{ is a median})$$

By SAS congruency

$$\Delta ABD \cong \Delta EDC$$

$$AB = CE \quad \text{(By CPCT)}$$

Similarly, $PQ = RN$ and $\angle A = \angle 2$

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR} \quad \text{(Given)}$$

,

$$\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$$

$$\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$$

By SSS similarity, we have

$$\Delta AEC \sim \Delta PNR$$

$$\angle 3 = \angle 4$$

$$\angle 1 = \angle 2$$

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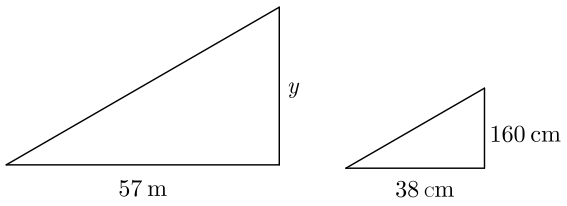
approximately 57 m behind a point directly below the helicopter. If he is 160 cm tall and casts a shadow of 38 cm at this time, what is the altitude of the helicopter?

Sol :

We draw a diagram of the situation as shown below.



Here we will use property of similar triangle.

$$\frac{\text{height of the helicopter}}{\text{height of the stewart}} = \frac{\text{shadow of Helicopter}}{\text{shadow of Stewart}}$$

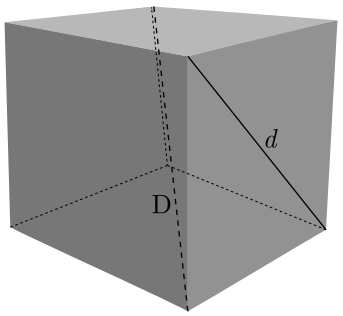
$$\frac{y}{1.6} = \frac{57}{0.38}$$

$$y = \frac{57 \times 1.6}{0.38}$$

$$= \frac{57 \times 160}{38}$$

$$= \frac{3 \times 160}{2} = 240 \text{ m}$$

16. The boxes used to ship some washing machines are perfect cubes with edges a. Find the length of the diagonal d of one side, and the length of the interior diagonal D (through the middle of the box).



Sol :

The diagonal of one side forms a 45-45-90 triangle with side a.

$$d^2 = a^2 + a^2 = 2a^2 \Rightarrow d = \sqrt{2}\, a$$

The diagonal through the centre of cube form a right triangle with side a and $\sqrt{2}\, a$

$$D^2 = a^2 + d^2$$

$$= a^2 + 2a^2 = 3a^2$$

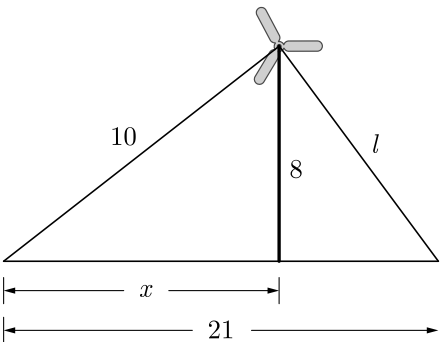
$$D = a\sqrt{3}$$

17. Windmill : A windmill is a device for tapping the energy of the wind by means of sails mounted on a rotating shaft. Windmills still used today, mainly in parts of the world which have traditionally relied on them, are powered by electricity or water.

Anil is constructing a 8 m tall windmill supported by two wires. One wire must be 10 m long and the distance between the wires must be 21 m. Anil wanted to know what length to cut for the other wire

Sol :

We draw a diagram of the situation as shown below.



$$x^2 + 8^2 = 10^2$$

$$x^2 = \sqrt{10^2 - 8^2} = 6^2$$

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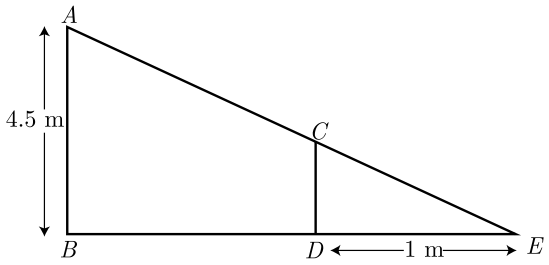
(v) Which similarity criterion is used in solving the above problem

Sol :

(i) As per question statement we make the diagram at following.



At $t = 2$ seconds, $BD = 2 \times 1 = 2\,\text{m}$

$DE = 1$ m

Since, $ABE \sim \Delta CDE$

$$\frac{AB}{CD} = \frac{BE}{DE} = \frac{BD+DE}{DE}$$

or, $$\frac{4.5}{CD} = \frac{2+1}{1} = 3$$

$$CD = \frac{4.5}{3} = 1.5 \text{ m} = 150 \text{ cm}$$

(ii) At point where shadow is equal to her height,

$$CD = DE = 1.5 \text{ m}$$

$$\frac{AB}{CD} = \frac{BE}{DE} = \frac{BD+DE}{DE}$$

$$\frac{4.5}{1.5} = \frac{BD+1.5}{1.5}$$

$$4.5 = BD + 1.5$$

$$BD = 4.5 - 1.5 = 3 \text{ m}$$

Time to reach at BD, $t = \frac{3}{1} = 3$ sec

(iii) As calculated in part (ii) we have $BD = 3$ m

(iv) After 4 sec, $BD = 1 \times 4 = 4$

$$\frac{AB}{CD} = \frac{BE}{DE}$$

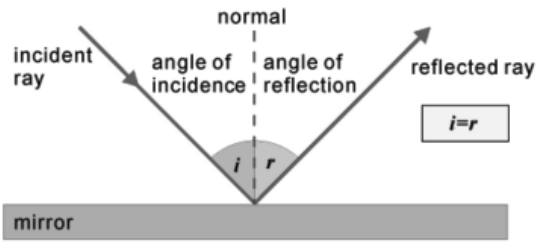
$$= \frac{BD+DE}{DE}$$

$$\frac{4.5}{1.5} = \frac{4+DE}{DE}$$

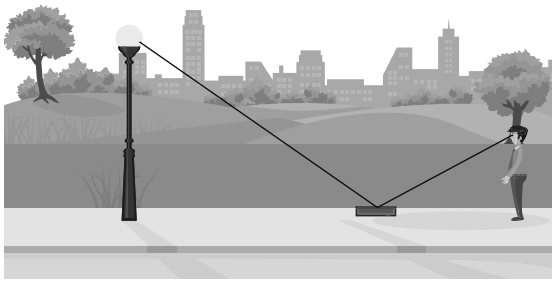
$$3DE = 4 + DE \Rightarrow DE = 2 \text{ m}$$

(v) We have used AA similarity criterion.

83. The law of reflection states that when a ray of light reflects off a surface, the angle of incidence is equal to the angle of reflection.



Ramesh places a mirror on level ground to determine the height of a pole (with traffic light fired on it). He stands at a certain distance so that he can see the top of the pole reflected from the mirror. Ramesh's eye level is 1.5 m above the ground. The distance of Ramesh and the pole from the mirror are 1.8 m and 6 m respectively.



(i) Which criterion of similarity is applicable to similar triangles?

(ii) What is the height of the pole?

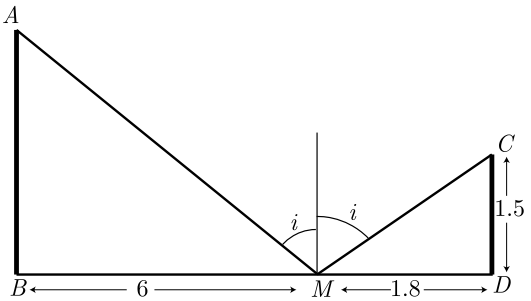
(iii) If angle of incidence is i , find tan i .

(iv) Now Ramesh move behind such that distance between pole and Ramesh is 13 meters. He place mirror between him and pole to see the reflection of light in right position. What is the distance between mirror and Ramesh ?

(v) What is the distance between mirror and pole?

Sol :

(i) Since angle of incidence and angle of reflection are the same, we draw the figure as given below.



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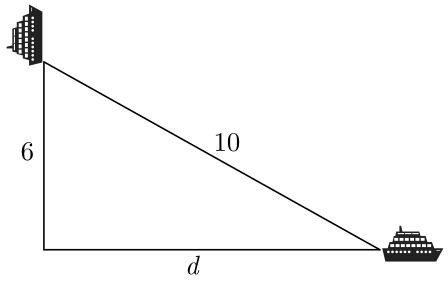
80. Two Ships : Two ships are cruising together on the open ocean at 6 nautical miles per hour. One of them turns to make a angle 90° with the first and increases speed, heading for port. Assuming the first ship continues traveling at 6 knots, find the speed of the other ship if they are 10 mi apart after 1 hr.

Sol :

We draw a diagram of the situation after 1 hour as shown below.



We have $d = \sqrt{10^2 - 6^2}$

$= 8$ mile

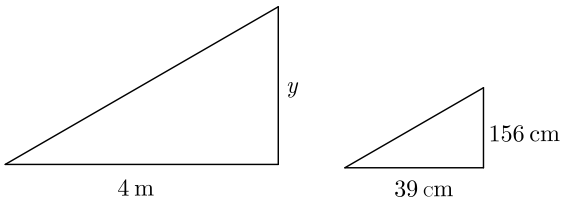
Since it travel 8 miles in hour, its speed is 8 nautical mile/hour.

81. Statue of a Pineapple : The Big Pineapple is a heritage-listed tourist attraction at Nambour Connection Road, Woombye, Sunshine Coast Region, Queensland, Australia. It was designed by Peddle Thorp and Harvey, Paul Luff, and Gary Smallcombe and Associates. It is also known as Sunshine Plantation. It was added to the Queensland Heritage Register on 6 March 2009.

Ishita last year visited Nambour and wanted to find the height of a statue of a pineapple. She measured the pineapple's shadow and her own shadow. Her height is 156 cm and casts a shadow of 39 cm. The length of shadow of pineapple is 4 m. What is the height of the pineapple?

Sol :

We draw a diagram of the situation as shown below.



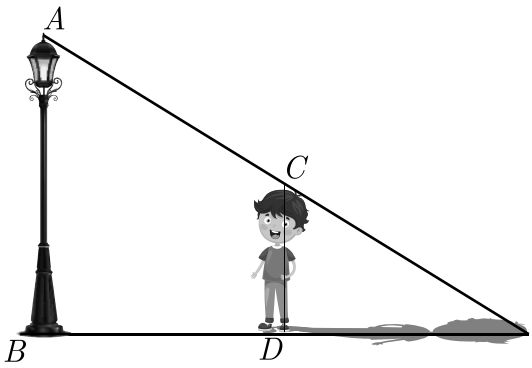
$$\frac{\text{height of the pinaple}}{\text{height of Ishita}} = \frac{\text{shadow of pineaple}}{\text{shadow of Ishita}}$$

$$\frac{y}{1.56} = \frac{4}{0.39}$$

$$y = \frac{4 \times 1.56}{0.39}$$

$$= 4 \times 4 = 16 \text{ m}$$

82. Rohan is very intelligent in maths. He always try to relate the concept of maths in daily life. One day he is walking away from the base of a lamp post at a speed of 1 m/s. Lamp is 4.5 m above the ground.



(i) If after 2 second, length of shadow is 1 meter, what is the height of Rohan ?

(ii) What is the minimum time after which his shadow will become larger than his original height?

(iii) What is the distance of Rohan from pole at this point ?

(iv) What will be the length of his shadow after 4 seconds?

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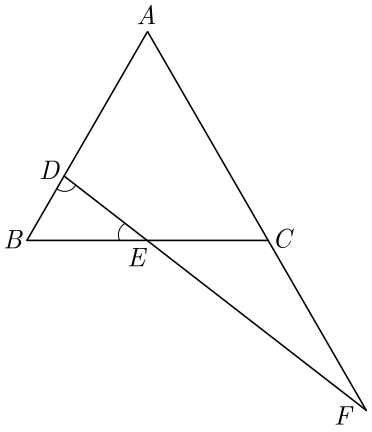
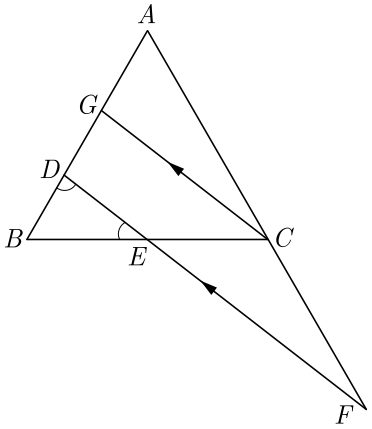


Sol : [Board 2019 Comp.]

We have redrawn the given figure as below. Here $CG \,||\, FD$.



We have $\angle BED = \angle BDE$

Since E is mid-point of BC,

$$BE = BD = EC \qquad ...(1)$$

In ΔBCG, $DE \,||\, FG$

From (1) we have

$$\frac{BD}{DG} = \frac{BE}{EC} = 1$$

$$BD = DG = EC = BE$$

In ΔADF, $CG \,||\, FD$

By BPT $\quad \frac{AG}{GD} = \frac{AC}{CF}$

$$\frac{AG + GD}{GD} = \frac{AF + CF}{CF}$$

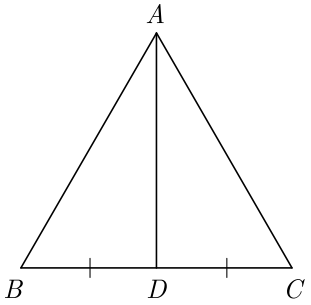
, $\quad \frac{AD}{GD} = \frac{AF}{CF}$

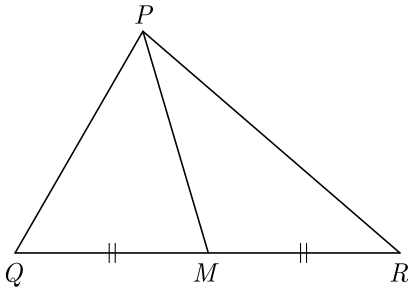
Thus $\quad \frac{AF}{CF} = \frac{AD}{BE}$

66. In $\Delta ABC, AD$ is the median to BC and in $\Delta PQR, PM$ is the median to QR. If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. Prove that $\Delta ABC \sim \Delta PQR$.

Sol : [Board Term-1 2012, 2013]

As per given condition we have drawn the figure below.





In ΔABC AD is the median, therefore

$$BC = 2BD$$

and in ΔPQR, PM is the median,

$$QR = 2QM$$

Given, $\quad \frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$

or, $\quad \frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM}$

In triangles ABD and PQM,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

By SSS similarity we have

$$\Delta ABD \sim \Delta PQM$$

By CPST we have

$$\angle B = \angle Q,$$

In ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

By SAS similarity we have

$$\angle B = \angle Q,$$

Thus $\quad \Delta ABC \sim \Delta PQR.$ Hence Proved.

67. In the right triangle, B is a point on AC such that $AB + AD = BC + CD$. If $AB = x, BC = h$ and $CD = d$, then find x (in term of h and d).

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Now $\angle AMB = \angle CMD$

Also, $\angle ABM = \angle CDM = 90^\circ$

So, by AA similarity criterion,

$$\Delta AMB \sim \Delta CDM$$

(ii) As $\Delta ABM \sim \Delta CDM$ we obtain,

$$\frac{AB}{CD} = \frac{BM}{DM}$$

$$\frac{AB}{1.8} = \frac{5}{1.5}$$

$$AB = \frac{6}{1.8} \times 1.5 = 5 \text{ m}$$

Thus, the height of the pole is 5 metres.

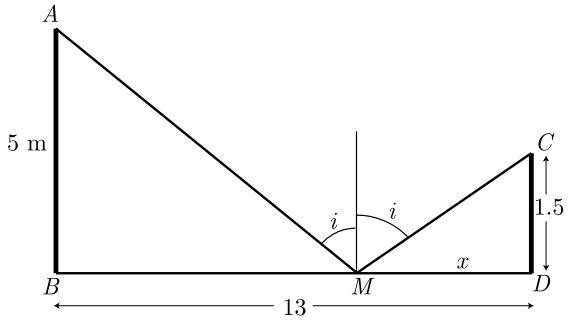
(iii) From the geometry of diagram we have

$$\angle MCD = i$$

$$\tan \angle MCD = \frac{MD}{CD}$$

$$\tan i = \frac{1.8}{1.5} = \frac{6}{5}$$

(iv) On the basis of given information we have drawn the figure as follows:



Once again due to AA similarity criterion,

$$\Delta AMB \sim \Delta CDM$$

$$\frac{5}{13 - x} = \frac{1.5}{x}$$

$$\frac{1}{13 - x} = \frac{0.3}{x}$$

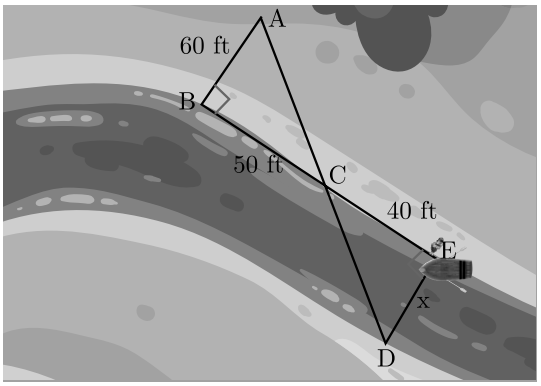
$$x = 3.9 - 0.3x$$

$$1.3x = 3.9 \Rightarrow x = 3$$

(v) Distance between mirror and pole,

$$= 13 - x = 13 - 3 = 10 \text{ m}$$

84. Tania is very intelligent in maths. She always try to relate the concept of maths in daily life. One day she plans to cross a river and want to know how far it is to the other side. She takes measurements on her side of the river and make the drawing as shown below.



(i) Which similarity criterion is used in solving the above problem ?

(ii) Consider the following statement :

$S_1 : \angle ACB = \angle DCE$

$S_2 : \angle BAC = \angle CDE$

Which of the above statement is/are correct.

(a) S_1 and S_2 both (b) S_1

(c) S_2 (d) None

(iii) Consider the following statement :

$$S_3 : \frac{AB}{DE} = \frac{CA}{CD}$$

$$S_4 : \frac{BC}{CE} = \frac{AB}{DE}$$

$$S_5 : \frac{CA}{CD} = \frac{DE}{AB}$$

Which of the above statements are correct ?

(a) S_3 and S_5 (b) S_4 and S_5

(c) S_3 and S_4 (d) All three

(iv) What is the distance x across the river?

(v) What is the approximate length of AD shown in the figure?

Sol :

(i) We have used AA similarity criterion.

(ii) Here, $\angle ABC = \angle DEC$ (90° each)

Since vertical opposite angle are equal,

$$\angle ACB = \angle DCE$$

Thus due to AA similarity criterion,

$$\Delta ABC \backsim \Delta DEC$$

and $\angle BAC = \angle CDE$

Therefore both are correct.

Thus (a) is correct option.

(iii) Since ΔABC and ΔDEC are similar triangle,

$$\frac{AB}{DE} = \frac{BC}{CE} = \frac{CA}{CD}$$

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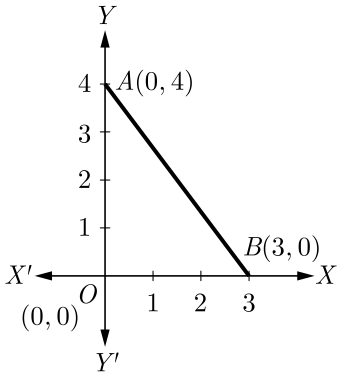
33. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is



(a) 5 (b) 12

(c) 11 (d) $7+\sqrt{5}$

Sol :

We have $OA = 4$

$OB = 3$

and $AB = \sqrt{3^2+4^2} = 5$

Now, perimeter of ΔAOB is the sum of the length of all its sides.

$$p = OA + OB + AB$$
$$= 4+3+5 = 12$$

Hence, the required perimeter of triangle is 12. However you can calculate perimeter direct from diagram.

Thus (b) is correct option.

34. The point which lies on the perpendicular bisector of the line segment joining the points $A(-2,-5)$ and $B(2,5)$ is

(a) (0, 0) (b) (0, 2)

(c) (2, 0) (d) $(-2,0)$

Sol :

We know that, the perpendicular bisector of the any line segment divides the line segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid-point of the line segment.

Mid-point of the line segment joining the points $A(-2,-5)$ and $B(2,5)$

$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0,0)$$

Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

Thus (a) is correct option.

35. If the point $P(2,1)$ lies on the line segment joining points $A(4,2)$ and $B(8,4)$, then

(a) $AP = \frac{1}{3}AB$ (b) $AP = PB$

(c) $PB = \frac{1}{3}AB$ (d) $AP = \frac{1}{2}AB$

Sol :

Let, $AP : AB = m : n$

Using section formula, we have,

$$4 = \frac{8m+2n}{m+n}$$

and $$2 = \frac{4m+n}{m+n}$$

Solving these as linear equation, we get,

$$m = 1 \text{ and } n = 2$$
$$\frac{AP}{AB} = \frac{1}{2}$$
$$AP = \frac{1}{2}AB$$

Thus (d) is correct option.

36. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6,5)$ and $R(-2,3)$, then the value of a is

(a) -4 (b) -12

(c) 12 (d) -6

Sol :

Since $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the points $Q(-6,5)$ and $R(-2,3)$,

$$\left(\frac{a}{3}, 4\right) = \left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$$
$$\left(\frac{a}{3}, 4\right) = (-4, 4)$$

Now $$\frac{a}{3} = -4 \Rightarrow a = -12$$

Thus (b) is correct option.

37. The perpendicular bisector of the line segment joining the points $A(1,5)$ and $B(4,6)$ cuts the y -axis at

(a) (0, 13) (b) $(0,-13)$

(c) (0, 12) (d) (13, 0)

Sol :

Let $P(0,b)$ be the required point. Since, any point on perpendicular bisector is equidistant from the end point of line segment.

i.e., $PA = PB$

$$\sqrt{(0-1)^2+(b-5)^2} = \sqrt{(0-4)^2+(b-6)^2}$$

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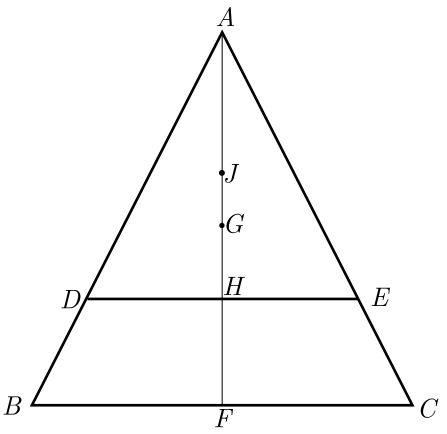

(i) What is the length of AH ?

(ii) What is the distance of point A from point G ?

(iii) What is the distance of point A from point J?

(iv) What is the distance GJ ?

Sol :

(i) $AF = h$

$HF = \frac{h}{4}$

$AH = AF - HF = h - \frac{h}{4} = \frac{3h}{4}$

(ii) Here AF is the median to BC from A in $\triangle ABC$ and G is centroid Thus $AG = \frac{2}{3}AF$

(iii) From part (ii)

$AG = \frac{2}{3}AF$

$AF = \frac{3}{2}AG$

Now AH is median to DE from A in $\triangle ADE$

Thus $AJ = \frac{2}{3}AH$ Since $DE \| BC$,

From part (i) $AH = \frac{3}{4}AF$

Thus $AJ = \frac{2}{3} \times \frac{3}{4}AF$

$= \frac{2}{3} \times \frac{3}{4} \times \frac{3}{2}AG = \frac{3}{4}AG$

(iv) $GJ = AG - AJ$

$= AG - \frac{3}{4}AG = \frac{1}{4}AG$

****

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Thus (c) is correct option.

23. If the points $A(4,3)$ and $B(x,5)$ are on the circle with centre $O(2,3)$, then the value of x is

(a) 0 (b) 1

(c) 2 (d) 3

Sol :

Since, A and B lie on the circle having centre O.

$$\begin{aligned} OA &= OB \\ \sqrt{(4-2)^2+(3-3)^2} &= \sqrt{(x-2)^2+(5-3)^2} \\ 2 &= \sqrt{(x-2)^2+4} \\ 4 &= (x-2)^2+4 \\ (x-2)^2 &= 0 \\ x &= 2 \end{aligned}$$

Thus (c) is correct option.

24. The point on the x-axis which is equidistant from the points $A(-2,3)$ and $B(5,4)$ is

(a) (0, 2) (b) (2, 0)

(c) (3, 0) (d) $(-2,0)$

Sol :

Let $P(x,0)$ be a point on x-axis such that,

$$\begin{aligned} AP &= BP \\ AP^2 &= BP^2 \\ (x+2)^2+(0-3)^2 &= (x-5)^2+(0+4)^2 \\ x^2+4x+4+9 &= x^2-10x+25+16 \\ 14x &= 28 \\ x &= 2 \end{aligned}$$

Hence required point is (2, 0).

Thus (b) is correct option.

25. C is the mid-point of PQ, if P is $(4,x)$, C is $(y,-1)$ and Q is $(-2,4)$, then x and y respectively are

(a) -6 and 1 (b) -6 and 2

(c) 6 and -1 (d) 6 and -2

Sol :

Since, $C(y,-1)$ is the mid-point of $P(4,x)$ and $Q(-2,4)$.

We have, $\frac{4-2}{2} = y \Rightarrow y = 1$

and $\frac{4+x}{2} = -1 \Rightarrow x = -6$

Thus (a) is correct option.

26. If $x-2y+k=0$ is a median of the triangle whose vertices are at points $A(-1,3)$, $B(0,4)$ and $C(-5,2)$,then the value of k is

(a) 2 (b) 4

(c) 6 (d) 8

Sol :

Coordinate of the centroid G of ΔABC

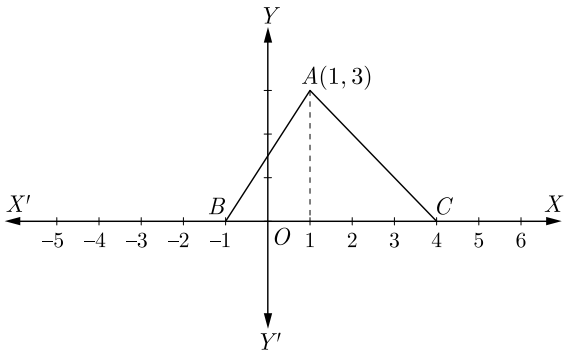
$$= \left(\frac{-1+0-5}{2}, \frac{3+4+2}{3}\right)$$
$$= (-2,3)$$

Since, G lies on the median, $x-2y+k=0$, it must satisfy the equation,

$$-2-6+k = 0 \Rightarrow k = 8$$

Thus (d) is correct option.

27. In the given figure, the area of ΔABC (in sq units) is



(a) 15 (b) 10

(c) 7.5 (d) 2.5

Sol :

From the given graph, it is clear that $A(1,3)$, $B(-1,0)$ and $C(4,0)$

Area of ΔABC

$$= \frac{1}{2}\left|[1(0-0)+(-1)(0-3)+4(3-0)]\right|$$

$$= \frac{1}{2}\left|[0+3+12]\right| = \frac{15}{2} = 7.5 \text{ sq units}$$

However you can calculate area direct by height and base.

$$A = \frac{1}{2}\times 5\times 3 = 7.5 \text{ sq units.}$$

Thus (c) is correct option.

28. The distance of the point $P(2,3)$ from the x-axis is

(a) 2 (b) 3

(c) 1 (d) 5

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Thus (a) is correct option.

6. The centroid of the triangle whose vertices are $(3,-7),(-8,6)$ and $(5,\ 10)$ is

(a) $(0,\ 9)$ (b) $(0,\ 3)$

(c) $(1,\ 3)$ (d) $(3,\ 5)$

Sol : [Board 2022 Term 1 Standard]

Centroid is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

i.e. $\left(\frac{3+(-8)+5}{3}, \frac{-7+6+10}{3}\right) = \left(\frac{0}{3}, \frac{9}{3}\right)$

$= (0,3)$

Thus (b) is correct option.

7. The ratio in which the point $(2, y)$ divides the join of $(-4,3)$ and $(6,\ 3)$, hence the value of y is

(a) $2:3, y=3$ (b) $3:2, y=4$

(c) $3:2, y=3$ (d) $3:2, y=2$

Sol : [Board 2022 Term 1 Basic]

Let the required ratio be $k:1$

Then, $2 = \frac{6k-4(1)}{k+1}$

or $k = \frac{3}{2}$

The required ratio is $\frac{3}{2}:1$ or $3:2$

Also, $y = \frac{3(3)+2(3)}{3+2} = 3$

Thus (c) is correct option.

8. The point P on x-axis equidistant from the points $A(-1,0)$ and $B(5,0)$ is

(a) $(2,\ 0)$ (b) $(0,\ 2)$

(c) $(3,\ 0)$ (d) $(-3,5)$

Sol : [Board 2020 OD Standard]

Let the position of the point P on x-axis be $(x,0)$, then

$$PA^2 = PB^2$$

$$(x+1)^2+(0)^2 = (5-x)^2+(0)^2$$

$$x^2+2x+1 = 25+x^2-10x$$

$$2x+10x = 25-1$$

$$12x = 24 \Rightarrow x = 2$$

Hence, the point $P(x,0)$ is $(2,\ 0)$.

Thus (a) is correct option.

Alternative :

You may easily observe that both point $A(-1,0)$ and $B(5,0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-1,0)$ and $B(5,0)$.

$$x = \frac{-1+5}{2} = 2$$

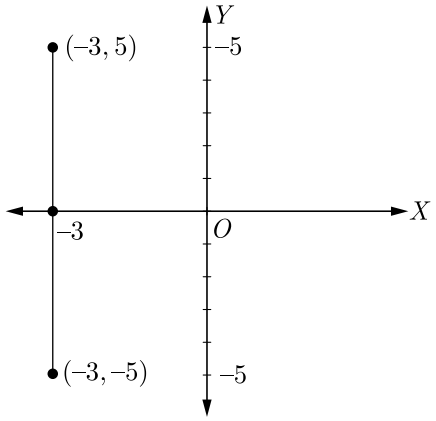
9. The co-ordinates of the point which is reflection of point $(-3,5)$ in x-axis are

(a) $(3,\ 5)$ (b) $(3,\ -5)$

(c) $(-3,-5)$ (d) $(-3,5)$

Sol : [Board 2020 OD Standard]

The reflection of point $(-3,5)$ in x- axis is $(-3,-5)$.



Thus (c) is correct option.

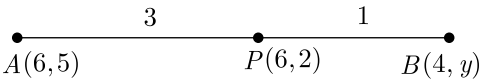
10. If the point $P\ (6,\ 2)$ divides the line segment joining $A(6,5)$ and $B(4,y)$ in the ratio $3:1$ then the value of y is

(a) 4 (b) 3

(c) 2 (d) 1

Sol : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below,



Here, $x_1 = 6,\ y_1 = 5$

and $x_2 = 4\ \ y_2 = y$

Now $y = \frac{my_2+ny_1}{m+n}$

$$2 = \frac{3\times y+1\times 5}{3+1}$$

$$2 = \frac{3y+5}{4}$$

$$3y+5 = 8$$

$$3y = 8-5 = 3 \Rightarrow y = 1$$

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Thus (d) is correct option.

11. The distance between the points $(a\cos\theta + b\sin\theta,\ 0)$, and $(0,\ a\sin\theta - b\cos\theta)$ is

(a) $a^2 + b^2$ (b) $a^2 - b^2$

(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

Sol : [Board 2020 Delhi Standard]

We have $x_1 = a\cos\theta + b\sin\theta$ and $y_1 = 0$

and $x_2 = 0$ and $y_2 = a\sin\theta - b\cos\theta$

$$\begin{aligned} d^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ &= (0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2 \\ &= (-1)^2(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 \\ &= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + \\ &\quad + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta \\ &= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) \\ &= a^2 \times 1 + b^2 \times 1 = a^2 + b^2 \end{aligned}$$

Thus $d^2 = a^2 + b^2$

$d = \sqrt{a^2 + b^2}$

Therefore (c) is correct option.

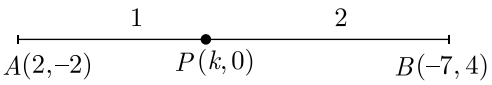
12. If the point $P(k,\ 0)$ divides the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ in the ratio $1 : 2$, then the value of k is

(a) 1 (b) 2

(c) -2 (d) -1

Sol : [Board 2020 Delhi Standard]

As per question statement figure is shown below.



$$k = \frac{1(-7) + 2(2)}{1 + 2} \qquad \left(x = \frac{mx_2 + nx_1}{m + n}\right)$$

$$= \frac{-7 + 4}{3} = \frac{-3}{3} = -1$$

Thus $k = -1$

Thus (d) is correct option.

13. The coordinates of a point A on y-axis, at a distance of 4 units from x-axis and below it are

(a) $(4,\ 0)$ (b) $(0,\ 4)$

(c) $(-4, 0)$ (d) $(0,\ -4)$

Sol : [Board 2020 Delhi Basic]

Because the point is 4 units down the x-axis i.e., co-ordinate is -4 and on y-axis abscissa is 0. So, the coordinates of point A is $(0, -4)$.

Thus (d) is correct option.

14. The distance of the point $(-12,\ 5)$ from the origin is

(a) 12 (b) 5

(c) 13 (d) 169

Sol :

The distance between the origin and the point (x, y) is $\sqrt{x^2 + y^2}$.

Therefore, the distance between the origin and point $(-12, 5)$

$$\begin{aligned} d &= \sqrt{(-12 - 0)^2 + (5 - 0)^2} \\ &= \sqrt{144 + 25} = \sqrt{169} \\ &= 13 \text{ units} \end{aligned}$$

Thus (c) is correct option.

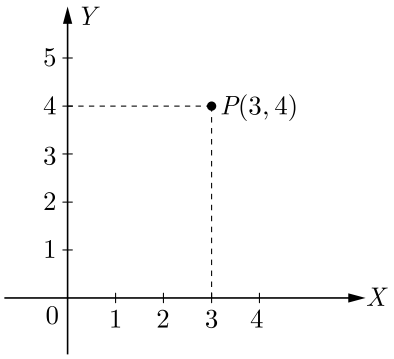
15. Distance of point $P(3, 4)$ from x-axis is

(a) 3 units (b) 4 units

(c) 5 units (d) 1 units

Sol : [Board 2020 Delhi Basic]

Point $P(3, 4)$ is 4 units from the x-axis and 3 units from the y-axis.



Thus (b) is correct option.

16. The distance of the point $P(-3, -4)$ from the x -axis (in units) is

(a) 3 (b) -3

(c) 4 (d) 5

Sol : [Board 2020 SQP Standard]

Point $P(-3, -4)$ is 4 units from the x-axis and 3 units from the y-axis.

Thus (c) is correct option.

17. If $A(\frac{m}{3},\ 5)$ is the mid-point of the line segment joining the points $Q(-6, 7)$ and $R(-2, 3)$, then the value of m is

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(a) -12 (b) -4

(c) 12 (d) -6

Sol : [Board 2020 SQP Standard]

Given points are $Q(-6,7)$ and $R(-2,3)$

Mid point $A(\frac{m}{3}, 5) = \left(\frac{-6-2}{2}, \frac{7+3}{2}\right)$

$= (-4, 5)$

Equating, $\frac{m}{3} = -4 \Rightarrow m = -12$

Thus (a) is correct option.

18. The mid-point of the line-segment AB is $P(0,4)$, if the coordinates of B are $(-2, 3)$ then the coordinates of A are

(a) $(2, 5)$ (b) $(-2,-5)$

(c) $(2, 9)$ (d) $(-2,11)$

Sol : [Board 2020 OD Basic]

Let point A be (x, y).

Now using mid-point formula,

$$(0,4) = \left(\frac{x-2}{2}, \frac{y+3}{2}\right)$$

Thus $0 = \frac{x-2}{2} \Rightarrow x = 2$

and $4 = \frac{y+3}{2} \Rightarrow y = 5$

Hence point A is $(2, 5)$.

Thus (a) is correct option.

19. x-axis divides the line segment joining $A(2,-3)$ and $B(5,6)$ in the ratio

(a) 2 : 3 (b) 3 : 5

(c) 1 : 2 (d) 2 : 1

Sol : [Board 2020 OD Basic]

Let point $P(x, 0)$ on x-axis divide the segment joining points $A(2, -3)$ and $B(5, 6)$ in ratio $k : 1$, then

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$0 = \frac{6k-3}{k+1}$$

$$6k = 3 \Rightarrow k = \frac{1}{2}$$

Therefore ratio is 1 : 2.

Thus (c) is correct option.

20. The point which divides the line segment joining the points $(8,-9)$ and $(2, 3)$ in the ratio 1 : 2 internally lies in the

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

Sol : [Board 2020 SQP Standard]

We have $x_1 = 8$, $y_1 = -9$, $x_2 = 2$ and $y_2 = 3$.

and $m_1 : m_2 = 1:2$

Let the required point be $P(x, y)$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$= \frac{1 \times 2 + 2 \times 8}{1+2} = 6$$

and $y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

$$= \frac{1 \times 3 + 2(-9)}{1+2} = -5$$

Thus $(x, y) = (6, -5)$ and this point lies in IV quadrant.

Thus (d) is correct option.

21. If the centre of a circle is $(3, 5)$ and end points of a diameter are $(4, 7)$ and $(2, y)$, then the value of y is

(a) 3 (b) -3

(c) 7 (d) 4

Sol : [Board 2020 Delhi Basic]

Since, centre is the mid-point of end points of the diameter.

$$(3, 5) = \left(\frac{4+2}{2}, \frac{7+y}{2}\right)$$

Comparing both the sides, we get

$$5 = \frac{7+y}{2}$$

$$7 + y = 10 \Rightarrow y = 3$$

Thus (a) is correct option.

22. If the distance between the points $A(4, p)$ and $B(1,0)$ is 5 units then the value(s) of p is(are)

(a) 4 only (b) -4 only

(c) ± 4 (d) 0

Sol : [Board 2020 Delhi Basic]

Given, points are $A(4, p)$ and $B(1,0)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y^2 - y_1)^2}$$

$$5 = \sqrt{(1-4)^2 + (0-p)^2}$$

$$25 = 9 + p^2$$

$$p^2 = 25 - 9 = 16$$

$$p = \pm 4$$

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CHAPTER 7

Co-ordinate Geometry

OBJECTIVE QUESTIONS

1. If the distance between the points $(3,-5)$ and $(x,-5)$ is 15 units, then the values of x are:

(a) 12, –18 (b) –12, 18

(c) 18, 5 (d) –9, –12

Sol : [Board 2024 OD Standard]

Given, points are $(3,-5)$ and $(x,-5)$.

$$\begin{aligned} d^2 &= (x_2-x_1)^2+(y^2-y_1)^2 \\ 15^2 &= (-5-(-5))^2+(x-3)^2 \\ 225 &= 0+(x-3)^2 \\ 225 &= (x-3)^2 \\ \pm 15 &= (x-3) \\ x-3 &= 15 \\ x &= 15+3=18 \\ x-3 &= -15 \\ x &= -15+3=-12 \end{aligned}$$

Thus answer is –12, 18.

Thus (b) is correct option.

2. AD is a median of ΔABC with vertices $A(5,-6)$, $B(6,4)$ and $C(0,0)$. Length AD is equal to :

(a) $\sqrt{68}$ units (b) $2\sqrt{15}$ units

(c) $\sqrt{101}$ units (d) 10 units

Sol : [Board 2024 OD Standard]

Mid-point of BC,

$$\begin{aligned} D &= \left(\frac{6+0}{2}, \frac{4+0}{2}\right) \\ &= (3,2) \end{aligned}$$

Now length of AD,

$$\begin{aligned} AD &= \sqrt{(3-5)^2+(2-(-6))^2} \\ &= \sqrt{2^2+8^2} \\ &= \sqrt{68} \end{aligned}$$

Thus (a) is correct option.

3. The centre of a circle is at (2, 0). If one end of a diameter is at (6, 0), then the other end is at

(a) (0, 0) (b) (4, 0)

(c) $(-2,0)$ (d) $(-6,0)$

Sol : [Board 2024 OD Standard]

Let (x, y) be the other end of diameter. Since, centre is the mid-point of end points of the diameter.

$$(2, 0) = \left(\frac{x+6}{2}, \frac{y+0}{2}\right)$$

Comparing both the sides, we get

$$2 = \frac{x+6}{2} \Rightarrow x=-2$$

$$0 = \frac{y+0}{2} \Rightarrow y=0$$

Thus other end of diameter is $(-2,0)$.

Thus (c) is correct option.

4. The distance of the point $(-6, 8)$ from x-axis is

(a) 6 units (b) -6 units

(c) 8 units (d) 10 units

Sol : [Board 2023 OD Standard]

The distance of the point $(-6, 8)$ from the y-axis is the absolute value of its x-coordinate, which is 8.

Thus (c) is correct option.

5. The point of intersection of the line represented by $3x-y=3$ and y-axis is given by

(a) (0, –3) (b) (0, 3)

(c) (2, 0) (d) (–2, 0)

Sol : [Board 2023 OD Standard]

We have $3x-y=3$

At the y-axis, value of x is 0, thus substitute value of $x=0$ in given equation we have,

$$\begin{aligned} 3\times 0-y &= 3 \\ -y &= 3 \\ y &= -3 \end{aligned}$$

Hence, the line $3x-y=3$ cuts y axis at point (0, – 3).

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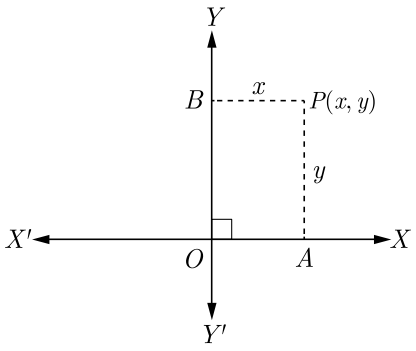
Sol :

We know that, if (x, y) is any point on the cartesian plane in first quadrant, then x is perpendicular distance from y-axis and y is perpendicular distance from x-axis.

Distance of the point $P(2, 3)$ from the x-axis is 3.



Thus (b) is correct option.

29. The distance between the points $A(0, 6)$ and $B(0, -2)$ is

(a) 6 (b) 8

(c) 4 (d) 2

Sol :

Distance between the points (x_1, y_1) and (x_2, y_2) is given as,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0,\ y_1 = 6$ and $x_2 = 0,\ y_2 = -2$

Distance between $A(0, 6)$ and $B(0, -2)$

$$AB = \sqrt{(0 - 0)^2 + (-2 - 6)^2}$$
$$= \sqrt{0 + (-8)^2} = \sqrt{8^2} = 8$$

Thus (b) is correct option.

30. The distance of the point $P(-6, 8)$ from the origin is

(a) 8 (b) $2\sqrt{7}$

(c) 10 (d) 6

Sol :

Distance between the points (x, y) and origin is given as,

$$d = \sqrt{x^2 + y^2}$$

Distance between $P(-6, 8)$ and origin is,

$$PO = \sqrt{(6)^2 + (-8)^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100} = 10$$

Thus (c) is correct option.

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31. The distance between the points (0, 5) and $(-5, 0)$ is

(a) 5 (b) $5\sqrt{2}$

(c) $2\sqrt{5}$ (d) 10

Sol :

Distance between the points (x_1, y_1) and (x_2, y_2) is given as,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

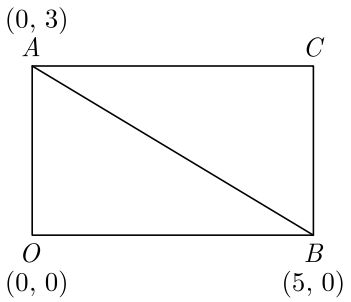
Here, $x_1 = 0,\ y_1 = 5$ and $x_2 = -5,\ y_2 = 0$

Distance between the points (0, 5) and $(-5, 0)$

$$d = \sqrt{[-5 - 0]^2 + [0 - (-5)]^2}$$
$$= \sqrt{5^2 + 5^2}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$

Thus (b) is correct option.

32. If $AOBC$ is a rectangle whose three vertices are $A(0, 3)$, $O(0, 0)$ and $B(5, 0)$, then the length of its diagonal is



(a) 5 (b) 3

(c) $\sqrt{34}$ (d) 4

Sol :

Length of the diagonal is AB which is the distance between the points $A(0, 3)$ and $B(5, 0)$.

Distance between the points (x_1, y_1) and (x_2, y_2) is given as,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0,\ y_1 = 3$, and $x_2 = 5,\ y_2 = 0$

Distance between the points $A(0, 3)$ and $B(5, 0)$

$$AB = \sqrt{(5 - 0)^2 + (0 - 3)^2}$$
$$= \sqrt{25 + 9}$$
$$= \sqrt{34}$$

Hence, the required length of its diagonal is $\sqrt{34}$.

Thus (c) is correct option.

Here S_5 is not correct because $\frac{AB}{DE} = \frac{CA}{CD}$

Thus (c) is correct option.

(iv) We have $\frac{AB}{DE} = \frac{BC}{CE}$

$$\frac{60}{x} = \frac{50}{40} \Rightarrow x = 48 \text{ ft}$$

(v) $AC = \sqrt{60^2 + 50^2}$

$= \sqrt{6100} = 71.8$

$CD = \sqrt{40^2 + 48^2}$

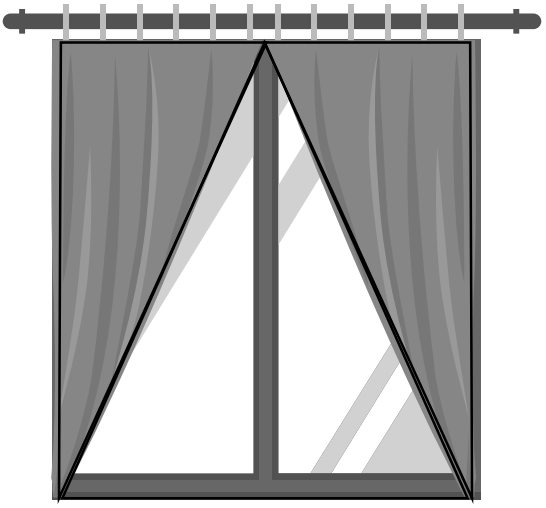
$= \sqrt{3904} = 62.5$

$AD = AC + CD = 71.8 + 62.5$

$= 140.6$ (Approx) ≈ 140

Thus (c) is correct option.

85. Rani wants to make the curtains for her window as shown in the figure. The window is in the shape of a rectangle, whose width and height are in the ratio $2:3$. The area of the window is 9600 square cm.



(i) What is the shape of the window that is uncovered?

(ii) What will be the ratio of two sides of each curtain (other than hypotenuse) ?

(iii) What are the dimensions of the window ?

(iv) What will be the perimeter of the window ?

(v) How much window area is covered by the curtains?

Sol :

(i) It is isosceles triangle.

(ii) Let $2x$ be the width of window, then $3x$ will be height of window because ratio is 2 : 3. Now width of single curtain will be x because it is half of window. Length of single curtain is equal the height of window. Thus ratio is $\frac{x}{3x} = \frac{1}{3}$

(iii) Area, $9600 = 2x \times 3x = 6x^2$

$x^2 = \frac{9600}{6} = 1600$

$x = \sqrt{1600} = 40$ cm

Width, $2x = 80$ cm

Length, $3x = 120$ cm

(iv) Perimeter,

$P = 2(80 + 120) = 400$ cm

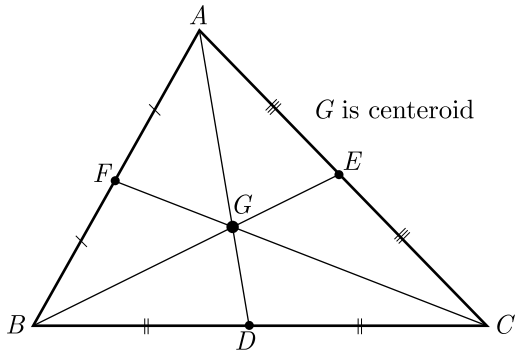
(v) Area of both curtains $= 2 \times \left(\frac{1}{2} \times 40 \times 120\right)$

$= 40 \times 120 = 4800$

$$\text{Window area} = \frac{4800}{9600} \times 100 = 50\%$$

86. The centroid is the centre point of the object. It is also defined as the point of intersection of all the three medians. The median is a line that joins the midpoint of a side and the opposite vertex of the triangle. The centroid of the triangle separates the median in the ratio of 2 : 1. It can be found by taking the average of x- coordinate points and y-coordinate points of all the vertices of the triangle.

See the figure given below



Here D, E and F are mid points of sides BC, AC and AB in same order. G is centroid, the centroid divides the median in the ratio 2 : 1 with the larger part towards the vertex. Thus $AG:GD = 2:1$

On the basis of above information read the question below.

If G is Centroid of $\triangle ABC$ with height h and J is centroid of $\triangle ADE$. Line DE parallel to BC, cuts the $\triangle ABC$ at a height $\frac{h}{4}$ from BC. $HF = \frac{h}{4}$.

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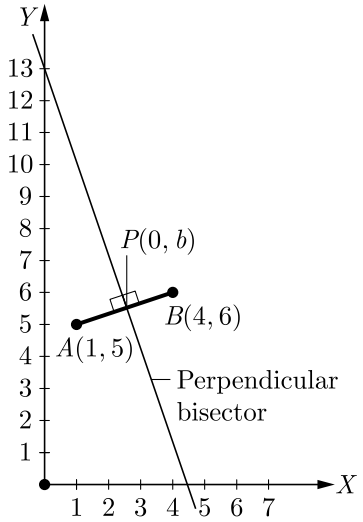
$$1 + b^2 - 10b + 25 = 16 + b^2 - 12b + 36$$
$$2b = 26 \Rightarrow b = 13$$



Thus (a) is correct option.

38. **Assertion :** The value of y is 6, for which the distance between the points $P(2,-3)$ and $Q(10, y)$ is 10.

Reason : Distance between two given points $A(x_1, y_1)$ *and* $B(x_2, y_2)$ is given,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

$$PQ = 10$$
$$PQ^2 = 100$$
$$(10-2)^2 + (y+3)^2 = 100$$
$$(y+3)^2 = 100 - 64 = 36$$
$$y + 3 = \pm 6$$
$$y = -3 \pm 6$$
$$y = 3, -9$$

Assertion (A) is false but reason (R) is true.

Thus (s) is correct option.

ONE MARK QUESTIONS

39. Find a point P on x-axis equidistant from the points $A(-1, 0)$ and $B(5, 0)$.

Sol : [Board 2020 OD Standard]

Let the position of the point P on x-axis be $(x, 0)$, then

$$PA^2 = PB^2$$
$$(x+1)^2 + (0)^2 = (5-x)^2 + (0)^2$$
$$x^2 + 2x + 1 = 25 + x^2 - 10x$$
$$2x + 10x = 25 - 1$$
$$12x = 24 \Rightarrow x = 2$$

Hence, the point $P(x, 0)$ is (2, 0).

Alternative :

You may easily observe that both point $A(-1, 0)$ and $B(5, 0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-1, 0)$ and $B(5, 0)$.

$$x = \frac{-1+5}{2} = 2$$

PRACTICE

- Write the coordinates of a point P on x-axis which is equidistant from the points $A(-2, 0)$ and $B(6, 0)$.
  [Board 2019 OD]
  Ans : $(-4, 0)$

- Find the point on the x-axis which is equidistant from the points $A(-2, 3)$ and $B(5, 4)$?
  [Board 2009]
  Ans : (2, 0)

- Find the point on the x-axis which is equidistant from the points $(2, -5)$ and $(-2, 9)$.
  [Board Term-2 2012]
  Ans : $(-7, 0)$.

40. Find the point on y-axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$.

Sol : [Board 2019 Delhi]

We have point $A = (5, -2)$ and $B = (-3, 2)$

Let $C(0, a)$ be point on y-axis.

According to question, point C is equidistant from A and B.

Thus $AC = BC$

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and $B(3,-8)$?

[Board Term-2 Delhi Compt. 2017]

Ans : $2:7$

Find the ratio in which $(11,15)$ divides the line segment joining the points $(15,5)$ and $(9,20)$.

[Board Term-2 2014]

Ans : $2:1$

75. Show that the points (a,a), $(-a,-a)$ and $\left(-\sqrt{3}\,a,\sqrt{3}\,a\right)$ are the vertices of an equilateral triangle.

Sol : [Board Term-2 Foreign 2015]

Let $A(a,a), B(-a,-a)$ and $C\left(-\sqrt{3}\,a,\sqrt{3}\,a\right)$.

Now $AB = \sqrt{(a+a)^2+(a+a)^2}$

$= \sqrt{4a^2+4a^2} = 2\sqrt{2}\,a$

$BC = \sqrt{\left(-a+\sqrt{3}\,a\right)^2+\left(-a-\sqrt{3}\,a\right)^2}$

$= \sqrt{a^2-2\sqrt{3}\,a^2+3a^2+a^2+2\sqrt{3}\,a^2+3a^2}$

$= 2\sqrt{2}\,a$

$AC = \sqrt{\left(a+\sqrt{3}\,a\right)^2+\left(a-\sqrt{3}\,a\right)^2}$

$= \sqrt{a^2+2\sqrt{3}\,a^2+3a^2+a^2-2\sqrt{3}\,a^2+3a^2}$

$= 2\sqrt{2}\,a$

Since $AB = BC = AC$, therefore ABC is an equilateral triangle.

76. If the mid-point of the line segment joining $A\left[\frac{x}{2},\frac{y+1}{2}\right]$ and $B(x+1,y-3)$ is $C(5,-2)$, find x, y.

Sol : [Board Term-2 OD 2012, Delhi 2014]

If the mid-point of the line segment joining $A\left[\frac{x}{2},\frac{y+1}{2}\right]$ and $B(x+1,y-3)$ is $C(5,-2)$, then at mid point,

$$\frac{\frac{x}{2}+x+1}{2} = 5$$

$$\frac{3x}{2}+1 = 10$$

$$3x = 18 \Rightarrow x = 6$$

Also $$\frac{\frac{y+1}{2}+y-3}{2} = -2$$

$$\frac{y+1}{2}+y-3 = -4$$

$$y+1+2y-6 = -8 \Rightarrow y = -1$$

77. Show that $A(6,4)$, $B(5,-2)$ and $C(7,-2)$ are the vertices of an isosceles triangle.

Sol : [Board Term-2 2012]

We have $A(6,4), B(5,-2), C(7,-2)$.

Now $AB = \sqrt{(6-5)^2+(4+2)^2}$

$= \sqrt{1^2+6^2} = \sqrt{37}$

$BC = \sqrt{(5-7)^2+(-2+2)^2}$

$= \sqrt{(-2)^2+0^2} = 2$

$CA = \sqrt{(7-6)^2+(-2-4)^2}$

$= \sqrt{1^2+6^2} = \sqrt{37}$

$AB = BC = \sqrt{37}$

Since two sides of a triangle are equal in length, triangle is an isosceles triangle.

78. Show that $A(-1,0)$, $B(3,1)$, $C(2,2)$ and $D(-2,1)$ are the vertices of a parallelogram $ABCD$.

Sol : [Board Term-2 2012]

Mid-point of AC,

$$\left(\frac{-1+2}{2},\ \frac{0+2}{2}\right) = \left(\frac{1}{2},1\right)$$

Mid-point of BD,

$$\left(\frac{3-2}{2},\ \frac{1+1}{2}\right) = \left(\frac{1}{2},1\right)$$

Here Mid-point of AC = Mid-point of BD

Since diagonals of a quadrilateral bisect each other, $ABCD$ is a parallelogram.

79. If $P(2,-1)$, $Q(3,4)$, $R(-2,3)$ and $S(-3,-2)$ be four points in a plane, show that $PQRS$ is a rhombus but not a square.

Sol : [Board Term-2 OD 2012]

We have $P(2,-1)$, $Q(3,4)$, $R(-2,3)$, $S(-3,-2)$

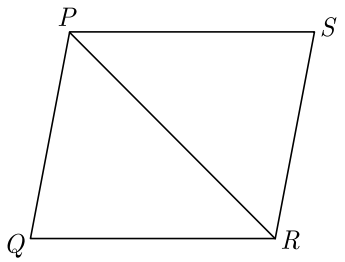
$PQ = \sqrt{1^2+5^2} = \sqrt{26}$

$QR = \sqrt{5^2+1^2} = \sqrt{26}$

$RS = \sqrt{1^2+5^2} = \sqrt{26}$

$PS = \sqrt{5^2+1^2} = \sqrt{26}$

Since all the four sides are equal, $PQRS$ is a rhombus.



Now $PR = \sqrt{1^2+5^2} = \sqrt{26}$

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the coordinates of point A is $(0, -4)$.

46. What is the distance of the point $(-12,\ 5)$ from the origin ?

Sol :

The distance between the origin and the point (x, y) is $\sqrt{x^2+y^2}$.

Therefore, the distance between the origin and point $(-12, 5)$

$$\begin{aligned} d &= \sqrt{(-12-0)^2+(5-0)^2} \\ &= \sqrt{144+25} = \sqrt{169} \\ &= 13 \text{ units} \end{aligned}$$

47. Find the ratio in which x-axis divides the line segment joining $A(2, -3)$ and $B(5, 6)$.

Sol : [Board 2020 OD Basic]

Let point $P(x,\ 0)$ on x-axis divide the segment joining points $A(2, -3)$ and $B(5,\ 6)$ in ratio $k : 1$, then

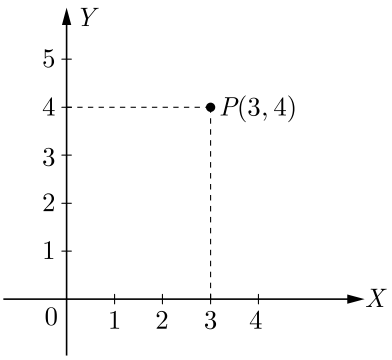
$$\begin{aligned} y &= \frac{m_1y_2+m_2y_1}{m_1+m_2} \\ 0 &= \frac{6k-3}{k+1} \\ 6k &= 3 \Rightarrow k = \frac{1}{2} \end{aligned}$$

Therefore ratio is $1 : 2$.

48. What is the distance of point $P(3, 4)$ from x-axis?

Sol : [Board 2020 Delhi Basic]

Point $P(3, 4)$ is 4 units from the x-axis and 3 units from the y-axis.



PRACTICE

✎ What is the distance of the point $P(-3, -4)$ from the x-axis?

[Board 2020 SQP Standard]

Ans : 4

49. If $A(\frac{m}{3},\ 5)$ is the mid-point of the line segment joining the points $Q(-6, 7)$ and $R(-2, 3)$, then what is the value of m ?

Sol : [Board 2020 SQP Standard]

Given points are $Q(-6, 7)$ and $R(-2, 3)$

$$\begin{aligned} \text{Mid point} \quad A(\tfrac{m}{3},\ 5) &= \left(\frac{-6-2}{2},\ \frac{7+3}{2}\right) \\ &= (-4, 5) \end{aligned}$$

$$\text{Equating,} \qquad \frac{m}{3} = -4 \Rightarrow m = -12$$

PRACTICE

✎ If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6, 5)$ and $R(-2, 3)$, then what is the value of a?

[Board Term-2 2014]

Ans : -12

✎ Find the value of a, for which point $P\left(\frac{a}{3}, 2\right)$ is the midpoint of the line segment joining the points $Q(-5, 4)$ and $R(-1, 0)$.

[Board Term-2 SQP 2016]

Ans : -9

50. The mid-point of the line-segment AB is $P(0, 4)$, if the coordinates of B are $(-2,\ 3)$ then find the co-ordinates of A.

Sol : [Board 2020 OD Basic]

Let point A be $(x,\ y)$.

Now using mid-point formula,

$$(0, 4) = \left(\frac{x-2}{2},\ \frac{y+3}{2}\right)$$

$$\text{Thus} \qquad 0 = \frac{x-2}{2} \Rightarrow x = 2$$

$$\text{and} \qquad 4 = \frac{y+3}{2} \Rightarrow y = 5$$

Hence point A is $(2,\ 5)$.

51. If the centre of a circle is $(3,\ 5)$ and end points of a diameter are $(4,\ 7)$ and $(2,\ y)$, what is the value of y ?

Sol : [Board 2020 Delhi Basic]

Since, centre is the mid-point of end points of the diameter.

$$(3,\ 5) = \left(\frac{4+2}{2},\ \frac{7+y}{2}\right)$$

Comparing both the sides, we get

$$5 = \frac{7+y}{2}$$

$$7+y = 10 \Rightarrow y = 3$$

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TWO MARKS QUESTIONS

67. A line intersects y-axis and x-axis at point P and Q, respectively. If $R(2,5)$ is the mid-point of line segment PQ, then find the coordinates of P and Q.

Sol : [Board 2023 OD Standard]

Let line intersect y-axis at point $P(0,k)$ and x -axis at point $Q(h,\ 0)$,

Since $R(2,5)$ is mid point of PQ, we have

$$(2,\ 5) = \left(\frac{0+h}{2},\ \frac{k+0}{2}\right)$$

$$(2,\ 5) = \left(\frac{h}{2},\ \frac{k}{2}\right)$$

Thus $h=4$ and $k=10$ and points are $P(0,10)$ and $Q(4,\ 0)$.

68. Find a relation between x and y such that the point $P(x,y)$ is equidistant from the points $A(-5,3)$ and $B(7,2)$.

Sol : [Board Term-2 SQP 2016]

Let $P(x,y)$ is equidistant from $A(-5,3)$ and $B(7,2)$, then we have

$$\begin{aligned} AP &= BP \\ \sqrt{(x+5)^2+(y-3)^2} &= \sqrt{(x-7)^2+(y-2)^2} \\ (x+5)^2+(y-3)^2 &= (x-7)^2+(y-2)^2 \\ 10x+25-6y+9 &= -14x+49-4y+4 \\ 24x+34 &= 2y+53 \\ 24x-2y &= 19 \end{aligned}$$

Thus $24x-2y-19=0$ is the required relation.

69. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2,\ -5)$ and $R(-3,\ 6)$, find the co-ordinates of P.

Sol : [Board Term-2 2016]

Let the point P be $(2y,y)$. Since $PQ = PR$, we have

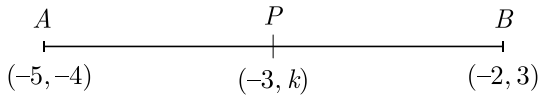
$$\begin{aligned} \sqrt{(2y-2)^2+(y+5)^2} &= \sqrt{(2y+3)^2+(y-6)^2} \\ (2y-2)^2+(y+5)^2 &= (2y+3)^2+(y-6)^2 \\ -8y+4+10y+25 &= 12y+9-12y+36 \\ 2y+29 &= 45 \text{ or } y = 8 \end{aligned}$$

Hence, coordinates of point P are $(16,8)$.

70. Find the ratio in which the point $(-3,k)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$.Also find the value of k.

Sol : [Board Term-2 Foreign 2016]

As per question, line diagram is shown below.



Let AB be divides by P in ratio $n:1$.

x co-ordinate for section formula

$$\begin{aligned} -3 &= \frac{(-2)n+1(-5)}{n+1} \\ -3(n+1) &= -2n-5 \\ -3n-3 &= -2n-5 \\ 5-3 &= 3n-2n \\ 2 &= n \end{aligned}$$

Ratio $\quad \frac{n}{1} = \frac{2}{1}$ or $2:1$

Now, y co-ordinate,

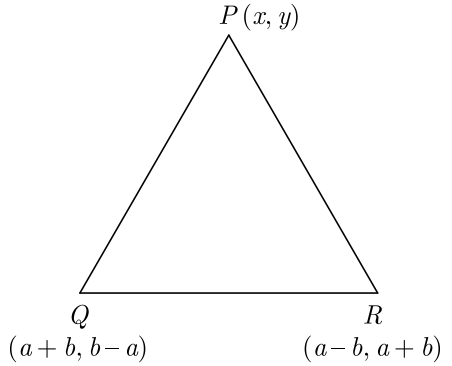
$$\begin{aligned} k &= \frac{2(3)+1(-4)}{2+1} \\ &= \frac{6-4}{3} = \frac{2}{3} \end{aligned}$$

71. If the point $P(x,y)$ is equidistant from the points $Q(a+b,b-a)$ and $R(a-b,a+b)$, then prove that $bx=ay$.

Sol : [Board Term-2 Delhi 2012, OD 2016]

We have $\quad |PQ| = |PR|$

$$\sqrt{\left[x-(a+b)\right]^2+\left[y-(b-a)\right]^2} = \sqrt{\left[x-(a-b)\right]^2+\left[y-(b+a)\right]^2}$$



$$\begin{aligned} &\left[x-(a+b)\right]^2+\left[y-(b-a)\right]^2 \\ &\quad = \left[x-(a-b)\right]^2+\left[y-(a+b)\right]^2 \\ &-2x(a+b)-2y(b-a) = -2x(a-b)-2y(a+b) \\ &2x(a+b)+2y(b-a) = 2x(a-b)+2y(a+b) \\ &2x(a+b-a+b)+2y(b-a-a-b) = 0 \end{aligned}$$

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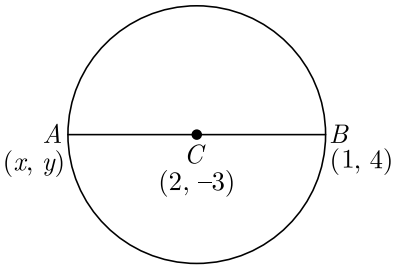

Let the co-ordinates of point A be (x, y).

x-coordinate of C, $\frac{x+1}{2} = 2$

$$x+1 = 4 \Rightarrow x = 3$$

and y-coordinate of C, $\frac{y+4}{2} = -3$

$$y+4 = -6 \Rightarrow y = -10$$

Hence, coordinates of point A are $(3, -10)$.

PRACTICE

Find the coordinates of a point A, where AB is diameter of the circle whose centre is $(2, -3)$ and B is the point $(3, 4)$.

[Board 2019 Delhi]

Ans : $(1, -10)$

60. Find the perpendicular distance of $A(5,12)$ from the y-axis.

Sol : [Board Term-2 2011]

Perpendicular from point $A(5,12)$ on y-axis touch it at $(0,12)$.

Distance between $(5,12)$ and $(0,12)$ is,

$$d = \sqrt{(0-5)^2+(12-12)^2}$$
$$= \sqrt{25}$$
$$= 5 \text{ units.}$$

61. If the centre and radius of circle is $(3,4)$ and 7 units respectively,, then what it the position of the point $A(5,8)$ with respect to circle?

Sol : [Board Term-2 2013]

Distance of the point, from the centre,

$$d = \sqrt{(5-3)^2+(8-4)^2}$$
$$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Since $2\sqrt{5}$ is less than 7, the point lies inside the circle.

62. Find the perimeter of a triangle with vertices $(0,4),(0,0)$ and $(3,0)$.

Sol : [Board Term-2, 2011]

We have $A(0,4), B(0,0)$, and $C(3,0)$.

$$AB = \sqrt{(0-2)^2+(0-4)^2} = \sqrt{16} = 4$$
$$BC = \sqrt{(3-0)^2+(0-0)^2} = \sqrt{9} = 3$$
$$CA = \sqrt{(0-3)^2+(4-0)^2}$$
$$= \sqrt{9+16} = \sqrt{25} = 5$$

Thus perimeter of triangle is $4+3+5 = 12$

63. Locate a point Q on line segment AB such that $BQ = \frac{5}{7} \times AB$. What is the ratio of line segment in which AB is divided?

Sol : [Board Term-2 2013]

We have $BQ = \frac{5}{7}AB$

$$\frac{BQ}{AB} = \frac{5}{7} \Rightarrow \frac{AB}{BQ} = \frac{7}{5}$$
$$\frac{AB-BQ}{BQ} = \frac{7-5}{5}$$
$$\frac{AQ}{BQ} = \frac{2}{5}$$

Thus $AQ : BQ = 2:5$

64. Find the distance of the point $(-4,-7)$ from the y-axis.

Sol : [Board Term-2 2013]

Perpendicular from point $A(-4,-7)$ on y-axis touch it at $(0,-7)$.

Distance between $(-4,-7)$ and $(0,-7)$ is

$$d = \sqrt{(0+4)^2+(-7+7)^2}$$
$$= \sqrt{4^2+0} = \sqrt{16} = 4 \text{ units}$$

65. If the distance between the points $(4,k)$ and $(1,0)$ is 5, then what can be the possible values of k.

Sol : [Board Term-2 2017]

Using distance formula we have

$$\sqrt{(4-1)^2+(k-0)^2} = 5$$
$$3^2+k^2 = 25$$
$$k^2 = 25-9$$
$$= 16$$
$$k = \pm 4$$

66. Find the coordinates of the point on y-axis which is nearest to the point $(-2,5)$.

Sol : [Board Term-2 SQP 2017]

Point $(0,5)$ on y-axis is nearest to the point $(-2,5)$.

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52. If the distance between the points $A(4, p)$ and $B(1, 0)$ is 5 units then what are the values of p?

Sol : [Board 2020 Delhi Basic]

Given, points are $A(4, p)$ and $B(1, 0)$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y^2 - y_1)^2} \\ 5 &= \sqrt{(1-4)^2 + (0-p)^2} \\ 25 &= 9 + p^2 \\ p^2 &= 25 - 9 = 16 \\ p &= \pm 4 \end{aligned}$$

53. If the points $A(4, 3)$ and $B(x, 5)$ are on the circle with centre $O(2, 3)$, then what is the value of x?

Sol :

Since, A and B lie on the circle having centre O.

$$\begin{aligned} OA &= OB \\ \sqrt{(4-2)^2 + (3-3)^2} &= \sqrt{(x-2)^2 + (5-3)^2} \\ 2 &= \sqrt{(x-2)^2 + 4} \\ 4 &= (x-2)^2 + 4 \\ (x-2)^2 &= 0 \Rightarrow x = 2 \end{aligned}$$

54. If three points (0, 0), $(3, \sqrt{3})$ and $(3, \lambda)$ form an equilateral triangle, then what is the value of λ?

Sol :

Let the given points are $A(0, 0)$, $B(3, \sqrt{3})$ and $C(3, \lambda)$.

Since, ΔABC is an equilateral triangle, therefore

$$\begin{aligned} AB &= AC \\ \sqrt{(3-0)^2 + (\sqrt{3}-0)^2} &= \sqrt{(3-0)^2 + (\lambda - 0)^2} \\ 9 + 3 &= 9 + \lambda^2 \\ \lambda^2 &= 3 \\ \lambda &= \pm\sqrt{3} \end{aligned}$$

55. The co-ordinate of the point dividing the line segment joining the points $A(1, 3)$ and $B(4, 6)$ in the ratio 2 : 1 is ..... .

Sol : [Board 2020 OD Basic]

Let point $P(x, y)$ divides the line segment joining the points $A(1, 3)$ and $B(4, 6)$ in the ratio 2 : 1.

Using section formula we have

$$\begin{aligned} (x, y) &= \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right) \\ (x, y) &= \left(\frac{2\times 4 + 1 \times 1}{2+1}, \frac{2\times 6 + 1 \times 3}{2+1}\right) \\ &= \left(\frac{8+1}{3}, \frac{12+3}{3}\right) \\ &= \left(\frac{9}{3}, \frac{15}{3}\right) \\ &= (3, 5) \end{aligned}$$

56. Find the distance of a point $P(x, y)$ from the origin.

Sol : [Board 2018]

Distance between origin (0, 0) and point $P(x, y)$ is

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(x-0)^2 + (y-0)^2} \\ &= \sqrt{x^2 + y^2} \end{aligned}$$

Distance between P and origin is $\sqrt{x^2 + y^2}$.

57. If the mid-point of the line segment joining the points $A(3, 4)$ and $B(k, 6)$ is $P(x, y)$ and $x + y - 10 = 0$, find the value of k.

Sol : [Board 2020 OD Standard]

If $P(x, y)$ is mid point of $A(3, 4)$ and $B(k, 6)$, then we have

$$\frac{3+k}{2} = x$$

and $$y = \frac{4+6}{2} = \frac{10}{2} = 5$$

Substituting above value in $x + y - 10 = 0$ we have

$$\begin{aligned} \frac{3+k}{2} + 5 - 10 &= 0 \\ \frac{3+k}{2} &= 5 \\ 3 + k &= 10 \Rightarrow k = 10 - 3 = 7 \end{aligned}$$

58. The ordinate of a point A on y-axis is 5 and B has co-ordinates $(-3, 1)$. Find the length of AB.

Sol : [Board Term-2 2014]

We have $A(0,5)$ and $B(-3,1)$.

Distance between A and B,

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3-0)^2 + (1-5)^2} \\ &= \sqrt{9 + 16} = \sqrt{25} = 5 \end{aligned}$$

59. Find the coordinates of a point A, where AB is diameter of a circle whose centre is $(2, -3)$ and B is the point $(1, 4)$.

Sol : [Board 2019 Delhi]

As per question we have shown the figure below. Since, AB is the diameter, centre C must be the mid point of the diameter of AB.

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$$2x(2b)+2y(-2a) = 0$$
$$xb-ay = 0$$
$$bx = ay \qquad \text{Hence Proved}$$

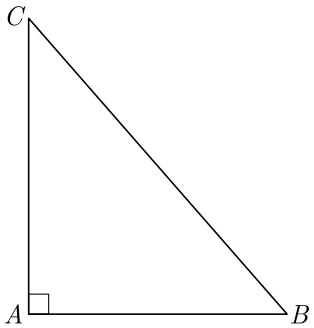
72. Prove that the point $(3,0)$, $(6,4)$ and $(-1,3)$ are the vertices of a right angled isosceles triangle.

Sol : [Board Term-2 OD 2016]

We have $A(3,0)$, $B(6,4)$ and $C(-1,3)$

Now
$$AB^2 = (3-6)^2+(0-4)^2$$
$$= 9+16 = 25$$
$$BC^2 = (6+1)^2+(4-3)^2$$
$$= 49+1 = 50$$
$$CA^2 = (-1-3)^2+(3-0)^2$$
$$= 16+9 = 25$$
$$AB^2 = CA^2 \text{ or, } AB = CA$$

Hence triangle is isosceles.



Also, $25+25 = 50$

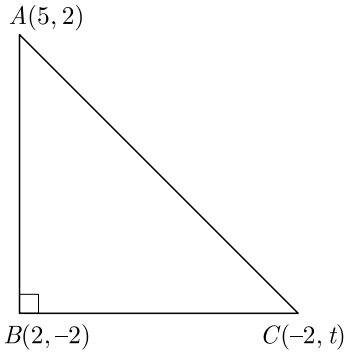
or, $AB^2+CA^2 = BC^2$

Since Pythagoras theorem is verified, therefore triangle is a right angled triangle.

73. If $A(5,2)$, $B(2,-2)$ and $C(-2,t)$ are the vertices of a right angled triangle with $\angle B = 90^\circ$, then find the value of t.

Sol : [Board Term-2 Delhi 2015]

As per question, triangle is shown below.



Now
$$AB^2 = (2-5)^2+(-2-2)^2 = 9+16 = 25$$
$$BC^2 = (-2-2)^2+(t+2)^2 = 16+(t+2)^2$$
$$AC^2 = (5+2)^2+(2-t)^2 = 49+(2-t^2)$$

Since ΔABC is a right angled triangle
$$AC^2 = AB^2+BC^2$$
$$49+(2-t)^2 = 25+16+(t+2)^2$$
$$49+4-4t+t^2 = 41+t^2+4t+4$$
$$53-4t = 45+4t$$
$$8t = 8$$
$$t = 1$$

PRACTICE

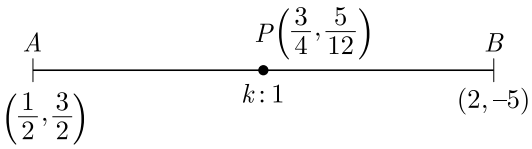
- The points $A(4,7)$, $B(p,3)$ and $C(7,3)$ are the vertices of a right triangle, right-angled at B. Find the value of p.
  [Board Term-2 OD 2015]
  Ans : 7 or 4

- If $A(4,3)$, $B(-1,y)$, and $C(3,4)$ are the vertices of a right triangle ABC, right angled at A, then find the value of y.
  [Board Term-2 OD 2015]
  Ans : -2

74. Find the ratio in which the point $P(\frac{3}{4},\frac{5}{12})$ divides the line segment joining the point A $(\frac{1}{2},\frac{3}{2})$ and $(2,-5)$.

Sol : [Board Term-2 Delhi 2015]

Let P divides AB in the ratio k:1. Line diagram is shown below.



Now
$$\frac{k(2)+1(\frac{1}{2})}{k+1} = \frac{3}{4}$$
$$8k+2 = 3k+3$$
$$k = \frac{1}{5}$$

Thus required ratio is $\frac{1}{5}$:1 or 1:5.

PRACTICE

- In what ratio does the point $P(-4,6)$ divides the line segment joining the points $A(-6,10)$

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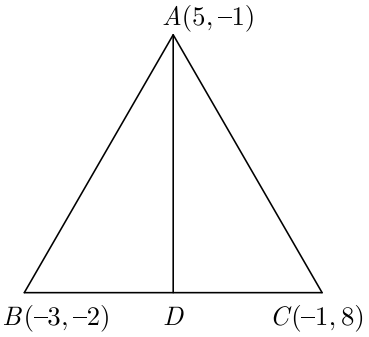


Since D is mid-point of BC, co-ordinates of D,

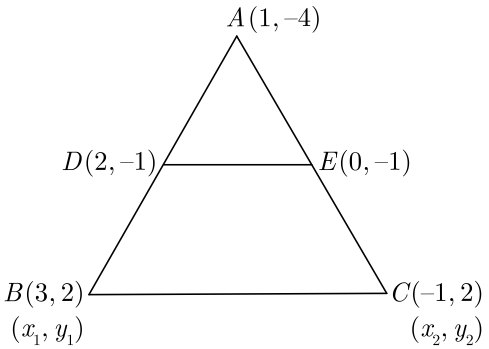
$$(x_1,\ y_2) = \left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$$
$$= (-2,\ 3)$$
$$AD = \sqrt{(5+2)^2+(-1-3)^2}$$
$$= \sqrt{(7)^2+(4)^2}$$
$$= \sqrt{49+16} = \sqrt{65} \text{ units}$$

Thus length of median is $\sqrt{65}$ units.

84. Find the mid-point of side BC of ΔABC, with $A(1,-4)$ and the mid-points of the sides through A being $(2,-1)$ and $(0,-1)$.

Sol : [Board Term-2 2012]

Assume co-ordinates of B and C are (x_1, y_1) and (x_2, y_2) respectively. As per question, triangle is shown below.



Now $$2 = \frac{1+x_1}{2} \Rightarrow x_1 = 3$$

and $$-1 = \frac{-4+y_1}{2} \Rightarrow y_1 = 2$$

$$0 = \frac{1+x_2}{2} \Rightarrow x = -1$$

$$-1 = \frac{-4+y_2}{2} \Rightarrow y_2 = 2$$

Thus $$B(x_1, y_1) = (3,2),$$
$$C(x_2, y_2) = (-1,2)$$

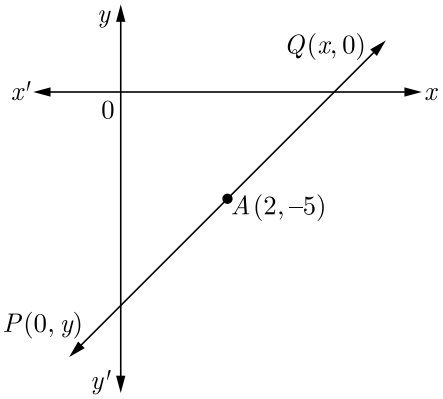
So, mid-point of BC is $\left(\frac{3-1}{2}, \frac{2+2}{2}\right) = (1,2)$

85. A line intersects the y-axis and x-axis at the points P and Q respectively. If $(2,-5)$ is the mid-point of PQ, then find the coordinates of P and Q.

Sol : [Board Term-2 OD 2017]

Let coordinates of P be $(0,y)$ and of Q be $(x,0)$. $A(2,-5)$ is mid point of PQ.

As per question, line diagram is shown below.



Using section formula,

$$(2,-5) = \left(\frac{0+x}{2} + \frac{y+0}{2}\right)$$
$$2 = \frac{x}{2} \Rightarrow x = 4$$

and $$-5 = \frac{y}{2} \Rightarrow y = -10$$

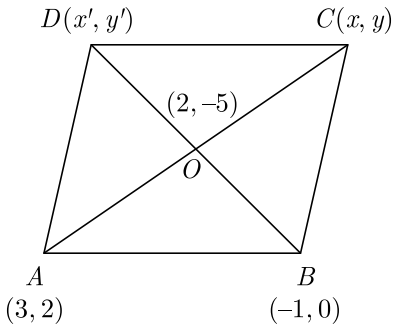
Thus P is $(0,-10)$ and Q is $(4,0)$

86. If two adjacent vertices of a parallelogram are $(3,2)$ and $(-1,0)$ and the diagonals intersect at $(2,-5)$ then find the co-ordinates of the other two vertices.

Sol : [Board Term-2 Foreign 2017]

Let two other co-ordinates be (x,y) and (x',y') respectively using mid-point formula.

As per question parallelogram is shown below.



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$$= \sqrt{4^2+4^2} = \sqrt{32}$$

$$PQ^2 + QR^2 = 2 \times 26 = 52 \neq \left(\sqrt{32}\right)^2$$

Since ΔPQR is not a right triangle, $PQRS$ is a rhombus but not a square.

80. If $(3,2)$ and $(-3,2)$ are two vertices of an equilateral triangle which contains the origin, find the third vertex.

Sol : [Board Term-2 OD 2012]

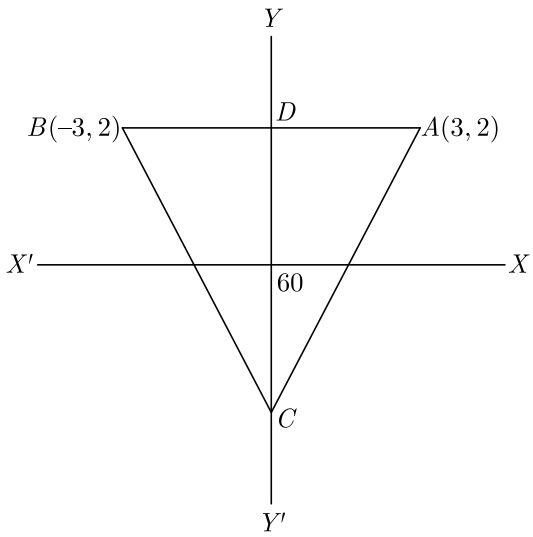
We have $A(3,2)$ and $B(-3,2)$.

It can be easily seen that mid-point of AB is lying on y-axis. Thus AB is equal distance from x-axis everywhere.

Also $OD \perp AB$

Hence 3^{rd} vertex of ΔABC is also lying on y-axis.

The digram of triangle should be as given below.



Let $C(x,y)$ be the coordinate of 3^{rd} vertex of ΔABC.

Now $AB^2 = (3+3)^2 + (2-2)^2 = 36$

$BC^2 = (x+3)^2 + (y-2)^2$

$AC^2 = (x-3)^2 + (y-2)^2$

Since $AB^2 = AC^2 = BC^2$

$$(x+3)^2 + (y-2)^2 = 36 \quad (1)$$

$$(x-3)^2 + (y-2)^2 = 36 \quad (2)$$

Since $P(x,y)$ lie on y-axis, substituting $x=0$ in (1) we have

$$3^2 + (y-2)^2 = 36 - 9 = 27$$

$$(y-2)^2 = 36 - 9 = 27$$

Taking square root both side

$$y - 2 = \pm 3\sqrt{3}$$

$$y = 2 \pm 3\sqrt{3}$$

Since origin is inside the given triangle, coordinate of C below the origin,

$$y = 2 - 3\sqrt{3}$$

Hence Coordinate of C is $\left(0,\ 2-3\sqrt{3}\right)$

81. Find a so that $(3,a)$ lies on the line represented by $2x-3y-5=0$. Also, find the co-ordinates of the point where the line cuts the x-axis.

Sol : [Board Term-2 2012]

Since $(3,a)$ lies on $2x-3y-5=0$, it must satisfy this equation. Therefore

$$2 \times 3 - 3a - 5 = 0$$

$$6 - 3a - 5 = 0$$

$$1 = 3a$$

$$a = \frac{1}{3}$$

Line $2x-3y-5=0$ will cut the x-axis at $(x,0)$. and it must satisfy the equation of line.

$$2x - 5 = 0 \Rightarrow x = \frac{5}{2}$$

Hence point is $\left(\frac{5}{2},0\right)$.

82. If $\left(1,\frac{p}{3}\right)$ is the mid point of the line segment joining the points $(2,0)$ and $\left(0,\frac{2}{9}\right)$, then show that the line $5x+3y+2=0$ passes through the point $(-1,3p)$.

Sol :

Since $\left(1,\frac{p}{3}\right)$ is the mid point of the line segment joining the points $(2,0)$ and $\left(0,\frac{2}{9}\right)$, we have

$$\frac{p}{3} = \frac{0+\frac{2}{9}}{2} = \frac{1}{9}$$

$$p = \frac{1}{3}$$

Now the point $(-1,3p)$ is $(-1,1)$.

The line $5x+3y+2=0$, passes through the point $(-1,1)$ as $5(-5)+3(1)+2=0$

83. If the vertices of ΔABC are $A(5,-1), B(-3,-2), C(-1,8)$, Find the length of median through A.

Sol : [Board Term-2 2012]

Let AD be the median. As per question, triangle is shown below.

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Using distance formula we have

$$\sqrt{(0-5)^2+(a+2)^2} = \sqrt{(0+3)^2+(a-2)^2}$$
$$\sqrt{25+a^2+4+4a} = \sqrt{9+a^2+4-4a}$$
$$25+a^2+4+4a = 9+a^2+4-4a$$
$$8a = -16 \Rightarrow a = -2$$

Hence, point on y-axis is $(0-2)$.

PRACTICE

✍ Find the co-ordinates of the point on the $y-$ axis which is equidistant from the points $A(5,3)$ and $B(1,-5)$

[Board Term-2 OD Compt. 2017]

Ans : $(0, \frac{1}{2})$

41. Find the distance between the points $(a\cos\theta + b\sin\theta,\ 0)$, and $(0,\ a\sin\theta - b\cos\theta)$?

Sol : [Board 2020 Delhi Standard]

We have $x_1 = a\cos\theta + b\sin\theta$ and $y_1 = 0$

and $x_2 = 0$ and $y_2 = a\sin\theta - b\cos\theta$

$$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$
$$= (0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2$$
$$= (-1)^2(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$
$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$
$$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$$
$$= a^2 \times 1 + b^2 \times 1 = a^2 + b^2$$

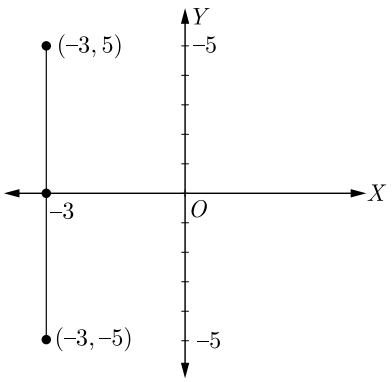
Thus $d^2 = a^2 + b^2$

$$d = \sqrt{a^2+b^2}$$

42. Find the co-ordinates of the point which is reflection of point $(-3,5)$ in x-axis.

Sol : [Board 2020 OD Standard]

The reflection of point $(-3,5)$ in x-axis is $(-3,-5)$



43. If $A(4,-1), B(5,3), C(2,y)$ and $D(1,1)$ are the vertices of a parallelogram $ABCD$, find y.

Sol : [Board Term-2, 2012]

Diagonals of a parallelogram bisect each other.

Mid-points of AC and BD are same.

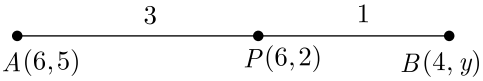
Thus $\left(3, \frac{-1+y}{2}\right) = (3,2)$

$$\frac{-1+y}{2} = 2 \Rightarrow y = 5$$

44. If the point P (6, 2) divides the line segment joining $A(6,5)$ and $B(4,y)$ in the ratio $3:1$ then what is the value of y?

Sol : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below,



Here, $x_1 = 6,\ y_1 = 5$

and $x_2 = 4\ \ y_2 = y$

Now $y = \frac{my_2 + ny_1}{m+n}$

$$2 = \frac{3 \times y + 1 \times 5}{3+1}$$
$$2 = \frac{3y+5}{4}$$
$$3y+5 = 8$$
$$3y = 8-5 = 3 \Rightarrow y = 1$$

PRACTICE

✍ If the point $P(k,\ 0)$ divides the line segment joining the points $A(2,-2)$ and $B(-7,4)$ in the ratio 1 : 2, then what is the value of k?

[Board 2020 Delhi Standard]

Ans : -1

✍ If the point $C(k,\ 4)$ divides the line segment joining two points $A(2,\ 6)$ and $B(5,1)$ in ratio 2 : 3, the value of k is ..... .

[Board 2020 Delhi Basic]

Ans : $\frac{16}{5}$

45. Find the coordinates of a point A on y-axis, at a distance of 4 units from x-axis and below it.

Sol : [Board 2020 Delhi Basic]

Because the point is 4 units down the x-axis i.e., co-ordinate is -4 and on y-axis abscissa is 0. So,

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Now $2 = \frac{x+3}{2} \Rightarrow x = 1$

and $-5 = \frac{2+y}{2} \Rightarrow y = -12$

Again, $\frac{-1+x'}{2} = 2 \Rightarrow x' = 5$

and $\frac{0+y'}{2} = -5 \Rightarrow y' = -10$

Hence, coordinates of $C(1,-12)$ and $D(5,-10)$

87. Prove that the points $(2,-2),(-2,1)$ and $(5,2)$ are the vertices of a right angled triangle. Also find the area of this triangle.

Sol : [Board Term-2 Foreign 2016]

We have $A(2,-2), B(-2,1)$ and $(5,2)$

Now using distance formula we get

$$AB^2 = (2+2)^2 + (-2-1)^2$$
$$= 16 + 9 = 25$$
$$AB^2 = 25 \Rightarrow AB = 5.$$

Thus $AB = 5$.

Similarly $BC^2 = (-2-5)^2 + (1-2)^2$

$$= 49 + 1 = 50$$
$$BC^2 = 50 \Rightarrow BC = 5\sqrt{2}$$
$$AC^2 = (2-5)^2 + (-2-2)^2$$
$$= 9 + 16 = 25$$
$$AC^2 = 25$$
$$AC = 5$$

Clearly $AB^2 + AC^2 = BC^2$

$$25 + 25 = 50$$

Hence the triangle is right angled,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$= \frac{1}{2} \times 5 \times 5 = \frac{25}{2} \text{ sq unit.}$$

THREE MARKS QUESTIONS

88. Find the ratio in which the line segment joining the points $A(6,3)$ and $B(-2,-5)$ is divided by x-axis.

Sol : [Board 2023 OD Standard]

If point (x,y) divides the line joining the point $(x_1\ y_1)$ and $(x_2\ y_2)$ in the ratio $m:n$ then,

$$(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Here $x_1 = 6,\ y_1 = 3,\ x_2 = -2,\ y_2 = -5$

Since point lies on x-axis we have $y = 0$.

Thus $\frac{my_2+my_1}{m+n} = 0$

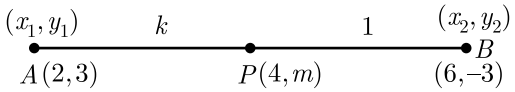
$$\frac{-5m+3n}{m+n} = 0$$
$$-5m+3n = 0$$
$$-5m = -3n$$
$$\frac{m}{n} = \frac{3}{5}$$

Thus ratio, $m:n$ is 3 : 5.

89. Find the ratio in which $P(4,m)$ divides the segment joining the points $A(2,3)$ and $B(6,-3)$. Hence find m.

Sol : [Board 2018]

Let $P(x,y)$ be the point which divide AB in $k:1$ ratio.



Now $x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$

$$4 = \frac{k(6)+1(2)}{k+1}$$
$$4k+4 = 6k+2$$
$$6k-4k = 4-2$$
$$2k = 2 \Rightarrow k = 1$$

Thus point P divides the line segment AB in $1:1$ ratio.

Now $y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

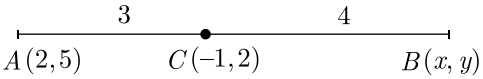
$$m = \frac{1\times(-3)+1(3)}{1+1}$$
$$= \frac{-3+3}{2} = 0$$

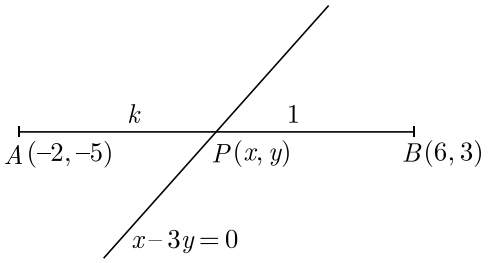
Thus $m = 0$.

90. If the point $C(-1,2)$ divides internally the line segment joining $A(2,5)$ and $B(x,y)$ in the ratio 3 :4 find the coordinates of B.

Sol : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.





Using section formula, we get

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$= \frac{k \times 6 + 1 \times (-2)}{k+1}$$

$$x = \frac{6k-2}{k+1} \quad ...(1)$$

and $$y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{k \times 3 + 1 \times (-5)}{k+1}$$

$$y = \frac{3k-5}{k+1} \quad ...(2)$$

The point $P(x, y)$ lies on the line, hence it satisfies the equation of the given line.

$$\frac{6k-2}{k+1} - 3\left(\frac{3k-5}{k+1}\right) = 0$$

$$6k-2-3(3k-5) = 0$$

$$6k-2-9k+15 = 0$$

$$-3k+13 = 0 \Rightarrow k = \frac{13}{3}$$

Hence, the required ratio is 13 : 3.

Now, substituting value of k in x and y, we get

$$x = \frac{6 \times \frac{13}{3} - 2}{\frac{13}{3} + 1}$$

$$= \frac{78-6}{16} = \frac{72}{16} = \frac{9}{2}$$

$$y = \frac{3 \times \frac{13}{3} - 5}{\frac{13}{3} + 1}$$

$$= \frac{8 \times 3}{16} = \frac{24}{16} = \frac{3}{2}$$

Hence, the co-ordinates of point of intersection

$$P(x, y) = \left(\frac{9}{2}, \frac{3}{2}\right)$$

PRACTICE

✎ Determine the ratio in which the straight line $x - y - 2 = 0$ divides the line segment joining $(3, -1)$ and $(8, 9)$.

[Board Term-2, 2012]

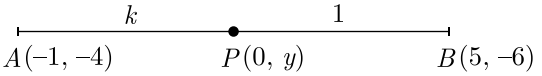
Ans : 2:3

115. Find the ratio in which the y-axis divides the line segment joining the points $(-1, -4)$ and $(5, -6)$. Also find the coordinates of the point of intersection.

Sol : [Board 2019 OD]

Let points $P(0, y)$ divides the line joining the point $A(-1, -4)$ and $B(5, -6)$ in ratios $k:1$.

As per given information in question we have drawn figure below.



Section formula is given by

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \quad ...(1)$$

and $$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \quad ...(2)$$

Here, $m_1 = k$ and $m_2 = 1$,

$x_1 = -1$ and $x_2 = 5$

$y_1 = -4$ and $y_2 = -6$

Now $$0 = \frac{k \times 5 + 1 \times (-1)}{k+1}$$

$$5k - 1 = 0 \Rightarrow k = \frac{1}{5}$$

Substitute value of k in eq (2), we get

$$y = \frac{k(-6) + 1(-4)}{k+1}$$

$$= \frac{\frac{1}{5}(-6) + 1(-4)}{\frac{1}{5} + 1}$$

$$= \frac{-26}{6} = \frac{-13}{3}$$

Hence, value of k is $\frac{1}{5}$ and required point is $\left(0, -\frac{13}{3}\right)$

PRACTICE

✎ Find the ratio in which y-axis divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$. Also find the co-ordinates of the point of division.

[Board Term-2 Delhi 2016]

Ans : 5:1, $\left(0, -\frac{13}{3}\right)$

116. If $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$ are the vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides.

Sol : [Board 2018]

As per information given in question we have drawn the figure below.

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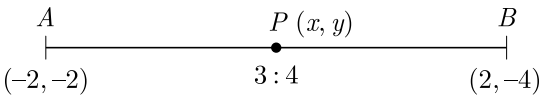
Sol : [Board Term-2 OD 2017]

We have $AP = \frac{3}{7}AB \Rightarrow AP : PB = 3:4$

As per question, line diagram is shown below.



Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

Applying section formula we get

$$x = \frac{3 \times 2 + 4 \times (-2)}{3+4} = -\frac{2}{7}$$

$$y = \frac{3 \times (-4) + 4 \times (-2)}{3+4} = -\frac{20}{7}$$

Hence P is $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

PRACTICE

✎ Find the co-ordinate of a point P on the line segment joining $A(1,2)$ and $B(6,7)$ such that $AP = \frac{2}{5}AB$.

[Board Term-2 OD 2015]

Ans : (3, 4)

94. If the distance of $P(x, y)$ from $A(6, 2)$ and $B(-2, 6)$ are equal, prove that $y = 2x$.

Sol : [Board Term-2, 2015]

We have $P(x, y), A(6, 2), B(-2, 6)$

Now $PA = PB$

$$PA^2 = PB^2$$

$$(x-6)^2 + (y-2)^2 = (x+2)^2 + (y-6)^2$$

$$-12x + 36 - 4y + 4 = 4x + 4 - 12y + 36$$

$$-12x - 4y = 4x - 12y$$

$$12y - 4y = 4x + 12x$$

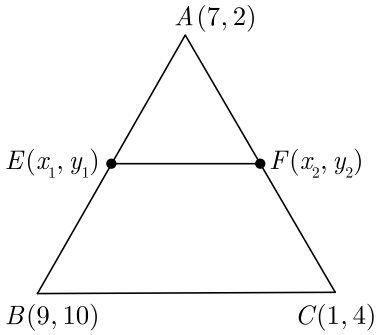
$$8y = 16x$$

$$y = 2x$$ Hence Proved

95. The co-ordinates of the vertices of ΔABC are $A(7, 2)$, $B(9, 10)$ and $C(1, 4)$. If E and F are the mid-points of AB and AC respectively, prove that $EF = \frac{1}{2}BC$.

Sol : [Board Term-2 2015]

Let the mid-points of AB and AC be $E(x_1, y_1)$ and $F(x_2, y_2)$. As per question, triangle is shown below.



Co-ordinates of point E,

$$(x_1, y_1) = \left(\frac{9+7}{2}, \frac{10+2}{2}\right) = (8, 6)$$

Co-ordinates of point F,

$$(x_2, y_2) = \left(\frac{7+1}{2}, \frac{2+4}{2}\right) = (4, 3)$$

Length, $EF = \sqrt{(8-4)^2 + (6-3)^2}$

$$= \sqrt{4^2 + 3^2} = 5 \text{ units} \quad ...(1)$$

Length $BC = \sqrt{(9-1)^2 + (10-4)^2}$

$$= \sqrt{8^2 + 6^2} = 10 \text{ units} \quad ...(2)$$

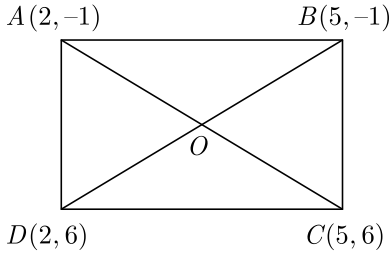
From equation (1) and (2) we get

$$EF = \frac{1}{2}BC$$ Hence proved.

96. Prove that the diagonals of a rectangle $ABCD$, with vertices $A(2, -1), B(5, -1), C(5, 6)$ and $D(2, 6)$ are equal and bisect each other.

Sol : [Board Term-2 2014]

As per question, rectangle $ABCD$, is shown below.



Now $AC = \sqrt{(5-2)^2 + (6+1)^2}$

$$= \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$$

$$BD = \sqrt{(5-2)^2 + (-1-6)^2}$$

$$= \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$$

Since $AC = BD = \sqrt{58}$ the diagonals of rectangle $ABCD$ are equal.

Mid-point of AC, $= \left(\frac{2+5}{2}, \frac{-1+6}{2}\right)$

$$= \left(\frac{7}{2}, \frac{5}{2}\right)$$

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Using section formula,

$$-1 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$-1 = \frac{3 \times x + 4 \times 2}{3+4}$$

$$= \frac{3x+8}{7}$$

$$3x+8 = -7$$

$$3x = -15 \Rightarrow x = -5$$

and

$$2 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$2 = \frac{3y + 4 \times 5}{3+4}$$

$$= \frac{3y+20}{7}$$

$$3y+20 = 14$$

$$3y = 14 - 20 = -6$$

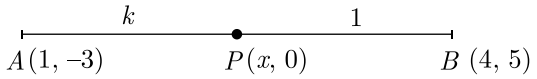
$$y = -2$$

Hence, the coordinates of $B(x, y)$ is $(-5, -2)$.

91. Find the ratio in which the segment joining the points $(1, -3)$ and $(4, 5)$ is divided by x-axis? Also find the coordinates of this point on x-axis.

Sol : [Board 2019 Delhi]

Let the required ratio be $k:1$ and the point on x -axis be $(x, 0)$.



Here, $(x_1, y_1) = (1, -3)$

and $(x_2, y_2) = (4, 5)$

Using section formula y coordinate, we obtain,

$$y = \frac{my_2 + ny_1}{m+n}$$

$$0 = \frac{k \times 5 + 1 \times 1(-3)}{k+1}$$

$$0 = 5k - 3$$

$$5k = 3 \Rightarrow k = \frac{3}{5}$$

Hence, the required ratio is $\frac{3}{5}$ i.e $3:5$.

Now, again using section formula for x, we obtain

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$x = \frac{k \times (4) + 1 \times 1}{k+1}$$

$$= \frac{\frac{3}{5}(4)+1}{\frac{3}{5}+1}$$

$$= \frac{12+5}{3+5} = \frac{17}{8}$$

Co-ordinate of P is $\left(\frac{17}{8}, 0\right)$.

PRACTICE

- In what ratio does the x-axis divide the line segment joining the points $(-4, -6)$ and $(-1, 7)$? Find the coordinates of the point of division.

  [Board Term-2 SQP 2017]

  Ans : $6:7$, $(-\frac{34}{13}, 0)$

- Find the ratio in which the line segment joining the points $A(3, -3)$ and $B(-2, 7)$ is divided by x-axis. Also find the co-ordinates of point of division.

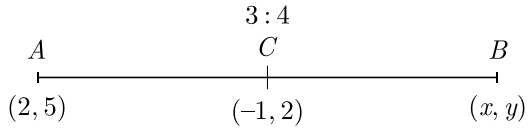
  [Board Term-2 Delhi 2014]

  Ans : $\frac{3}{7}$, $(\frac{3}{2}, 0)$

92. If the point $C(-1, 2)$ divides internally the line segment joining the points $A(2, 5)$ and $B(x, y)$ in the ratio $3:4$, find the value of $x^2 + y^2$.

Sol : [Board Term-2 Foreign 2016]

As per question, line diagram is shown below.



We have $\frac{AC}{BC} = \frac{3}{4}$

Applying section formula for x co-ordinate,

$$-1 = \frac{3x + 4(2)}{3+4}$$

$$-7 = 3x + 8 \Rightarrow x = -5$$

Similarly applying section formula for y co-ordinate,

$$2 = \frac{3y + 4(5)}{3+4}$$

$$14 = 3y + 20 \Rightarrow y = -2$$

Thus (x, y) is $(-5, -2)$.

Now $x^2 + y^2 = (-5)^2 + (-2)^2$

$$= 25 + 4 = 29$$

93. If the co-ordinates of points A and B are $(-2, -2)$ and $(2, -4)$ respectively, find the co-ordinates of P such that $AP = \frac{3}{7}AB$, where P lies on the line segment AB.

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Thus $x_2 = \dfrac{-\frac{1}{3}+(-3)}{2}$

$= \dfrac{-10}{6} = \dfrac{-5}{3}$

$y_2 = \dfrac{0+4}{2} = 2$

Thus co-ordinates of Q is $(-\frac{5}{2}, 2)$.

PRACTICE

- If the line segment joining the points $A(2,1)$ and $B(5,-8)$ is trisected at the points P and Q, find the coordinates P.
  [Board Term-2 OD Compt. 2017]
  Ans : $(3,-2)$.

- Find the co-ordinates of the points which divide the line segment joining the points $(5,7)$ and $(8,10)$ in 3 equal parts.
  [Board Term-2 OD Compt. 2017]
  Ans : $P(6,8)$, $Q(7,9)$

- Find the co-ordinates of the points of trisection of the line segment joining the points $(3,-2)$ and $(-3,-4)$.
  [Board Term-2 Foreign 2017]
  Ans : $(1,-\frac{8}{3})$ and $(-1,-\frac{10}{3})$

- Find the coordinates of the point which divide the line segment joining $A(2,-3)$ and $B(-4,-6)$ into three equal parts.
  [Board Term-2 SQP 2016]
  Ans : $(0,-4)$ and $(-2,-5)$

100. If (a,b) is the mid-point of the segment joining the points $A(10,-6)$ and $B(k,4)$ and $a-2b=18$, find the value of k and the distance AB.

Sol : [Board Term-2, 2012]

We have $A(10,-6)$ and $B(k,4)$.

If $P(a,b)$ is mid-point of AB, then we have

$(a,b) = \left(\dfrac{k+10}{2}, \dfrac{-6+4}{2}\right)$

$a = \dfrac{k+10}{2}$ and $b = -1$

From given condition we have

$a-2b = 18$

Substituting value $b=-1$ we obtain

$a+2 = 18 \Rightarrow a = 16$

$a = \dfrac{k+10}{2} = 16 \Rightarrow k = 22$

$P(a,b) = (16,1)$

$AB = \sqrt{(22-10)^2+(4+6)^2}$

$= 2\sqrt{61}$ units

101. Find the ratio in which the line $2x+3y-5=0$ divides the line segment joining the points $(8,-9)$ and $(2,1)$. Also find the co-ordinates of the point of division.

Sol : [Board Term-2, 2012]

Let a point $P(x,y)$ on line $2x+3y-5=0$ divides AB in the ratio $k:1$.

Now $x = \dfrac{2k+8}{k+1}$

and $y = \dfrac{k-9}{k+1}$

Substituting above value in line $2x+3y-5=0$ we have

$2\left(\dfrac{2k+8}{k+1}\right)+3\left(\dfrac{k-9}{k+1}\right)-5 = 0$

$4k+16+3k-27-5k-5 = 0$

$2k-16 = 0$

$k = 8$

Thus ratio is 8 : 1.

Substituting the value $k=8$ we get

$x = \left(\dfrac{2\times 8+8}{8+1}\right) = \dfrac{8}{3}$

$y = \left(\dfrac{8-9}{8+1}\right) = -\dfrac{1}{9}$

Thus $P(x,y) = \left(\dfrac{8}{3}, -\dfrac{1}{9}\right)$

102. Find the area of the rhombus of vertices $(3,0),(4,5),(-1,4)$ and $(-2,-1)$ taken in order.

Sol : [Board Term-2, 2012]

We have $A(3,0)$, $B(4,5)$, $C(-1,4)$, $D(-2,-1)$

Diagonal AC, $d_1 = \sqrt{(3+1)^2+(0-4)^2}$

$= \sqrt{16+16} = \sqrt{32}$

$= \sqrt{16\times 2} = 4\sqrt{2}$

Diagonal BD, $d_2 = \sqrt{(4+2)^2+(5+1)^2}$

$= \sqrt{36+36} = \sqrt{72}$

$= \sqrt{36\times 2} = 6\sqrt{2}$

Area of rhombus $= \dfrac{1}{2}\times d_1 \times d_2$

$= \dfrac{1}{2}4\sqrt{2}\times 6\sqrt{2}$

$= 24$ sq. unit.

NODIA

103. Find the ratio in which the line joining points $(a+b, b+a)$ and $(a-b, b-a)$ is divided by the point (a, b).

Sol : [Board Term-2, 2013]

Let $A(a+b, b+a), B(a-b, b-a)$ and $P(a, b)$ and P divides AB in $k:1$, then we have

$$\begin{aligned} a &= \frac{k(a-b)+1(a+b)}{k+1} \\ a(k+1) &= k(a-b)+a+b \\ ak+a &= ak-bk+a+b \\ bk &= b \\ k &= 1 \end{aligned}$$

Thus (a, b) divides $A(a+b, b+a)$ and $B(a-b, b-a)$ in 1:1 internally.

104. Find the co-ordinates of a point on the $x-$axis which is equidistant from the points $A(2,-5)$ and $B(-2, 9)$.

Sol : [Board Term-2 Delhi Compt. 2017]

Let the point P on the x axis be $(x, 0)$. Since it is equidistant from the given points $A(2, -5)$ and $B(-2, 9)$

$$\begin{aligned} PA &= PB \\ PA^2 &= PB^2 \\ (x-2)^2+[0-(-5)]^2 &= (x-(-2))^2+(0-9)^2 \\ x^2-4x+4+25 &= x^2+4x+4+81 \\ -4x+29 &= 4x+85 \\ x &= -\frac{56}{8} = -7 \end{aligned}$$

Hence the point on x axis is $(-7, 0)$

105. Show that ΔABC with vertices $A(-2, 0), B(0, 2)$ and $C(2, 0)$ is similar to ΔDEF with vertices $D(-4, 0), F(4, 0)$ and $E(0, 4)$.

Sol : [Board Term-2 Delhi 2017, Foreign 2017]

Using distance formula

$$\begin{aligned} AB &= \sqrt{(0+2)^2+(2-0)^2} = \sqrt{4+4} \\ &= 2\sqrt{2} \text{ units} \\ BC &= \sqrt{(2-0)^2+(0-2)^2} = \sqrt{4+4} \\ &= 2\sqrt{2} \text{ units} \\ CA &= \sqrt{(-2-2)^2+(0-0)^2} = \sqrt{16} \\ &= 4 \text{ units} \end{aligned}$$

and $$DE = \sqrt{(0+4)^2+(4-0)^2} = \sqrt{32}$$

$$\begin{aligned} &= 4\sqrt{2} \text{ units} \\ EF &= \sqrt{(4-0)^2+(0-4)^2} = \sqrt{32} \\ &= 4\sqrt{2} \text{ units} \\ FD &= \sqrt{(-4-4)^2+(0-0)^2} = \sqrt{64} \\ &= 8 \text{ units} \end{aligned}$$

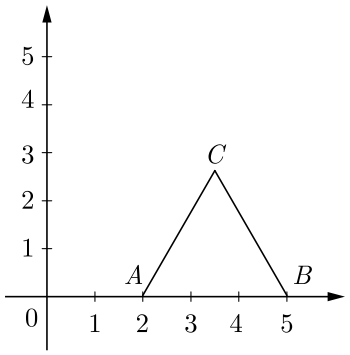
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{2\sqrt{2}}{4\sqrt{2}} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{4}{8} = \frac{1}{2}$$

Since ratio of the corresponding sides of two similar Δs is equal, we have

$$\Delta ABC \sim \Delta DEF$$ Hence Proved.

106. In the given figure ΔABC is an equilateral triangle of side 3 units. Find the co-ordinates of the other two vertices.



Sol : [Board Term-2 Foreign 2017]

The co-ordinates of B will be $(2+3, 0)$ or $.(5, 0)$

Let co-ordinates of C be (x, y). Since triangle is equilateral, we have

$$\begin{aligned} AC^2 &= BC^2 \\ (x-2)^2(y-0)^2 &= (x-5)^2+(y-0)^2 \\ x^2+4-4x+y^2 &= x^2+25-10x+y^2 \\ 6x &= 21 \\ x &= \frac{7}{2} \end{aligned}$$

and $$\begin{aligned} (x-2)^2+(y-0)^2 &= 9 \\ \left(\frac{7}{2}-2\right)^2+y^2 &= 9 \\ \frac{9}{4}+y^2 &= 9 \text{ or, } y^2 = 9-\frac{9}{4} \\ y^2 &= \frac{27}{4} = \frac{3\sqrt{3}}{2} \end{aligned}$$

Hence C is $\left(\frac{7}{2}, \frac{3\sqrt{3}}{2}\right)$.

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107. If the distances of $P(x,y)$ from $A(5,1)$ and $B(-1,5)$ are equal, then prove that $3x=2y$.

Sol : [Board Term-2 OD 2016]

Since $P(x,y)$ is equidistant from the given points $A(5,1)$ and $B(-1,5)$,

$$PA = PB$$
$$PA^2 = PB^2$$

Using distance formula,

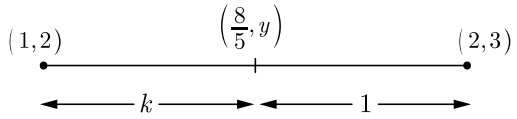
$$(5-x)^2+(1-y)^2 = (-1-x)^2+(5-y)^2$$
$$(5-x)^2+(1-y)^2 = (1+x)^2+(5-y)^2$$
$$25-10x+1-2y = 1+2x+25-10y$$
$$-10x-2y = 2x-10y$$
$$8y = 12x$$
$$3x = 2y$$ Hence proved.

FIVE MARKS QUESTIONS

108. Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points $(1,2)$ and $(2,3)$. Also, find the value of y.

Sol : [Board 2024 OD Standard]

Let the point $\left(\frac{8}{5},y\right)$ divides the line segment joining the points $(1,2)$ and $(2,3)$ in the ratio $k:1$.



x co-ordinate for section formula,

$$\frac{8}{5} = \frac{2\times k+1\times 1}{k+1}$$
$$\frac{8}{5} = \frac{2k+1}{k+1}$$
$$8k+8 = 10k+5$$
$$3 = 2k$$
$$k = \frac{3}{2}$$

Thus required ratio is 3 : 2.

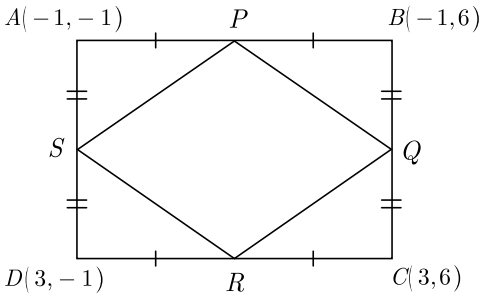
Now y co-ordinate for section formula,

$$y = \frac{k(3)+1(2)}{k+1}$$
$$= \frac{\frac{3}{2}\times 3+2}{\frac{3}{2}+1}$$
$$= \frac{\frac{9}{2}+2}{\frac{5}{2}}$$
$$= \frac{9+2\times 2}{5}$$
$$y = \frac{13}{5}$$

109. $ABCD$ is a rectangle formed by the points $A(-1,-1)$, $B(-1,6)$, $C(3,6)$ and $D(3,-1)$. P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively. Show that diagonals of the quadrilateral $PQRS$ bisect each other.

Sol : [Board 2024 OD Standard]

As per question we have shown the diagram below.



Now
$$P = \left(\frac{-1-1}{2}, \frac{-1+6}{2}\right)$$
$$= \left(-1, \frac{5}{2}\right)$$
$$Q = \left(\frac{3-1}{2}, \frac{6+6}{2}\right)$$
$$= (1,6)$$
$$R = \left(\frac{3+3}{2}, \frac{6-1}{2}\right)$$
$$= \left(3, \frac{5}{2}\right)$$
$$S = \left(\frac{-1+3}{2}, \frac{-1-1}{2}\right)$$
$$= (1,-1)$$

Mid point of diagonal PR,

$$= \left(\frac{-1+3}{2}, \frac{\frac{5}{2}+\frac{5}{2}}{2}\right)$$
$$= \left(1, \frac{5}{2}\right)$$

Mid point of diagonal QS,

$$= \left(\frac{1+1}{2}, \frac{6-1}{2}\right)$$
$$= \left(1, \frac{5}{2}\right)$$

Here, midpoint of PR is same as mid point of QS, and PR and QS bisect each other.

So, diagonals of quadrilateral $PQRS$ bisect each other.

NODIA

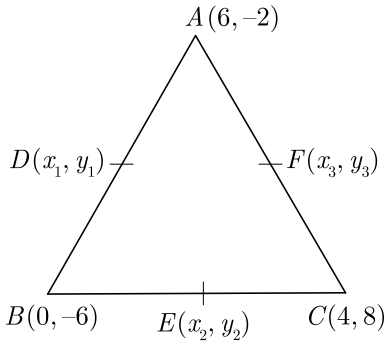
Mid-point of BD, $= \left(\frac{2+5}{2}, \frac{6-1}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$

Since the mid-point of diagonal AC and mid-point of diagonal BD is same and equal to $(\frac{7}{5}, \frac{5}{2})$. Hence they bisect each other.

97. The vertices of ΔABC are $A(6,-2), B(0,-6)$ and $C(4,8)$. Find the co-ordinates of mid-points of AB, BC and AC.

Sol : [Board Term-2, 2014]

Let mid-point of AB, BC and AC be $D(x_1, y_1)$, $E(x_2, y_2)$ and $F(x_2, y_3)$. As per question, triangle is shown below.



Using section formula, the co-ordinates of the points D, E, F are

For D, $x_1 = \frac{6+0}{2} = 3$

$y_1 = \frac{-2-6}{2} = -4$

For E, $x_2 = \frac{0+4}{2} = 2$

$y_2 = \frac{-6+8}{2} = 1$

For F, $x_3 = \frac{4+6}{2} = 5$

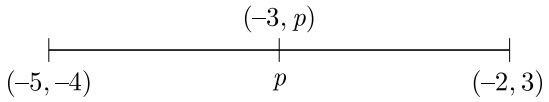
$y_3 = \frac{-2+8}{2} = 3$

The co-ordinates of the mid-points of AB, BC and AC are $D(3,-4)$, $E(2,1)$ and $F(5,3)$ respectively.

98. Find the ratio in which the point $(-3, p)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$.Hence find the value of p.

Sol : [Board Term-2, 2012]

As per question, line diagram is shown below.



Let $X(-3, p)$ divides the line joining of $A(-5,-4)$ and $B(-2,3)$ in the ratio $k:1$.

The co-ordinates of p are $\left[\frac{-2k-5}{k+1}, \frac{3k-4}{k+1}\right]$

But co-ordinates of P are $(-3, p)$. Therefore we get

$$\frac{-2k-5}{k+1} = -3 \Rightarrow k = 2$$

and $$\frac{3k-4}{k+1} = p$$

Substituting $k = 2$ gives

$$p = \frac{2}{3}$$

Hence ratio of division is 2:1 and $p = \frac{2}{3}$

PRACTICE

- Find the ratio in which the point $p(m, 6)$ divides the line segment joining the points $A(-4, 3)$ and $B(2, 8)$. Also find the value of m.

  [Board Term-2, 2012]

  Ans : $3 : 2, -\frac{2}{5}$

- In what ratio does the point $(\frac{24}{11}, y)$ divides the line segment joining the points $P(2,-2)$ and $Q(3,7)$? Also find the value of y.

  [Board Term-2 SQP 2012]

  Ans : $2 : 9, -\frac{4}{11}$

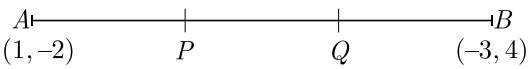
99. Find the co-ordinates of the points of trisection of the line segment joining the points $A(1,-2)$ and $B(-3,4)$.

Sol : [Board Term-2, 2012]

Let $P(x_1, y_1), Q(x_2, y_2)$ divides AB into 3 equal parts.

Thus P divides AB in the ratio of 1:2.

As per question, line diagram is shown below.



Now $x_1 = \frac{1(-3)+2(1)}{1+3}$

$= \frac{-3+2}{3} = \frac{-1}{3}$

$y_1 = \frac{1(4)+2(-2)}{1+2}$

$= \frac{4-4}{3} = 0$

Co-ordinates of P is $(-\frac{1}{3}, 0)$.

Here Q is mid-point of PB.

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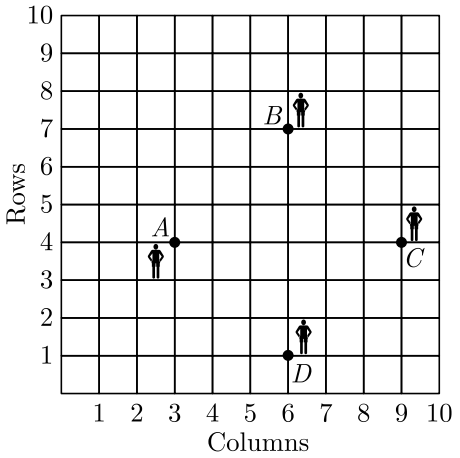
Distance travelled by Aditya is $\sqrt{68}$ units and Aditya travels more distance.

(ii) Since, D is mid-point of town A and town B

$$D = \left(\frac{1+4}{2}, \frac{7+2}{2}\right)$$
$$= \left(\frac{5}{2}, \frac{9}{2}\right)$$

112. In a classroom, 4 friends are seated at the points A, B, C, and D as shown in Figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, Don't you think $ABCD$ is a square? Chameli disagrees. Using distance formula, find which of them is correct.



Sol : [Board 2020 Delhi Basic]

Coordinates of points A, B, C, D are $A(3,4)$, $B(6,7)$, $C(9,4)$ and $D(6,1)$.

Distance formula,

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Now $AB = \sqrt{(3-6)^2+(4-7)^2}$
$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$BC = \sqrt{(6-9)^2+(7-4)^2}$
$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$CD = \sqrt{(9-6)^2+(4-1)^2}$
$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$DA = \sqrt{(6-3)^2+(1-4)^2}$
$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

Now $AC = \sqrt{(3-9)^2+(4-4)^2}$
$= \sqrt{36+0} = 6$ units

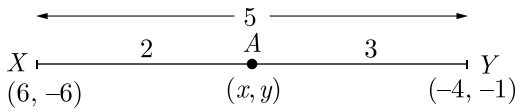
$DB = \sqrt{(6-6)^2+(1-7)^2}$
$= \sqrt{0+36} = 6$ units

Since, $AB = BC = CD = DA$ and $AC = DB$, $ABCD$ is a square and Champa is right.

113. Point A lies on the line segment XY joining $X(6,-6)$ and $Y(-4,-1)$ in such a way that $\frac{XA}{XY} = \frac{2}{5}$. If point A also lies on the line $3x+k(y+1)=0$, find the value of k.

Sol : [Board 2019 OD]

As per given information in question we have drawn the figure given below.



We use section formula for point $A(x,y)$.

Here, $m_1 = 2$, $m_2 = 3$, $x_1 = 6$, $x_2 = -4$, $y_1 = -6$ and $y_2 = -1$

Now $x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$
$= \frac{2\times(-4)+3(6)}{2+3}$
$= \frac{-8+18}{5}$
$= \frac{10}{5} = 2$

and $y = \frac{m_1y_2+m_2y_2}{m_1+m_2}$
$= \frac{2\times(-1)+3(-6)}{2+3}$
$= \frac{-2-18}{5}$
$= \frac{-20}{5} = -4$

Hence, coordinates of point A is $(2,-4)$.

Since point A also lies on the line $3x+k(y+1)=0$, its coordinates must satisfies this line.

Thus $3(2)+k(-4+1) = 0$
$6+(-3k) = 0$
$3k = 6 \Rightarrow k = 2$

Hence, value of k is 2.

114. Find the ratio in which the line $x-3y=0$ divides the line segment joining the points $(-2,-5)$ and $(6,3)$. Find the coordinates of the point of intersection.

Sol : [Board 2019 OD]

Let $k:1$ be the ratio in which line $x-3y=0$ divides the line segment at $p(x,\ y)$.

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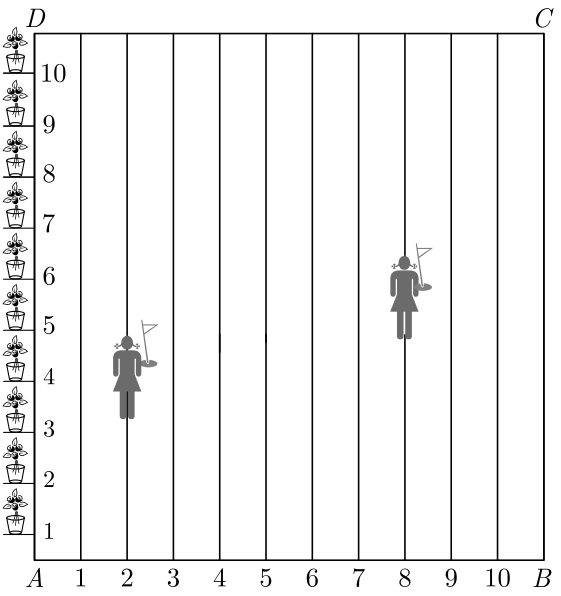
110. To conduct Sports Day activities, in your rectangular school ground $ABCD$, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in Figure. Niharika runs ¼th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th distance AD on the eighth line and posts a red flag.

(i) What is the distance between the two flags?

(ii) If Rashmi has to post a blue flag exactly half way between the line segment joining the two flags, where should she post the blue flag?



Sol : [Board 2020 Delhi Basic]

We assume A as origin (0, 0), AB as x-axis and AD as y-axis.

Niharika runs in the 2nd line with green flag and distance covered (parallel to AD),

$$= \frac{1}{4} \times 100 = 25 \text{ m}$$

Thus co-ordinates of green flag are (2, 25) and we label it as P i.e., $P(2, 25)$.

Similarly, Preet runs in the eighth line with red flag and distance covered (parallel to AD),

$$= \frac{1}{5} \times 100 = 20 \text{ m}$$

Co-ordinates of red flag are (8, 20) and we label it as Q i.e., $Q(8, 20)$

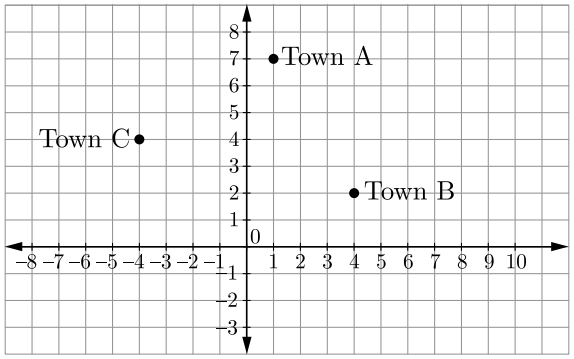
(i) Now, using distance formula, distance between green flag and red flag,

$$PQ = \sqrt{(8-2)^2 + (20-25)^2}$$
$$= \sqrt{6^2 + (-5)^2} = \sqrt{36+25}$$
$$= \sqrt{61} \text{ m}$$

(ii) Also, Rashmi has to post a blue flag the mid-point of PQ, therefore by using mid-point formula, we obtain $\left(\frac{2+8}{2}, \frac{25+20}{2}\right)$ i.e. $\left(5, \frac{45}{2}\right)$

Hence, the blue flag is in the fifth line, at a distance of $\frac{45}{2}$ i.e., 22.5 m along the direction parallel to AD.

111. Two friends Seema and Aditya work in the same office at Delhi. In the Christmas vacations, both decided to go to their hometown represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation answer the following questions:



(i) Who will travel more distance, Seema or Aditya, to reach to their hometown?

(ii) Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the points represented by Town A and Town B. Find the coordinates of the point represented by the point D.

Sol : [Board 2020 SQP Standard]

From the given figure, the coordinates of points A, B and C are $(1, 7), (4, 2)$ and $(-4, 4)$ respectively.

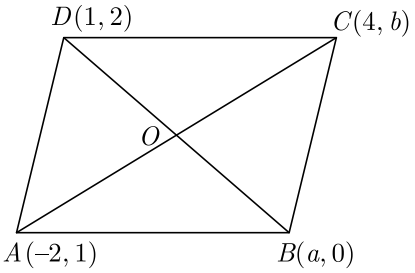
(i) Distance travelled by seema

$$CA = \sqrt{(-4-1)^2 + (4-7)^2}$$
$$= \sqrt{(-5)^2 + (-3)^2}$$
$$= \sqrt{25+9} = \sqrt{34} \text{ units}$$

Thus distance travelled by seema is $\sqrt{34}$ units.

Similarly, distance travelled by Aditya

$$CB = \sqrt{(4+4)^2 + (4-2)^2}$$
$$= \sqrt{8^2 + 2^2} = \sqrt{64+4}$$
$$= \sqrt{68} \text{ units}$$



Here $ABCD$ is a parallelogram and diagonals AC and BD bisect each other. Therefore mid point of BD is same as mid point of AC.

$$\left(\frac{a+1}{2}, \frac{2}{2}\right) = \left(\frac{-2+4}{2}, \frac{b+1}{2}\right)$$

$$\frac{a+1}{2} = 1 \Rightarrow a = 1$$

and $$\frac{b+1}{2} = 1 \Rightarrow b = 1$$

Now $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(1+2)^2 + (0-1)^2}$$

$$= \sqrt{9+1} = \sqrt{10} \text{ unit}$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4-1)^2 + (1-0)^2}$$

$$= \sqrt{9+1} = \sqrt{10} \text{ unit}$$

Since $ABCD$ is a parallelogram,

$$AB = CD = \sqrt{10} \text{ unit}$$

$$BC = AD = \sqrt{10} \text{ unit}$$

Therefore length of sides are $\sqrt{10}$ units each.

117. If $P(9a-2, -b)$ divides the line segment joining $A(3a+1, -3)$ and $B(8a, 5)$ in the ratio 3:1. find the values of a and b.

Sol : [Board Term-2 SQP 2016]

Using section formula we have

$$9a - 2 = \frac{3(8a) + 1 + (3a+1)}{3+1} \quad ...(1)$$

$$-b = \frac{3(5) + 1(-3)}{3+1} \quad ...(2)$$

Form (2) $$-b = \frac{15-3}{4} = 3$$

$$b = -3$$

From (1), $$9a - 2 = \frac{24a + 3a + 1}{4}$$

$$4(9a-2) = 27a + 1$$

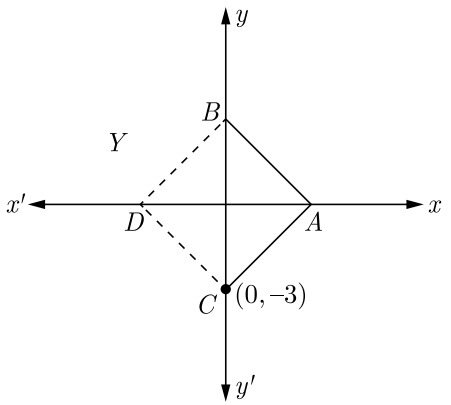
$$36a - 8 = 27a + 1$$

$$9a = 9 \Rightarrow a = 1$$

118. The base BC of an equilateral triangle ABC lies on y-axis. The co-ordinates of point C are $(0,3)$. The origin is the mid-point of the base. Find the co-ordinates of the point A and B. Also find the co-ordinates of another point D such that $BACD$ is a rhombus.

Sol : [Board Term-2 Foreign 2015]

As per question, diagram of rhombus is shown below.



Co-ordinates of point B are $(0,3)$.

Thus $BC = 6$ unit

Let the co-ordinates of point A be $(x,0)$.

Now $AB = \sqrt{x^2 + 9}$

Since $AB = BC$, thus we have

$$x^2 + 9 = 36$$

$$x^2 = 27 \Rightarrow x = \pm 3\sqrt{3}$$

Co-ordinates of point A is $(3\sqrt{3}, 0)$.

Since $ABCD$ is a rhombus,

$$AB = AC = CD = DB$$

Thus co-ordinate of point D is $(-3\sqrt{3}, 0)$.

119. The co-ordinates of vertices of ΔABC are $A(0,0)$, $B(0,2)$ and $C(2,0)$. Prove that ΔABC is an isosceles triangle. Also find its area.

Sol : [Board Term-2 Delhi 2014]

Using distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ we have

$$AB = \sqrt{(0-0)^2 + (0-2)^2} = \sqrt{4} = 2$$

$$AC = \sqrt{(0-2)^2 + (0-0)^2} = \sqrt{4} = 2$$

$$BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$$

Clearly, $AB = AC \neq BC$

Thus ΔABC is an isosceles triangle.

Now, $AB^2 + AC^2 = 2^2 + 2^2 = 4 + 4 = 8$

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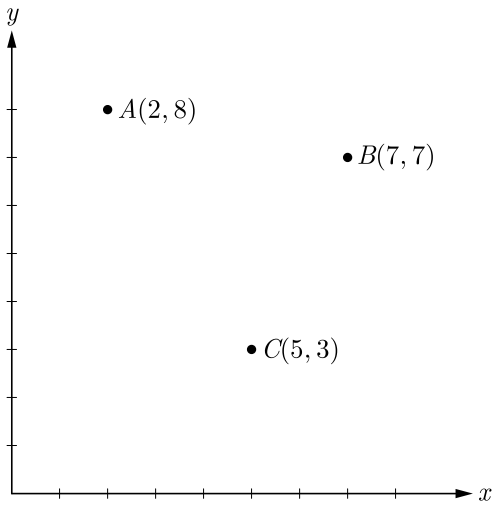
Three friends Amar, Bandhu and Chakradev lives in societies represented by the points A, B and C respectively. They all work in offices located in a same building represented by the point O . Since they all go to same building everyday, they decided to do carpooling to save money on petrol. Based on the above information, answer the following questions.



Sol :

(i) Which society is nearest to the office?

(ii) What is the distance between A and C?

(iii) Find the least distance between AB, OA and BC ?

(iv) Find the best route to go to the office ?

(v) If Bandhu and Chakradev planned to meet at a club situated at the mid-point of the line joining the points B and C, find the coordinates of this point.

Sol :

Coordinates of A, B and C are (2, 8), (7, 7) and (5, 3) respectively. Coordinates of point O are (0, 0).

(i) From the graph, it is clear that society C is nearest to the office.

(ii) Distance between A and C,

$$= \sqrt{(5-2)^2 + (3-8)^2}$$
$$= \sqrt{3^2 + 5^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

(iii) By distance formula we have,

$$AB = \sqrt{(7-2)^2 + (7-8)^2} = \sqrt{26} \text{ units}$$
$$OA = \sqrt{2^2 + 8^2} = 2\sqrt{17} \text{ units}$$
$$BC = \sqrt{(7-5)^2 + (7-3)^2} = 2\sqrt{5} \text{ units}$$

Distance BC is the least.

(iv) From graph it is clear that $ABCO$ is best route to go to office.

(v) Let this point is represented by D. Coordinates of D,

$$= \left(\frac{5+7}{2}, \frac{3+7}{2}\right) = (6, 5)$$

136. To conduct sports day activities, in a rectangular shaped school ground $ABCD$, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AB, as shown in figure. Nishtha runs $\frac{1}{4}$th the distance AB on the 2nd line and posts a green flag. Suman runs $\frac{1}{5}$th the distance AB on the 8th line and posts a red flag.

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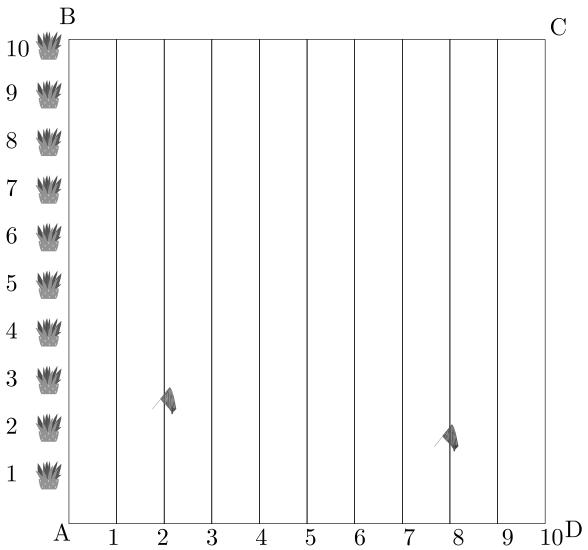

(i) What is the position of green flag ?

(ii) What is the position of red flag ?

(iii) What is the distance between both the flags?

(iv) What is the distance of red flag from point A ?

(v) If Rakhi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

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$= \sqrt{(4)^2+(3)^2} = 5$ units

Diagonal, $AC = \sqrt{(5-4)^2+(-1-0)^2}$

$= \sqrt{1^2+1^2} = \sqrt{2}$ units

Diagonal $BD = \sqrt{(8-1)^2+(3+4)^2}$

$= \sqrt{(7)^2+(7)^2} = 7\sqrt{2}$ units

As the length of all the sides are equal but the length of the diagonals are not equal. Thus $ABCD$ is not square but a rhombus.

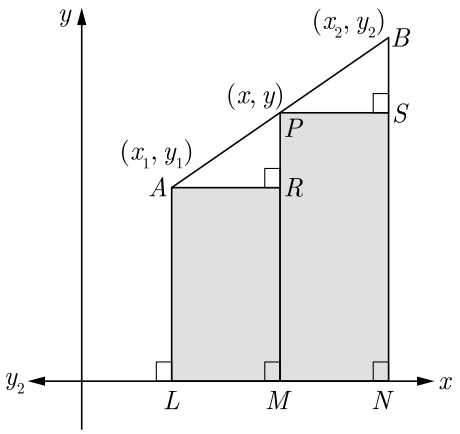
124. (i) Derive section formula.

(ii) In what ratio does $(-4, 6)$ divides the line segment joining the point $A(-6, 4)$ and $B(3, -8)$

Sol : [Board Term-2 Delhi 2014]

(i) Section Formula : Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points. Let $P(x, y)$ be a point on line, joining A and B, such that P divides it in the ratio $m_1 : m_2$.

Now $(x, y) = \left(\frac{m_2x_1 + m_1x_2}{m_1+m_2}, \frac{m_2y_1 + m_1y_2}{m_1+m_2}\right)$



Proof : Let AB be a line segment joining the points.

$A(x_1, y_1)$, $B(x_2, y_2)$.

Let P divides AB in the ratio $m_1 : m_2$. Let P have co-ordinates (x, y).

Draw $AL, PM, PN, \perp$ to x-axis

It is clear form figure, that

$AR = LM = OM - OL = x - x_1$

$PR = PM - RM = y - y_1$.

also, $PS = ON - OM = x_2 - x$

$BS = BN - SN = y_2 - y$

Now $\Delta APR \sim \Delta PBS$ [AAA]

Thus $\frac{AR}{PS} = \frac{PR}{BS} = \frac{AP}{PB}$

and $\frac{AR}{PS} = \frac{AP}{PB}$

$\frac{x - x_1}{x_2 - x} = \frac{m_1}{m_2}$

$m_2x - m_2x_1 = m_1x_2 - m_1x$

$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$

Now $\frac{PR}{BS} = \frac{AP}{PB}$

$\frac{y - y_2}{y_2 - y} = \frac{m_1}{m_2}$

$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$

Thus co-ordinates of P are $\left(\frac{m_2x_1 + m_1x_2}{m_1+m_2}, \frac{m_2y_1 + m_1y_2}{m_1+m_2}\right)$

(ii) Assume that $(-4, 6)$ divides the line segment joining the point $A(-6, 4)$ and $B(3, -8)$ in ratio $k : 1$

Using section formula for x co-ordinate we have

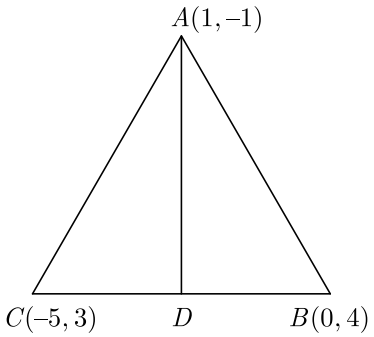
$-4 = \frac{k(3) - 6}{k+1}$

$-4k - 4 = 3k - 6 \Rightarrow k = \frac{2}{7}$

125. $(1, -1)$,$(0, 4)$ and $(-5, 3)$ are vertices of a triangle. Check whether it is a scalene triangle, isosceles triangle or an equilateral triangle. Also, find the length of its median joining the vertex $(1, -1)$ the mid-point of the opposite side.

Sol : [Board Term-2, 2015]

Let the vertices of ΔABC be $A(1, -1)$, $B(0, 4)$ and $C(-5, 3)$. Let $D(x, y)$ be mid point of BC. Now the triangle is shown below.



Using distance formula, we get

$AB = \sqrt{(1-0)^2+(-1-4)^2}$

$= \sqrt{1+5^2} = \sqrt{26}$

$BC = \sqrt{(-5-0)^2+(3-4)^2}$

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$$= \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(-5-1)^2+(3+1)^2}$$

$$= \sqrt{36+16} = 2\sqrt{13}$$

Since $AB = BC \neq AC$, triangle ΔABC is isosceles.

Now, using mid-section formula, the co-ordinates of mid-point of BC are

$$x = \frac{-5+0}{2} = -\frac{5}{2}$$

$$y = \frac{3+4}{2} = \frac{7}{2}$$

$$D(x,y) = \left(-\frac{5}{2}, \frac{7}{2}\right)$$

Length of median AD,

$$AD = \sqrt{\left(\frac{-5}{2}-1\right)^2+\left(\frac{7}{2}+1\right)^2}$$

$$= \sqrt{\left(\frac{-7}{2}\right)^2+\left(\frac{9}{2}\right)^2}$$

$$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2} \quad \text{square unit}$$

Thus length of median AD is $\frac{\sqrt{130}}{2}$ units.

126. Point $(-1, y)$ and $B(5,7)$ lie on a circle with centre $O(2,-3y)$. Find the values of y. Hence find the radius of the circle.

Sol : [Board Term-2 Delhi 2014]

Since, $A(-1, y)$ and $B(5,7)$ lie on a circle with centre $O(2,-3y)$, OA and OB are the radius of circle and are equal. Thus

$$OA = OB$$

$$\sqrt{(-1-2)^2+(y+3y)^2} = \sqrt{(5-2)^2+(7+3y)^2}$$

$$9+16y^2 = 9y^2+42y+58$$

$$y^2-6y-7 = 0$$

$$(y+1)(y-7) = 0$$

$$y = -1, 7$$

When $y = -1$, centre is $O(2,-3y) = (2,3)$ and radius

$$OB = \left|\sqrt{(5-2)^2+(7-3)^2}\right|$$

$$= \sqrt{9+16} = 5 \text{ unit}$$

When $y = 7$, centre is $O(2,-3y) = (2,-21)$ and radius

$$OB = \left|\sqrt{(2-5)^2+(-21-7)^2}\right|$$

$$= \left|\sqrt{9+784}\right| = \sqrt{793} \text{ unit}$$

COMPETENCEY BASED QUESTIONS

127. BEST : The Brihanmumbai Electricity Supply and Transport (BEST) is a civic transport and electricity provider public body based in Mumbai. BEST operates one of India's largest fleets of buses. The bus transport service covers the entire city and also extends its operations outside city limits into neighbouring urban areas. In addition to buses, it also operates a ferry service in the northern reaches of the city.

There are two routes to travel from source A to destination B by using BEST bus service. First bus reaches at B via point C and second bus reaches from A to B directly. If coordinates of A, B and C are $(-2,-3)$, $(2,3)$ and $(3,2)$ respectively, then by which bus do you want to travel from A to B using BEST services. (Assume that both buses have same speed).

Sol :

Here the coordinates of A, B and C are $(-2,-3)$, $(2,3)$ and $(3,2)$ respectively.

Now, $AB = \sqrt{(-2-2)^2+(-3-3)^2}$

$$= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$$

$$AC = \sqrt{(-2-3)^2+(-3-2)^2}$$

$$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

$$BC = \sqrt{(2-3)^2+(3-2)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

Distance travelled by first bus

$$= AC + BC$$

$$= 5\sqrt{2}+\sqrt{2} = 6\sqrt{2}$$

Distance travelled by second bus

$$= AB = 2\sqrt{13}$$

But $6\sqrt{2} > 2\sqrt{13}$

Hence, by second bus we would like to travel.

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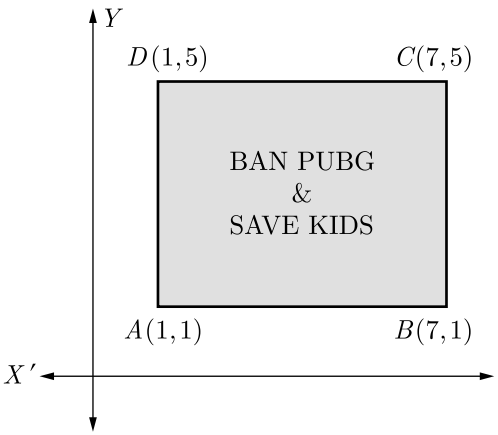
128. Use of mobile screen for long hours makes your eye sight weak and give you headaches. Children who are addicted to play "PUBG" can get easily stressed out. To raise social awareness about ill effects of playing PUBG, a school decided to start 'BAN PUBG' campaign, in which students are asked to prepare campaign board in the shape of a rectangle. One such campaign board made by class X student of the school is shown in the figure.



Based on the above information, answer the following questions :

(i) Find the coordinates of the point of intersection of diagonals AC and BD.

(ii) Find the length of the diagonal AC.

(iii) Find the area of the campaign Board $ABCD$.

(iv) Find the ratio of the length of side AB to the length of the diagonal AC.

Sol :

We have a rectangle $ABCD$ where $A(1,1)$, $B(7,1)$, $C(7,5)$ and $D(1,5)$.

(i) Coordinates of the point of intersection of diagonals AC and BD

$$(x, y) = \left(\frac{7+1}{2}, \frac{1+5}{2}\right)$$
$$= (4,3)$$

(ii) Length of diagonal AC,

$$l = \sqrt{(7-1)^2 + (5-1)^2}$$
$$= \sqrt{16+36}$$
$$= \sqrt{52}$$
$$= \sqrt{13} \text{ units}$$

(iii) Area of the campaign Board

$$AB = 6 \text{ units} = CD$$
$$BC = 4 \text{ units} = AD$$

Area of campaign board $ABCD$,

$$\text{Area} = 6 \times 4$$
$$= 24 \text{ sq. units}$$

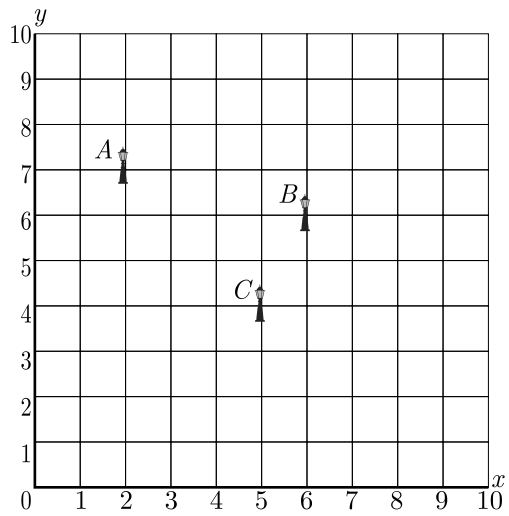
(iv) Ratio of the length of side AB to the length of the diagonal AC,

$$AC = \frac{AB}{AC}$$
$$= \frac{6}{\sqrt{(5-1)^2 + (7-1)^2}}$$
$$= \frac{6}{\sqrt{16+36}} = \frac{6}{\sqrt{52}}$$
$$= \frac{6}{2\sqrt{13}} = \frac{3\sqrt{13}}{13}$$
$$= 3\sqrt{13}:13$$

129. Resident Welfare Association (RWA) of a Gulmohar Society in Delhi have installed three electric poles A, B and C in a society's common park. Despite these three poles, some parts of the park are still in dark.

So, RWA decides to have one more electric pole D in the park.

The park can be modelled as a coordinate systems given below.



On the basis of the above information, answer any four of the following questions:

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points A and B such that $AD : DB = 4 : 3$. What are the coordinates of point D ?

Sol :

(i) It may be seen easily from figure that coordinates of point A are $(-2, 2)$.

(ii) $OA = \sqrt{(0+2)^2 + (0-2)^2} = 2\sqrt{2}$

(iii) It may be seen easily from figure that coordinates of point A are $(-1, -2)$.

$$AB = \sqrt{(-2+1)^2 + (2+2)^2}$$
$$= \sqrt{1+4^2} = \sqrt{17}$$

(iv) It may be seen easily from figure that coordinates of point A are $(0, 3)$.

$$BC = \sqrt{(-1-3)^2 + (-2-0)^2}$$
$$= \sqrt{4^2+4} = 2\sqrt{5}$$

(v) We have $A(-2, 2)$ and $B(-1, -2)$

$$\frac{m}{n} = \frac{4}{3}$$

$$x = \frac{mx_2 + nx_1}{m+n}$$
$$= \frac{-1(4) + 3(-2)}{4+3} = \frac{-10}{7}$$

$$y = \frac{my_2 + hy_1}{m+n}$$
$$= \frac{-2(4) + 3(2)}{4+3} = \frac{-2}{7}$$

133. Satellite Images : Satellite images are images of Earth collected by imaging satellites operated by governments and businesses around the world. Satellite imaging companies sell images by licensing them to governments and businesses such as Apple Maps and Google Maps. It should not be confused for astronomy images collected by space telescope.

Barun lives in Jaipur in Vaishali. Satellite image of his colony is shown in given figure. In this view, his house is pointed out by a flag, which is situated at the point of intersection of x and y - axes. If he goes 2 cm east and 3 cm north from the house, then he reaches to a grocery store, If he goes 4 cm west and 6 cm south from the house, then he reaches to his office. If he goes 6 cm east and 8 cm south from the house, then he reaches to a food court. If he goes 6 cm west and 8 cm north from the house, he reaches to a his kid's school.

Based on the above information, answer the following questions.

(i) Find the distance between grocery store and food court.

(ii) Find the distance of the school from the house.

(iii) If the grocery store and office lie on a line, what is the ratio of distance of house from grocery store to that from office ?

(iv) Find the ratio of distances of house from school to food court.

(v) What shape is formed by the coordinates of positions of school, grocery store, food court and office?

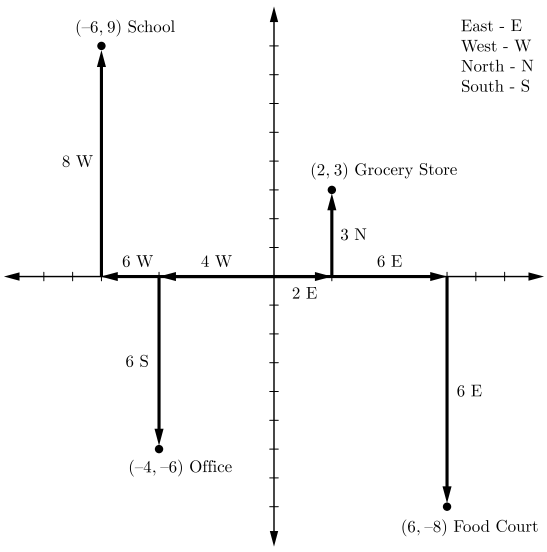
Sol :

Consider the house is at origin $(0, 0)$, then coordinates of grocery store, office, food court and school are respectively $(2, 3)$, $(-4, -6)$, $(6, -8)$ and $(-6, 8)$.

(i) Grocery store is at $(2, 3)$ and food court is at $(6, -8)$.

Distance between grocery store and food court,

$$= \sqrt{(6-2)^2 + (-8-3)^2}$$
$$= \sqrt{4^2 + 11^2}$$
$$= \sqrt{16 + 121}$$
$$= \sqrt{137} \text{ cm}$$



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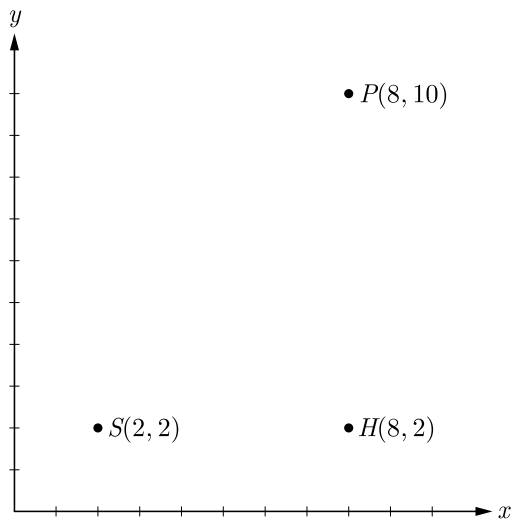
age, people often lose motor functions. Performing day to day activities becomes a daunting task. In an old age home, the association helps with daily activities. One of the factors that make old age homes attractive to elders is the companionship. The steady security in an old age home gives them protection from intruders and helps them live a safe and secure life.

A school decides to take the students for school picnic to an amusement park. However, the students have to first visit an old age home and then move towards the park. If the point S, P and H represent the school, the park and the old age home respectively, find the additional distance covered by the students had they travelled directly from school to the park. ? (Note : All distance are in km)



Sol :

Using the distance formula, we get

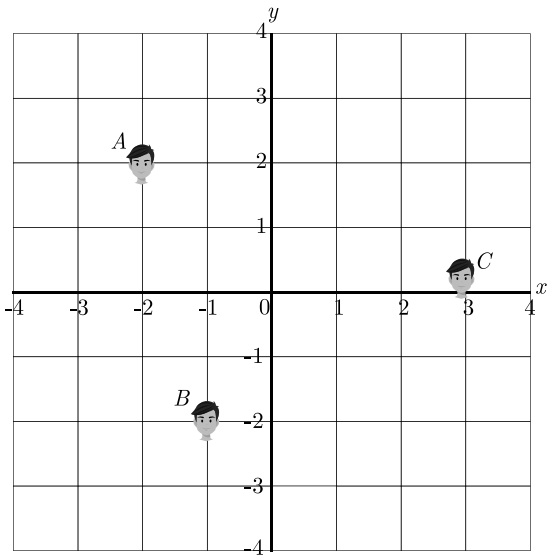
$$SP = \sqrt{(8-2)^2 + (10-2)^2}$$
$$= \sqrt{(6)^2 + (8)^2}$$
$$= \sqrt{36+64} = \sqrt{100}$$
$$SP = 10\,\text{km}$$
$$SH = \sqrt{(8-2)^2 + (2-2)^2}$$
$$= \sqrt{(6)^2 + (0)^2}$$
$$= \sqrt{36+0} = \sqrt{36}$$
$$SH = 6\,\text{km}$$
$$HP = \sqrt{(8-8)^2 + (10-2)^2}$$
$$\sqrt{(0)^2 + (8)^2}$$
$$= \sqrt{0+64} = \sqrt{64}$$
$$HP = 8\,\text{km}$$

Distance between school and park $= SP = 10\,\text{km}$

Distance between school and park via the old age home $= SH + HP = 6 + 8 = 14\,\text{km}$

Additional distance travelled while going to the old age home $= 14 - 10 = 4\,\text{km}$.

132. Ajay, Bhigu and Colin are fast friend since childhood. They always want to sit in a row in the classroom. But teacher doesn't allow them and rotate the seats row-wise everyday. Bhigu is very good in maths and he does distance calculation everyday. He consider the centre of class as origin and marks their position on a paper in a co-ordinate system. One day Bhigu make the following diagram of their seating position.



(i) What are the coordinates of point A?

(ii) What is the distance of point A from origin ?

(iii) What is the distance between A and B ?

(iv) What is the distance between B and C ?

(v) A point D lies on the line segment between

NODIA

also, $BC^2 = (2\sqrt{2})^2 = 8$

$AB^2 + AC^2 = BC^2$

Thus ΔABC is an isosceles right angled triangle.

Now, area of ΔABC

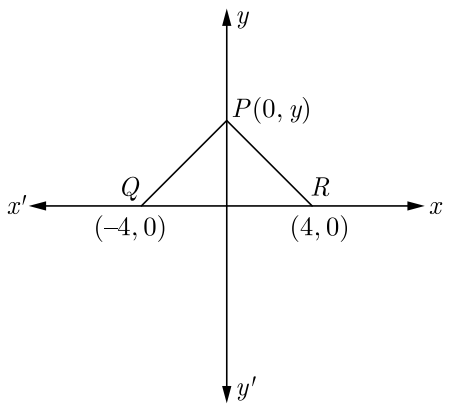
$$\Delta_{ABC} = \frac{1}{2}\text{base} \times \text{height}$$

$$= \frac{1}{2} \times 2 \times 2 = 2 \text{ sq. units.}$$

120. The base QR of an equilateral triangle PQR lies on x-axis. The co-ordinates of point Q are $(-4,0)$ and the origin is the mid-point of the base. find the co-ordinates of the point P and R.

Sol : [Board Term-2 Delhi 2017, Foreign 2015]

As per question, line diagram is shown below.



Co-ordinates of point R is $(4,0)$.

Thus $QR = 8$ units

Let the co-ordinates of point P be $(0,y)$

Since $PQ = QR$

$$(-4-0)^2 + (0-y)^2 = 64$$

$$16 + y^2 = 64$$

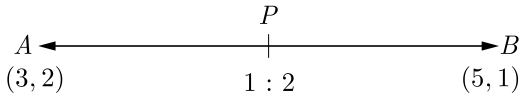
$$y = \pm 4\sqrt{3}$$

Coordinates of P are $(0,4\sqrt{3})$ or $(0,-4\sqrt{3})$

121. The line segment joining the points $A(3,2)$ and $B(5,1)$ is divided at the point P in the ratio 1:2 and P lies on the line $3x-18y+k=0$. Find the value of k.

Sol : [Board Term-2 Delhi 2012]

Let co-ordinates of P be (x_1, y_1) and it divides line AB in the ratio 1:2.



$$x_1 = \frac{mx_2 + nx_1}{m+n} = \frac{1 \times 5 + 2 \times 3}{1+2} = \frac{11}{3}$$

$$y_2 = \frac{my_2 + ny_1}{m+n} = \frac{1 \times 2 + 2 \times 2}{1+2} = \frac{5}{3}$$

Since point $P(x_1, y_1)$ lies on line ,$3x-18y+k=0$, so co-ordinates of P must satisfy the equation of line.

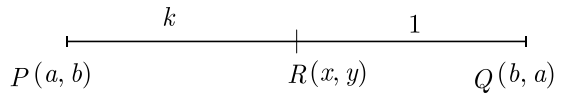
$$3 \times \frac{11}{3} - 18 \times \frac{5}{3} + k = 0$$

$$k = 19$$

122. If $R(x,y)$ is a point on the line segment joining the points $P(a,b)$ and $Q(b,a)$, then prove that $x+y=a+b$.

Sol : [Board Term-2, OD 2012]

As per question line is shown below.



Let point $R(x,y)$ divides the line joining P and Q in the ratio $k:1$, then we have

$$x = \frac{kb+a}{k+1}$$

and $$y = \frac{ka+b}{k+1}$$

Adding, $$x+y = \frac{kb+a+ka+b}{k+1}$$

$$= \frac{k(a+b)+(a+b)}{k+1}$$

$$= \frac{(k+1)(a+b)}{k+1} = a+b$$

$x+y = a+b$ Hence Proved

123. The vertices of quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)$ and $D(1,-4)$. Prove that $ABCD$ is a rhombus.

Sol : [Board Term-2 Delhi 2015]

The vertices of the quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)$ $D(1,-4)$.

Now $AB = \sqrt{(8-5)^2+(3+1)^2}$

$$= \sqrt{3^2+4^2} = 5 \text{ units}$$

$BC = \sqrt{(8-4)^2+(3-0)^2}$

$$= \sqrt{4^2+3^2} = 5 \text{ units}$$

$CD = \sqrt{(4-1)^2+(0+4)^2}$

$$= \sqrt{(3)^2+(4)^2} = 5 \text{ units}$$

$AD = \sqrt{(5-1)^2+(-1+4)^2}$

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(ii) House is at (0, 0) and school is at $(6, -8)$.
Distance of the school from the house,

$$= \sqrt{(-6)^2 + 8^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100} = 10 \text{ cm}$$

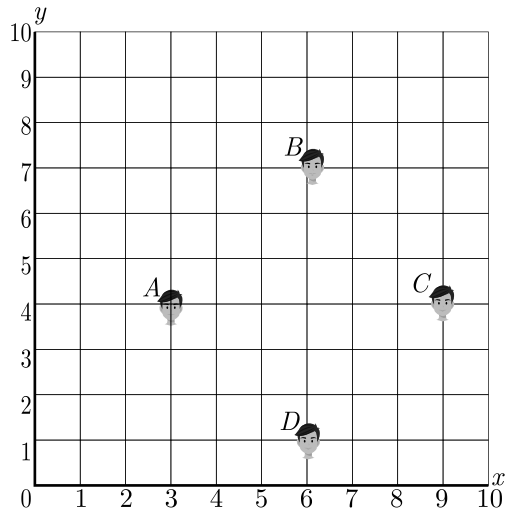
(iii) Let O divides EG in the ratio $k : 1$, then

$$0 = \frac{2k-4}{k+1}$$
$$2k = 4$$
$$k = 2$$

Thus, O divides EG in the ratio $2 : 1$
Hence, required ratio $= OG : OE$ i.e., $1 : 2$
(iv) Since, (0, 0) is the mid-point of $(-6, 8)$ and $(6, -8)$, therefore both school and food court are at equal distances from the house. Hence, required ratio is $1 : 1$.
(v) Mid-points of grocery store and office is $\left(\frac{2-4}{2}, \frac{3-6}{2}\right)$, i.e., $\left(-1, \frac{-3}{2}\right)$
Since mid-point are not same, diagonals does not bisect each other. Hence, they from a quadrilateral.

134. Morning assembly is an integral part of the school's schedule. Almost all the schools conduct morning assemblies which include prayers, information of latest happenings, inspiring thoughts, speech, national anthem, etc. A good school is always particular about their morning assembly schedule. Morning assembly is important for a child's development. It is essential to understand that morning assembly is not just about standing in long queues and singing prayers or national anthem, but it's something beyond just prayers. All the activities carried out in morning assembly by the school staff and students have a great influence in every point of life. The positive effects of attending school assemblies can be felt throughout life.

Have you noticed that in school assembly you always stand in row and column and this make a coordinate system. Suppose a school have 100 students and they all assemble in prayer in 10 rows as given below.



Here A, B, C and D are four friend Amar, Bharat, Colin and Dravid.

(i) What is the distance between A and B ?

(ii) What is the distance between C and D ?

(iii) What is the distance between A and C ?

(iv) What is the distance between D and B ?

Sol :

(i) We have $A(3, 4)$ and $B(6, 7)$.

$$AB = \sqrt{(6-3)^2 + (7-4)^2} = 2\sqrt{3}$$

(ii) We have $C(9, 4)$ and $D(6, 1)$.

$$CD = \sqrt{(9-6)^2 + (4-1)^2} = 2\sqrt{3}$$

(iii) We have $A(3, 4)$ and $C(9, 4)$

$$AC = \sqrt{(3-9)^2 + (4-4)^2} = 6$$

(iv) We have $B(6, 7)$ and $D(6, 1)$.

$$BD = \sqrt{(6-6)^2 + (7-1)^2} = 6$$

135. Carpooling : It is the sharing of car journeys so that more than one person travels in a car, and prevents the need for others to have to drive to a location themselves. By having more people using one vehicle, carpooling reduces each person's travel costs such as: fuel costs, tolls, and the stress of driving. Carpooling is also a more environmentally friendly and sustainable way to travel as sharing journeys reduces air pollution, carbon emissions, traffic congestion on the roads, and the need for parking spaces.

NODIA

(i) What is the position of the pole C?

(ii) What is the distance of the pole B from the corner O of the park ?

(iii) Find the position of the fourth pole D so that four points A, B C and D form a parallelogram .

(iv) What is the distance between poles A and C ?

(v) What is the distance between poles B and ?

Sol :

(i) From the given digram we can easily get that position of the pole C (5, 4).

(ii) Coordinates of B are (6, 6).

$$\text{Distance from origin} = \sqrt{(6-0)^2+(6-0)^2}$$
$$= \sqrt{36+36} = 6\sqrt{2} \text{ units}$$

(iii) If $ABCD$ is a parallelogram, the diagonals bisects each other. Here AC and BD are diagonals.

$$\text{Mid-point of } AC = \left(\frac{2+5}{2}, \frac{7+4}{2}\right) = (3.5, 5.5)$$

Now, mid-point of diagonal, BD will be $(3.5, 5.5)$ also. Let, the coordinates of D be (x, y)

$$\text{Now} \quad \frac{6+x}{2} = 3.5$$

$$\text{and} \quad \frac{6+y}{2} = 5.5$$

$$x = 1 \text{ and } y = 5$$

(iv) Coordinates of A are (2, 7) and coordinates of C are (5, 4).

Distance between pole A and C ,

$$AC = \sqrt{(5-2)^2+(4-7)^2}$$
$$= \sqrt{9+9} = 3\sqrt{2} \text{ units}$$

(v) Coordinates of B are (6, 6) and coordinates of D are (1, 5).

Distance between pole B and D ,

$$BD = \sqrt{(6-1)^2+(6-5)^2}$$
$$= \sqrt{5^2+1^2}$$
$$= \sqrt{25+1} = \sqrt{26} \text{ units}$$

130. Birla Science Museum is the first Science and Technology Museum of the country, established in 1954. It houses exhibits and displays on science and technology where visitors can interact with the exhibits to make the understanding of science and technology easy and entertaining.

Birla Science Museum has set aside a children's room having planets and stars painted on the ceiling. Suppose an imaginary coordinate system is placed on the ceiling in the room with the centre of the ceiling at (0, 0,). Three particular stars are located at $S(-10, 5)$, $T(3, -8)$ and $R(-7, -4)$, where the coordinates represent the distance in feet from the center of the room.

(i) Find the distance between each pair of stars.

(ii) Which star is farthest from the center of the room?

Sol :

(i) Distance formula is given by

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Thus

$$ST = \sqrt{\left[3-(-10)\right]^2+(-8-5)^2}$$
$$= \sqrt{13^2+(-13)^2}$$
$$= \sqrt{338} = 13\sqrt{2}$$

$$TR = \sqrt{(-7-3)^2+\left[-4-(-8)\right]^2}$$
$$= \sqrt{(-10)^2+(4)^2}$$
$$= \sqrt{116} = 4\sqrt{29}$$

$$SR = \sqrt{\left[-7-(-10)\right]^2+(-4-5)^2}$$
$$= \sqrt{(3)^2+(-9)^2}$$
$$= \sqrt{90} = 3\sqrt{10}$$

So, $ST = 13\sqrt{2}$ feet

$TR = 4\sqrt{29}$ feet

and $SR = 3\sqrt{10}$ feet

(ii) Let $O = (0, 0)$.

$$OS = \sqrt{(-10-0)^2+(5-0)^2} = \sqrt{125}$$

$$OT = \sqrt{(3-0)^2+(-8-0)^2} = \sqrt{73}$$

$$OR = \sqrt{(-7-0)^2+(-4-0)^2} = \sqrt{65}$$

The star located at S is farthest from the center of the room.

131. Old Age Home : An old age homes is a place where old people live and are cared for when they are too old to look after themselves. With advancing

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Sol :

(i) $\frac{1}{4}$ th of the AD corresponds to $y = \frac{100}{4} = 25$ and 2nd line corresponds to $x = 2$. Thus coordinates of green flag point are $(2, 25)$.

(ii) $\frac{1}{5}$ th of the AD corresponds to $y = \frac{100}{5} = 20$ and 8th line corresponds to $x = 8$. Thus coordinates of red point are $(8, 20)$.

(iii) $d = \sqrt{(8-2)^2 + (20-25)^2}$

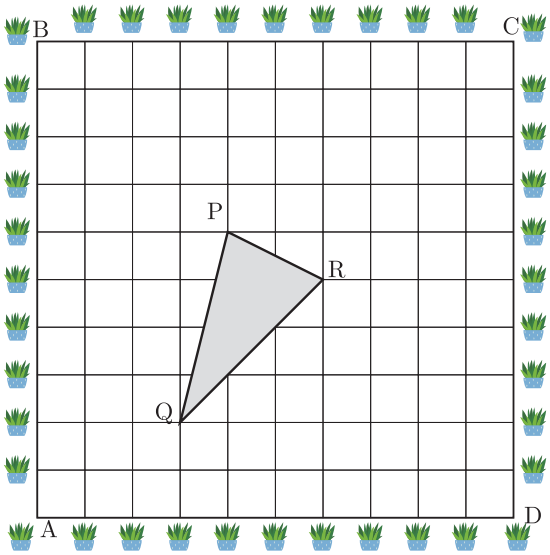
$= \sqrt{6^2 + 5^2} = \sqrt{61}$

(iv) $d = \sqrt{8^2 + 20^2} = 4\sqrt{29}$

(v) Mid point of green flag and red flag

$= \left(\frac{2+8}{2}, \frac{25+20}{2}\right) = (5, 22.5)$

137. A garden is in the shape of rectangle. Gardener grew sapling of Ashoka tree on the boundary of garden at the distance of 1 meter from each other. He want to decorate the garden with rose plants. He choose triangular region inside the park to grow rose plants. On the above situation, gardener took help from the students of class 10th. They made a chart for it which looks as the above figure.



(i) If A is taken as origin, What are the coordinates of triangle PQR ?

(ii) If C is taken as origin, what is the co-ordinate of point P ?

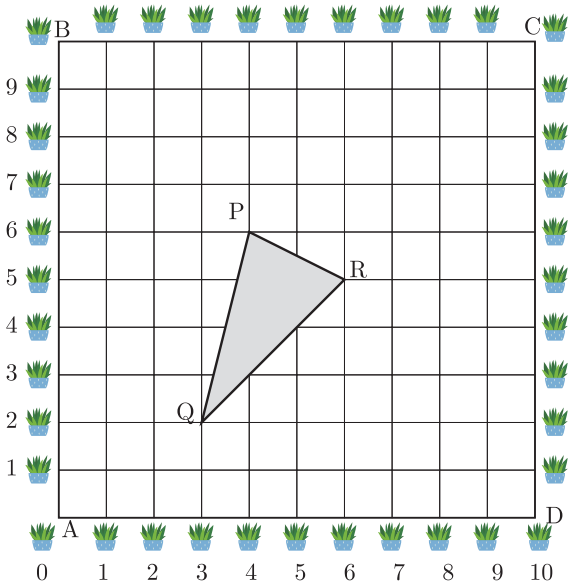
(iii) If B is taken as origin, what are the co-ordinate of P ?

(iv) What is distance between P and Q if origin is taken A?

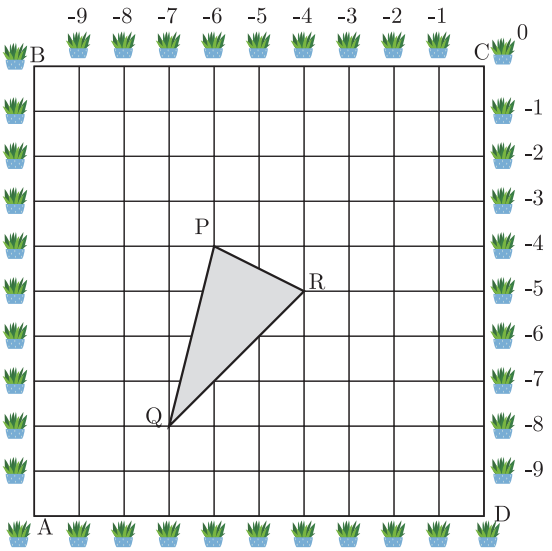
(v) What is distance between P and Q if origin is taken B?

Sol :

(i) In following figure we have shown the co-ordinate taking A as origin.



(ii) In following figure we have shown the co-ordinate taking C as origin.



(iii) In following figure we have shown the co-ordinate taking B as origin.

NODIA

$$\sec 5A = \sec(60^\circ - A)$$
$$5A = 60^\circ - A$$
$$6A = 60^\circ \Rightarrow A = 10^\circ$$

46. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $A > B$, then the value of A is ..... .

Sol : [Board 2020 Delhi Basic]

We have $\tan(A+B) = \sqrt{3}$
$= \tan 60^\circ$
Hence, $A + B = 60^\circ$ ...(1)
Again, $\tan(A-B) = \frac{1}{\sqrt{3}}$
$= \tan 30^\circ$
$A - B = 30^\circ$ ...(2)
Adding equation (1) and (2) we get
$2A = 90^\circ \Rightarrow A = 45^\circ$

47. The value of $\left(\sin^2\theta + \frac{1}{1+\tan^2\theta}\right) =$ ..... .

Sol : [Board 2020 Delhi Standard]

$$\sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta}$$
$$= \sin^2\theta + \cos^2\theta = 1$$

48. The value of $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) =$ ..... .

Sol : [Board 2020 Delhi Standard]

$$(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$$
$$= \sec^2\theta(1-\sin^2\theta)$$
$$= \sec^2\theta \times \cos^2\theta$$
$$= \frac{1}{\cos^2\theta} \times \cos^2\theta = 1$$

49. Prove that
$(1+\tan A - \sec A) \times (1+\tan A + \sec A) = 2\tan A$

Sol : [Board 2020 Delhi Basic]

LHS $= (1+\tan A - \sec A) \times (1+\tan A + \sec A)$
$= (1+\tan A)^2 - \sec^2 A$
$= 1 + \tan^2 A + 2\tan A - \sec^2 A$
$= \sec^2 A + 2\tan A - \sec^2 A$
$= 2\tan A$ = RHS

50. If $\tan A = \cot B$, then find the value of $(A+B)$.

Sol : [Board 2020 OD Standard]

We have $\tan A = \cot B$
$\tan A = \tan(90^\circ - B)$
$A = 90^\circ - B$
Thus $A + B = 90^\circ$

51. If $x = 3\sin\theta + 4\cos\theta$ and $y = 3\cos\theta - 4\sin\theta$ then prove that $x^2 + y^2 = 25$.

Sol : [Board 2020 OD Basic]

We have $x = 3\sin\theta + 4\cos\theta$
and $y = 3\cos\theta - 4\sin\theta$
$x^2 + y^2 = (3\sin\theta + 4\cos\theta)^2 + (3\cos\theta - 4\sin\theta)^2$
$= (9\sin^2\theta + 16\cos^2\theta + 24\sin\theta\cos\theta) +$
$+ (9\cos^2\theta + 16\sin^2\theta - 24\sin\theta\cos\theta)$
$= 9(\sin^2\theta + \cos^2\theta) + 16(\sin^2\theta + \cos^2\theta)$
$= 9 + 16 = 25$

52. If $\cos A = \frac{2}{5}$, find the value of $4 + 4\tan^2 A$.

Sol : [Board SQP 2018]

$4 + 4\tan^2 A = 4(1 + \tan^2 A)$
$4\sec^2 A = \frac{4}{\cos^2 A} = \frac{4}{\left(\frac{2}{5}\right)^2} = 4 \times \frac{25}{4} = 25$

53. Evaluate $\sin^2 60^\circ - 2\tan 45^\circ - \cos^2 30^\circ$

Sol : [Board 2019 OD]

$\sin^2 60^\circ - 2\tan 45^\circ - \cos^2 30^\circ$
$= \left(\frac{\sqrt{3}}{2}\right)^2 - 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$
$= \frac{3}{4} - 2 - \frac{3}{4} = -2$

54. If $\sin\theta + \sin^2\theta = 1$ then prove that $\cos^2\theta + \cos^4\theta = 1$.

Sol : [Board 2020 OD Basic]

We have $\sin\theta + \sin^2\theta = 1$
$\sin\theta + (1 - \cos^2\theta) = 1$
$\sin\theta - \cos^2\theta = 0$
$\sin\theta = \cos^2\theta$
Squaring both sides, we get
$\sin^2\theta = \cos^4\theta$
$1 - \cos^2\theta = \cos^4\theta$
$\cos^4\theta + \cos^2\theta = 1$ Hence Proved

55. In a triangle ABC, write $\cos\left(\frac{B+C}{2}\right)$ in terms of

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CHAPTER 8

Introduction of Trigonometry

OBJECTIVE QUESTIONS

1. If $\sec\theta - \tan\theta = m$, then the value of $\sec\theta + \tan\theta$ is

(a) $1 - \frac{1}{m}$ (b) $m^2 - 1$

(c) $\frac{1}{m}$ (d) $-m$

Sol : [Board 2024 OD Standard]

$$\sec^2\theta - \tan^2\theta = 1$$
$$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$$
$$\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta} = \frac{1}{m}$$

Thus (c) is correct option.

2. For $\theta = 30^\circ$, the value of $2\sin\theta\cos\theta$ is

(a) 1 (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3}{2}$

Sol : [Board 2024 OD Standard]

$$2\sin\theta\cos\theta = 2\sin 30^\circ \cos 30^\circ$$
$$= 2 \times \tfrac{1}{2} \times \tfrac{\sqrt{3}}{2}$$
$$= \tfrac{\sqrt{3}}{2}$$

Thus (b) is correct option.

3. If θ is an acute angle of a right angled triangle, then which of the following equation is not true ?

(a) $\sin\theta\cot\theta = \cos\theta$

(b) $\cos\theta\tan\theta = \sin\theta$

(c) $\text{cosec}^2\theta - \cot^2\theta = 1$

(d) $\tan^2\theta - \sec^2\theta = 1$

Sol : [Board 2023 OD Standard]

For the given acute angle θ,

$$\tan^2\theta + 1 = \sec^2\theta$$

So, $$\sec^2\theta - \tan^2\theta = 1$$

but in option (d) is incorrect.

We can verify that all other three option are correct.

NODIA

Thus (d) is correct option.

4. $\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$, in simplified form is :

(a) $\tan^2\theta$ (b) $\sec^2\theta$

(c) 1 (d) –1

Sol : [Board 2023 OD Standard]

$$\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta} = \cot^2\theta - \text{cosec}^2\theta$$
$$= \cot^2\theta - (1 + \cot^2\theta)$$
$$= \cot^2\theta - 1 - \cot^2\theta = -1$$

Thus (d) is correct option.

5. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(a) 1 (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Sol : [Board 2022 Term-1 Standard]

Given, $\sin\theta - \cos\theta = 0$

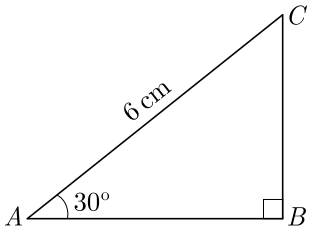
$$\sin\theta = \cos\theta$$
$$\sin\theta = \sin(90^\circ - \theta)$$
$$\theta = 90^\circ - \theta \Rightarrow \theta = 45^\circ$$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$$
$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Thus (c) is correct option.

6. In the adjoining figure, the length of BC is



$$= \frac{4\left(\frac{3}{4}\right)-1}{4\left(\frac{3}{4}\right)+1} = \frac{3-1}{3+1}$$
$$= \frac{2}{4} = \frac{1}{2}$$

Thus (c) is correct option.

19. If $x = p\sec\theta$ and $y = q\tan\theta$, then

(a) $x^2 - y^2 = p^2q^2$

(b) $x^2q^2 - y^2p^2 = pq$

(c) $x^2q^2 - y^2p^2 = \frac{1}{p^2q^2}$

(d) $x^2q^2 - y^2p^2 = p^2q^2$

Sol :

We know, $\sec^2\theta - \tan^2\theta = 1$

Substituting $\sec\theta = \frac{x}{p}$ and $\tan\theta = \frac{y}{q}$ in above equation we have

$$\left(\frac{x}{p}\right)^2 - \left(\frac{y}{q}\right)^2 = 1$$
$$x^2q^2 - y^2p^2 = p^2q^2$$

Thus (d) is correct option.

20. If $b\tan\theta = a$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ is

(a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$

(c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

Sol :

We have $\tan\theta = \frac{a}{b}$

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\frac{\sin\theta}{\cos\theta} - b}{a\frac{\sin\theta}{\cos\theta} + b}$$
$$= \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2-b^2}{a^2+b^2}$$

Thus (d) is correct option.

21. $(\cos^4A - \sin^4A)$ is equal to

(a) $1 - 2\cos^2A$ (b) $2\sin^2A - 1$

(c) $\sin^2A - \cos^2A$ (d) $2\cos^2A - 1$

Sol :

$$\cos^4A - \sin^4A = (\cos^2A)^2 - (\sin^2A)^2$$
$$= (\cos^2A - \sin^2A)(\cos^2A + \sin^2A)$$
$$= (\cos^2A - \sin^2A)(1)$$
$$= \cos^2A - (1 - \cos^2A)$$
$$= 2\cos^2A - 1$$

Thus (d) is correct option.

22. If $\sec 5A = \text{cosec}(A + 30^\circ)$, where $5A$ is an acute angle, then the value of A is

(a) 15° (b) 5°

(c) 20° (d) 10°

Sol :

We have, $\sec 5A = \text{cosec}(A + 30^\circ)$

$$\sec 5A = \sec[90^\circ - (A + 30^\circ)]$$
$$\sec 5A = \sec(60^\circ - A)$$
$$5A = 60^\circ - A$$
$$6A = 60^\circ \Rightarrow A = 10^\circ$$

Thus (d) is correct option.

23. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, than $x^2 + y^2$ is equal to

(a) 0 (b) 1/2

(c) 1 (d) 3/2

Sol :

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$$
$$x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$$
$$x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$$
$$x\sin\theta = \sin\theta\cos\theta$$
$$\Rightarrow x = \cos\theta$$

Now, $x\sin\theta = y\cos\theta$

$$\cos\theta\sin\theta = y\cos\theta$$
$$y = \sin\theta$$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

Thus (c) is correct option.

24. If $\tan 2A = \cot(A - 18^\circ)$, where $2A$ is an acute angle, then the value of A is

(a) 12° (b) 18°

(c) 36° (d) 48°

Sol :

We have $\tan 2A = \cot(A - 18^\circ)$

$$\cot(90^\circ - 2A) = \cot(A - 18^\circ)$$
$$90^\circ - 2A = A - 18^\circ$$
$$90^\circ + 18^\circ = A + 2A$$
$$3A = 108^\circ \Rightarrow A = \frac{108^\circ}{3} = 36^\circ$$

Thus (c) is correct option.

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$= \sec(15^\circ - \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \tan(55^\circ + \theta)$

$= 0$

13. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to

(a) $\frac{b}{\sqrt{b^2 - a^2}}$ (b) $\frac{b}{a}$

(c) $\frac{\sqrt{b^2 - a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2 - a^2}}$

Sol :

We have $\sin\theta = \frac{a}{b} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$\text{Base} = \sqrt{b^2 - a^2}$

So, $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

$= \frac{\sqrt{b^2 - a^2}}{b}$

Thus (c) is correct option.

14. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

(a) $\cos\beta$ (b) $\cos 2\beta$

(c) $\sin\alpha$ (d) $\sin 2\alpha$

Sol :

Given, $\cos(\alpha + \beta) = 0 = \cos 90^\circ \quad [\cos 90^\circ = 0]$

$\alpha + \beta = 90^\circ$

$\alpha = 90^\circ - \beta$

Now, $\sin(\alpha - \beta) = \sin(90^\circ - \beta - \beta)$

$= \sin(90^\circ - 2\beta) = \cos 2\beta$

Thus (b) is correct option.

15. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90^\circ$, then the value of $\tan 5\alpha$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) 1 (d) 0

Sol :

We have $\cos 9\alpha = \sin\alpha \quad$ where $9\alpha < 90^\circ$

$\sin(90^\circ - 9\alpha) = \sin\alpha$

$90^\circ - 9\alpha = \alpha$

$10\alpha = 90^\circ \Rightarrow \alpha = 9^\circ$

$\tan 5\alpha = \tan(5 \times 9^\circ)$

$= \tan 45^\circ = 1 \quad [\tan 45^\circ = 1]$

Thus (c) is correct option.

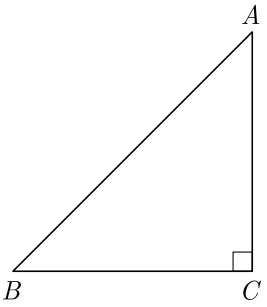
16. If ΔABC is right angled at C, then the value of $\cos(A + B)$ is

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Sol :

We know that in ΔABC,



$\angle A + \angle B + \angle C = 180^\circ$

But right angled at C i.e., $\angle C = 90^\circ$, thus

$\angle A + \angle B + 90^\circ = 180^\circ$

$A + B = 90^\circ$

$\cos(A + B) = \cos 90^\circ = 0$

Thus (a) is correct option.

17. If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is

(a) 0° (b) 30°

(c) 60° (d) 90°

Sol :

Given, $\sin\alpha = \frac{1}{2} = \sin 30^\circ \Rightarrow \alpha = 30^\circ$

and $\cos\beta = \frac{1}{2} = \cos 60^\circ \Rightarrow \beta = 60^\circ$

$\alpha + \beta = 30^\circ + 60^\circ = 90^\circ$

Thus (d) is correct option.

18. If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Sol :

Given, $4\tan\theta = 3$

$\tan\theta = \frac{3}{4} \quad$ ...(i)

$\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1} = \frac{4\tan\theta - 1}{4\tan\theta + 1}$

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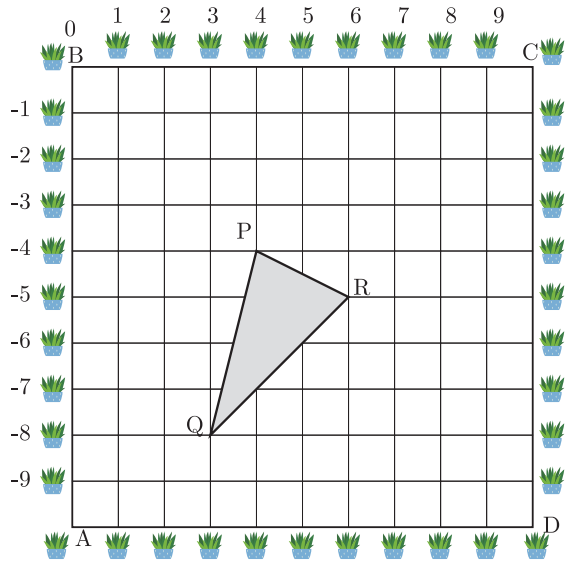

(iv) $PQ = \sqrt{(4-3)^2+(6-2)^2} = \sqrt{17}$

(v) Distance not depend on origin. In this case this is $\sqrt{17}$.

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25. If $\tan\theta+\sin\theta=m$ and $\tan\theta-\sin\theta=n$, then m^2-n^2 is equal to

(a) $\sqrt{mn}$ (b) $\sqrt{\frac{m}{n}}$

(c) $4\sqrt{mn}$ (d) None of these

Sol :

Given, $\tan\theta+\sin\theta=m$ and $\tan\theta-\sin\theta=n$

$$\begin{aligned}m^2-n^2 &= (\tan\theta+\sin\theta)^2-(\tan\theta-\sin\theta)^2\\ &= 4\tan\theta\sin\theta\\ &= 4\sqrt{\tan^2\theta\sin^2\theta}\\ &= 4\sqrt{\sin^2\theta\frac{\sin^2\theta}{\cos^2\theta}}\\ &= 4\sqrt{\sin^2\theta\frac{(1-\cos^2\theta)}{\cos^2\theta}}\\ &= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta}\\ &= 4\sqrt{\tan^2\theta-\sin^2\theta}\\ &= 4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)}\\ &= 4\sqrt{mn}\end{aligned}$$

Thus (c) is correct option.

26. If $f(x)=\cos^2x+\sec^2x$, then $f(x)$

(a) ≥ 1 (b) ≤ 1

(c) ≥ 2 (d) ≤ 2

Sol : (c) ≥ 2

Given, $$\begin{aligned}f(x) &= \cos^2x+\sec^2x\\ &= \cos^2x+\sec^2x-2+2\\ &= \cos^2x+\sec^2x-2\cos x\cdot\sec x+2\\ &= (\cos x-\sec x)^2+2\end{aligned}$$

Square of any expression is always greater than equal to zero.

$$f(x)\geq 2$$ Hence proved.

Thus (c) is correct option.

27. **Assertion (A):** For $0<\theta\leq 90^\circ$, $\operatorname{cosec}\theta-\cot\theta$ and $\operatorname{cosec}\theta+\cot\theta$ are reciprocal of each other.

Reason (R): $\operatorname{cosec}^2\theta-\cot^2\theta=1$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Sol : [Board 2023 OD Standard]

According to trigonometry identity

$$\begin{aligned}\operatorname{cosec}^2\theta-\cot^2\theta &= 1\\ (\operatorname{cosec}\theta+\cot\theta)(\operatorname{cosec}\theta-\cot\theta) &= 1\\ (\operatorname{cosec}\theta-\cot\theta) &= \frac{1}{(\operatorname{cosec}\theta+\cot\theta)}\end{aligned}$$

Thus assertion is True.

Reason : It is a Trigonometric Identity which is used in assertion. Thus reason is also true and correct explanation of assertion.

Thus (a) is correct option.

28. **Assertion :** $\sin^2 67^\circ+\cos^2 67^\circ=1$

Reason : For any value of $\theta, \sin^2\theta+\cos^2\theta=1$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $$\begin{aligned}\sin^2\theta+\cos^2\theta &= 1\\ \sin^2 67^\circ+\cos^2 67^\circ &= 1\end{aligned}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

29. **Assertion :** The value of $\sin\theta=\frac{4}{3}$ is not possible.

Reason : Hypotenuse is the largest side in any right angled triangle.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

$$\sin\theta=\frac{P}{H}=\frac{4}{3}$$

Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion

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(a) $2\sqrt{3}$ cm (b) $3\sqrt{3}$ cm

(c) $4\sqrt{3}$ cm (d) 3 cm

Sol : [Board 2022 Term-1 Basic]

In ΔABC, $\sin 30^\circ = \dfrac{BC}{AC}$

$$\frac{1}{2} = \frac{BC}{6}$$

$$BC = 3 \text{ cm}$$

Thus (d) is correct option.

7. If $\sin A = \dfrac{1}{2}$, then the value of $\cot A$ is

(a) $\sqrt{3}$ (b) $\dfrac{1}{\sqrt{3}}$

(c) $\dfrac{\sqrt{3}}{2}$ (d) 1

Sol : [Board 2022 Term-1 SQP Standard]

We have $\sin A = \dfrac{1}{2}$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{2}$$

Now, $\text{Base} = \sqrt{2^2 - 1^2} = \sqrt{3}$

So, $\cot A = \dfrac{\text{Base}}{\text{Perpendicular}}$

$$= \frac{\sqrt{3}}{1} = \sqrt{3}$$

Hence, the required value of $\cot A$ is $\sqrt{3}$.
Thus (a) is correct option.

8. Given that $\sin\alpha = \dfrac{\sqrt{3}}{2}$ and $\cos\beta = 0$, then the value of $\beta - \alpha$ is

(a) 0° (b) 90°

(c) 60° (d) 30°

Sol : [Board 2022 Term-1 SQP Basic]

We have $\sin\alpha = \dfrac{\sqrt{3}}{2}$

$$\sin\alpha = \sin 60^\circ \Rightarrow \alpha = 60^\circ \quad ...(1)$$

and $\cos\beta = 0$

$$\cos\beta = \cos 90^\circ \Rightarrow \beta = 90^\circ \quad ...(2)$$

Now, $\beta - \alpha = 90^\circ - 60^\circ = 30^\circ$

Thus (d) is correct option.

9. If ΔABC is right angled at C, then the value of $\sec(A + B)$ is

(a) 0 (b) 1

(c) $\dfrac{2}{\sqrt{3}}$ (d) not defined

Sol : [Board 2020 SQP Standard]

We have $\angle C = 90^\circ$

Since, $\angle A + \angle B + \angle C = 180^\circ$

$$\angle A + \angle B = 180^\circ - \angle C$$

$$= 180^\circ - 90^\circ = 90^\circ$$

Now, $\sec(A + B) = \sec 90^\circ$ not defined

Thus (d) is correct option.

10. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, $(\theta \neq 90^\circ)$ then the value of $\tan\theta$ is

(a) $\sqrt{2} - 1$ (b) $\sqrt{2} + 1$

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

Sol : [Board 2020 SQP Standard]

We have $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

Dividing both sides by $\cos\theta$, we get

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \sqrt{2}\frac{\cos\theta}{\cos\theta}$$

$$\tan\theta + 1 = \sqrt{2}$$

$$\tan\theta = \sqrt{2} - 1$$

Thus (a) is correct option.

11. If $\cos A = \dfrac{4}{5}$, then the value of $\tan A$ is

(a) $\dfrac{3}{5}$ (b) $\dfrac{3}{4}$

(c) $\dfrac{4}{3}$ (d) $\dfrac{5}{3}$

Sol :

We have $\cos A = \dfrac{4}{5}$

We know that, $\cos A = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{4}{5}$

$$\text{Perpendicular} = \sqrt{5^2 - 4^2}$$

$$= \sqrt{25 - 16} = 3$$

Now, $\tan A = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{3}{4}$

Thus (b) is correct option.

12. The value of the expression

$\operatorname{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot(35^\circ - \theta)$ is

(a) -1 (b) 0

(c) 1 (d) $\dfrac{3}{2}$

Sol :

Given :

$\operatorname{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot(35^\circ - \theta)$

$\operatorname{cosec}[90^\circ - (15^\circ - \theta)] - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot[90^\circ - (55^\circ + \theta)]$

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37. If $\sin\theta = \frac{a}{b}$, then find the value of $\cos\theta$?

Sol : [Board Term-1 2012]

We have $\sin\theta = \frac{a}{b} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

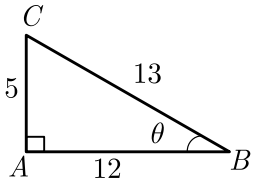
$\text{Base} = \sqrt{b^2 - a^2}$

So, $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{b^2 - a^2}}{b}$

38. If $\sin\theta = \frac{5}{13}$, then the value of $\tan\theta$ is ..... .

Sol : [Board 2020 OD Basic]

From $\sin\theta = \frac{5}{13}$ we can draw the figure as given below.



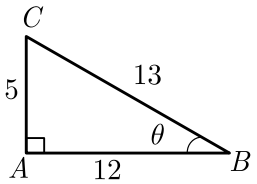
Now, $\tan\theta = \frac{AC}{BC} = \frac{5}{12}$

39. If $\cot\theta = \frac{12}{5}$, then the value of $\sin\theta$ is ..... .

Sol : [Board 2020 Delhi Basic]

Given, $\cot\theta = \frac{12}{5} \Rightarrow \tan\theta = \frac{5}{12}$

From $\tan\theta = \frac{5}{12}$ we can draw the figure as given below.



So, $\sin\theta = \frac{AC}{CB} = \frac{5}{13}$

40. If $\cos(\alpha+\beta) = 0$, then find $\sin(\alpha-\beta)$ in terms of β.

Sol : [Board Term-1 2016]

We have, $\cos(\alpha+\beta) = 0 = \cos 90^\circ \quad [\cos 90^\circ = 0]$

$\alpha + \beta = 90^\circ$

$\alpha = 90^\circ - \beta$

Now, $\sin(\alpha-\beta) = \sin(90^\circ - \beta - \beta)$

$= \sin(90^\circ - 2\beta) = \cos 2\beta$

Thus (b) is correct option.

41. If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then find the value of $(\alpha+\beta)$.

Sol : [Board Term-1 2014]

$\sin\alpha = \frac{1}{2} = \sin 30^\circ \Rightarrow \alpha = 30^\circ$

and $\cos\beta = \frac{1}{2} = \cos 60^\circ \Rightarrow \beta = 60^\circ$

$\alpha + \beta = 30^\circ + 60^\circ = 90^\circ$

42. The value of the $(\tan^2 60^\circ + \sin^2 45^\circ)$ is ..... .

Sol : [Board 2020 OD Basic]

$\tan^2 60^\circ + \sin^2 45^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= 3 + \frac{1}{2} = \frac{7}{2}$

43. If $\sin\theta - \cos\theta = 0$, then find the value of $(\sin^4\theta + \cos^4\theta)$

Sol : [Board 2007]

We have, $\sin\theta - \cos\theta = 0$

$\sin\theta = \cos\theta$

$\sin\theta = \sin(90^\circ - \theta)$

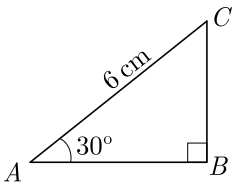
$\theta = 90^\circ - \theta \Rightarrow \theta = 45^\circ$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

44. In the adjoining figure, what is the length of BC?



Sol : [Board 2009]

In ΔABC, $\sin 30^\circ = \frac{BC}{AC}$

$\frac{1}{2} = \frac{BC}{6} \Rightarrow BC = 3\text{ cm}$

45. If $\sec 5A = \text{cosec}(A + 30^\circ)$, where $5A$ is an acute angle, then what is the value of A?

Sol : [Board 2010]

We have, $\sec 5A = \text{cosec}(A + 30^\circ)$

$\sec 5A = \sec[90^\circ - (A - 30^\circ)]$

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(A).

Thus (a) is correct option.

30. **Assertion :** The value of $\sec^2 10^\circ - \cot^2 80^\circ$ is 1.

Reason : The value of $\sin 30^\circ = \frac{1}{2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have,

$$\sec^2 10^\circ - \cot^2 80^\circ = \sec^2 10^\circ - \cot^2(90^\circ - 10^\circ)$$
$$= \sec^2 10^\circ - \tan^2 10^\circ = 1$$

Also, $\sin 30^\circ = \frac{1}{2}$

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A) Thus (b) is correct option.

ONE MARK QUESTIONS

31. Evaluate : $2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ$

Sol : [Board 2024 OD Standard]

$$x\ \ 2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ$$
$$= 2\sqrt{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} + 2\sqrt{3} \times \frac{\sqrt{3}}{2}$$
$$= 1 + 3$$
$$= 4$$

32. Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = 0$, then what is the value of $\beta - \alpha$?

Sol : [Board 2020 SQP Standard]

We have $\sin\alpha = \frac{\sqrt{3}}{2}$

$$\sin\alpha = \sin 60^\circ \Rightarrow \alpha = 60^\circ \quad ...(1)$$

and $\cos\beta = 0$

$$\cos\beta = \cos 90^\circ \Rightarrow \beta = 90^\circ \quad ...(2)$$

Now, $\beta - \alpha = 90^\circ - 60^\circ = 30^\circ$

33. If ΔABC is right angled at C, then find the value of $\sec(A+B)$.

Sol : [Board 2020 SQP Standard]

We have $\angle C = 90^\circ$

Since, $\angle A + \angle B + \angle C = 180^\circ$

$$\angle A + \angle B = 180^\circ - \angle C$$
$$= 180^\circ - 90^\circ = 90^\circ$$

Now, $\sec(A+B) = \sec 90^\circ$ not defined

PRACTICE

If ΔABC is right angled at C, then find the value of $\cos(A+B)$.

[Board Term-1 2015]

Ans : 0

34. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, $(\theta \neq 90^\circ)$ then what is the value of $\tan\theta$?

Sol : [Board 2020 SQP Standard]

We have $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

Dividing both sides by $\cos\theta$, we get

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \sqrt{2}\frac{\cos\theta}{\cos\theta}$$
$$\tan\theta + 1 = \sqrt{2}$$
$$\tan\theta = \sqrt{2} - 1$$

35. If $\cos A = \frac{4}{5}$, then find the value of $\tan A$.

Sol : [Board Term-1 2011]

We have $\cos A = \frac{4}{5}$

We know that, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$

$$\text{Perpendicular} = \sqrt{5^2 - 4^2}$$
$$= \sqrt{25 - 16} = 3$$

Now, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

36. If $\sin A = \frac{1}{2}$, then what is the value of $\cot A$?

Sol : [Board Term-1 2017]

We have $\sin A = \frac{1}{2}$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{2}$$

Now, $\text{Base} = \sqrt{2^2 - 1^2} = \sqrt{3}$

So, $\cot A = \frac{\text{Base}}{\text{Perpendicular}}$

$$= \frac{\sqrt{3}}{1} = \sqrt{3}$$

Hence, the required value of $\cot A$ is $\sqrt{3}$.

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angle A.

Sol : [Board Term-1 2016]

In a triangle $A+B+C = 180^\circ$

$$B+C = 180^\circ - A$$

Thus $$\cos\left(\frac{B+C}{2}\right) = \cos\left[\frac{180^\circ - A}{2}\right]$$

$$= \cos\left[90 - \frac{A}{2}\right]$$

$$= \sin\frac{A}{2}$$

56. If $\sec\theta \cdot \sin\theta = 0$, then find the value of θ.

Sol : [Board Term-1 2016]

We have $\sec\theta \cdot \sin\theta = 0$

$$\frac{1}{\cos\theta} \cdot \sin\theta = 0$$

$$\frac{\sin\theta}{\cos\theta} = 0$$

$$\tan\theta = 0 = \tan 0^\circ \quad \text{Thus } \theta = 0^\circ$$

57. If $\tan 2A = \cot(A+60^\circ)$, find the value of A where $2A$ is an acute angle.

Sol : [Board Term-1 2016]

We have $\tan 2A = \cot(A+60^\circ)$

$$\cot(90^\circ - 2A) = \cot(A+60^\circ)$$

$$90^\circ - 2A = A + 60^\circ$$

$$3A = 30^\circ \Rightarrow A = 10^\circ$$

58. Prove that $1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = \operatorname{cosec}\alpha$

Sol : [Board 2020 OD Standard]

$$1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = 1 + \frac{\operatorname{cosec}^2\alpha - 1}{1+\operatorname{cosec}\alpha}$$

$$= 1 + \frac{(1+\operatorname{cosec}\alpha)(\operatorname{cosec}\alpha - 1)}{1+\operatorname{cosec}\alpha}$$

$$= 1 + \operatorname{cosec}\alpha - 1$$

$$= \operatorname{cosec}\alpha \qquad \text{Hence Proved}$$

59. If $\tan(3x+30^\circ) = 1$ then find the value of x.

Sol : [Board Term-1 2015]

We have $\tan(3x+30^\circ) = 1 = \tan 45^\circ$

$$3x + 30^\circ = 45^\circ$$

$$x = 5^\circ$$

60. What happens to value of $\cos\theta$ when θ increases from 0° to 90°.

Sol : [Board Term-1 2015]

$\cos\theta$ decreases from 1 to θ.

61. If A and B are acute angles and $\sin A = \cos B$, then find the value of $A+B$.

Sol : [Board Term-1 2016]

We have $\sin A = \cos B$

$$\sin A = \sin(90^\circ - B)$$

$$A = 90^\circ - B$$

$$A + B = 90^\circ$$

62. If $k+1 = \sec^2\theta(1+\sin\theta)(1-\sin\theta)$, then find the value of k.

Sol : [Board Term-1 2015]

We have $k+1 = \sec^2\theta(1+\sin\theta)(1-\sin\theta)$

$$= \sec^2\theta(1-\sin^2\theta)$$

$$= \sec^2\theta.\cos^2\theta$$

$$= \sec^2\theta \times \frac{1}{\sec^2\theta}$$

$$k+1 = 1 \Rightarrow k = 1-1 = 0$$

Thus $k = 0$

63. Find the value of $\sin^2 41^\circ + \sin^2 49^\circ$

Sol : [Board Term-1 2012]

We have

$$\sin^2 41 + \sin^2 49 = \sin^2(90^\circ - 49^\circ) + \sin^2 49^\circ$$

$$= \cos^2 49 + \sin^2 49^\circ$$

$$= 1$$

64. Prove that $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta} = 1$.

Sol : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$$

$$= \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1 = \text{RHS}$$

65. Prove that : $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

Sol : [Board 2018]

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

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$$\sin^2 60^\circ - 2\tan 45^\circ - \cos^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 - 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{3}{4} - 2 - \frac{3}{4} = -2$$

95. If $\tan(3x + 30^\circ) = 1$ then find the value of x.

Sol : [Board Term-1 2015]

We have $\tan(3x + 30^\circ) = 1 = \tan 45^\circ$

$3x + 30^\circ = 45^\circ$

$x = 5^\circ$

96. What happens to value of $\cos\theta$ when θ increases from 0° to 90°.

Sol : [Board Term-1 2015]

$\cos\theta$ decreases from 1 to 0.

97. If A and B are acute angles and $\sin A = \cos B$, then find the value of $A + B$.

Sol : [Board Term-1 2016]

We have $\sin A = \cos B$

$\sin A = \sin(90^\circ - B)$

$A = 90^\circ - B$

$A + B = 90^\circ$

PRACTICE

If A and B are acute angles and $\operatorname{cosec} A = \sec B$, then find the value of $A + B$.
[Board Term-1 2013]
Ans : 90°

TWO MARKS QUESTIONS

98. If $A = 60^\circ$ and $B = 30^\circ$, verify that :
$\sin(A + B) = \sin A \cos B + \cos A \sin B$

Sol : [Board 2024 OD Standard]

$\text{LHS} = \sin(A + B) = \sin(60^\circ + 30^\circ)$

$= \sin 90^\circ = 1$

$= 1$

$\text{RHS} = \sin A \cos B + \cos A \sin B$

$= \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{4}{4}$

$= 1 = \text{L.H.S.}$

$\text{LHS} = \text{RHS}$ Hence verified

99. If $\tan 2A = \cot(A - 18^\circ)$, where $2A$ is an acute angle, find the value of A.

Sol : [Board 2018]

We have $\tan 2A = \cot(A - 18^\circ)$

$\cot(90^\circ - 2A) = \cot(A - 18^\circ)$

Thus $90^\circ - 2A = A - 18^\circ$

$3A = 90^\circ + 18^\circ = 108^\circ$

$A = 36^\circ$

PRACTICE

If $\cos 2A = \sin(A - 15^\circ)$, find A.
[Board Term-1 2015]
Ans : 35°

100. Evaluate :
$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

Sol : [Board Term-1 2016]

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$
$$= \frac{3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 + 2 - 1}{(1)^2}$$
$$= \frac{3 \times \frac{1}{3} + 3 + 2 - 1}{1}$$
$$= 1 + 3 + 2 - 1 = 5$$

101. Find the value of $\cos 2\theta$, if $2\sin 2\theta = \sqrt{3}$.

Sol : [Board Term-1 2012]

We have $2\sin 2\theta = \sqrt{3}$

$\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$

$2\theta = 60^\circ$

Hence, $\cos 2\theta = \cos 60^\circ = \frac{1}{2}$.

102. Find the value of $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$ is it equal to $\sin 90^\circ$ or $\cos 90^\circ$?

Sol : [Board Term-1 2016]

$\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

NODIA

If $4\cos\theta = 11\sin\theta$, find the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$.

[Board Term-1 2012]

Ans : $\frac{93}{149}$

73. Evaluate :

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

Sol : [Board Term-1 2016]

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

$$= \frac{3\times\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 + 2 - 1}{(1)^2}$$

$$= \frac{3\times\frac{1}{3} + 3 + 2 - 1}{1}$$

$$= 1 + 3 + 2 - 1 = 5$$

74. If $\sin(A+B) = 1$ and $\sin(A-B) = \frac{1}{2}$, $0 \le A+B < 90^0$ and $A > B$, then find A and B.

Sol : [Board Term-1 2016]

We have $\sin(A+B) = 1 = \sin 90^\circ$

$$A + B = 90^\circ \quad ...(1)$$

and $\sin(A-B) = \frac{1}{2} = \sin 30^\circ$

$$A - B = 30^\circ \quad ...(2)$$

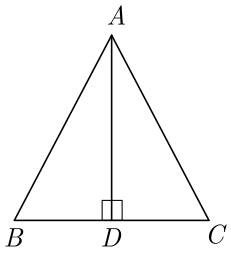
Solving eq. (1) and (2), we obtain

$$A = 60^\circ \text{ and } B = 30^\circ$$

75. Find $\operatorname{cosec} 30^\circ$ and $\cos 60^\circ$ geometrically.

Sol : [Board Term-1 2015]

Let a triangle ABC with each side equal to $2a$ as shown below.



In ΔABC, $\angle A = \angle B = \angle C = 60^\circ$

Now we draw AD perpendicular to BC, then

$$\Delta BDA \cong \Delta CDA$$

$$BD = CD$$

$$\angle BAD = CAD = 30^\circ \quad \text{by } CPCT$$

$$AD = \sqrt{3}a$$

In ΔBDA, $\operatorname{cosec} 30^\circ = \frac{AB}{BD} = \frac{2a}{a} = 2$

and $\cos 60^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

76. Evaluate : $\frac{\sin 90^\circ}{\cos 45^\circ} + \frac{1}{\operatorname{cosec} 30^\circ}$

Sol : [Board Term-1 2013]

We have

$$\frac{\sin 90^\circ}{\cos 45^\circ} + \frac{1}{\operatorname{cosec} 30^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} + \frac{1}{2}$$

$$= \sqrt{2} + \frac{1}{2} = \frac{2\sqrt{2}+1}{2}$$

NODIA

77. If $\tan(A+B) = \sqrt{3}$, $\tan(A-B) = \frac{1}{\sqrt{3}}$ $0^\circ < A+B \le 90^\circ$, then find A and B.

Sol : [Board Term-1 2012]

We have $\tan(A+B) = \sqrt{3} = \tan 60^\circ$

$$A + B = 60^\circ \quad ...(1)$$

Also $\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$$A - B = 30^\circ \quad ...(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^\circ$$

$$A = \frac{90^\circ}{2} = 45^\circ$$

Substituting this value of A in equation (1), we get

$$B = 60^\circ - A = 60^\circ - 45^\circ = 15^\circ$$

Hence, $A = 45^\circ$ and $B = 15^\circ$

78. If $\cos(A-B) = \frac{\sqrt{3}}{2}$ and $\sin(A+B) = \frac{\sqrt{3}}{2}$, find A and B, where $(A+B)$ and $(A-B)$ are acute angles.

Sol : [Board Term-1 2012]

We have $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$$A - B = 30^\circ \quad ...(1)$$

Also $\sin(A+B) = \frac{\sqrt{3}}{2} = \sin 60^\circ$

$$A + B = 60^\circ \quad ...(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^\circ \Rightarrow A = 45^\circ$$

Substituting this value of A in equation (1), we get $B = 15^\circ$

79. Find the value of $\cos 2\theta$, if $2\sin 2\theta = \sqrt{3}$.

Sol : [Board Term-1 2012]

(i) $\tan\theta = \frac{AC}{AD} = \frac{1.5}{2.6} = \frac{15}{26}$

(ii) $\sec\theta + \operatorname{cosec}\theta = \frac{CD}{AD} + \frac{CD}{AC}$

$= \frac{3}{2.6} + \frac{3}{1.5} = \frac{41}{13}$

108. Prove that : $\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1+\cot\theta}{\sin\theta}$

Sol : [Board 2020 Delhi Standard]

$$\text{LHS} = \frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$$
$$= \frac{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} + 1}$$
$$= \frac{\sin\theta(\cos\theta + 1 - \sin\theta)}{\sin\theta(\cos\theta - 1 + \sin\theta)}$$
$$= \frac{\sin\theta\cos\theta + \sin\theta - \sin^2\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$
$$= \frac{\sin\theta\cos\theta + \sin\theta - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$
$$= \frac{\sin\theta(\cos\theta + 1) - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$
$$= \frac{(1+\cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$
$$= \frac{1+\cos\theta}{\sin\theta} = \text{RHS}$$

109. If $\sin\theta + \cos\theta = \sqrt{2}$ prove that $\tan\theta + \cot\theta = 2$

Sol : [Board 2020 OD Standard, SQP 2018]

We have $\sin\theta + \cos\theta = \sqrt{2}$

Squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$
$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$
$$1 + 2\sin\theta\cos\theta = 2$$
$$2\sin\theta\cos\theta = 1$$
$$\sin\theta\cos\theta = \frac{1}{2} \qquad \ldots(1)$$

Now $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$
$$= \frac{1}{\frac{1}{2}} = 2 = \text{RHS}$$

PRACTICE

If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

[Board 2020 SQP Standard]

Ans : Proof

110. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or ½.

Sol : [Board 2020 OD Standard]

We have, $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing by $\sin^2\theta$ on both sides, we get

$$\frac{1}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$$
$$\frac{1}{\sin^2\theta} + 1 = 3\cot\theta$$
$$\operatorname{cosec}^2\theta + 1 = 3\cot\theta$$
$$1 + \cot^2\theta + 1 = 3\cot\theta$$
$$\cot^2\theta - 3\cot\theta + 2 = 0$$
$$\cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$$
$$\cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$$
$$(\cot\theta - 2)(\cot\theta - 1) = 0$$
$$\cot\theta = 1 \text{ or } 2$$
$$\tan\theta = 1 \text{ or } ½.$$

111. Prove that $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

Sol : [Board 2019 Delhi Standard]

$$\text{LHS} = (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$$
$$= (\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta) +$$
$$+ (\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$$
$$= (\sin^2\theta + \cos^2\theta) + (\operatorname{cosec}^2\theta + \sec^2\theta)$$
$$+ 2\sin\theta \times \frac{1}{\sin\theta} + 2\cos\theta \times \frac{1}{\cos\theta}$$
$$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2 + 2$$
$$= 7 + \tan^2\theta + \cot^2\theta = \text{RHS}$$

112. Prove that $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

Sol : [Board 2019 Delhi]

$$\text{LHS} = (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$$
$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$
$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$
$$= \frac{(\sin A + \cos A - 1)(\cos A + \sin A + 1)}{\sin A\cos A}$$
$$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A\cos A}$$
$$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}{\sin A\cos A}$$
$$= \frac{1 + 2\sin A\cos A - 1}{\sin A\cos A}$$

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We have

$$\begin{aligned}\sec^4\theta - \sec^2\theta &= \sec^2\theta(\sec^2\theta - 1)\\ &= \sec^2\theta(\tan^2\theta)\\ &= (1+\tan^2\theta)\tan^2\theta\\ &= \tan^2\theta + \tan^4\theta\end{aligned}$$

Hence Proved.

88. If $\sin A = \frac{\sqrt{3}}{2}$, find the value of $2\cot^2 A - 1$.

Sol : [Board Term-1 2012]

Using $\cot^2\theta = -1 + \operatorname{cosec}^2\theta$ we have

$$\begin{aligned}2\cot^2 A - 1 &= 2(\operatorname{cosec}^2 A - 1) - 1\\ &= \frac{2}{\sin^2 A} - 3\\ &= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} - 3\\ &= \frac{8}{3} - 3 = \frac{-1}{3}\end{aligned}$$

Thus $2\cot^2 A - 1 = \frac{-1}{3}$

89. Find the value of θ, if, $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4; \theta \le 90^\circ$

Sol : [Board Term-1 2015]

We have

$$\begin{aligned}\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} &= 4\\ \frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} &= 4\\ \frac{\cos\theta[1+\sin\theta+1-\sin\theta]}{1-\sin^2\theta} &= 4\\ \frac{\cos\theta(2)}{\cos^2\theta} &= 4\\ \frac{1}{\cos\theta} &= 2\\ \cos\theta &= \frac{1}{2}\\ \cos\theta &= \cos 60^\circ\end{aligned}$$

Thus $\theta = 60^\circ$.

90. Prove that : $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

Sol : [Board Term-1 2012]

$$\begin{aligned}\sqrt{\frac{1-\cos A}{1+\cos A}} &= \sqrt{\frac{1-\cos A}{1+\cos A}\times\frac{1-\cos A}{1-\cos A}}\\ &= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}}\\ &= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}\\ &= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}\\ &= \operatorname{cosec} A - \cot A \quad \text{Hence Proved.}\end{aligned}$$

91. If $\sin\theta - \cos\theta = \frac{1}{2}$, then find the value of $\sin\theta + \cos\theta$.

Sol : [Board Term-1 2013]

We have $\sin\theta - \cos\theta = \frac{1}{2}$

Squaring both sides, we get

$$\begin{aligned}(\sin\theta - \cos\theta)^2 &= \left(\frac{1}{2}\right)^2\\ \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta &= \frac{1}{4}\\ 1 - 2\sin\theta\cos\theta &= \frac{1}{4}\\ 2\sin\theta\cos\theta &= 1 - \frac{1}{4} = \frac{3}{4}\end{aligned}$$

Again, $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$$\begin{aligned}&= 1 + 2\sin\theta\cos\theta\\ &= 1 + \frac{3}{4} = \frac{7}{4}\end{aligned}$$

Thus $\sin\theta + \cos\theta = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$

92. If θ be an acute angle and $5\operatorname{cosec}\theta = 7$, then evaluate $\sin\theta + \cos^2\theta - 1$.

Sol : [Board Term-1 2012]

We have

$$\begin{aligned}5\operatorname{cosec}\theta &= 7\\ \operatorname{cosec}\theta &= \frac{7}{5}\\ \sin\theta &= \frac{5}{7}\\ \sin\theta + \cos^2\theta - 1 &= \sin\theta - (1-\cos^2\theta)\\ &= \sin\theta - \sin^2\theta\end{aligned}$$

$[\sin^2\theta + \cos^2\theta = 1]$

$$\begin{aligned}&= \frac{5}{7} - \left(\frac{5}{7}\right)^2\\ &= \frac{35-25}{49} = \frac{10}{49}\end{aligned}$$

93. The value of the $(\tan^2 60^\circ + \sin^2 45^\circ)$ is ..... .

Sol : [Board 2020 OD Basic]

$$\begin{aligned}\tan^2 60^\circ + \sin^2 45^\circ &= (\sqrt{3})^2 + \left(\frac{1}{\sqrt{2}}\right)^2\\ &= 3 + \frac{1}{2} = \frac{7}{2}\end{aligned}$$

94. Evaluate $\sin^2 60^\circ - 2\tan 45^\circ - \cos^2 30^\circ$

Sol : [Board 2019 OD]

NODIA

We have $2\sin 2\theta = \sqrt{3}$

$$\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

$$2\theta = 60^\circ$$

Hence, $\cos 2\theta = \cos 60^\circ = \frac{1}{2}$.

80. Find the value of $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$ is it equal to $\sin 90^\circ$ or $\cos 90^\circ$?

Sol : [Board Term-1 2016]

$\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

It is equal to $\sin 90^\circ = 1$ but not equal to $\cos 90^\circ$ as $\cos 90^\circ = 0$.

81. If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^\circ < \theta < 90^\circ$, find the value of θ.

Sol : [Boar Term-1, 2012]

We have

$$\sqrt{3}\sin\theta - \cos\theta = 0 \text{ and } 0^\circ < \theta < 90^\circ$$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$= \tan 30^\circ \qquad \left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\theta = 30^\circ$$

82. Evaluate : $\dfrac{\cos 45^\circ}{\sec 30^\circ} + \dfrac{1}{\sec 60^\circ}$

Sol : [Board Term-1 Delhi 2012]

We have $\dfrac{\cos 45^\circ}{\sec 30^\circ} + \dfrac{1}{\sec 60^\circ} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \dfrac{1}{2}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{1}{2} = \frac{\sqrt{6}+2}{4}$$

83. If $\sin\phi = \frac{1}{2}$, show that $3\cos\phi - 4\cos^3\phi = 0$.

Sol : [Board 2010]

We have $\sin\phi = \frac{1}{2}$

$$\phi = 30^\circ$$

Now substituting this value of θ in LHS we have

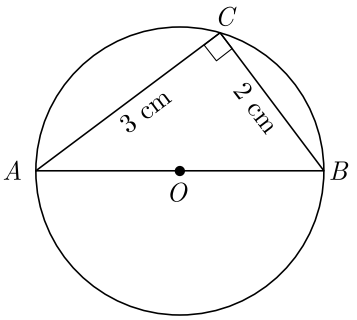
$$3\cos\phi - 4\cos^3\phi = 3\cos 30^\circ - 4\cos^3 30^\circ$$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$= 0 \qquad \text{Hence Proved}$$

84. In the given figure, AOB is a diameter of a circle with centre O, find $\tan A \tan B$.



Sol : [Board Term-1 2012]

In ΔABC, $\angle C$ is a angle in a semi-circle, thus

$$\angle C = 90^\circ$$

$$\tan A = \frac{BC}{AC} = \frac{2}{3}$$

and $\tan B = \dfrac{AC}{BC} = \dfrac{3}{2}$

$$\tan A \tan B = \frac{2}{3} \times \frac{3}{2} = 1$$

85. Express the trigonometric ratio of $\sec A$ and $\tan A$ in terms of $\sin A$.

Sol : [Board Term-1 2015]

We have $\sec A = \dfrac{1}{\cos A} = \dfrac{1}{\sqrt{1-\sin^2 A}}$

and $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{\sin A}{\sqrt{1-\sin^2 A}}$

86. Prove that : $\dfrac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = 1$

Sol : [Board Term-1 2015]

$$\frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{1 - 2\sin^2\theta\cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta}$$

$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\cos^2\theta} = 1$$

87. Prove that : $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

Sol : [Board Term-1 2015]

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$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

It is equal to $\sin 90^\circ = 1$ but not equal to $\cos 90^\circ$ as $\cos 90^\circ = 0$.

103. If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^\circ < \theta < 90^\circ$, find the value of θ.

Sol : [Boar Term-1, 2012]

We have

$$\begin{aligned}\sqrt{3}\sin\theta - \cos\theta &= 0 \text{ and } 0^\circ < \theta < 90^\circ\\ \sqrt{3}\sin\theta &= \cos\theta\\ \frac{\sin\theta}{\cos\theta} &= \frac{1}{\sqrt{3}}\\ \tan\theta &= \frac{1}{\sqrt{3}}\\ &= \tan 30^\circ \qquad \left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]\\ \theta &= 30^\circ\end{aligned}$$

104. Evaluate : $\dfrac{\cos 45^\circ}{\sec 30^\circ} + \dfrac{1}{\sec 60^\circ}$

Sol : [Board Term-1 2012]

We have $\dfrac{\cos 45^\circ}{\sec 30^\circ} + \dfrac{1}{\sec 60^\circ} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \dfrac{1}{2}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{1}{2} = \frac{\sqrt{6}+2}{4}$$

THREE MARKS QUESTIONS

105. Prove that : $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1+\cos A}$

Sol : [Board 2023 OD Standard]

We have $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

$$\text{LHS} = \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\cos A + 1}{1}$$

$$\text{LHS} = \cos A + 1$$

$$\text{RHS} = \frac{\sin^2 A}{1-\cos A}$$

$$= \frac{1-\cos^2 A}{1-\cos A}$$

$$= \frac{(1+\cos A)(1-\cos A)}{1-\cos A}$$

$$= 1 + \cos A$$

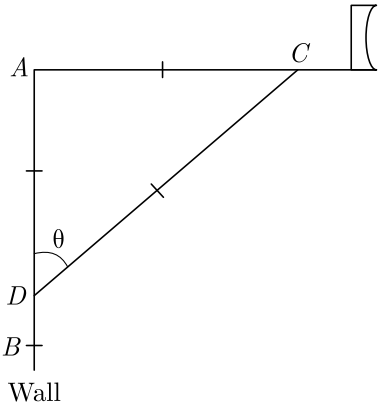
$$= \text{RHS}$$

106. Show that : $\dfrac{\cos^2(45^\circ+\theta) + \cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)} = 1$

Sol : [Board 2020 OD Standard]

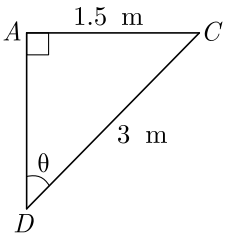
$$\begin{aligned}\text{LHS} &= \frac{\cos^2(45^\circ+\theta) + \cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}\\ &= \frac{\cos^2(45^\circ+\theta) + \sin^2(90^\circ-45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(90^\circ-30^\circ+\theta)}\\ &= \frac{\cos^2(45^\circ+\theta) + \sin^2(45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(60^\circ+\theta)}\\ &= \frac{1}{1} = 1 = \text{RHS}\end{aligned}$$

107. The rod of TV disc antenna is fixed at right angles to wall AB and a rod CD is supporting the disc as shown in Figure. If $AC = 1.5$ m long and $CD = 3$ m, find (i) $\tan\theta$ (ii) $\sec\theta + \text{cosec}\,\theta$.



Sol : [Board 2020 Delhi Standard]

From the given information we draw the figure as below



In right angle triangle ΔCAD, applying Pythagoras theorem,

$$\begin{aligned}AD^2 + AC^2 &= DC^2\\ AD^2 + (1.5)^2 &= (3)^2\\ AD^2 &= 9 - 2.25 = 6.75\\ AD &= \sqrt{6.75} = 2.6\text{ m (Approx)}\end{aligned}$$

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$$= \tan A \frac{[1-2(1-\cos^2 A)]}{(2\cos^2 A-1)}$$
$$= \tan A \frac{[1-2+2\cos^2 A)]}{(2\cos^2 A-1)}$$
$$= \tan A \frac{(2\cos^2 A-1)}{(2\cos^2 A-1)}$$
$$= \tan A \quad \text{Hence Proved]}$$

66. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Sol : [Board 2020 OD Standard]

$$\tan^4\theta + \tan^2\theta = \tan^2\theta(1+\tan^2\theta)$$
$$= \tan^2\theta \times \sec^2\theta$$
$$= (\sec^2\theta - 1)\sec^2\theta$$
$$= \sec^4\theta - \sec^2\theta \quad \text{Hence Proved}$$

67. Prove that : $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

Sol : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$$
$$= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$
$$= \frac{2}{1-\sin^2\theta}$$
$$= 2\sec^2\theta = \text{RHS}$$

68. If $\sqrt{2}\sin\theta = 1$, find the value of $\sec^2\theta - \text{cosec}^2\theta$.

Sol : [Board Term-1 2012]

We have $\sqrt{2}\sin\theta = 1$

$$\sin\theta = \frac{1}{\sqrt{2}} = \sin 45^\circ$$

Thus $\theta = 45^\circ$

Now $\sec^2\theta - \text{cosec}^2\theta = \sec^2 45^\circ - \text{cosec}^2 45^\circ$

$$= \left(\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2 = 0$$

69. Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$.

Sol : [Board Term-1 2013]

$$\text{LHS} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$
$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$
$$= \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \sec\theta - \tan\theta = \text{RHS} \quad \text{Hence Proved}$$

70. Prove that : $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$

Sol : [Board 2020 OD Basic]

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2\theta}{\sec^2\theta}$$
$$= \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}$$
$$= \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$$
$$= \cos^2\theta - \sin^2\theta \quad \text{Hence Proved}$$

71. Prove that $\frac{\text{cosec}\,\theta}{\text{cosec}\,\theta - 1} + \frac{\text{cosec}\,\theta}{\text{cosec}\,\theta + 1} = 2\sec^2\theta$.

Sol : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{\text{cosec}\,\theta}{\text{cosec}\,\theta - 1} + \frac{\text{cosec}\,\theta}{\text{cosec}\,\theta + 1}$$
$$= \text{cosec}\,\theta\left[\frac{1}{\text{cosec}\,\theta - 1} + \frac{1}{\text{cosec}\,\theta + 1}\right]$$
$$= \text{cosec}\,\theta\left[\frac{\text{cosec}\,\theta + 1 + \text{cosec}\,\theta - 1}{(\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1)}\right]$$
$$= \text{cosec}\,\theta\left(\frac{2\,\text{cosec}\,\theta}{\text{cosec}^2\theta - 1}\right)$$
$$= \frac{2\,\text{cosec}^2\theta}{\text{cosec}^2\theta - 1} = \frac{2\,\text{cosec}^2\theta}{\cot^2\theta}$$
$$= \frac{2 \times \frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{2}{\cos^2\theta}$$
$$= 2\sec^2\theta = \text{RHS} \quad \text{Hence Proved}$$

72. If $5\tan\theta = 3$,then what is the value of $\left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$?

Sol : [Board 2020 Delhi Basic]

We have $5\tan\theta = 3 \Rightarrow \tan\theta = \frac{3}{5}$

Dividing numerator and denominator by $\cos\theta$ we have

$$\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta} = \frac{5\frac{\sin\theta}{\cos\theta} - 3}{4\frac{\sin\theta}{\cos\theta} + 3} = \frac{5\tan\theta - 3}{4\tan\theta + 3}$$
$$= \frac{5 \times \frac{3}{5} - 3}{4 \times \frac{3}{5} + 3} = \frac{3-3}{\frac{12}{5} + 3} = 0$$

PRACTICE

- If $4\tan\theta = 3$, then find $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$.
  [Board Term-1 2013]
  Ans : $\frac{1}{2}$

- If $4\tan\theta = 3$, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$
  [Board 2018]
  Ans : $\frac{13}{11}$

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$= 2 = \text{RHS}$

113. Prove that $\dfrac{\sin A - \cos A - 1}{\sin A + \cos A - 1} = \dfrac{1}{\sec A - \tan A}$

Sol : [Board 2019 Delhi]

$$\text{LHS} = \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$$

$$= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{1 + \sin A}{1 + \sin A}$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\sin A + \cos A - 1 + \sin^2 A + \cos A \sin A - \sin A}$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{-1 + \cos A + (1 - \cos^2 A) + \sin A \cos A}$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\cos A(1 - \cos A + \sin A)}$$

$$= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

$$= \frac{(\sec A + \tan A)}{(\sec A - \tan A)} \times (\sec A - \tan A)$$

$$= \frac{\sec^2 A - \tan^2 A}{\sec A - \tan A}$$

$$= \frac{1}{\sec A - \tan A} = \text{RHS}$$

114. Prove that :

$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

Sol : [Board 2020 Delhi Standard]

$$\text{LHS} = 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta] + -3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$$

$$= -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) + 1$$

$$= -(\sin^2\theta + \cos^2\theta)^2 + 1$$

$$= -1 + 1 = 0 = \text{RHS}$$

115. Prove that

$\dfrac{\tan^2 A}{\tan^2 A - 1} + \dfrac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \dfrac{1}{1 - 2\cos^2 A}$

Sol : [Board 2019 Delhi]

$$\text{LHS} = \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A \sin^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A}$$

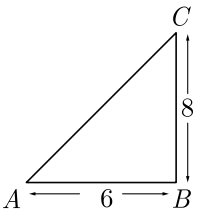
$$= \frac{1}{1 - 2\cos^2 A}$$

$$= \text{RHS}$$

116. If in a triangle ABC right angled at B, $AB = 6$ units and $BC = 8$ units, then find the value of $\sin A \cos C + \cos A \sin C$.

Sol : [Board Term-1 2016]

As per question statement figure is shown below.



We have $AC^2 = 8^2 + 6^2 = 100$

$AC = 10$ cm

Now $\sin A = \dfrac{BC}{AC} = \dfrac{8}{10}$;

$\cos A = \dfrac{AB}{AC} = \dfrac{6}{10}$

and $\sin C = \dfrac{AB}{AC} = \dfrac{6}{10}$;

$\cos C = \dfrac{BC}{AC} = \dfrac{8}{10}$

Thus $\sin A \cos C + \cos A \sin C$

$$= \frac{8}{10} \times \frac{8}{10} + \frac{6}{10} \times \frac{6}{10}$$

$$= \frac{64}{100} + \frac{36}{100} = \frac{100}{100} = 1$$

117. Evaluate : $\dfrac{5\cos^2 60^\circ + 4\cos^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 60^\circ}$

Sol : [Board Term-1 OD 2013]

$$\frac{5\cos^2 60^\circ + 4\cos^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 60^\circ}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + 3 - 1}{\frac{1}{4} + \frac{1}{4}}$$

$$= \frac{\frac{5}{4} + 2}{\frac{1}{2}} = \frac{\frac{13}{4}}{\frac{1}{2}} = \frac{13}{2}$$

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118. If $\cos(40^\circ + x) = \sin 30^\circ$, find the value of x.

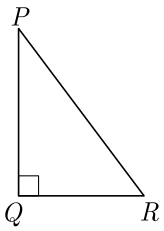
Sol : [Board Term-1 2015]

We have

$$\begin{aligned}\cos(40^\circ - x) &= \sin 30^\circ\\ \cos(40^\circ + x) &= \sin(90^\circ - 60^\circ)\\ \cos(40^\circ + x) &= \cos 60^\circ\\ 40^\circ + x &= 60^\circ\\ x &= 60^\circ - 40^\circ = 20^\circ\end{aligned}$$

Thus $x = 20^\circ$.

119. In the given $\angle PQR$, right-angled at Q, $QR = 9$ cm and $PR - PQ = 1$ cm. Determine the value of $\sin R + \cos R$.



Sol : [Board Term-1 2015]

Using Pythagoras theorem we have

$$\begin{aligned}PQ^2 + QR^2 &= PR^2\\ PQ^2 + 9^2 &= (PQ+1)^2\\ PQ^2 + 81 &= (PQ+1)^2\\ PQ^2 + 81 &= PQ^2 + 1 + 2PQ\\ PQ &= 40\end{aligned}$$

Since $PR - PQ = 1$, thus,

$$\begin{aligned}PR &= 1 + 40 = 41\\ \sin R + \cos R &= \frac{40}{41} + \frac{9}{41} = \frac{49}{41}\end{aligned}$$

120. Verify : $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{\sin\theta}{1+\cos\theta}$, for $\theta = 60^\circ$

Sol : [Board Term-1 Delhi 2013]

$$\begin{aligned}\text{LHS} &= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos 60^\circ}{1+\cos 60^\circ}}\\ &= \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}}\\ &= \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}} = \frac{1}{\sqrt{3}} \qquad \left(\cos 60^\circ = \frac{1}{2}\right)\\ \text{RHS} &= \frac{\sin\theta}{1+\cos\theta} = \frac{\sin 60^\circ}{1+\cos 60^\circ}\\ &= \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}\\ \text{RHS} &= \text{LHS}\end{aligned}$$

Hence, relation is verified for $\theta = 60^\circ$.

121. If $\tan A + \cot A = 2$, then find the value of $\tan^2 A + \cot^2 A$.

Sol : [Board Term-1 2015]

We have $\tan A + \cot A = 2$

Squaring both sides, we have

$$\begin{aligned}(\tan A + \cot A)^2 &= (2)^2\\ \tan^2 A + \cot^2 A + 2\tan A\cot A &= 4\\ \tan^2 A + \cot^2 A + 2\tan A \times \frac{1}{\tan A} &= 4\\ \tan^2 A + \cot^2 A + 2 &= 4\\ \tan^2 A + \cot^2 A &= 4 - 2\\ \tan^2 A + \cot^2 A &= 2\end{aligned}$$

122. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Sol : [Board Term-1 2011]

We have $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

We have $\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$$\begin{aligned}&= (\sqrt{2}-1)\cos\theta\\ &= \frac{(\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}+1)}\cos\theta\end{aligned}$$

Thus $\sin\theta = \dfrac{1}{\sqrt{2}+1}\cos\theta$

$$\begin{aligned}(\sqrt{2}+1)\sin\theta &= \cos\theta\\ \sqrt{2}\sin\theta + \sin\theta &= \cos\theta\\ \cos\theta - \sin\theta &= \sqrt{2}\sin\theta\end{aligned}$$ Hence proved.

123. Prove that : $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \sin A + \cos A$.

Sol : [Board Term-1 2013, 2011]

$$\begin{aligned}\text{LHS} &= \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}\\ &= \frac{\cos A}{1-\left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1-\left(\frac{\cos A}{\sin A}\right)}\\ &= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}\\ &= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}\\ &= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}\end{aligned}$$

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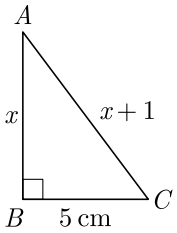
$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$
$$= \cos A + \sin A$$
$$= \sin A + \cos A$$
$$= \text{RHS} \qquad \text{Hence proved.}$$

124. In ΔABC, $\angle B = 90^\circ$, $BC = 5$ cm, $AC - AB = 1$, Evaluate : $\frac{1+\sin C}{1+\cos C}$.

Sol : [Board Term-1 2011]

As per question we have drawn the figure given below.



We have $AC - AB = 1$

Let $AB = x$, then we have

$$AC = x + 1$$

Now $AC^2 = AB^2 + BC^2$

$$(x+1)^2 = x^2 + 5^2$$
$$x^2 + 2x + 1 = x^2 + 25$$
$$2x = 24$$
$$x = \frac{24}{2} = 12 \text{ cm}$$

Hence, $AB = 12$ cm and $AC = 13$ cm

Now $\sin C = \frac{AB}{AC} = \frac{12}{13}$

$$\cos C = \frac{BC}{AC} = \frac{5}{13}$$

Now $\frac{1+\sin C}{1+\cos C} = \frac{1+\frac{12}{13}}{1+\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$

125. Prove that : $\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A} = \cos A - \sin A$

Sol : [Board Term-1 2016]

$$\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A}$$
$$= \frac{\cos A}{1+\frac{\sin A}{\cos A}} - \frac{\sin A}{1+\frac{\cos A}{\sin A}}$$
$$= \frac{\cos^2 A}{\cos A + \sin A} - \frac{\sin^2 A}{\sin A + \cos A}$$
$$= \frac{\cos^2 A - \sin^2 A}{(\sin A + \cos A)}$$
$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A + \cos A}$$
$$= \cos A - \sin A \qquad \text{Hence Proved.}$$

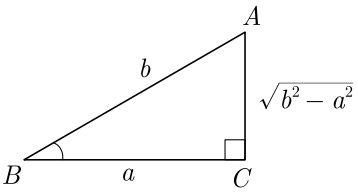
126. If $b\cos\theta = a$, then prove that $\text{cosec}\,\theta + \cot\theta = \sqrt{\frac{b+a}{b-a}}$.

Sol : [Board Term-1 2015]

We have $b\cos\theta = a$

or, $\cos\theta = \frac{a}{b}$

Now consider the triangle shown below.



$$AC^2 = AB^2 - BC^2$$

or, $\cos\theta = \frac{a}{b}$

$$AC = \sqrt{b^2 - a^2}$$

Now $\text{cosec}\,\theta = \frac{b}{\sqrt{b^2-a^2}}, \cot\theta = \frac{a}{\sqrt{b^2-a^2}}$

$$\text{cosec}\,\theta + \cot\theta = \frac{b+a}{\sqrt{b^2-a^2}} = \sqrt{\frac{b+a}{b-a}}$$

127. Prove that : $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \tan\theta$

Sol : [Bard Term-1 2015]

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$
$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$
$$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$$
$$= \tan\theta$$

128. When is an equation called 'an identity'. Prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$.

Sol : [Board Term-1 2015]

Equations that are true no matter what value is plugged in for the variable. On simplifying an identity equation, one always get a true statement. Consider the triangle shown below.

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$$\cos\theta = \frac{\sqrt{5}}{\sqrt{6}}$$

(1) $\dfrac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \dfrac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$

$$= \frac{\cot^2\theta - \tan^2\theta}{2+\cot^2\theta+\tan^2\theta}$$

$$= \frac{\left(\sqrt{5}\right)^2 - \left(\frac{1}{5}\right)^2}{2+\left(\sqrt{5}\right)^2+\left(\frac{1}{\sqrt{5}}\right)^2}$$

$$= \frac{5-\frac{1}{5}}{2+5+\frac{1}{5}}$$

$$= \frac{25-1}{35+1} = \frac{24}{36} = \frac{2}{3}$$

(2) $\sin^2\theta + \cos^2\theta = \left(\dfrac{1}{\sqrt{6}}\right)^2 + \left(\dfrac{\sqrt{5}}{\sqrt{6}}\right)^2$

$$= \frac{1}{6} + \frac{5}{6} = \frac{6}{6}$$

$$= 1 \qquad \text{Hence proved.}$$

172. If $\sec\theta + \tan\theta = p$, show that $\sec\theta - \tan\theta = \dfrac{1}{p}$, Hence, find the values of $\cos\theta$ and $\sin\theta$.

Sol : [Board Term-1 2015]

We have $\sec\theta + \tan\theta = p \qquad (1)$

Now $\dfrac{1}{p} = \dfrac{1}{\sec\theta+\tan\theta} \times \dfrac{(\sec\theta-\tan\theta)}{(\sec\theta-\tan\theta)}$

$$= \frac{\sec\theta-\tan\theta}{\sec^2\theta-\tan^2\theta} = \sec\theta - \tan\theta$$

or $\dfrac{1}{p} = \sec\theta - \tan\theta \qquad (2)$

Solving $\sec\theta + \tan\theta = p$ and $\sec\theta - \tan\theta = \dfrac{1}{p}$,

$$\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right) = \frac{p^2+1}{2p}$$

Thus $\cos\theta = \dfrac{2p}{p^2+1}$

and $\tan\theta = \dfrac{1}{2}\left(p - \dfrac{1}{p}\right) = \dfrac{p^2-1}{2p}$

and $\sin\theta = \tan\theta\cos\theta = \dfrac{p^2-1}{p^2+1}$

173. Prove that : $(\operatorname{cosec}\theta + \cot\theta)^2 = \dfrac{\sec\theta+1}{\sec\theta-1}$

Sol :

$(\operatorname{cosec}\theta + \cot\theta)^2 = \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta.\cot\theta$

$$= \left(\frac{1}{\sin\theta}\right)^2 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 + \frac{2\times 1}{\sin\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} + \frac{2\cos\theta}{\sin^2\theta}$$

$$= \frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}$$

$$= \frac{\sec\theta+1}{\sec\theta-1} \qquad \text{Hence Prove.}$$

174. Prove that :

$(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A \operatorname{cosec} A)^2$

Sol : [Board Term-1 2014]

LHS $= (\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$

$$= \left(\sin A + \frac{1}{\cos A}\right)^2 + \left(\cos A + \frac{1}{\sin A}\right)^2$$

$$= \sin^2 A + \frac{1}{\cos^2 A} + 2\frac{\sin A}{\cos A} + \cos^2 A + \frac{1}{\sin^2 A} + 2\frac{\cos A}{\sin A}$$

$$= \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} + 2\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} + 2\left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right)$$

$$= 1 + \frac{1}{\sin^2 A \cos^2 A} + \frac{2}{\sin A \cos A}$$

$$= \left(1 + \frac{1}{\sin A \cos A}\right)^2$$

$$= (1 + \sec A \operatorname{cosec} A)^2 \qquad \text{Hence Proved}$$

175. If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$
$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
Prove that each of the side is equal to ± 1.

Sol : [Board Term-1 2012]

We have

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

Multiply both sides by

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

or, $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \times$

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$

or, $(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= (\sec A - \tan A)^2(\sec A - \tan B)^2(\sec C - \tan C)^2$

or, $1 = \left[(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)\right]^2$

or, $(\sec A - \tan A)(\sec B - \tan B)(\sec C + \tan C) = \pm 1$

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$$= x^2 - 2x\frac{1}{4x} + \frac{1}{16x^2}$$

$$\tan^2\theta = \left(x - \frac{1}{4x}\right)^2$$

Taking square root both sides we obtain

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

Now $\tan\theta = x - \frac{1}{4x}$ (2)

or $\tan\theta = -\left(x - \frac{1}{4x}\right) = -x + \frac{1}{4x}$ (3)

Adding (1) and (2) we have

$$\tan\theta + \sec\theta = 2x$$

Adding (1) and (3) we have

$$\sec\theta + \tan\theta = \frac{1}{4x} + \frac{1}{4x} = \frac{1}{2x}$$ Hence proved.

134. Prove that : $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2\theta - 1}$

Sol : [Board Term-1 2011]

$$\text{LHS} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)}$$

$$= \frac{1 + 1}{\sin^2\theta - 1 + \sin^2\theta}$$

$$= \frac{2}{2\sin^2\theta - 1}$$

$$= \text{RHS}$$ Hence Proved.

135. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, prove that $x^2 + y^2 = 1$.

Sol : [Board Term-1 2011]

We have $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ (1)

and $x\sin\theta = y\cos\theta$

or, $x = \frac{y\cos\theta}{\sin\theta}$ (2)

Eliminating x from equation (1) and (2) we obtain,

$$\frac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$$

$$y(\sin^2\theta + \cos^2\theta) = \sin\theta$$

$$y = \sin\theta$$ ...(3)

Substituting this value of y in equation (2) we have,

$$x = \cos\theta$$ (4)

Squaring and adding equation (3) and (4), we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$$ Hence Proved.

136. Prove that $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$

Sol : [Board Term-1 2011]

$$X = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$$

$$= (1 - \sin\theta\cos\theta)$$

$$Y = \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$

$$= (1 + \sin\theta\cos\theta)$$

Now given expression

$$X + Y = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= (1 - \sin\theta\cos\theta) + (1 + \sin\theta\cos\theta)$$

$$= 2 - \sin\theta\cos\theta + \sin\theta\cos\theta$$

$$= 2 = \text{RHS}$$ Hence Proved.

137. Express : $\sin A, \tan A$ and $\text{cosec}\, A$ in terms of $\sec A$.

Sol : [Board Term-1 2011]

(1) $\sin^2 A + \cos^2 A = 1$

$$\sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \frac{1}{\sec^2 A}}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

(2) $\tan A = \frac{\sin A}{\cos A} = \sin A \sec A$

$$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \times \sec A$$

$$= \sqrt{\sec^2 A - 1}$$

(3) $\text{cosec}\, A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

138. Show that : $\frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta)\tan(30^\circ - \theta)} = 1$

Sol : [Board 2020 OD Standard]

$$\text{LHS} = \frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta)\tan(30^\circ - \theta)}$$

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$$= \frac{3+2\sqrt{3}+1}{(\sqrt{3})^2-(1)^2} = \frac{4+2\sqrt{3}}{2}$$
$$= 2+\sqrt{3}$$

Hence $\tan 75^\circ = 2+\sqrt{3}$

(ii) $\tan 90^\circ = \tan(60^\circ+30^\circ)$

$$= \frac{\tan 60^\circ + \tan 30^\circ}{1-\tan 60^\circ \tan 30^\circ}$$
$$= \frac{\sqrt{3}+\frac{1}{\sqrt{3}}}{1-\sqrt{3}\times\frac{1}{\sqrt{3}}} = \frac{\frac{3+1}{\sqrt{3}}}{0}$$

Hence, $\tan 90^\circ = \infty$

148. Evaluate :

$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}$

Sol : [Board Term-1 2013]

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}$$
$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24}$$
$$= \frac{1}{4}\left(\frac{1}{2}\right) + 4\left(\frac{1}{3}\right) + \frac{1}{2} + \frac{1}{24} = \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$$
$$= \frac{3+32+12+1}{24} = \frac{48}{24} = 2$$

149. Evaluate :

$4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45 - \sin^2 90^\circ)$

Sol : [Board Term-1 2013]

$$4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45 - \sin^2 90^\circ)$$
$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$
$$= 4\left[\frac{1}{16}+\frac{1}{16}\right] - 3\left[\frac{1}{2}-1\right]$$
$$= 4\left(\frac{2}{16}\right) - 3\left(-\frac{1}{2}\right)$$
$$= \frac{1}{2}+\frac{3}{2} = \frac{4}{2} = 2$$

150. If $15\tan^2\theta + 4\sec^2\theta = 23$, then find the value of $(\sec\theta + \operatorname{cosec}\theta)^2 - \sin^2\theta$.

Sol : [Board Term-1 2012]

We have $15\tan^2\theta + 4\sec^2\theta = 23$

$$15\tan^2\theta + 4(\tan^2\theta + 1) = 23$$
$$15\tan^2\theta + 4\tan^2\theta + 4 = 23$$
$$19\tan^2\theta = 19$$
$$\tan\theta = 1 = \tan 45^\circ$$

Thus $\theta = 45^\circ$

Now, $(\sec\theta + \operatorname{cosec}\theta)^2 - \sin^2\theta$

$$= (\sec 45^\circ + \operatorname{cosec} 45^\circ)^2 - \sin^2 45^\circ$$
$$= \left(\sqrt{2}+\sqrt{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= (2\sqrt{2})^2 - \frac{1}{2} = 8 - \frac{1}{2} = \frac{15}{2}$$

151. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$.

Sol : [Board 2009]

We have $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

Let $\cot\theta = x$, then we have

$$\sqrt{3}x^2 - 4x + \sqrt{3} = 0$$
$$\sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$$
$$(x-\sqrt{3})(\sqrt{3}x - 1) = 0$$
$$x = \sqrt{3} \text{ or } \frac{1}{\sqrt{3}}$$

Thus $\cot\theta = \sqrt{3}$ or $\cot\theta = \frac{1}{\sqrt{3}}$

Therefore $\theta = 30^\circ$ or $\theta = 60^\circ$

If $\theta = 30^\circ$, then

$$\cot^2 30^\circ + \tan^2 30^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$
$$= 3 + \frac{1}{3} = \frac{10}{3}$$

If $\theta = 60^\circ$, then

$$\cot^2 60^\circ + \tan^2 60^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$
$$= \frac{1}{3} + 3 = \frac{10}{3}.$$

152. Evaluate the following :

$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$$

Sol : [Board Term-1 2012]

$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$$
$$= \frac{2\left(\frac{1}{2}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2(1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$
$$= \frac{\frac{2}{4}+4-2}{\frac{1}{4}+\frac{1}{2}} = \frac{10}{3}$$

153. Prove that : $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$.

Sol : [Board Term-1 2012]

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

Thus $\sec A = \dfrac{1}{\cos A} = \dfrac{4}{\sqrt{7}}$

143. Prove that: $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$

Sol : [Board 2019 OD]

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + 1 + \tan\theta)}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^2\theta + 1 + \tan\theta}{\tan\theta}$$

$$= \tan\theta + \cot\theta + 1$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} + 1$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} + 1$$

$$= \frac{1}{\sin\theta\cos\theta} + 1$$

$$= \text{cosec}\,\theta\sec\theta + 1$$

$$= 1 + \sec\theta\,\text{cosec}\,\theta \qquad \text{Hence Proved}$$

144. Prove that: $\dfrac{\sin\theta}{\cot\theta + \text{cosec}\,\theta} = 2 + \dfrac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$

Sol : [Board 2019 OD]

$$\text{LHS} = \frac{\sin\theta}{\cot\theta + \text{cosec}\,\theta}$$

$$= \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}} = \frac{\sin^2\theta}{\cos\theta + 1}$$

$$= \frac{1 - \cos^2\theta}{\cos\theta + 1}$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta + 1}$$

$$= 1 - \cos\theta \qquad \ldots(1)$$

Now, $\text{RHS} = 2 + \dfrac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$

$$= 2 + \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}} = 2 + \frac{\sin^2\theta}{\cos\theta - 1}$$

$$= 2 + \frac{1 - \cos^2\theta}{\cos\theta - 1} = 2 - \frac{(\cos^2\theta - 1)}{(\cos\theta - 1)}$$

$$= 2 - \frac{(\cos\theta - 1)(\cos\theta + 1)}{\cos\theta - 1}$$

$$= 2 - (\cos\theta + 1) = 1 - \cos\theta$$

$$= \text{LHS} \qquad \text{Hence Proved}$$

145. Find A and B if $\sin(A+2B) = \frac{\sqrt{3}}{2}$ and $\cos(A+4B) = 0$, where A and B are acute angles.

Sol : [Board 2019 OD]

We have $\sin(A+2B) = \dfrac{\sqrt{3}}{2}$

$$\sin(A+2B) = \sin 60^\circ \qquad \left(\sin 60^\circ = \tfrac{\sqrt{3}}{2}\right)$$

$$A + 2B = 60^\circ \qquad \ldots(1)$$

Also, given $\cos(A+4B) = 0$

$$\cos(A+4B) = \cos 90^\circ \qquad (\cos 90^\circ = 0)$$

$$A + 4B = 90^\circ \qquad \ldots(2)$$

Subtracting equation (2) from equation (1) we get

$$-2B = -30^\circ \Rightarrow B = 15^\circ$$

From equation (1) we have

$$A + 2(15^\circ) = 60^\circ$$

$$A = 60^\circ - 30^\circ = 30^\circ$$

Hence angle $A = 30^\circ$ and angle $B = 15^\circ$.

146. Evaluate :

$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

Sol : [Board Term-1 2015]

$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times (1)^2 \times (\sqrt{3})^2 - 2 \times 1 \times 1^2 \times 1$$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 3 - 2$$

$$= \frac{1}{6} + \frac{3}{2} - 2 = \frac{1 + 9 - 12}{6} = -\frac{2}{6} = -\frac{1}{3}$$

147. Given that

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$,

find the values of $\tan 75^\circ$ and $\tan 90^\circ$ by taking suitable values of A and B.

Sol : [Board Term-1 2012]

We have $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(i) $\tan 75^\circ = \tan(45^\circ + 30^\circ)$

$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

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$$= 2 \times \frac{1}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta$$ Hence Proved

159. Prove that : $\dfrac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \dfrac{\sec\theta+1}{\sec\theta-1}$.

Sol : [Board 2010]

$$\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{\sin\theta}{\cos\theta}-\sin\theta}$$

$$= \frac{\sin\theta\left(\frac{1}{\cos\theta}+1\right)}{\sin\theta\left(\frac{1}{\cos\theta}-1\right)}$$

$$= \frac{\sec\theta+1}{\sec\theta-1}$$

Hence Proved.

160. Prove that : $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A+1} = 2\sec^2 A$

Sol : [Board Term-1 2011]

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}$$

$$= \frac{\operatorname{cosec}^2 A+\operatorname{cosec} A+\operatorname{cosec}^2 A-\operatorname{cosec} A}{(\operatorname{cosec} A-1)(\operatorname{cosec} A+1)}$$

$$= \frac{2\operatorname{cosec}^2 A}{\operatorname{cosec}^2 A-1} = \frac{2\operatorname{cosec}^2 A}{\cot^2 A}$$

$$= \frac{\frac{2}{\sin^2 A}}{\frac{\cos^2 A}{\sin^2 A}} = \frac{2}{\sin^2 A}\times\frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{2}{\cos^2 A} = 2\sec^2 A$$ Hence Proved.

161. If $\operatorname{cosec}\theta+\cot\theta = p$, then prove that $\cos\theta = \dfrac{p^2-1}{p^2+1}$.

Sol : [Board Term-1 2016]

$$\frac{p^2-1}{p^2+1} = \frac{(\operatorname{cosec}\theta+\cot\theta)^2-1}{(\operatorname{cosec}\theta+\cot\theta)^2+1}$$

$$= \frac{\operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta-1}{\operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta+1}$$

$$= \frac{1+\cot^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta-1}{\operatorname{cosec}^2\theta+\operatorname{cosec}^2\theta-1+2\operatorname{cosec}\theta\cot\theta+1}$$

$$= \frac{2\cot\theta(\cot\theta+\operatorname{cosec}\theta)}{2\operatorname{cosec}\theta(\operatorname{cosec}\theta+\cot\theta)}$$

$$= \frac{\cos\theta}{\sin\theta}\times\sin\theta = \cos\theta$$

162. Prove that : $\dfrac{\cos^2\theta}{1-\tan\theta}+\dfrac{\sin^3\theta}{\sin\theta-\cos\theta} = 1+\sin\theta\cos\theta$.

Sol : [Board 2007]

$$\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta} = \frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}$$

$$= \frac{\cos^3\theta}{\cos\theta-\sin\theta}-\frac{\sin^3\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$$

$$= \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta)}{(\cos\theta-\sin\theta)}$$

$$= 1+\sin\theta\cos\theta$$ Hence Proved

163. If $a\cos\theta+b\sin\theta = m$ and $a\sin\theta-b\cos\theta = n$, prove that $m^2+n^2 = a^2+b^2$

Sol : [Board Term-1 2012]

We have

$$m^2 = a^2\cos^2\theta+2ab\sin\theta\cos\theta+b^2\sin^2\theta \quad ...(1)$$

and $n^2 = a^2\sin^2\theta-2ab\sin\theta\cos\theta+b^2\cos^2\theta$ ...(2)

Adding equations (1) and (2) we get

$$m^2+n^2 = a^2(\cos^2\theta+\sin^2\theta)+b^2(\cos^2\theta+\sin^2\theta)$$

$$= a^2(1)+b^2(1)$$

$$= a^2+b^2$$

164. If $\cos\theta+\sin\theta = p$ and $\sec\theta+\operatorname{cosec}\theta = q$, prove that $q(p^2-1) = 2p$

Sol : [Board Term-1 2014]

We have $\cos\theta+\sin\theta = p$ and $\sec\theta+\operatorname{cosec}\theta = q$

$$q(p^2-1) = (\sec\theta+\operatorname{cosec}\theta)[(\cos\theta+\sin\theta)^2-1]$$

$$= (\sec\theta+\operatorname{cosec}\theta)(\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta-1)$$

$$= (\sec\theta+\operatorname{cosec}\theta)[1+2\sin\theta\cos\theta-1]$$

$$= \left(\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$

$$= \left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right)2\sin\theta\cos\theta$$

$$= 2(\sin\theta+\cos\theta) = 2p$$ Hence Proved.

165. If $x = r\sin A\cos C$, $y = r\sin A\sin C$ and $z = r\cos A$, then prove that $x^2+y^2+z^2 = r^2$

Sol : [Board Term-1 2011]

Since, $x^2 = r^2\sin^2 A\cos^2 C$

$y^2 = r^2\sin^2 A\sin^2 C$

and $z^2 = r^2\cos^2 A$

$$x^2+y^2+z^2 = r^2\sin^2 A\cos^2 C+r^2\sin^2 A\sin^2 C+r^2\cos^2 A$$

$$= r^2\sin^2 A(\cos^2 C+\sin^2 C)+r^2\cos^2 A$$

$$= r^2\sin^2 A+r^2\cos^2 A$$

$$= r^2(\sin^2 A+\cos^2 A)$$

$$= r^2$$ Hence Proved.

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$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1-\tan\theta)\tan\theta}$$
$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$
$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$
$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)}$$
$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$
$$= \tan\theta + 1 + \cot\theta$$

Hence Proved.

154. In an acute angled triangle ABC if $\sin(A+B-C) = \frac{1}{2}$ and $\cos(B+C-A) = \frac{1}{\sqrt{2}}$ find $\angle A, \angle B$ and $\angle C$.

Sol : [Board 2009]

We have $\sin(A+B-C) = \frac{1}{2} = \sin 30^\circ$

$$A + B - C = 30^\circ \quad ...(1)$$

and $\cos(B+C-A) = \frac{1}{\sqrt{2}} = \cos 45^\circ$

$$B + C - A = 45^\circ \quad ...(2)$$

Adding equation (1) and (2), we get

$$2B = 75^\circ \Rightarrow B = 37.5^\circ$$

Subtracting equation (2) from equation (1) we get,

$$2(A - C) = -15^\circ$$
$$A - C = -7.5^\circ \quad ...(3)$$

Now $A + B + C = 180^\circ$

$$A + C = 180^\circ - 37.5^\circ = 142.5^\circ \quad ...(4)$$

Adding equation (3) and (4), we have

$$2A = 135^\circ \Rightarrow A = 67.5^\circ$$

and, $C = 75^\circ$

Hence, $\angle A = 67.5^\circ, \angle B = 37.5^\circ, \angle C = 75^\circ$

155. Prove that $b^2x^2 - a^2y^2 = a^2b^2$, if :

(1) $x = a\sec\theta, y = b\tan\theta$, or

(2) $x = a\,\text{cosec}\,\theta, y = b\cot\theta$

Sol : [Board Term-1 2016]

(1) We have $x = a\sec\theta, y = b\tan\theta$,

$$\frac{x^2}{a^2} = \sec^2\theta, \ \frac{y^2}{b^2} = \tan^2\theta$$

or, $$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1$$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

(ii) We have $x = a\,\text{cosec}\,\theta, y = b\cot\theta$

$$\frac{x^2}{a^2} = \text{cosec}^2\theta, \ \frac{y^2}{b^2} = \cot^2\theta$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \text{cosec}^2\theta - \cot\theta = 1$$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

156. If $\text{cosec}\,\theta - \cot\theta = \sqrt{2}\cot\theta$, then prove that $\text{cosec}\,\theta + \cot\theta = \sqrt{2}\,\text{cosec}\,\theta$.

Sol : [Board Term-1 2015]

We have $\text{cosec}\,\theta - \cot\theta = \sqrt{2}\cot\theta$

Squaring both sides we have

$$\text{cosec}^2\theta + \cot^2\theta - 2\,\text{cosec}\,\theta\cot\theta = 2\cot^2\theta$$
$$\text{cosec}^2\theta - \cot^2\theta = 2\,\text{cosec}\,\theta\cot\theta$$
$$(\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 2\,\text{cosec}\,\theta\cot\theta$$
$$(\text{cosec}\,\theta + \cot\theta)\sqrt{2}\cot\theta = 2\,\text{cosec}\,\theta\cot\theta$$
$$\text{cosec}\,\theta + \cot\theta = \sqrt{2}\,\text{cosec}\,\theta$$

Hence Proved.

157. Prove that :

$$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta + \sin\theta)^2} = \frac{\sec\theta\,\text{cosec}\,\theta - 1}{\text{cosec}\,\theta + \sec\theta}$$

Sol : [Board Term-1 2015]

$$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta + \sin\theta)^2}$$
$$= \frac{\frac{\cos^3\theta}{\sin^3\theta} \times \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\frac{\sin^3\theta}{\cos^3\theta} \times \cos^3\theta}{(\cos\theta + \sin\theta)^2}$$
$$= \frac{\cos^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\sin^3\theta}{(\cos\theta + \sin\theta)^2}$$
$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)^2}$$
$$= \frac{1 - \sin\theta\cos\theta}{\cos\theta + \sin\theta} = \frac{\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}}{\frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta}}$$
$$= \frac{\text{cosec}\,\theta\sec\theta - 1}{\text{cosec}\,\theta + \sec\theta}$$ Hence Proved

158. Prove that : $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\,\text{cosec}\,\theta.$

Sol : [Board 2008]

$$\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$$
$$= \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{(\sec\theta + 1)(\sec\theta - 1)}}$$
$$= \frac{2\sec\theta}{\sqrt{\sec^2\theta - 1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta}$$
$$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$$

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$$= \frac{\cos^2(45^\circ+\theta)+\sin^2(90^\circ-45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(90^\circ-30^\circ+\theta)}$$
$$= \frac{\cos^2(45^\circ+\theta)+\sin^2(45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(60^\circ+\theta)}$$
$$= \frac{1}{1} = 1 = \text{RHS}$$

FIVE MARKS QUESTIONS

139. Prove that: $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\text{cosec}\,\theta$

Sol : [Board 2024 OD Standard]

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$$
$$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$
$$= \frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta(1-\tan\theta)}$$
$$= \frac{\tan^2\theta}{\tan\theta-1}-\frac{1}{\tan\theta(\tan\theta-1)}$$
$$= \frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$$
$$= \frac{(\tan\theta-1)(\tan^2\theta+1+\tan\theta)}{\tan\theta(\tan\theta-1)}$$
$$= \frac{\tan^2\theta+1+\tan\theta}{\tan\theta}$$
$$= \tan\theta+\cot\theta+1$$
$$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}+1$$
$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}+1$$
$$= \frac{1}{\sin\theta\cos\theta}+1$$
$$= \text{cosec}\,\theta\sec\theta+1$$
$$= 1+\sec\theta\,\text{cosec}\,\theta$$
$$= \text{RHS}$$

Hence Proved

140. If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$.

Sol : [Board 2020 Delhi Standard]

We have $\sin\theta+\cos\theta = \sqrt{3}$

Squaring both the sides, we get

$$(\sin\theta+\cos\theta)^2 = (\sqrt{3})^2$$
$$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta = 3$$
$$1+2\sin\theta\cos\theta = 3$$
$$2\sin\theta\cos\theta = 3-1 = 2$$
$$\sin\theta\cos\theta = 1 \quad ...(1)$$

Now $\tan\theta+\cot\theta = \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

or $\tan\theta+\cot\theta = \frac{1}{\sin\theta\cos\theta}$

Substituting the value of $\sin\theta\cos\theta$ from equation (1) we have

$$\tan\theta+\cot\theta = \frac{1}{1}$$

Hence, $\tan\theta+\cot\theta = 1$

141. If $\sec\theta = x+\frac{1}{4x}, x\neq 0$ find $(\sec\theta+\tan\theta)$.

Sol : [Board 2019 Delhi]

We have $\sec\theta = x+\frac{1}{4x}$ ...(1)

Since, $\tan^2\theta = \sec^2\theta-1$

Substituting value of $\sec\theta$ we have

$$\tan^2\theta = \left(x+\frac{1}{4x}\right)^2-1$$
$$= x^2+\frac{2x}{4x}+\frac{1}{16x^2}-1$$
$$= x^2+\frac{1}{16x^2}-\frac{1}{2}$$
$$= \left(x-\frac{1}{4x}\right)^2$$
$$\tan\theta = \pm\left(x-\frac{1}{4x}\right)$$

When $\sec\theta = x+\frac{1}{4x}$ and $\tan\theta = x-\frac{1}{4x}$ we have

$$\sec\theta+\tan\theta = \left(x+\frac{1}{4x}\right)+\left(x-\frac{1}{4x}\right) = 2x$$

When $\sec\theta = x+\frac{1}{4x}$ and $\tan\theta = -\left(x-\frac{1}{4x}\right)$ we have

$$\sec\theta+\tan\theta = \left(x+\frac{1}{4x}\right)+\left\{-\left(x-\frac{1}{4x}\right)\right\}$$
$$= x+\frac{1}{4x}-x+\frac{1}{4x}$$
$$= \frac{2}{4x} = \frac{1}{2x}$$

142. If $\sin A = \frac{3}{4}$ calculate $\sec A$.

Sol : [Board 2019 OD]

We have $\sin A = \frac{3}{4}$

Now $\cos^2 A = 1-\sin^2 A$

$$\cos^2 A = 1-\left(\frac{3}{4}\right)^2 = 1-\frac{9}{16} = \frac{7}{16}$$
$$\cos A = \frac{\sqrt{7}}{4}$$

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166. Prove that: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta$.

Sol : [Board Term-1 2017]

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)}\times\frac{(1+\sin\theta)}{(1+\sin\theta)}}+\sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)}\times\frac{(1-\sin\theta)}{(1-\sin\theta)}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}}+\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$=\frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}$$

$$=\frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$

$$=\frac{2}{\cos\theta}=2\sec\theta$$ Hence Proved

167. Prove that

$(1-\sin\theta+\cos\theta)^2=2(1+\cos\theta)(1-\sin\theta)$.

Sol : [Board Term-1 2013]

$$(1-\sin\theta+\cos\theta)^2$$

$$=1+\sin^2\theta+\cos^2\theta-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$

$$=1+1-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$

$$=2+2\cos\theta-2\sin\theta-2\sin\theta\cos\theta$$

$$=2(1+\cos\theta)-2\sin\theta(1+\cos\theta)$$

$$=(1+\cos\theta)(2-2\sin\theta)$$

$$=2(1+\cos\theta)(1-\sin\theta)$$ Hence Proved

168. Prove that : $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta-1}=\sec\theta+\tan\theta$

Sol : [Board Term-1 2012]

$$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)-(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)(1-\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$$

$$=\tan\theta+\sec\theta$$ Hence Proved

169. Prove that :

$(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2=7+\tan^2\theta+\cot^2\theta\cot^2\theta$

Sol : [Board Term-1 2012]

$$(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2$$

$$=\sin^2\theta+\operatorname{cosec}^2\theta+2\sin\theta\operatorname{cosec}\theta+\cos^2\theta$$

$$+\sec^2\theta+2\cos\theta\sec\theta$$

$$=(\sin^2\theta+\cos^2\theta)+\operatorname{cosec}^2\theta+2+\sec^2\theta+2$$

$$=1+(1+\cot^2\theta)+2+(1+\tan^2\theta)+2$$

$$=7+\tan^2\theta+\cot^2\theta$$ Hence Proved

170. If $\sin\theta=\frac{c}{\sqrt{c^2+d^2}}$ and $d>0$, find the value of $\cos\theta$ and $\tan\theta$.

Sol : [Board Term-1 2013]

We have $\sin\theta=\frac{c}{\sqrt{c^2+d^2}}$

Now $\cos^2\theta=1-\sin^2\theta$

$$=1-\left(\frac{c}{\sqrt{c^2+d^2}}\right)^2$$

$$=1-\frac{c^2}{c^2+d^2}$$

$$=\frac{c^2+d^2-c^2}{c^2+d^2}=\frac{d^2}{c^2+d^2}$$

Thus $\cos\theta=\frac{d}{\sqrt{c^2+d^2}}$

Again, $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{c}{\sqrt{c^2+d^2}}}{\frac{d}{\sqrt{c^2+d^2}}}=\frac{c}{d}$

Thus $\tan\theta=\frac{c}{d}$

171. If $\tan\theta=\frac{1}{\sqrt{5}}$,

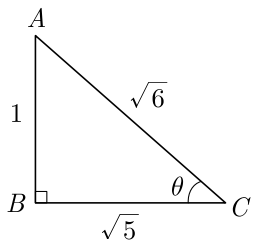
(1) Evaluate : $\frac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta}$

(2) Verify the identity : $\sin^2\theta+\cos^2\theta=1$

Sol : [Board Term-1 2012]

We have $\tan\theta=\frac{1}{\sqrt{5}}$

We draw the triangle as shown below and write all dimensions.



Now $\cot\theta=\frac{1}{\tan\theta}=\sqrt{5}$

$\sin\theta=\frac{1}{\sqrt{6}}$

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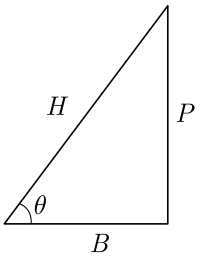

Let $\tan\theta = \frac{P}{B}$ and $\sec\theta = \frac{H}{B}$

$$H^2 = P^2 + B^2$$

Now $1 + \tan^2\theta = 1 + \left(\frac{P}{B}\right)^2 = 1 + \frac{P^2}{B^2}$

$$= \frac{B^2 + P^2}{B^2} = \frac{H^2}{B^2}$$

$$= \left(\frac{H}{B}\right)^2 = \sec^2\theta$$ Hence Proved.

129. Prove that : $(\cot\theta - \operatorname{cosec}\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Sol : [Board Term-1 2015]

$$\cot\theta - \operatorname{cosec}\theta = \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}$$

$$(\cot\theta - \operatorname{cosec}\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$

$$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$[[\sin^2\theta + \cos^2\theta = 1]]$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$ Hence Proved.

130. Prove that :
$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

Sol : [Board Term-1 2015]

$$\text{LHS} = (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \left(\frac{1}{\sin\theta\cos\theta}\right)$$

$$[\sin^2\theta + \cos^2\theta = 1]$$

$$= \cos\theta\sin\theta \times \frac{1}{\sin\theta\cos\theta} = 1$$

131. Prove that : $\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

Sol : [Board Term-1 2013]

We have

$$\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = \operatorname{cosec}^2\theta\left[\frac{1}{\frac{1}{\sin\theta} - 1} - \frac{1}{\frac{1}{\sin\theta} + 1}\right]$$

$$= \operatorname{cosec}^2\theta\left[\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta}\right]$$

$$= \frac{1}{\sin^2\theta}\sin\theta\left[\frac{(1+\sin\theta) - (1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}\right]$$

$$= \frac{1}{\sin\theta}\left[\frac{2\sin\theta}{1-\sin^2\theta}\right]$$

$$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$$ Hence Proved

132. Prove that :

$$\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}.$$

Sol : [Board Term-1 2011]

$$\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$$

$$\frac{1}{\operatorname{cosec} A - \cot A} + \frac{1}{\operatorname{cosec} A + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A}$$

$$\frac{1}{\operatorname{cosec} A - \cot A} + \frac{1}{\operatorname{cosec} A + \cot A} = \frac{2}{\sin A}$$

$$\frac{\operatorname{cosec} A + \cot A + \operatorname{cosec} A - \cot A}{(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot a)} = \frac{2}{\sin A}$$

$$\frac{2\operatorname{cosec} A}{\operatorname{cosec}^2 A - \cot^2 A} = \frac{2}{\sin A}$$

$$\frac{2\frac{1}{\sin A}}{1} = \frac{2}{\sin A}$$

$$\frac{2}{\sin A} = \frac{2}{\sin A}$$ Hence Proved.

133. If $\sec\theta = x + \frac{1}{4x}$ prove that $\sec\theta + \tan\theta = 2x$ or, $\frac{1}{2x}$

Sol : [Board Term-1 2011]

We have $\sec\theta = x + \frac{1}{4x}$ (1)

Squaring both side we have

$$\sec^2\theta = x^2 + 2x\frac{1}{4x} + \frac{1}{16x^2}$$

$$1 + \tan^2\theta = x^2 + \frac{1}{2} + \frac{1}{16x^2}$$

$$\tan^2\theta = x^2 + \frac{1}{2} + \frac{1}{16x^2} - 1$$

$$= x^2 - \frac{1}{2} + \frac{1}{16x^2}$$

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176. If $4\sin\theta = 3$, find the value of x if

$$\sqrt{\frac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$$

Sol : [Board 2009]

We have $\sin\theta = \frac{3}{4}$

or, $\sin^2\theta = \frac{9}{16}$

Since $\sin^2\theta + \cos^2 = 1$, we have

$$\cos^2\theta = 1 - \sin^2\theta$$
$$= 1 - \frac{9}{16} = \frac{7}{16}$$
$$\cos\theta = \frac{\sqrt{7}}{4}$$

and $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{3}{\sqrt{7}}$

Thus $\sqrt{\frac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$

$$\sqrt{\frac{1}{\tan^2\theta}} + 2 \times \frac{\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$
$$\frac{1}{\tan\theta} + \frac{2\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$
$$\frac{\sqrt{7}}{3} + \frac{2\sqrt{7}}{3} - \frac{\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$
$$\frac{4\sqrt{7} - \sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$
$$\frac{3\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

Thus $x = \frac{4}{3}$

177. Prove that $\sec^2\theta + \text{cosec}^2\theta$ can never be less than 2.

Sol : [Board-Term 1 2011]

Let $\sec^2\theta + \text{cosec}^2\theta = x$

$$1 + \tan^2\theta + 1 + \cot^2\theta = x$$
$$2 + \tan^2\theta + \cot^2\theta = x$$
$$2 + \tan^2\theta + \cot^2\theta = x$$

$\tan^2\theta \geq 0$ and $\cot^2\theta \geq 0$

Thus $x > 2$

Thus $\sec^2\theta + \text{cosec}^2\theta > 2$

Hence $\sec^2\theta + \text{cosec}^2\theta$ can never be less than 2.

178. (a) Solve for ϕ, if $\tan 5\phi = 1$

(b) Solve for ϕ, if $\frac{\sin\phi}{1+\cos\phi} + \frac{1+\cos\phi}{\sin\phi} = 4$

Sol :

(a) $\tan 5\phi = 1$

$$\tan 5\phi = \tan 45^\circ$$
$$5\phi = 45^\circ$$

Thus $\phi = 9^\circ$

(b) $\frac{\sin\phi}{1+\cos\phi} + \frac{1+\cos\phi}{\sin\phi} = 4$

$$\frac{\sin^2\phi + (1+\cos\theta)^2}{\sin\phi(1+\cos\phi)} = 4$$
$$\frac{\sin^2\phi + 1 + 2\cos\phi + \cos^2\phi}{\sin\phi + \sin\phi\cos\phi} = 4$$
$$\frac{\sin^2\phi + \cos^2\phi + 1 + 2\cos\phi}{\sin\phi(1+\cos\phi)} = 4$$
$$\frac{2 + 2\cos\phi}{\sin\phi(1+\cos\phi)} = 4$$
$$\frac{2(1+\cos\phi)}{\sin\phi(1+\cos\phi)} = 4$$
$$\frac{2}{\sin\phi} = 4$$
$$\sin\phi = \frac{1}{2}$$
$$\sin\phi = \sin 30^\circ$$

Thus $\phi = 30^\circ$

179. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Sol : [Board-Term 1 2009]

We have $\tan A + \sin A = m$

and $\tan A - \sin A = n$

$$m^2 - n^2 = (\tan A + \sin A)^2 - (\tan A - \sin A)^2$$
$$= (\tan^2 A + \sin^2 A + 2\sin A\tan A) - (\tan^2 A + \sin^2 A - 2\sin A\tan A)$$
$$= \tan^2 A + \sin^2 A + 2\sin A\tan A - \tan^2 A - \sin^2 A + 2\sin A\tan A$$
$$= 4\sin A\tan A$$

$$4\sqrt{mn} = 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$$
$$= 4\sqrt{\tan^2 A - \sin^2 A}$$
$$= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A}$$
$$= 4\sqrt{\frac{\sin^2 A - \sin^2 A\cos^2 A}{\cos^2 A}}$$
$$= 4\sqrt{\frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A}}$$
$$= 4\sqrt{\frac{\sin^2 A \times \sin^2 A}{\cos^2 A}}$$
$$= 4\frac{\sin A \times \sin A}{\cos A}$$
$$= 4\sin A \times \frac{\sin A}{\cos A}$$

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$$b = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$$
$$= 10 \times 1.732 = 17.32 \text{ m}$$

(iii) $\tan 30^\circ = \frac{b}{c}$

$$\frac{1}{\sqrt{3}} = \frac{10\sqrt{3}}{c}$$
$$c = 10 \times 3 = 30 \text{ m}$$

(iv) $\sin 30^\circ = \frac{b}{d}$

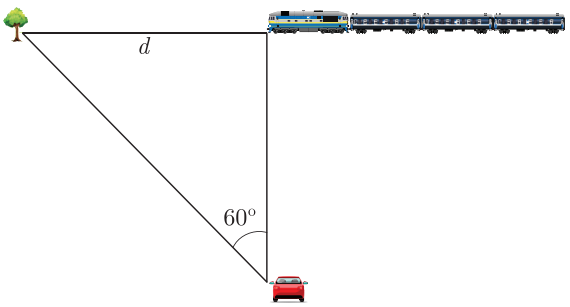
$$\frac{1}{2} = \frac{10\sqrt{3}}{d}$$
$$d = 20\sqrt{3}$$
$$= 20 \times 1.732 = 34.64 \text{ m}$$

196. Train Speed : While driving to their next gig, Sarthak and the boys get stuck in a line of cars at a railroad crossing as the gates go down. As the sleek, speedy express train approaches, Sarthak decides to pass the time estimating its speed.

He spots a large oak tree beside the track some distance away, and figures the angle of rotation from the crossing to the tree is about 60°. If their car is 40 meter from the crossing and it takes the train 3 sec to reach the tree, how fast is the train moving ?

Sol :

Let l be the length of train. We draw a diagram of the situation as shown below.



Now $\tan 60^\circ = \frac{l}{40}$

$$\sqrt{3} = \frac{l}{40}$$

Length of train, $l = 40\sqrt{3}$

$$= 40 \times 1.732 = 69.28 \text{ m}$$

Speed of train, $s = \frac{l}{t} = \frac{69.28}{3}$

$$= 23.09 \text{ m/sec}$$
$$= 23.09 \times \frac{3600}{1000}$$
$$= 83.124 \text{ km/hour}$$

Thus train is running at 83.124 km/hour speed.

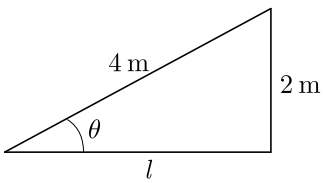
197. Ramp : A heavy-duty ramp is used to winch heavy appliances from street level up to a warehouse loading dock. If the ramp is 2 meter high and the incline is 4 meter long,

(i) What angle does the dock make with the street?

(ii) How long is the base of the ramp?

Sol :

Let θ be angle which the dock makes with the street and l be the length of ramp. We draw a diagram of the situation as shown below.



Now $\sin\theta = \frac{2}{4} = \frac{1}{2} = \sin 30^\circ$

Thus $\theta = 30^\circ$

Now $\tan\theta = \frac{2}{l}$

$$\tan 30^\circ = \frac{2}{l}$$
$$\frac{1}{\sqrt{3}} = \frac{2}{l}$$

Length of ramp, $l = 2\sqrt{3}$

$$= 2 \times 1.732 = 3.464 \text{ m}$$

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$$= \frac{2\left(\frac{1}{2}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2(1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{\frac{2}{4} + 4 - 2}{\frac{1}{4} + \frac{1}{2}} = \frac{10}{3}$$

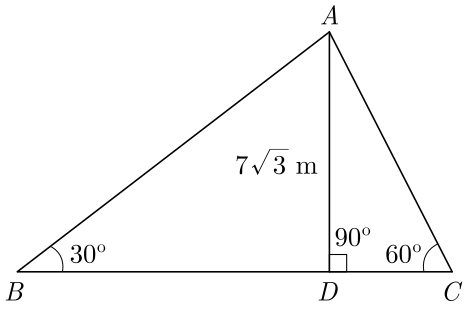
PRACTICE

Evaluate : $\dfrac{5\cos^2 60^\circ + 4\cos^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 60^\circ}$

[Board Term-1 2013]

Ans : $\frac{13}{2}$

187. In the given figure, if $AD = 7\sqrt{3}$ m, then find the value of BC.



Sol : [Board Term-2 2012]

Let $BD = x$ and $DC = y$

From ΔADB we get

$$\tan 30^\circ = \frac{7\sqrt{3}}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{7\sqrt{3}}{x}$$

$$x = 7\sqrt{3} \times \sqrt{3} = 21 \text{ m}$$

From ΔADC,

$$\tan 60^\circ = \frac{7\sqrt{3}}{y}$$

$$\sqrt{3} = \frac{7\sqrt{3}}{y} \Rightarrow y = 7 \text{ m}.$$

Now $BC = BD + DC$

$= 21 + 7 = 28$ m.

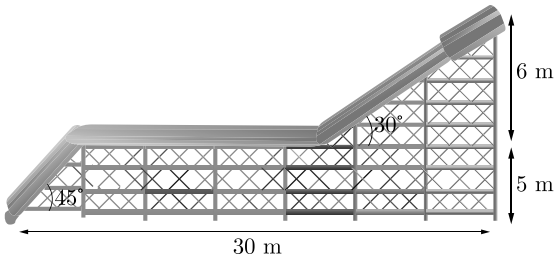
Hence, the value of BC is 28 m.

COMPETENCEY BASED QUESTIONS

188. Water Slide Design : Slide shown in the figure is part of a design for a water slide.

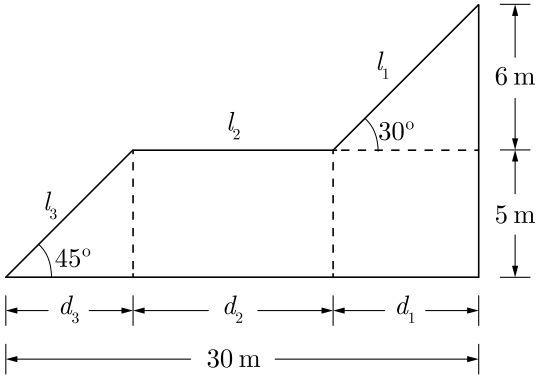
(i) Find the length of flat part of slide.

(ii) Find the total length of the slide.



Sol :

We draw a diagram of the situation as shown below. The water slide can be separated in 3 segment as shown in figure.



The length of slide is equal to the sum of l_1, l_2 and l_3.

For l_1, $\sin 30^\circ = \frac{6}{l_1}$

$$\frac{1}{2} = \frac{6}{l_1}$$

$$l_1 = 6 \times 2 = 12 \text{ m}$$

For d_1, $\tan 30^\circ = \frac{6}{d_1}$

$$\frac{1}{\sqrt{3}} = \frac{6}{d_1}$$

$$d_1 = 6 \times \sqrt{3}$$

$$= 6 \times 1.732 = 10.392 \text{ m}$$

For l_2, $l_2 = d_2$

For l_3, $\sin 45^\circ = \frac{5}{l_3}$

$$\frac{1}{\sqrt{2}} = \frac{5}{l_3}$$

$$l_1 = 5 \times \sqrt{2}$$

$$= 5 \times 1.414 = 7.07 \text{ m}$$

For d_3, $\tan 45^\circ = \frac{5}{d_3}$

$$1 = \frac{5}{d_3}$$

$$d_3 = 5 \text{ m}$$

Now $d_1 + d_2 + d_3 = 30$ m

Substituting d_1, d_2 and d_3 we have

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$= 4\sin A\tan A$

Thus $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved

180. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2+n^2)\cos^2\beta = n^2$.

Sol : [Board Term-12010]

We have $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$

$$m^2 = \frac{\cos^2\alpha}{\cos^2\beta} \text{ and } n^2 = \frac{\cos^2\alpha}{\sin^2\beta}$$

$$(m^2+n^2)\cos^2\beta = \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\left[\frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\frac{\sin^2\beta+\cos^2\beta}{\cos^2\beta\sin^2\beta}\cos^2\beta$$

$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha}{\sin^2\beta} = n^2 \quad \text{Hence Proved.}$$

181. If $7\operatorname{cosec}\phi - 3\cot\phi = 7$, prove that $7\cot\phi - 3\operatorname{cosec}\phi = 3$.

Sol : [Board Term-1 SQP 2015]

We have $7\operatorname{cosec}\phi - 3\cot\phi = 7$

$$7\operatorname{cosec}\phi - 7 = 3\cot\phi$$

$$7(\operatorname{cosec}\phi - 1) = 3\cot\phi$$

$$7(\operatorname{cosec}\phi - 1)(\operatorname{cosec}\phi + 1) = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7(\operatorname{cosec}^2\phi - 1) = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7\cot^2\phi = 3\cot\phi(\operatorname{cosec}\phi + 1)$$

$$7\cot\phi = 3(\operatorname{cosec}\phi + 1)$$

$7\cot\phi - 3\operatorname{cosec}\phi = 3$ Hence Proved

182. Prove that : $\frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$

Sol : [Board SQP 2018]

$$\text{LHS} = \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1}$$

$$= \frac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta(\cos\theta+1) - [(1-\cos\theta)(1+\cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1+\cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1+\cos\theta)(\cos\theta + \sin\theta - 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$= \operatorname{cosec}\theta + \cot\theta$ Hence Proved

183. Evaluate :

$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

Sol : [Board Term-1 2015]

$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times (1)^2 \times (\sqrt{3})^2 - 2 \times 1 \times 1^2 \times 1$$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 3 - 2$$

$$= \frac{1}{6} + \frac{3}{2} - 2 = \frac{1+9-12}{6} = -\frac{2}{6} = -\frac{1}{3}$$

184. Evaluate :

$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}$

Sol : [Board Term-1 2013]

$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}$

$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24}$$

$$= \frac{1}{4}\left(\frac{1}{2}\right) + 4\left(\frac{1}{3}\right) + \frac{1}{2} + \frac{1}{24}$$

$$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$$

$$= \frac{3+32+12+1}{24} = \frac{48}{24} = 2$$

185. Evaluate :

$4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45 - \sin^2 90^\circ)$

Sol : [Board Term-1 2013]

$4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45 - \sin^2 90^\circ)$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$$

$$= 4\left(\frac{2}{16}\right) - 3\left(-\frac{1}{2}\right) = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

186. Evaluate the following :

$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$$

Sol : [Board Term-1 2012]

$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ} = \frac{2\left(\frac{1}{2}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2(1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

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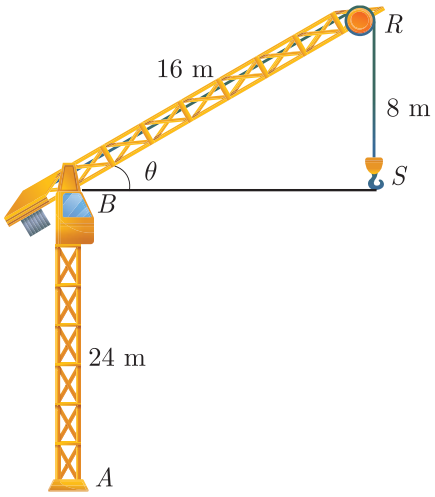
plane about B. A vertical cable, RS, carries a load S. The diagram shows current position of the jib, cable and load.



(i) What is the distance BS?

(ii) What is the angle that the jib, BR, makes with the horizontal ?

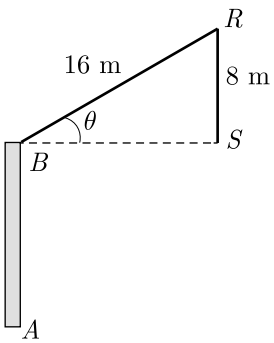
(iii) What is the measure of the angle BRS?

(iv) Now the jib BR, has been rotated and the length RS is increased. The load is now on the ground at a point 8 m from A. What is the angle through which the jib has been rotated ?

(v) What is the length by which RS has been increased?

Sol :

(i) As per information given in question we make the diagram as below.



$$16^2 = 8^2 + BS^2$$

$$BS = \sqrt{16^2 - 8^2} = 8\sqrt{3}$$

(ii) Let jib make θ with horizontal.

Now, $\sin\theta = \frac{8}{16} = \frac{1}{2} = \sin 30^\circ$

Thus $\theta = 30^\circ$

(iii) $\theta + 90^\circ + \angle BRS = 180^\circ$

$30^\circ + 90^\circ + \angle BSR = 180^\circ$

$\angle BSR = 180^\circ - 30^\circ - 90^\circ$

$= 60^\circ$

(iv) We make the digram as below on the information given.

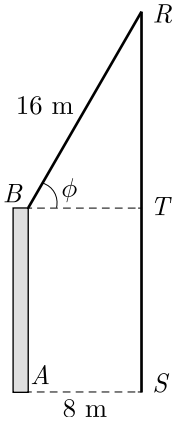
Here $BT = AS = 8$ m

In triangle ΔBTR,

$$\cos\phi = \frac{8}{16} = \frac{1}{2} = \cos 60^\circ$$

$$\phi = 60^\circ$$

$$\phi - \theta = 60^\circ - 30^\circ = 30^\circ$$



(v) $RT = 16^2 - 8^2 = 8\sqrt{3}$

$RS = RT + TS$

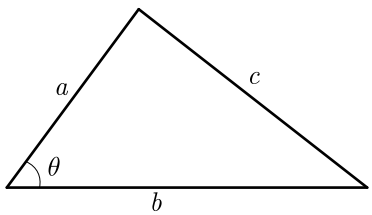
$= RT + AB$

$= 8\sqrt{3} + 24$

Increase in length $= 8\sqrt{3} + 24 - 8$

$= 8\sqrt{3} + 16 = 8(\sqrt{3} + 2)$

191. Area of a Triangle : Prove that the area A of the triangle given in the figure is $A = \frac{1}{2}ab\sin\theta$.



Sol :

We redraw the diagram of triangle as shown below.

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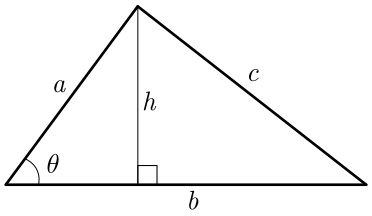


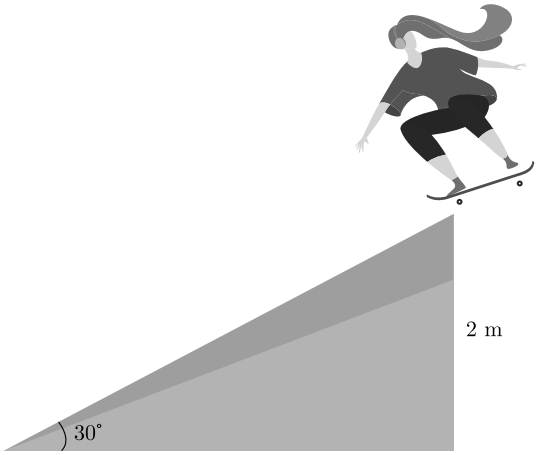
We have $\sin\theta = \frac{h}{a}$

$h = a\sin\theta$

$A = \frac{1}{2}bh = \frac{1}{2}b(a\sin\theta)$

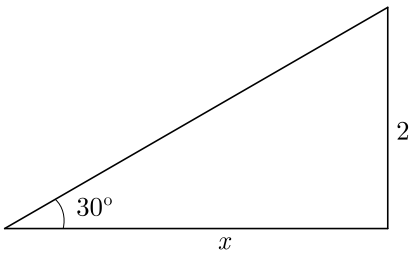
or $A = \frac{1}{2}ab\sin\theta$ Hence Proved

192. Width of a Ramp : A skateboarder wishes to build a jump ramp that is inclined at a 30° angle and that has a maximum height of 2 meter. Find the horizontal width of the ramp.



Sol :

Let x be the horizontal width of ramp. We draw a diagram of the situation as shown below.



$\tan 30^\circ = \frac{2}{x}$

$\frac{1}{\sqrt{3}} = \frac{2}{x}$

$x = 2\sqrt{3}$

$= 2 \times 1.732 = 3.464$ m

Thus horizontal width of ramp is 3.464 meter.

193. Fire brigade : The fire brigade is an organization which has the job of putting out fires; used especially to refer to the people who actually fight the fires.

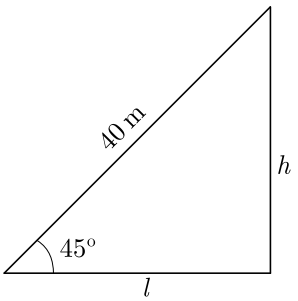
A fire brigade ladder-truck arrives at a highrise apartment complex where a fire has broken out. The maximum length the ladder extends is 40 meter and the angle of inclination is 45°. Assume the ladder is mounted at the top a 3 meter high truck.

(i) How high up the side of the building does the ladder reach?

(ii) What is the length of base of truck ?

Sol :

Let h be the height from truck where ladder can reach. We draw a diagram of the situation as shown below.



Now $\sin 45^\circ = \frac{h}{40}$

$\frac{1}{\sqrt{2}} = \frac{h}{40}$

$h = \frac{40}{\sqrt{2}} = 20\sqrt{2}$

$= 20 \times 1.414 = 28.28$

Now $\cos 45^\circ = \frac{l}{40}$

$\frac{1}{\sqrt{2}} = \frac{l}{40}$

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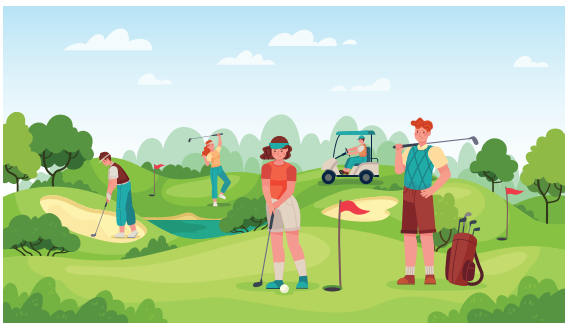
$$l = \frac{40}{\sqrt{2}} = 20\sqrt{2}$$
$$= 20 \times 1.414 = 28.28 \text{ m}$$

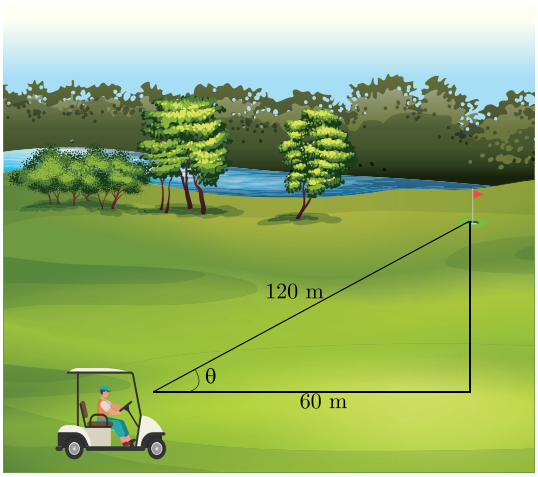
(i) Since height of fire truck is 3 m, ladder reachs upto $28.28 + 3 = 31.28$ m.

(ii) Length of base of truck is 28.28 m.

194. Golf : This a game played in an open field where the golfer plays his golf ball into a hole by using different types of clubs (golf instruments). In golf, a golfer plays a number of holes in a given order. 18 holes played in an order controlled by the golf course design, normally make up a game.



On your approach shot to the ninth green, the Global Positioning System (GPS) your cart is equipped with tells you the pin is 120 meter away. The distance plate states the straight line distance to the hole is 60 meter. Relative to a straight line between the plate and the hole, at what acute angle should you hit the shot?



Sol :

Here $\cos\theta = \frac{60}{120}$

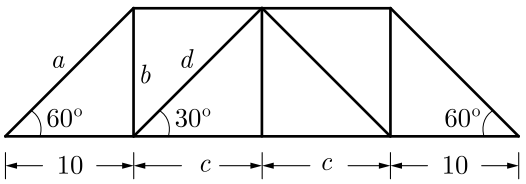
Here $\cos\theta = \frac{1}{2} = \cos 60^\circ$

Thus $\cos\theta = \cos 60^\circ$

Hence $\theta = 60^\circ$

195. Truss : A truss is a structure that consists of members organised into connected triangles so that the overall assembly behaves as a single object. Trusses are most commonly used in bridges, roofs and towers.

Consider the line diagram of truss shown below.



(i) Find the length a.

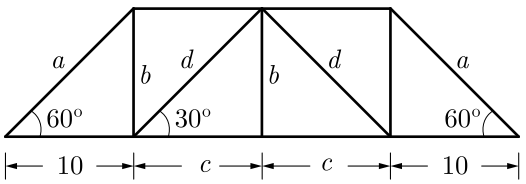
(ii) Find the length b.

(iii) Find the length c.

(iv) Find the length d.

Sol :

We redraw the diagram of truss as shown below.



(i) $\cos 60^\circ = \frac{10}{a}$

$\frac{1}{2} = \frac{10}{a}$

$a = 10 \times 2 = 20$ m

(ii) $\sin 60^\circ = \frac{b}{a}$

$\frac{\sqrt{3}}{2} = \frac{b}{20}$

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$$10.392 + l_2 + 5 = 30 \text{ m}$$
$$l_2 = 30 - 10.392 - 5 = 16.608 \text{ m}$$

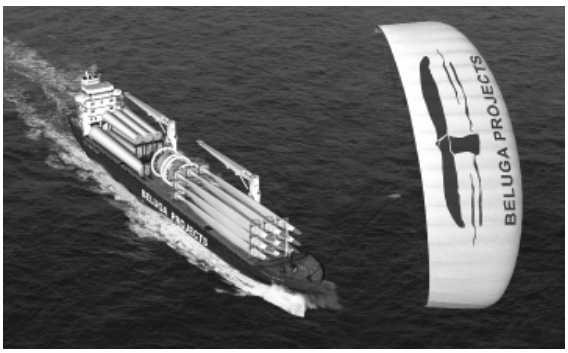
Now total length of water slide

$$l = l_1 + l_2 + l_3$$
$$= 12 + 14.608 + 7.07 = 33.678 \text{ m}$$

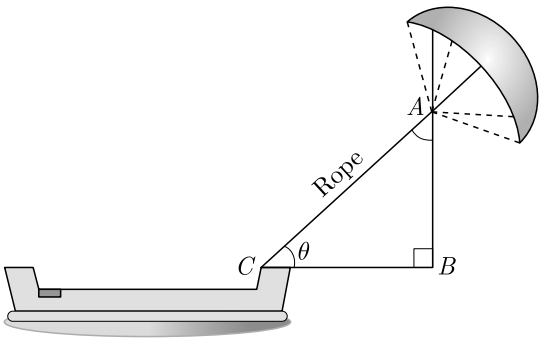
(i) Length of flat part is $l_2 = 14.608$ m

(ii) Length of full slide is $l = 33.678$ m

189. 'Skysails' is that genre of engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres – 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.



Based on the following figure related to sky sailing, answer the questions:



(i) In the given figure, if $\sin\theta = \cos(3\theta - 30^\circ)$, where θ and $3\theta - 30^\circ$ acute angles, then find the value of θ.

(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ calculated above and be at a vertical height of 200 m?

Sol : [Board 2020 SQP Standard]

(i) We have $\sin\theta = \cos(3\theta - 30^\circ)$

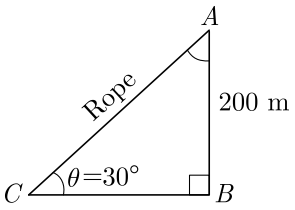
Writing $\sin\theta = \cos(90^\circ - \theta)$ we have

$$\cos(90^\circ - \theta) = \cos(3\theta - 30^\circ)$$

Comparing angle we have

$$90^\circ - \theta = 3\theta - 30^\circ$$
$$90^\circ + 30^\circ = 3\theta + \theta \Rightarrow 4\theta = 120^\circ$$
$$4\theta = 120^\circ \Rightarrow \theta = 30^\circ$$

(ii) We make the figure as given below.

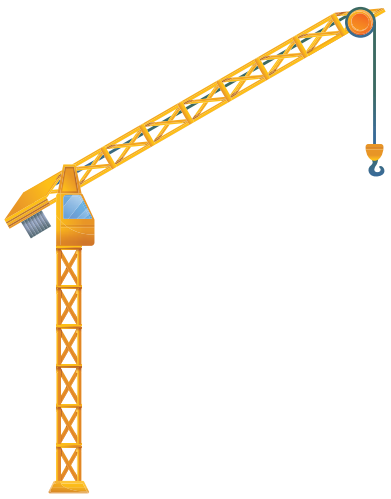


In triangle $\triangle ABC$ we have

$$\sin\theta = \frac{AB}{AC}$$
$$\sin 30^\circ = \frac{200}{AC}$$
$$\frac{1}{2} = \frac{200}{AC} \Rightarrow AC = 400 \text{ m}.$$

Thus length of rope is 400 mm

190. Tower cranes are a common fixture at any major construction site. They're pretty hard to miss -- they often rise hundreds of feet into the air, and can reach out just as far. The construction crew uses the tower crane to lift steel, concrete, large tools like acetylene torches and generators, and a wide variety of other building materials.



A crane stands on a level ground. It is represented by a tower AB, of height 24 m and a jib BR. The jib is of length 16 m and can rotate in a vertical

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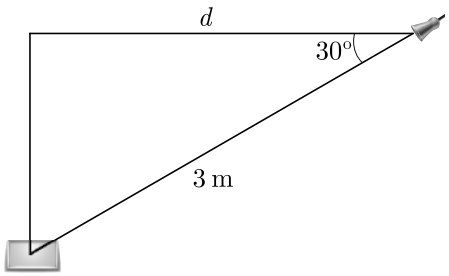
(i) The dock makes 30° with the street.

(ii) Base of ramp is 3.464 m long.

198. Placement of a Light : For best illumination of a piece of art, a lighting specialist for an art gallery recommends that a ceiling-mounted light be 3 meter from the piece of art and that the angle of depression of the light be 30°. How far from a wall should the light be placed so that the recommendations of the specialist are met? Notice that the art extends outward 10 cm from the wall.

Sol :

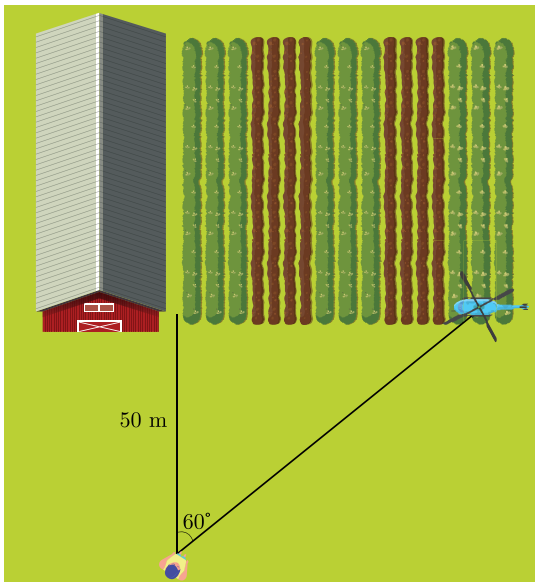
Let d be the distance of light from art on ceiling. We draw a diagram of the situation as shown below.



$$\cos 30^\circ = \frac{d}{3}$$
$$\frac{\sqrt{3}}{2} = \frac{d}{3}$$
$$d = \frac{3\sqrt{3}}{2}$$
$$= \frac{3 \times 1.732}{2}$$
$$= \frac{5.196}{2} = 2.598 \text{ m} = 2.6 \text{ m}$$

Since art is 10 cm or 0.1 meter extends from wall, distance of light from wall will be $2.6 + 0.1 = 2.7$ m.

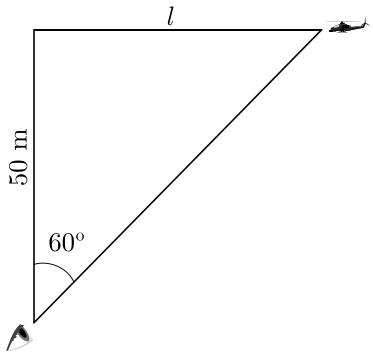
199. Crop Duster's Speed: While standing near the edge of a farmer's field, Vikram watches a crop duster dust the farmer's field for insect control. Curious as to the plane's speed during each drop, Vikram attempts an estimate using the angle of rotation from one end of the field to the other, while standing 50 meter from one corner.



Using a stopwatch he finds the plane makes each pass in 4.33 sec. If the angle of rotation was 60° , how fast is the plane flying as it applies the insecticide?

Sol :

Let l be the length of field. We draw a diagram of the situation as shown below.



Now $\tan 60^\circ = \frac{l}{50}$

$$\sqrt{3} = \frac{l}{50}$$
$$l = 50\sqrt{3}$$
$$= 50 \times 1.732 = 86.6 \text{ m}$$

Speed of plane $s = \frac{\text{Distance}}{\text{Time}} = \frac{l}{t}$

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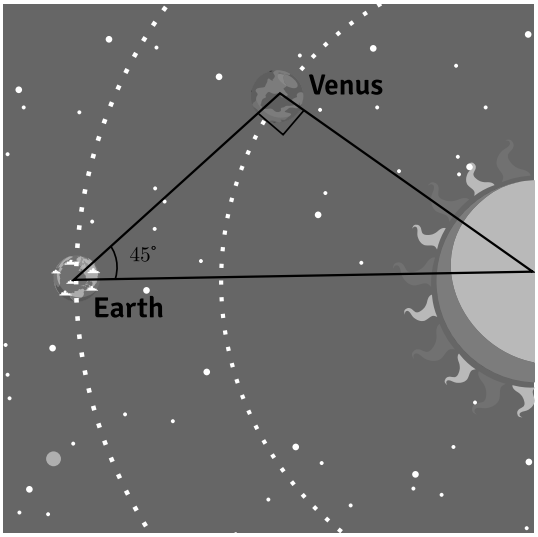
$$= \frac{86.6}{4.33} = 20 \text{ m/sec}$$

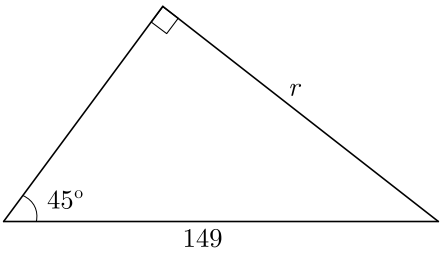
Thus plane is flying at 20 m/sec speed.

200. Astronomy : Venus rotates in a nearly circular orbit around the sun. The largest angle formed by Venus, Earth, and the sun is $45°$. The distance from Earth to the sun is approximately 149 million kilometers. See the following figure. What is the orbital radius r of Venus? Round to the nearest million kilometres.



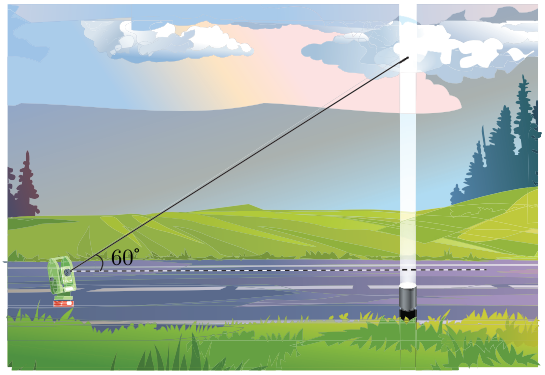
Sol :

We draw a diagram of the situation as shown below.



Now
$$\sin 45° = \frac{r}{149}$$
$$\frac{1}{\sqrt{2}} = \frac{r}{149}$$
$$r = \frac{149}{\sqrt{2}}$$
$$= \frac{149}{2} \times \sqrt{2}$$
$$= \frac{149 \times 1.414}{2}$$
$$= 105 \text{ million kilometre}$$

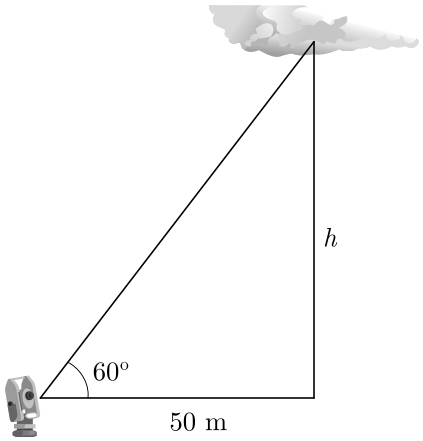
201. Cyclometer : Meteorologists find the height of a cloud using an instrument called a ceilometer. It consists of a light projector that directs a vertical light beam up to the cloud base and a light detector that scans the cloud to detect the light beam.



On July 2, 2021, at Midway Airport in Mumbai, a ceilometer was employed to find the height of the cloud cover. It was set up with its light detector 50 meter from its light projector. If the angle of elevation from the light detector to the base of the cloud was $60°$, what was the height of the cloud cover?

Sol :

Let h be the height of the cloud from ground. We draw a diagram of the situation as shown below.



$$\tan 60° = \frac{h}{50}$$
$$\sqrt{3} = \frac{h}{50}$$
$$h = 150 \times \sqrt{3}$$
$$= 150 \times 1.732$$
$$= 259.8 \text{ m}$$

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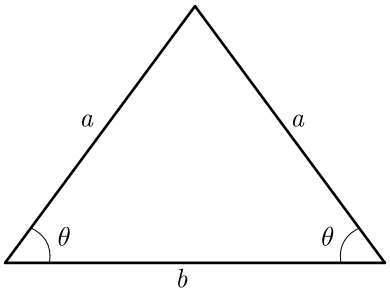
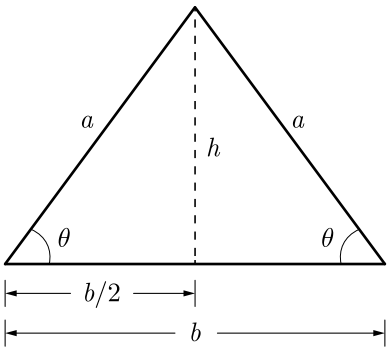
202. Area of an Isosceles Triangle : Consider the following isosceles triangle. The length of each of the two equal sides of the triangle is a, and each of the base angles has a measure of θ. Verify that the area of the triangle is $A = a^2 \sin\theta\cos\theta$.



Sol :

We draw a diagram of the situation as shown below.



Now $\cos\theta = \frac{\frac{b}{2}}{a}$

$$\frac{b}{2} = \cos\theta$$

$$b = 2\cos\theta$$

$$\sin\theta = \frac{h}{a}$$

$$h = a\sin\theta$$

$$A = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2}(2a\cos\theta)(a\sin\theta)$$

$$= a^2 \sin\theta\cos\theta$$

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CHAPTER 9

Some Applications of Trigonometry

OBJECTIVE QUESTIONS

1. If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the tower is

(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m

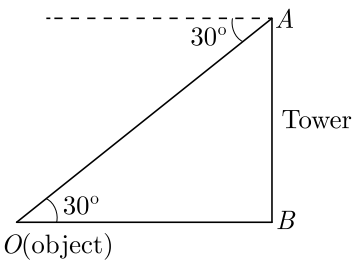
(c) $75\sqrt{3}$ m (d) 150 m

Sol : [Board Term-2 Foreign 2015]

We have $\tan 30^\circ = \frac{AB}{OB}$

$$\frac{1}{\sqrt{3}} = \frac{75}{OB}$$

$$OB = 75\sqrt{3} \text{ m}$$



Thus (c) is correct option.

2. If the height and length of the shadow of a man are equal, then the angle of elevation of the sun is,

(a) 45° (b) 60°

(c) 90° (d) 120°

Sol : [Board Term-2 OD 2012]

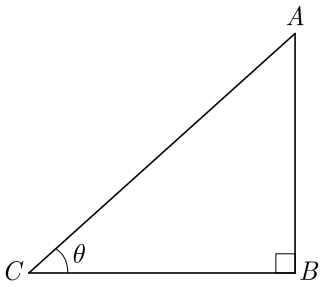
Let AB be the height of a man and BC be the shadow of a man.

$$AB = BC$$

In ΔABC, $\tan\theta = \frac{AB}{BC}$

$$\frac{AB}{AB} = \tan\theta$$

$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$



Thus (a) is correct option.

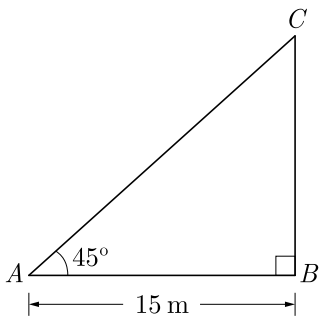
3. A tree casts a shadow 15 m long on the level of ground, when the angle of elevation of the sun is 45°. The height of a tree is

(a) 10 m (b) 14 m

(c) 8 m (d) 15 m

Sol : [Board Term-2 OD 2014]

Let BC be the tree of height h meter. Let AB be the shadow of tree.



In ΔABC, $CB = 90^\circ$

$$\frac{BC}{BA} = \tan 45^\circ$$

$$BC = AB = 15 \text{ m}$$

Thus (d) is correct option.

The ratio of the length of a rod and its shadow is $1:\sqrt{3}$ then the angle of elevation of the sun is

(a) 90° (b) 45°

(c) 30° (d) 75°

Sol : [Board Term-2 Delhi 2017]

Let AB be the rod of length h, BC be its shadow

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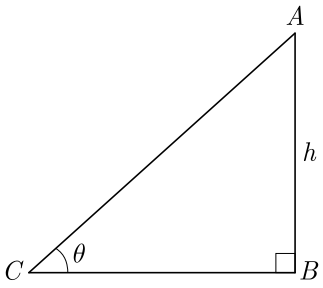
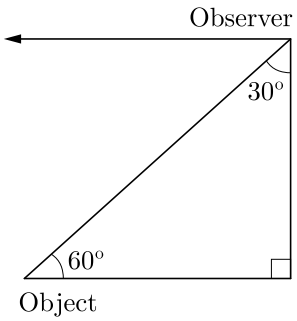
of length $\sqrt{3}\,h$, θ be the angle of elevation of the sun.



$$\text{In } \Delta ABC, \quad \frac{h}{\sqrt{3}\,h} = \tan\theta$$
$$\tan\theta = \frac{1}{\sqrt{3}}$$
$$\theta = 30^\circ$$

Thus (c) is correct option.

5. In the given figure, the positions of the observer and the object are mentioned, the angle of depression is



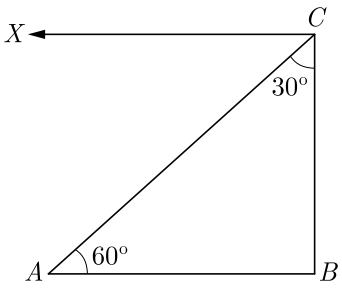
(a) 30° (b) 90°

(c) 60° (d) 45°

Sol : [Board 2018]

$$\angle XCA = \angle CAB$$
$$= 60^\circ$$

Hence, angle of depression $= 60^\circ$



Thus (c) is correct option.

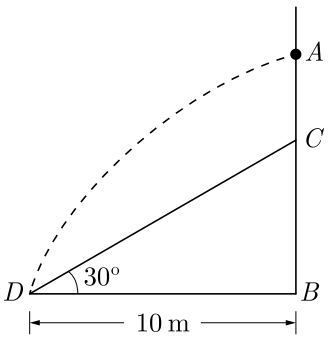
6. A tree is broken by the wind. The top struck the ground at an angle of 30° and at distance of 10 m from its root. The whole height of the tree is $(\sqrt{3} = 1.732)$

(a) $10\sqrt{3}$ m (b) $3\sqrt{10}$ m

(c) $20\sqrt{3}$ m (d) $3\sqrt{20}$ m

Sol : [Board 2010]

Let AB be the tree of height x, and AC be the broken part of tree.



Now $AC = CD$

$$\angle CDB = 30^\circ$$
$$BD = 10\text{ m}$$
$$\text{In } \Delta CDB, \quad \tan 30^\circ = \frac{CB}{DB} = \frac{CB}{10}$$
$$\frac{1}{\sqrt{3}} = \frac{CB}{10}$$
$$CB = \frac{10}{\sqrt{3}}$$
$$\text{Also,} \quad \cos 30^\circ = \frac{DB}{DC} = \frac{10}{DC}$$
$$DC = \frac{20}{\sqrt{3}} = AC$$

Height of tree,

$$AC + CB = \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}} = \frac{30}{\sqrt{3}}$$
$$= 10\sqrt{3}\text{ m}$$

Thus (a) is correct option.

7. A circle artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground, then the height of pole, if the angle made by the rope with the ground level is 30°, is

(a) 5 m (b) 10 m

(c) 15 m (d) 20 m

Sol : [Board 2019 OD]

Let AB be the vertical pole and CA be the 20 m long rope such that its one end A is tied from the

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$$BE = AC = 7\text{ m}$$

From $\triangle EBD$, we have

$$\tan 60^\circ = \frac{DE}{BE}$$

$$\frac{DE}{7} = \sqrt{3} \Rightarrow DE = 7\sqrt{3}\text{ m}$$

$$\text{Height of the tower} = (7 + 7\sqrt{3})$$

$$= 7(\sqrt{3} + 1)$$

$$= 7(1.732 + 1) = 7 \times 2.732$$

$$= 19.124\text{ m}$$

Thus (c) is correct option.

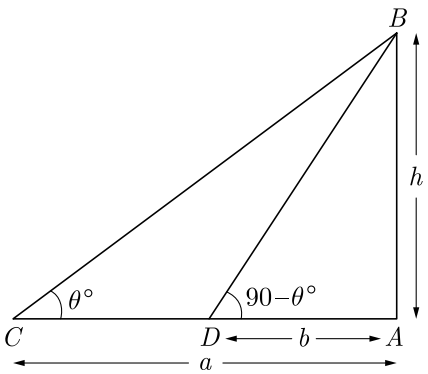
13. The angles of elevation of the top of a tower from the points P and Q at distance of a and b respectively from the base and in the same straight line with it, are complementary. The height of the tower is

(a) ab (b) $\sqrt{ab}$

(c) $\sqrt{\frac{a}{b}}$ (d) $\sqrt{\frac{b}{a}}$

Sol :

Let AB be the tower. Let C and D be two points at distance a and b respectively from the base of the tower.



In $\triangle ABC$, $\tan\theta = \frac{AB}{AC}$

$$\tan\theta = \frac{h}{a} \quad ...(1)$$

In $\triangle ABD$, $\tan(90^\circ - \theta) = \frac{AB}{AD}$

$$\cot\theta = \frac{h}{b} \quad ...(2)$$

From equation (1) and (2), we have

$$\tan\theta \times \cot\theta = \frac{h}{a} \times \frac{h}{b}$$

$$1 = \frac{h^2}{ab} \Rightarrow h = \sqrt{ab}$$

Thus (b) is correct option.

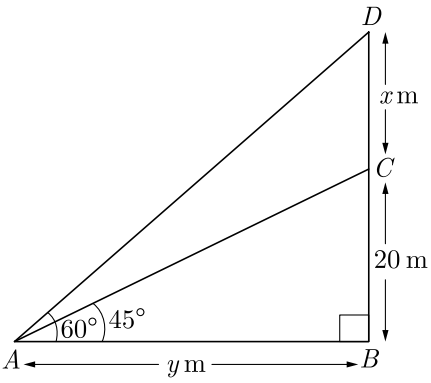
14. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively, then the height of the tower is

(a) 14.64 m (b) 28.64 m

(c) 38.64 m (d) 19.64 m

Sol :

Let the height of the building be BC, $BC = 20$ m and height of the tower be CD Let the point A be at a distance y from the foot of the building.



Now, in $\triangle ABC$, $\frac{BC}{AB} = \tan 45^\circ = 1$

$$\frac{20}{y} = 1 \Rightarrow y = 20\text{ m}$$

i.e. $AB = 20\text{ m}$

Now, in $\triangle ABC$, $\frac{BD}{AB} = \tan 60^\circ = \sqrt{3}$

$$\frac{BD}{AB} = \sqrt{3}$$

$$\frac{20 + x}{20} = \sqrt{3}$$

$$20 + x = 20\sqrt{3}$$

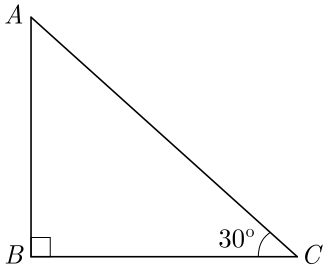
$$x = 20\sqrt{3} - 20$$

$$= 20 \times 0.732$$

$$= 14.64\text{ m}$$

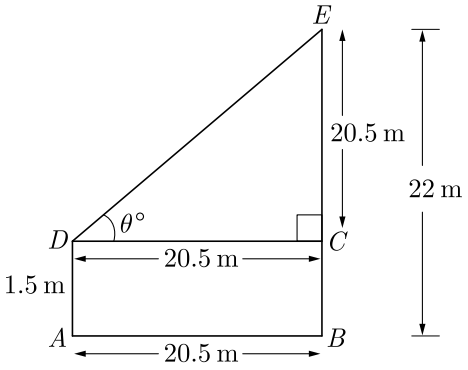
Thus (a) is correct option.

15. **Assertion :** In the figure, if $BC = 20$ m, then height AB is 11.56 m.



Sol :

Let $BE = 22\,\text{m}$ be the height of the tower and $AD = 1.5\,\text{m}$ be the height of the observer. The point D be the observer's eye. We draw $DC \parallel AB$ as shown below.



Then, $AB = 20.5\,\text{m} = DC$

and $EC = BE - BC = BE - AD$

$= 22 - 1.5 = 20.5\,\text{m} \quad [BC = AD]$

Let θ be the angle of elevation make by observer's eye to the top of the tower i.e. $\angle DCE$,

$$\tan\theta = \frac{P}{B} = \frac{CE}{DC} = \frac{20.5}{20.5}$$

$$\tan\theta = 1$$

$$\tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

Thus (b) is correct option.

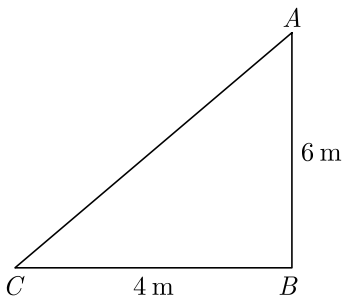
11. A 6 m high tree cast a 4 m long shadow. At the same time, a flag pole cast a shadow 50 m long. How long is the flag pole?

(a) 75 m (b) 100 m

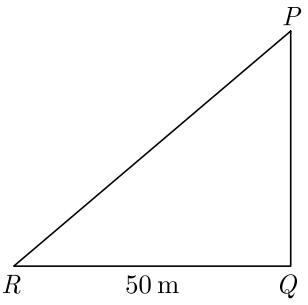
(c) 150 m (d) 50 m

Sol : (a) 75 m

Let AB be height of tree and BC its shadow.



Again, let PQ be height of pole and QR be its shadow. At the same time, the angle of elevation of tree and poles are equal i.e. $\triangle ABC \sim PQR$



Thus $\frac{AB}{BC} = \frac{PQ}{QR}$

$\frac{6}{4} = \frac{PQ}{50}$

$PQ = \frac{50 \times 6}{4} = 75\,\text{m}$

Thus (a) is correct option.

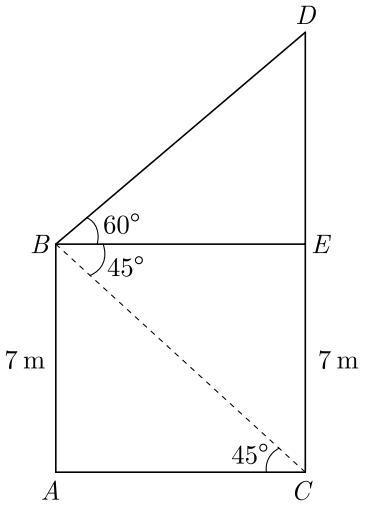
12. From the top of a 7 m high building the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°, then the height of the tower is

(a) 14.124 m (b) 17.124 m

(c) 19.124 m (d) 15.124 m

Sol :

Let AB be the building and CD be the tower. We draw $BE \perp CD$ as shown below.



Here $CE = AB = 7\,\text{m}$

$\angle EBD = 60^\circ$

and $\angle ACB = \angle CBE = 45^\circ$

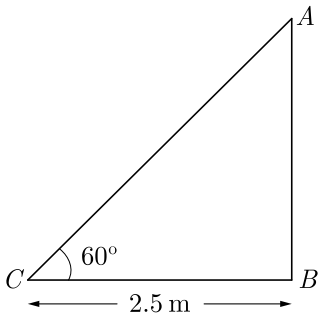
From $\triangle ACB$, we have

$$\cot 45^\circ = \frac{AC}{AB}$$

$$\frac{AC}{7} = 1 \Rightarrow AC = 7\,\text{m}$$

NODIA

As per given in question we have drawn figure below.



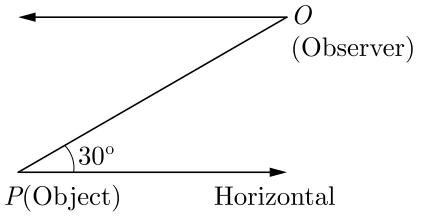
In ΔACB with $\angle C = 60^\circ$, we get

$$\cos 60^\circ = \frac{2.5}{AC}$$

$$\frac{1}{2} = \frac{2.5}{AC}$$

$$AC = 2 \times 2.5 = 5 \text{ m}$$

22. In the adjoining figure, the positions of observer and object are marked. The angle of depression is .....



Sol : [Board 2008]

30°

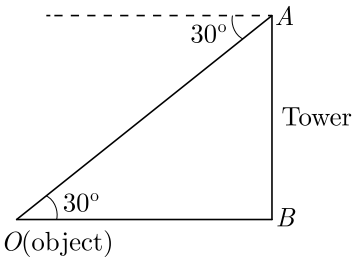
23. If the angle of depression of an object from a 75 m high tower is 30°, then what is the distance of the object from the tower?

Sol : [Board Term-2 OD 2017]

We have $\tan 30^\circ = \frac{AB}{OB}$

$$\frac{1}{\sqrt{3}} = \frac{75}{OB}$$

$$OB = 75\sqrt{3} \text{ m}$$

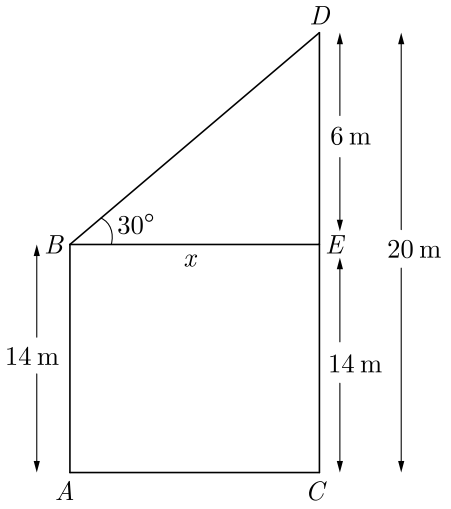


24. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then find the length of the wire.

Sol : [Board Term-2 Foreign 2014]

Height of big pole, $CD = 20 \text{ m}$

Height of small pole, $AB = 14 \text{ m}$



$$DE = CD - CE$$

$$= CD - AB \quad [AB = CE]$$

$$= 20 - 14 = 6 \text{ m}$$

In ΔBDE, $\sin 30^\circ = \frac{DE}{BD}$

$$\frac{1}{2} = \frac{6}{BD} \Rightarrow BD = 12 \text{ m}$$

Thus length of wire is 12 m.

25. If the height and length of the shadow of a man are equal, then find the angle of elevation of the sun.

Sol : [Board Term-2 OD 2016]

Let AB be the height of a man and BC be the shadow of a man.

$$AB = BC$$

In ΔABC, $\tan\theta = \frac{AB}{BC}$

$$\frac{AB}{AB} = \tan\theta$$

$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$

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Reason : $\tan\theta = \frac{AB}{BC} = \frac{\text{perpendicular}}{\text{base}}$ where θ is the

angle $\angle ACB$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $\tan 30^\circ = \frac{AB}{BC} = \frac{AB}{20}$

$$AB = \frac{1}{\sqrt{3}} \times 20$$

$$= \frac{20}{1.73} = 11.56 \text{ m}$$

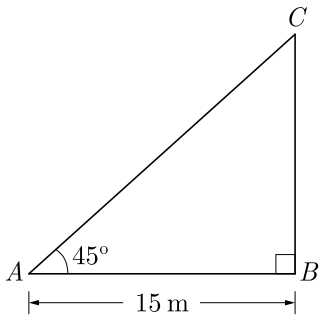
Both the assertion and reason are correct, reason is the correct explanation of the assertion.
Thus (a) is correct option.

ONE MARK QUESTIONS

16. A tree casts a shadow 15 m long on the level of ground, when the angle of elevation of the sun is 45°. Find the height of a tree.

Sol : [Board 2022 Term-2 Delhi]

Let BC be the tree of height h meter. Let AB be the shadow of tree.

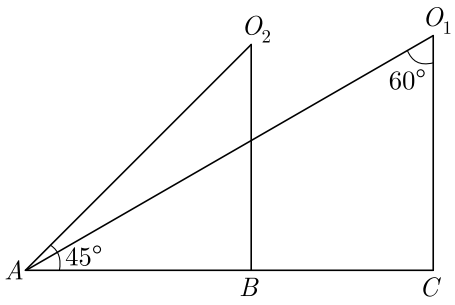


In ΔABC, $CB = 90^\circ$

$$\frac{BC}{BA} = \tan 45^\circ$$

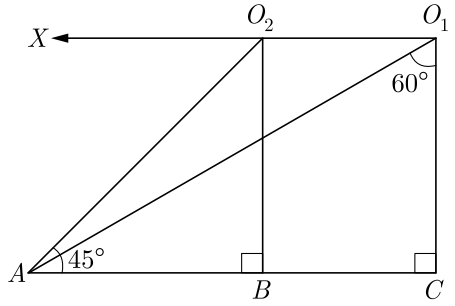
$$BC = AB = 15 \text{ m}$$

17. In Figure, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A are ..... .



Sol : [Board 2020 OD Standard]

Here we have drawn O_1X parallel to AC.



$$\angle AO_1X = 90^\circ - 60^\circ = 30^\circ$$

$$\angle AO_2X = \angle BAO_2 = 45^\circ$$

18. The ..... is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Sol :

line of sight

19. The ..... of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

Sol : [Board Term-2 Foreign 2015]

angle of elevation

20. The ..... of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.

Sol : [Board Term-2 Foreign 2015]

angle of depression

21. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

Sol : [Board Term-2 2011]

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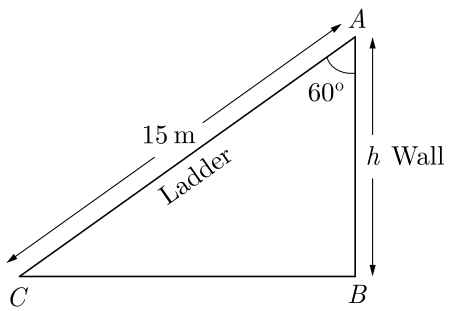
Sol : [Board Term-2 2014]

Let the height of wall be h. As per given in question we have drawn figure below.

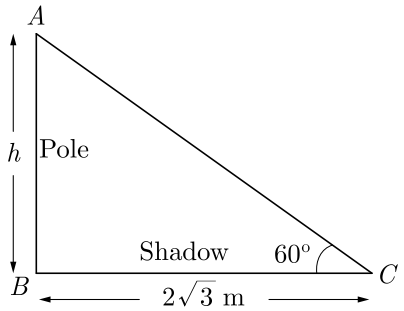


$$\frac{h}{15} = \cos 60^\circ$$
$$h = 15 \times \cos 60^\circ$$
$$= 15 \times \frac{1}{2} = 7.5 \text{ m}$$

30. A pole casts a shadow of length $2\sqrt{3}$ m on the ground, when the Sun's elevation is 60°. Find the height of the pole.

Sol : [Board Term-2 Foreign 2015]

Let the height of pole be h. As per given in question we have drawn figure below.

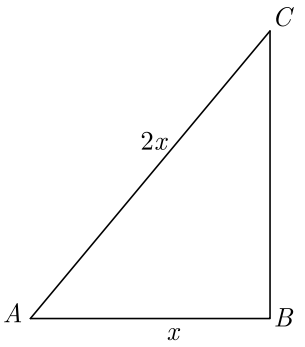


Now $$\frac{h}{2\sqrt{3}} = \tan 60^\circ$$
$$h = 2\sqrt{3} \tan 60^\circ$$
$$= 2\sqrt{3} \times \sqrt{3} = 6 \text{ m}$$

31. If the length of the ladder placed against a wall is twice the distance between the foot of the ladder and the wall. Find the angle made by the ladder with the horizontal.

Sol : [Board Term-2 2015]

Let the distance between the foot of the ladder and the wall is x, then length of the ladder will be $2x$. As per given in question we have drawn figure below.



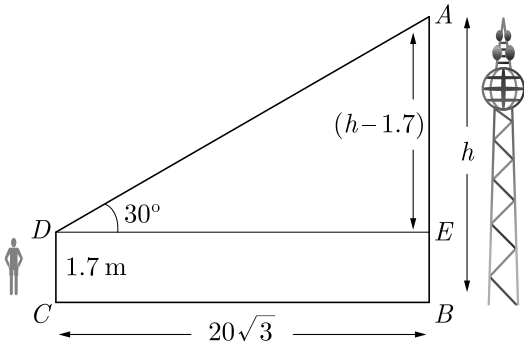
In ΔABC, $\angle B = 90^\circ$

$$\cos A = \frac{x}{2x} = \frac{1}{2} = \cos 60^\circ$$
$$A = 60^\circ$$

32. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of tower.

Sol : [Board Term-2 Foreign 2016]

Let height of the tower AB be h. As per given in question we have drawn figure below.



Here $AE = h - 1.7$

and $BC = DE = 20\sqrt{3}$

In ΔADE, $\angle E = 90^\circ$

$$\tan 30^\circ = \frac{h - 1.7}{20\sqrt{3}}$$
$$\frac{1}{\sqrt{3}} = \frac{h - 1.7}{20\sqrt{3}}$$
$$h - 1.7 = 20$$
$$h = 20 + 1.7 = 21.7 \text{ m}$$

33. In the given figure, AB is a 6 m high pole and DC is a ladder inclined at an angle of 60° to the horizontal and reaches up to point D of pole. If $AD = 2.54$ m, find the length of ladder. (use $\sqrt{3} = 1.73$)

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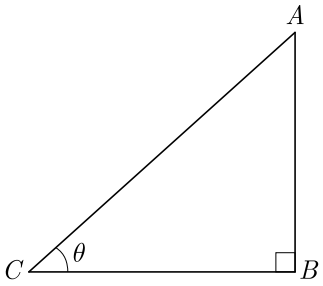
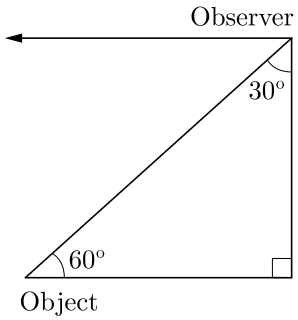


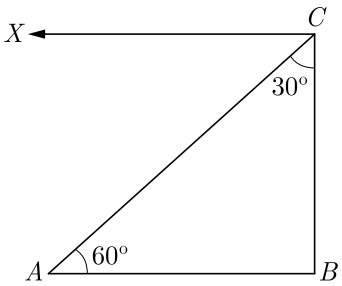
26. In the given figure, the positions of the observer and the object are mentioned, find the angle of depression?



Sol : [Board Term-2 OD 2015]

We have $\angle XCA = \angle CAB = 60^\circ$

Hence, angle of depression $= 60^\circ$



27. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. If the angle made by the rope with the ground level is 30°, then what is the height of pole?

Sol : [Board Term-2 Delhi 2013]

Let AB be the vertical pole and CA be the 20 m long rope such that its one end A is tied from the top of the vertical pole AB and the other end C is tied to a point C on the ground.

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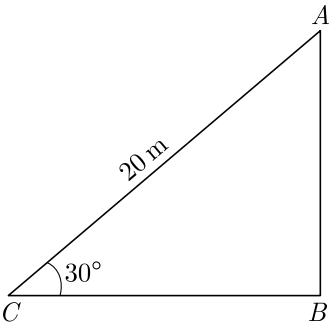


In $\triangle ABC$, we have

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{AC}$$

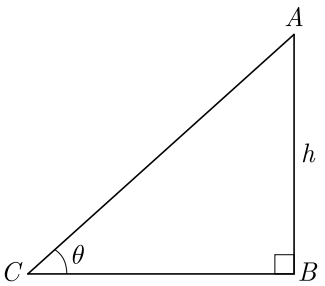
$$\frac{1}{2} = \frac{AB}{20} \Rightarrow AB = 10 \text{ m}$$

Hence, the height of the pole is 10 m.

28. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$ then what is the angle of elevation of the sun ?

Sol : [Board Term-2 Delhi 2012]

Let AB be the rod of length h, BC be its shadow of length $\sqrt{3}\,h$, θ be the angle of elevation of the sun.



In ΔABC, $\dfrac{h}{\sqrt{3}\,h} = \tan\theta$

$$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

PRACTICE

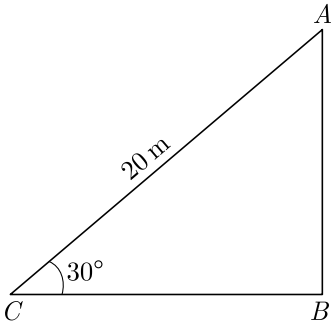
The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3} : 1$. What is the angle of elevation of the sun ?

[Board Term-2, 2016]

Ans : 60°

29. A ladder 15 m long leans against a wall making an angle of 60° with the wall. Find the height of the point where the ladder touches the wall.

top of the vertical pole AB and the other end C is tied to a point C on the ground.



In $\triangle ABC$, we have

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{20} \Rightarrow AB = 10\text{ m}$$

Hence, the height of the pole is 10 m.

Thus (b) is correct option.

8. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with level ground such that $\tan\theta = \frac{15}{8}$, then the height of kite is

(a) 75 m (b) 78.05 m

(c) 226 m (d) None of these

Sol : [Board 2019 Delhi]

Length of the string of the kite,

$$AB = 85\text{ m}$$

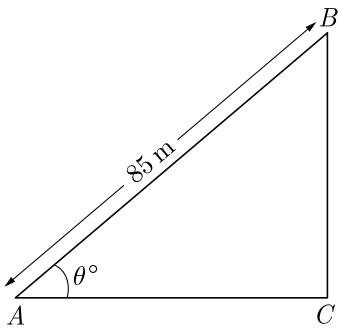
and

$$\tan\theta = \frac{15}{8}$$

$$\cot\theta = \frac{8}{15}$$

$$\text{cosec}^2\theta - 1 = \frac{64}{225}$$

$$\text{cosec}^2\theta = 1 + \frac{64}{225} = \frac{289}{225}$$



$$\text{cosec}\,\theta = \sqrt{\frac{289}{225}} = \frac{17}{15}$$

$$\sin\theta = \frac{15}{17}$$

In $\triangle ABC$, $\sin\theta = \frac{BC}{AB}$

$$\frac{15}{17} = \frac{BC}{85} \Rightarrow BC = 75\text{ m}$$

Thus height of kite is 75 m.

Thus (a) is correct option.

9. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is

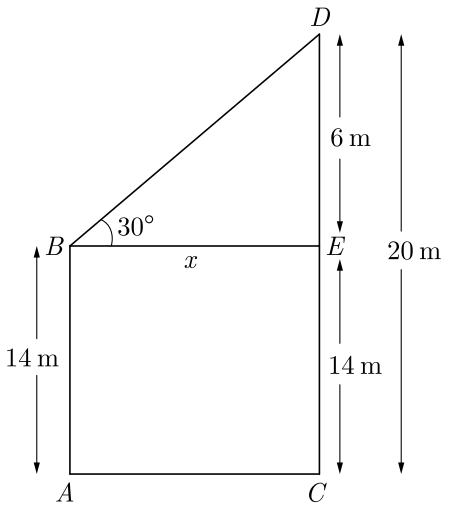
(a) 12 m (b) 10 m

(c) 8 m (d) 6 m

Sol : [Board 2018]

Height of big pole $CD = 20\text{ m}$

Height of small pole $AB = 14\text{ m}$



$$DE = CD - CE$$

$$= CD - AB \quad [AB = CE]$$

$$= 20 - 14 = 6\text{ m}$$

In $\triangle BDE$, $\sin 30^\circ = \frac{DE}{BD}$

$$\frac{1}{2} = \frac{6}{BD} \Rightarrow BD = 12\text{ m}$$

Thus length of wire is 12 m.

Thus (a) is correct option.

10. An observer, 1.5 m tall is 20.5 away from a tower 22 m high, then the angle of elevation of the top of the tower from the eye of observer is

(a) 30° (b) 45°

(c) 60° (d) 90°

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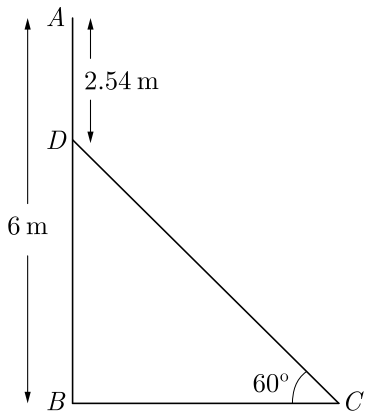


Sol : [Board Term-2 Delhi 2016]

We have $AD = 2.54$ m

$DB = 6 - 2.54 = 3.46$ m

In ΔBCD, $\angle B = 90^\circ$

$$\sin 60^\circ = \frac{BD}{DC}$$

$$\frac{\sqrt{3}}{2} = \frac{3.46}{DC}$$

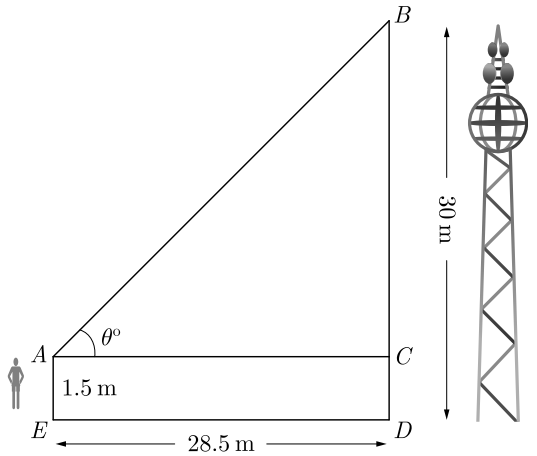
$$DC = \frac{3.46 \times 2}{\sqrt{3}} = \frac{3.46}{1.73} = 4$$

Thus length of ladder is 4 m.

34. An observer 1.5 m tall is 28.5 m away from a tower 30 m high. Find the angle of elevation of the top of the tower from his eye.

Sol : [Board Term-2 2012]

As per given in question we have drawn figure below.



Here $AE = 1.5$ m is height of observer and $BD = 30$ m m is tower.

Now $BC = 30 - 1.5$

$= 28.5$ m

In ΔBAC, $\tan\theta = \frac{BC}{AC}$

$$\tan\theta = \frac{28.5}{28.5}$$

$$= 1 = \tan 45^\circ$$

$$\theta = 45^\circ$$

Hence angle of elevation is 45°.

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PRACTICE

An observer, 1.5 m tall is 20.5 away from a tower 22 m high, then what is the angle of elevation of the top of the tower from the eye of observer?

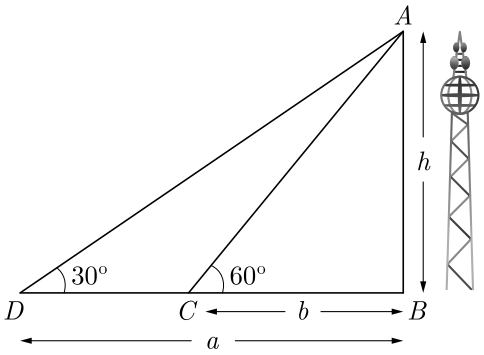
[Board 2010]

Ans : 45°

35. If the angles of elevation of the top of a tower from two points distant a and $b\,(a > b)$ from its foot and in the same straight line from it are respectively 30° and 60°, then find the height of the tower.

Sol : [Board Term-2 2014]

Let the height of tower be h. As per given in question we have drawn figure below.



From ΔABD, $\frac{h}{a} = \tan 30^\circ$

$$h = a \times \frac{1}{\sqrt{3}} = \frac{a}{\sqrt{3}} \quad \ldots(1)$$

From ΔABC, $\frac{h}{b} = \tan 60^\circ$

$$h = b \times \sqrt{3} = b\sqrt{3} \quad \ldots(2)$$

From (1) we get $a = \sqrt{3}\,h$

From (2) get $b = \frac{h}{\sqrt{3}}$

Thus $a \times b = \sqrt{3}\,h \times \frac{h}{\sqrt{3}}$

$$ab = h^2$$

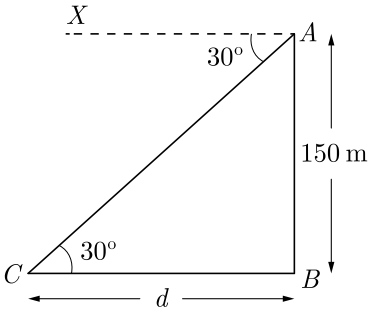
$$h = \sqrt{ab}$$

Hence, the height of the tower is $\sqrt{ab}$.

36. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. Find the distance of the car from the tower (in m).

Sol : [Board Term-2, 2014]

Let the distance of the car from the tower be d. As per given in question we have drawn figure below.



Due to alternate angles we have

$$\angle CAX = \angle ACB = 30^\circ$$

In ΔABC, $\angle B = 90^\circ$

$$\tan 30^\circ = \frac{150}{d}$$

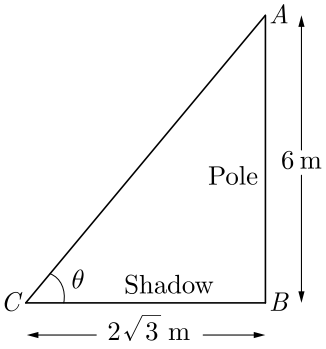
$$\frac{1}{\sqrt{3}} = \frac{150}{d}$$

Thus $d = 150\sqrt{3}$ m.

37. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then find the Sun's elevation.

Sol : [Board Term-2 2012]

Let the Sun's elevation be θ. As per given in question we have drawn figure below.



Length of pole is 6 m and length of shadow is $2\sqrt{3}$ m.

From ΔABC, we have

$$\tan\theta = \frac{AB}{BC} = \frac{6}{2\sqrt{3}}$$

$$= \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60^\circ$$

$$\theta = 60^\circ$$

Hence sun's elevation is 60°.

PRACTICE

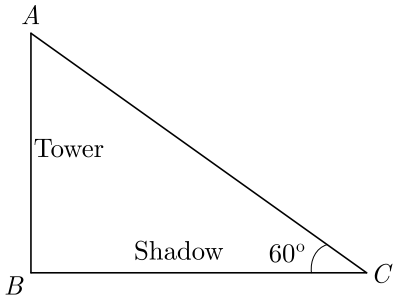
- If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ?
  [Board Term-2 OD 2017]
  Ans : 60°

- In figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the Sun's altitude.
  [Board Term-2 OD 2015]
  Ans : 30°

38. If the altitude of the sun is 60°, what is the height of a tower which casts a shadow of length 30 m ?

Sol : [Board Term-2 2011]

Let AB be the tower whose height be h. As per given in question we have drawn figure below.



Here shadow is $BC = 30$ m.

From ΔABC, we get

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{h}{30} = \sqrt{3} \Rightarrow h = 30\sqrt{3} \text{ m}$$

Hence, height of tower is $30\sqrt{3}$ m.

39. Find the angle of elevation of the top of the tower from the point on the ground which is 30 m away from the foot of the tower of height $10\sqrt{3}$ m.

Sol : [Board Term-2 2012]

Let the angle of elevation of top of the tower be θ. As per given in question we have drawn figure

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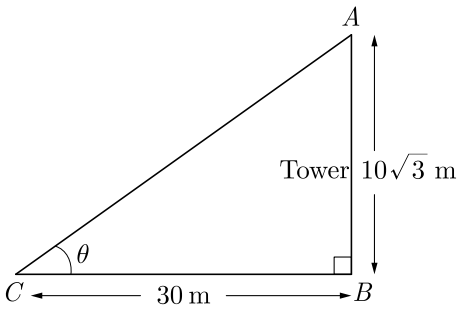
below.



From ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{10\sqrt{3}}{30}$$

$$= \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Thus $\theta = 30^\circ$

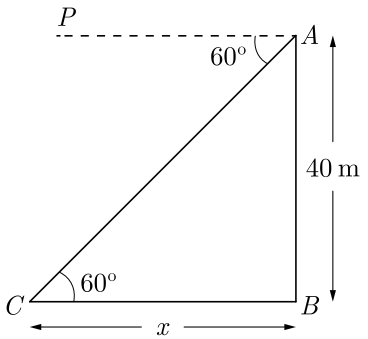
Hence angle of elevation is 30°.

TWO MARKS QUESTIONS

40. From the top of light house, 40 m above the water, the angle of depression of a small boat is 60°. Find how far the boat is from the base of the light house.

Sol : [Board Term-2 2015]

Let AB be the light house and C be the position of the boat. As per given in question we have drawn figure below.



Since $\angle PAC = 60^\circ \Rightarrow \angle ACB = 60^\circ$

Let $CB = x$. Now in ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{40}{x}$$

$$x = \frac{40}{\sqrt{3}} = \frac{40 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

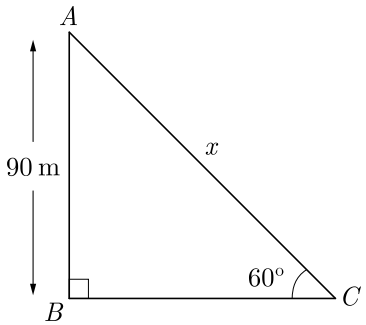
$$= \frac{40\sqrt{3}}{3} \text{ m}$$

Hence, the boat is $\frac{40\sqrt{3}}{3}$ m away from the foot of light house.

41. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string.

Sol : [Board Term-2 2011, 2014]

As per given in question we have drawn figure below.



In right ΔABC, we have

$$\sin 60^\circ = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{90}{x}$$

$$x = \frac{90 \times 2}{\sqrt{3}} = \frac{180}{\sqrt{3}} = \frac{3 \times 60}{\sqrt{3}}$$

$$= 60\sqrt{3} = 60 \times 1.732$$

Hence length of string is 103.92 m.

PRACTICE

✎ Find the length of kite string flying at 100 m above the ground with the elevation of 60°.

[Board Term-2 2012]

Ans : $\frac{200}{3}\sqrt{3}$

42. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Sol : [Board Term-2 2011]

Let the tree be AC and is broken at B. The broken part touches at the point D on the ground. As per

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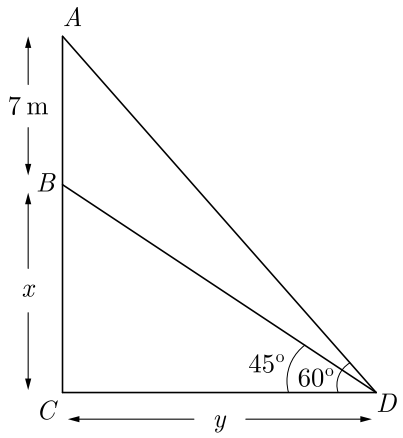


$$\frac{x}{y} = \tan 45^\circ = 1 \Rightarrow x = y$$

$$\frac{x+7}{x} = \tan 60^\circ = \sqrt{3}$$

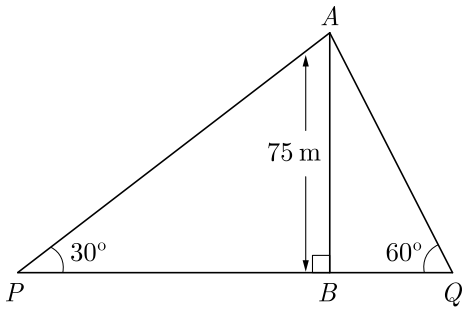
$$7 = (\sqrt{3}-1)x$$

$$x = \frac{7(\sqrt{3}+1)}{2} = \frac{7(2.73)}{2} = 9.6 \text{ m}$$

54. Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men. (Use $\sqrt{3} = 1.73$)

Sol : [Board Term-2 Foreign 2016]

Let AB be the building and the two men are at P and Q. As per given in question we have drawn figure below.



In ΔABP, $\tan 30^\circ = \frac{AB}{BP}$

$$\frac{1}{\sqrt{3}} = \frac{75}{BP}$$

$$BP = 75\sqrt{3} \text{ m}$$

In ΔABQ, $\tan 60^\circ = \frac{AB}{BQ}$

$$\sqrt{3} = \frac{75}{BQ}$$

$$BQ = \frac{75}{\sqrt{3}} = 25\sqrt{3}$$

Distance between the two men,

$$PQ = BP + BQ = 75\sqrt{3} + 25\sqrt{3}$$

$$= 100\sqrt{3} = 100 \times 1.73 = 173$$

PRACTICE

Two men standing on opposite sides of a tower measure the angles of elevation of he top of the tower as 30° and 60° respectively. If the height of the tower in 20 m, then find the distance between the two men.

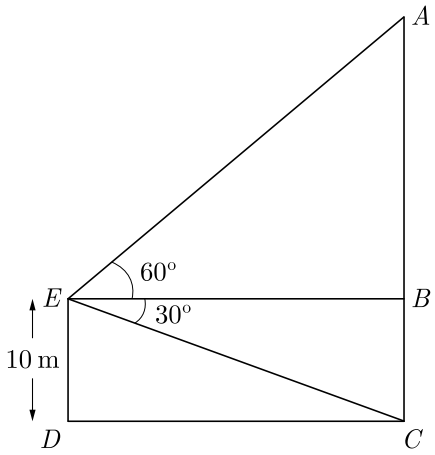
[Board Term-2 OD 2013]

Ans : $\frac{80}{3}\sqrt{3}$

55. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.

Sol : [Board Term-2 OD 2016]

As per given in question we have drawn figure below. Here AC is height of hill and man is at E. $ED = 10$ is height of ship from water level.



In ΔBCE, $BC = EC = 10$ m and

$$\angle BEC = 30^\circ$$

Now $\tan 30^\circ = \frac{BC}{BE}$

$$\frac{1}{\sqrt{3}} = \frac{10}{BE}$$

$$BE = 10\sqrt{3}$$

Since $BE = CD$, distance of hill from ship

$$CD = 10\sqrt{3} \text{ m} = 10 \times 1.732 \text{ m}$$

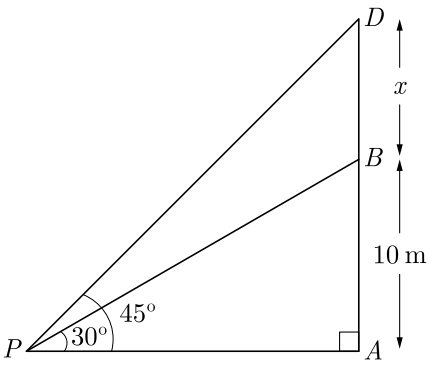
$$= 17.32 \text{ m}$$

NODIA

[Take $\sqrt{3} = 1.732$]

Sol : [Board Term-2 2011, Delhi 2012, 2013]

Let height of flagstaff be $BD = x$. As per given in question we have drawn figure below.



$$\tan 30^\circ = \frac{AB}{AP}$$
$$\frac{1}{\sqrt{3}} = \frac{10}{AP}$$
$$AP = 10\sqrt{3}$$

Distance of the building from P,

$$= 10 \times 1.732 = 17.32 \text{ m}$$

Now $\tan 45^\circ = \frac{AD}{AP}$

$$1 = \frac{10 + x}{17.32}$$
$$x = 17.32 - 10.00 = 7.32 \text{ m}$$

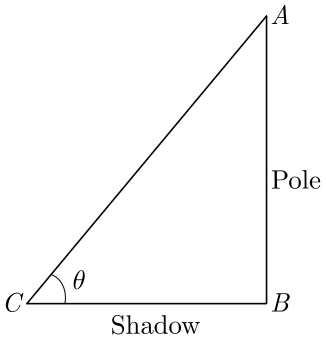
Hence, length of flagstaff is 7.32 m.

THREE MARKS QUESTIONS

46. Find the length of the shadow on the ground of a pole of height 18 m when angle of elevation θ of the sun is such that $\tan\theta = \frac{6}{7}$.

Sol : [Board 2023 OD Standard]

As per question we have drawn the figure below.



In right ΔABC, $\tan\theta = \frac{AB}{BC}$

Substitute $\tan\theta = \frac{6}{7}$ and height of pole $AB = 18$ m

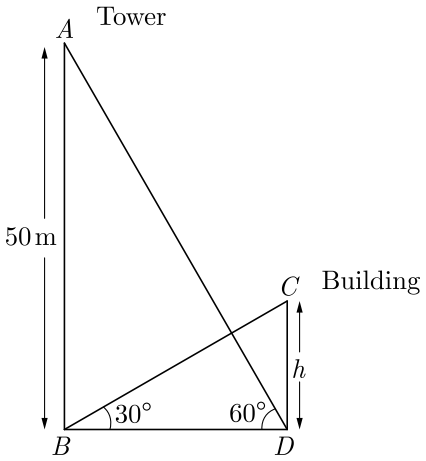
$$\frac{6}{7} = \frac{18}{BC}$$
$$BC = \frac{18 \times 7}{6} = 21 \text{ m}$$

Hence, the length of the shadow is 21 m.

47. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of a tower from the foot of the building is 60°. If the tower is 50 m high, then find the height of the building.

Sol : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below.



In ΔABD, $\tan 60^\circ = \frac{AB}{BD}$

$$\sqrt{3} = \frac{50}{BD}$$
$$BD = \frac{50}{\sqrt{3}}$$

Now in ΔBDC,

$$\tan 30^\circ = \frac{CD}{BD}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{\frac{50}{\sqrt{3}}} = \frac{h\sqrt{3}}{50}$$
$$3h = 50$$
$$h = \frac{50}{3} = 16.67$$

Hence, the height of the building is 16.67 m.

PRACTICE

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower

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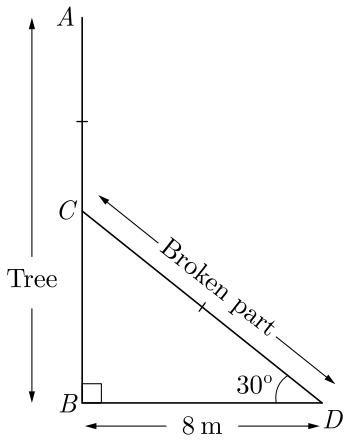
given in question we have drawn figure below.



In right ΔCBD, $\cos 30^\circ = \frac{BD}{CD}$

$$\frac{\sqrt{3}}{2} = \frac{8}{CD}$$

$$CD = \frac{16}{\sqrt{3}}$$

and $\tan 30^\circ = \frac{BC}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{BC}{8}$$

$$BC = \frac{8}{\sqrt{3}}$$

Height of tree,

$$BC + CD = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$$

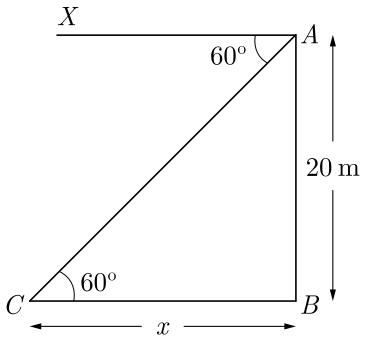
$$= \frac{24}{\sqrt{3}} = 8\sqrt{3}$$

Hence the height of the tree is $8\sqrt{3}$ m.

43. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. Take $\sqrt{3} = 1.732$

Sol : [Board Term-2 2011]

Let C be the point where the ball is lying. As per given in question we have drawn figure below.



Due to alternate angles we obtain

$$\angle XAC = \angle ACB = 60^\circ$$

In ΔABC, $\tan 60^\circ = \frac{AB}{BC}$

$$\sqrt{3} = \frac{20}{x}$$

$$x = \frac{20}{\sqrt{3}}$$

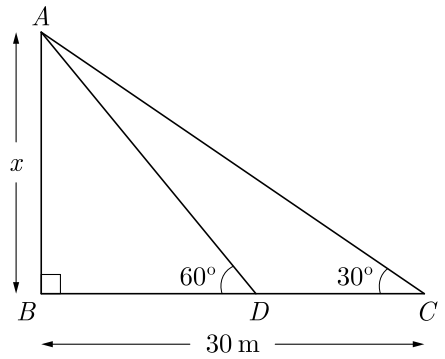
$$= 20\left(\frac{\sqrt{3}}{3}\right)$$

Hence, distance between ball and foot of tower is 11.53 m.

44. If the shadow of a tower is 30 m long, when the Sun's elevation is 30°. What is the length of the shadow, when Sun's elevation is 60° ?

Sol : [Board Term-2 2011]

As per given in question we have drawn figure below. Here AB is tower and BD is shadow at 60° and BC is shadow at 30° elevation.



In ΔABC, $\frac{AB}{BC} = \tan 30^\circ$

$$\frac{AB}{30} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

In ΔABD, $\frac{AB}{BD} = \tan 60^\circ$

$$\frac{10\sqrt{3}}{BD} = \tan 60^\circ = \sqrt{3}$$

$$BD = 10 \text{ m}$$

Hence the length of shadow is 10 m.

45. From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30° . A flag is hoisted at the top the of the building and the angle of elevation of the length of the flagstaff from P is 45°. Find the length of the flagstaff and distance of building from point P.

NODIA

We have $\tan 45^\circ = \dfrac{h-50}{x}$

$x = h - 50$ ...(1)

and $\tan 60^\circ = \dfrac{h}{x}$

$\sqrt{3} = \dfrac{h}{x}$

$x = \dfrac{h}{\sqrt{3}}$ ...(2)

From (1) and (2) we have

$h - 50 = \dfrac{h}{\sqrt{3}}$

$\sqrt{3}\,h - 50\sqrt{3} = h$

$\sqrt{3}\,h - h = 50\sqrt{3}$

$h(\sqrt{3}-1) = 50\sqrt{3}$

$h = \dfrac{50\sqrt{3}}{\sqrt{3}-1} = \dfrac{50(3+\sqrt{3})}{2}$

$= 25(3+\sqrt{3})$

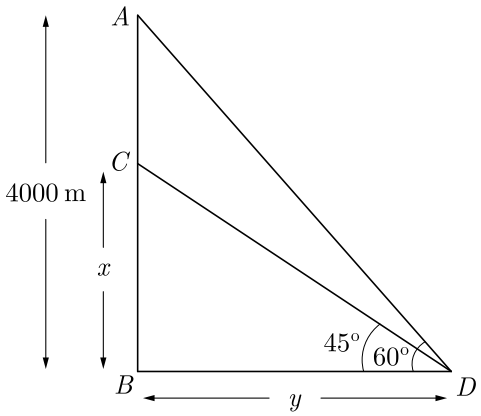
$= 75 + 25\sqrt{3} = 118.25$ m

Thus $h = 118.25$ m.

51. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. (Use $\sqrt{3} = 1.73$)

Sol : [Board Term-2 Foreign 2016]

Let the height first plane be $AB = 4000$ m and the height of second plane be $BC = x$ m. As per given in question we have drawn figure below.



Here $\angle BDC = \angle 45^\circ$ and $\angle BDA = 60^\circ$

In ΔCBD, $\dfrac{x}{y} = \tan 45^\circ = 1 \Rightarrow x = y$

and in ΔABD, $\dfrac{4000}{y} = \tan 60^\circ = \sqrt{3}$

$y = \dfrac{4000\sqrt{3}}{3}$

$= 2306.67$ m

Thus vertical distance between two,

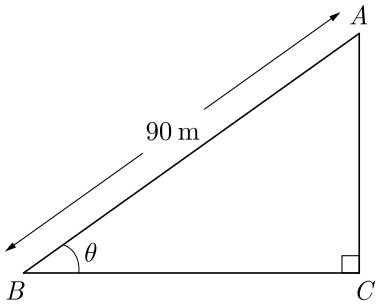
$4000 - y = 4000 - 2306.67$

$= 1693.33$ m

52. A boy, flying a kite with a string of 90 m long, which is making an angle θ with the ground. Find the height of the kite. (Given $\tan\theta = \frac{45}{8}$)

Sol : [Board Term-2 OD 2014]

Let A be the position of kite and AB be the string. As per given in question we have drawn figure below.



Since $\tan\theta = \dfrac{15}{8} = \dfrac{AC}{BC} = k$

Let AC be $15k$ and BC be $8k$. Now using Pythagoras Theorem

$AB = \sqrt{BC^2 + AC^2}$

$= \sqrt{(15k)^2 + (8k)^2} = 17k$

In ΔACB, $\dfrac{AC}{AB} = \sin\theta$

$\dfrac{AC}{90} = \dfrac{15k}{17k} = \dfrac{15}{17}$

$AC = \dfrac{15 \times 90}{17} = 79.41$ m

Hence, height of kite is 79.41 m.

53. A 7 m long flagstaff is fixed on the top of a tower standing on the horizontal plane. From point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60° and 45° respectively. Find the height of the tower correct to one place of decimal. (Use $\sqrt{3} = 1.73$)

Sol : [Board Term-2 Foreign 2016]

As per given in question we have drawn figure below. Here AB is flagstaff and BC is tower.

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is 60 m high, find the height of the building.
[Board 2020 Delhi Basic, Delhi 2013]
Ans : 20 m

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.
[Board Term-2 Delhi 2015]
Ans : $10\sqrt{3}$ m

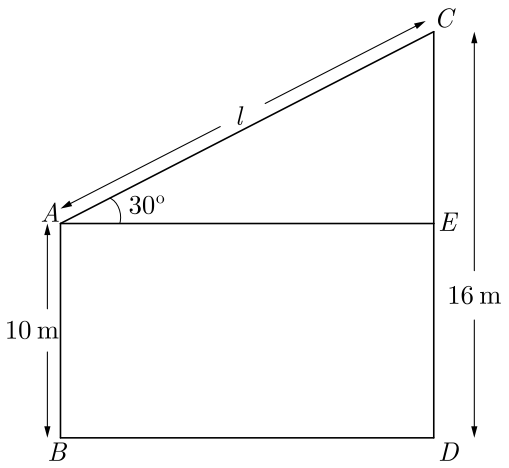
The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill.
[Board Term-2 2012]
Ans : 150 m

48. The top of two poles of height 16 m and 10 m are connected by a length l meter. If wire makes an angle of 30° with the horizontal, then find l.

Sol : [Board Term-2, 2012]

Let AB and CD be two poles, where $AB = 10$ m, $CD = 16$ m.

As per given in question we have drawn figure below.



Length $CE = CD - CE = CD - AB$

$= 16 - 10 = 6$ m.

From ΔAEC, $\sin 30^\circ = \frac{CE}{l}$

$\frac{1}{2} = \frac{CE}{l}$

$l = 2CE = 6 \times 2 = 12$ m.

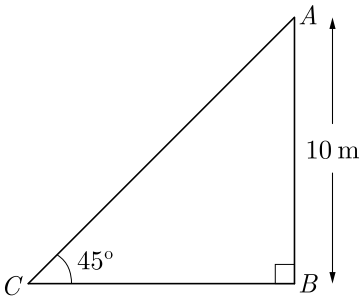
Hence, the value of l is 12 m.

49. An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire. [Use $\sqrt{2} = 1.414$]

Sol : [Board Term-2 2016]

Let OA be the electric pole and B be the point on the ground to fix the pole. Let BA be x.

As per given in question we have drawn figure below.



In ΔABC we have,

$\sin 45^\circ = \frac{AB}{AC}$

$\frac{1}{\sqrt{2}} = \frac{10}{AC}$

$AC = 10\sqrt{2} = 10 \times 1.414$

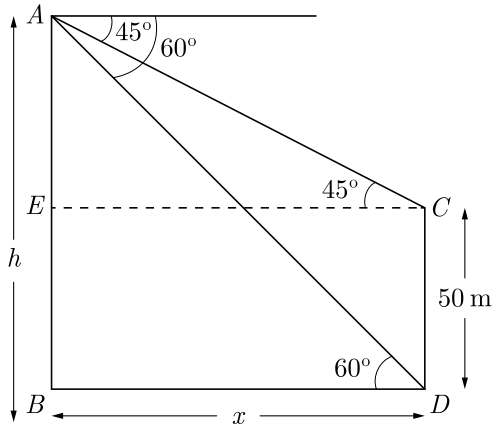
$= 14.14$ m

Hence, the length of wire is 14.14 m

50. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (Use $\sqrt{3} = 1.73$)

Sol : [Board Term-2 Delhi 2016]

As per given in question we have drawn figure below. Here AC is tower and DC is building.



NODIA

Now in ΔABE, $\angle AEB = 60^o$

where $AB = h$, $BE = 10\sqrt{3}$ m

and $\angle AEB = 60^o$

Thus $\tan 60^o = \frac{AB}{BE}$

$\sqrt{3} = \frac{AB}{10\sqrt{3}}$

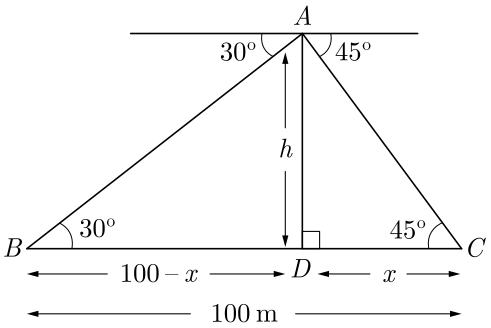
$AB = 10\sqrt{3} \times \sqrt{3} = 30$ m

Thus height of hill $AB + 10 = 40$ m

56. Two ships are approaching a light house from opposite directions. The angle of depression of two ships from top of the light house are 30^o and 45^o. If the distance between two ships is 100 m, Find the height of light-house.

Sol : [Board Term-2 Foreign 2014]

As per given in question we have drawn figure below. Here AD is light house of height h and BC is the distance between two ships.



We have $BC = 100$ m

In ΔADC, $\tan 45^o = \frac{h}{x} \Rightarrow h = x$

In ΔABD, $\tan 30^o = \frac{h}{100 - x}$

$\frac{1}{\sqrt{3}} = \frac{h}{100 - x}$

$100 - x = h\sqrt{3}$

$100 - h = h\sqrt{3}$ $\quad h = x$

$100 = h + h\sqrt{3} = h(1 + \sqrt{3})$

$h = \frac{100}{1 + \sqrt{3}}$

$= \frac{100}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)}$

$= \frac{100(\sqrt{3} - 1)}{3 - 1}$

$= 50(\sqrt{3} - 1)$

$= 50(1.732 - 1)$

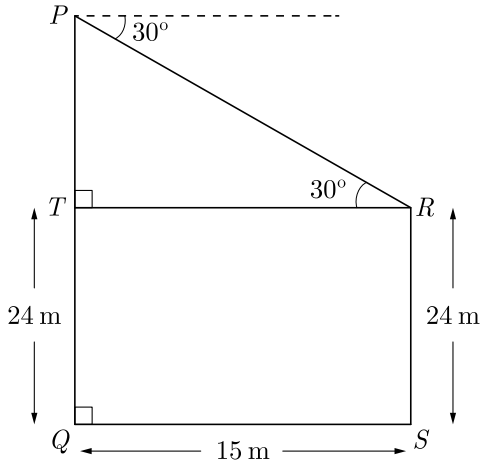
$= 50 \times 0.732$

Thus height of light house is 36.60 m.

57. The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is $30°$. If the height of the first of the pole is 24 m, find the height of the second pole. [Use $\sqrt{3} = 1.732$]

Sol : [Board Term-2 2013]

Let RS be first pole and PQ be second pole. As per given in question we have drawn figure below.



In right ΔPTR,

$\tan 30° = \frac{PT}{TR}$

$\frac{1}{\sqrt{3}} = \frac{PT}{15}$

$PT = \frac{15}{\sqrt{3}} = 5\sqrt{3}$

$= 5 \times 1.732 = 8.66$

$PQ = PT + TQ$

$= 8.66 + 24 = 32.66$ m

Thus height of the second pole is 32.66 m.

PRACTICE

The horizontal distance between two towers is 60 m. The angle of elevation of the top of the taller tower as seen from the top of the shorter one is 30^o. If the height of the taller tower is 150 m, then find the height of the shorter tower.

[Board Term-2 2015]

Ans :

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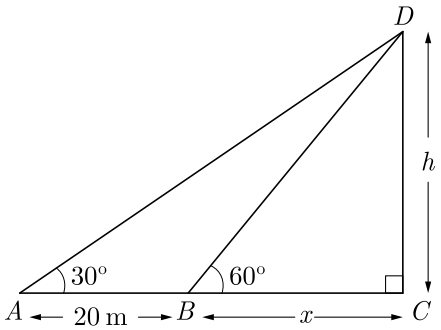
58. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metre towards the foot of the tower to a point B the angle of elevation increase to 60° . Find the height of the tower and the distance of the tower from the point A.

Sol : [Board Term-2 2012]

Let height of tower CD be h and distance BC be x.

As per given in question we have drawn figure below.



In right ΔDBC, $\dfrac{h}{x} = \tan 60^\circ$

$$h = \sqrt{3}\,x \qquad ...(1)$$

In right ΔADC,

$$\frac{h}{x+20} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x+20 \qquad ...(2)$$

Substituting the value of h from eq. (1) in eq. (2), we get

$$3x = x+20$$

$$x = 10 \text{ m} \qquad ...(3)$$

Thus $AC = 20 + x = = 30$ m.

and $h = \sqrt{3} \times 10 = 10\sqrt{3}$

$$= 10 \times 1.732$$

$$= 17.32 \text{ m}$$

Hence, height of tower is 17.32 m and distance of tower from point A is 30 m.

PRACTICE

A person observed the angle of elevation of the top of a tower as 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower.

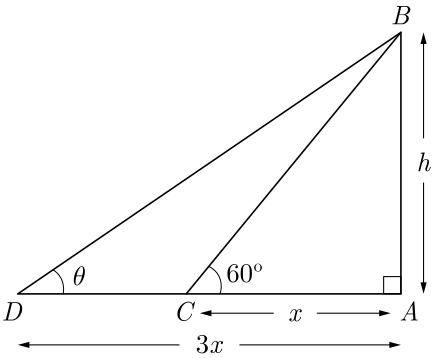
[Board Term-2 2012]

Ans : 43.3 m

59. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the sun is 60°. Find the angle of elevation of the sun at the of the longer shadow.

Sol : [Board Term-2 Foreign 2017]

Let AB be tower of height h, AC be the shadow at elevation of sun of 60°. As per given in question we have drawn figure below.



In right ΔBAC,

$$\frac{AB}{AC} = \tan 60^\circ$$

$$\frac{h}{x} = \sqrt{3}$$

$$h = x\sqrt{3}$$

In right ΔBAD,

$$\frac{AB}{AD} = \tan\theta$$

$$\frac{h}{3x} = \tan\theta$$

$$\frac{x\sqrt{3}}{3x} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

Thus $\theta = 30^\circ$.

60. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Sol : [Board Term-2 OD 2012]

Let CD be statue of 1.6 m and pedestal BC of height h. Let A be point on ground. As per given in question we have drawn figure below.

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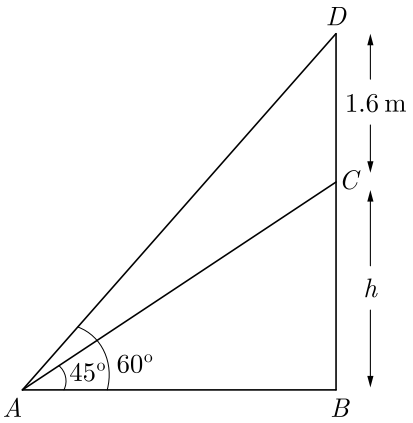


In right ΔABD,

$$\cot 60^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{h+1.6}$$

$$AB = \frac{h+1.6}{\sqrt{3}} \quad ...(1)$$

In right ΔABC,

$$\frac{AB}{BC} = \cot 45^\circ$$

$$1 = \frac{AB}{h}$$

$$AB = h \quad ...(2)$$

From (1) and (2), we get

$$h = \frac{h+1.6}{\sqrt{3}}$$

$$h\sqrt{3} = h + 1.6$$

$$h\sqrt{3} - h = 1.6$$

$$h(\sqrt{3}-1) = 1.6$$

$$h = \frac{1.6}{\sqrt{3}-1}$$

$$= \frac{1.6}{1.732-1}$$

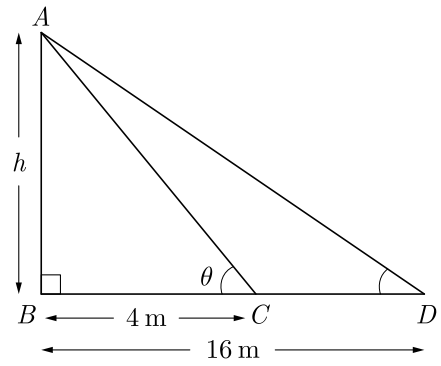
$$= \frac{1.6}{0.732} = 2.185 \text{ m}$$

Height of pedestal h is 2.2 m.

61. On a straight line passing through the foot of a tower, two C and D are at distance of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

Sol : [Board Term-2 OD 2017]

Let AB be tower of height h, C and D be the two point. As per given in question we have drawn figure below.



Since $\angle ACB$ and $\angle ADB$ are complementary,

$$\angle ACB = \theta \text{ and } \angle ADB = 90^\circ - \theta$$

Now, in right ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{h}{4} \quad ...(1)$$

In right ΔABD,

$$\tan(90-\theta) = \frac{AB}{BD} = \frac{h}{16}$$

$$\cot\theta = \frac{h}{16}$$

$$\tan\theta = \frac{16}{h} \quad ...(2)$$

From (1) and (2) we have

$$\frac{h}{4} = \frac{16}{h}$$

$$h^2 = 4 \times 16 = 64 = 8^2$$

$$h = 8 \text{ m}$$

Thus height of tower is 8 m.

FIVE MARKS QUESTIONS

62. From the top of a 15 m high building, the angle of elevation of the top of a tower is found to be 30°. From the bottom of the same building, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower and the distance between tower and the building.

Sol : [Board 2024 OD Standard]

As per given in question we have drawn figure below.

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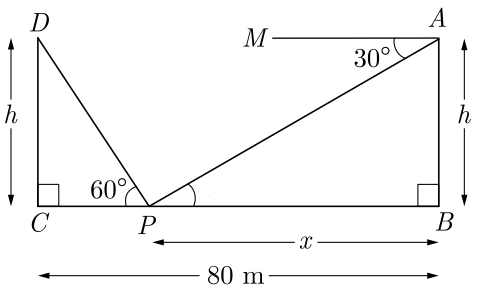
60° and the angle of depression from the top of the other pole of point P is 30°. Find the heights of the poles and the distance of the point P from the poles.

Sol : [Board 2019 OD]

Let the distance between pole AB and point P be x. As per given in question we have drawn figure below.



Here distance between pole CD and P is $80 - x$.

In right angle triangle ΔABP, $\angle APB = 30^\circ$

$$\tan 30^\circ = \frac{h}{x}$$

$$h = \frac{x}{\sqrt{3}} \quad ...(1)$$

In angle triangle ΔCDP,

$$\tan 60^\circ = \frac{CD}{CP} = \frac{CD}{CB - PB}$$

$$\sqrt{3} = \frac{h}{80 - x}$$

$$h = 80\sqrt{3} - x\sqrt{3} \quad ...(2)$$

Comparing (1) and (2) we have

$$\frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$$

$$x = 80 \times 3 - x \times 3$$

$$4x = 240$$

$$x = \frac{240}{4} = 60 \text{ m}$$

Substituting this value of x in (1) we have

$$h = \frac{60}{\sqrt{3}} = 20\sqrt{3} = 34.64 \text{ m}$$

Hence, height of the pole AB and CD is 34.64 m

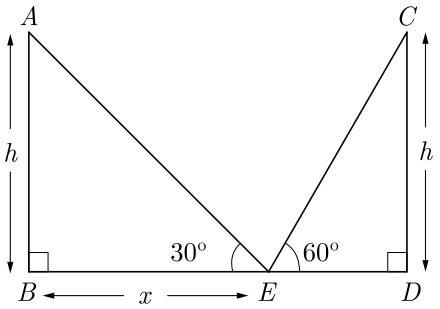
Distance of point P from pole AB is 20 m.

Distance of point P from pole CD is 60 m.

70. Two poles of equal heights are standing opposite to each other on either side of a road, which is 80 m wide. From a point between them on the road, angles of elevation of their top are 30° and 60°. Find the height of the poles and distance of point from poles.

Sol : [Board 2019 Delhi, OD 2011]

Let the distance between pole AB and man E be x. As per given in question we have drawn figure below.



Here distance between pole CD and man is $80 - x$

In right angle triangle ΔABE,

$$\tan 30^\circ = \frac{h}{x}$$

$$h = \frac{x}{\sqrt{3}} \quad ...(1)$$

In angle triangle ΔCDE,

$$\tan 60^\circ = \frac{h}{80 - x}$$

$$\sqrt{3} = \frac{h}{80 - x}$$

$$h = 80\sqrt{3} - x\sqrt{3} \quad ...(2)$$

Comparing (1) and (2) we have

$$\frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$$

$$x = 80 \times 3 - x \times 3$$

$$4x = 240$$

$$x = \frac{240}{4} = 60 \text{ m}$$

Substituting this value of x in (1) we have

$$h = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

Hence, height of the pole is 34.64 m

71. Amit, standing on a horizontal plane, find a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, find the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

Sol : [Board 2019 OD]

As per given information in question we have drawn the figure given below.

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and also the horizontal distance between the building and the tower.

[Board Term-2 SQP 2018]

Ans : 118.3 m

✍ From the top of a building 60 m high the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. Find the height of the tower.

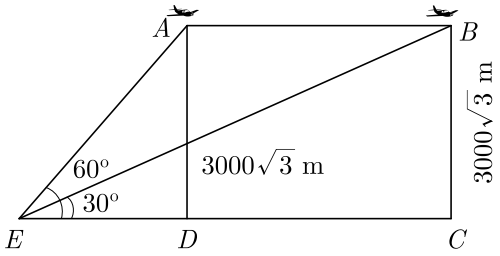
[Board Term-2 2011, 2012, OD 2014]

Ans : 40 m

64. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

Sol : [Board 2020 SQP Standard, 2014]

As per given in question we have drawn figure below. Here



$$\angle AED = 60^\circ, \angle BED = 30^\circ$$

$$AD = BC = 3000\sqrt{3} \text{ m}$$

Let the speed of the aeroplane be x.

$$AB = DC \times 30 \times x$$

$$= 30x \text{ m} \quad ...(1)$$

In right ΔAED, we have

$$\tan 60^\circ = \frac{AD}{DE}$$

$$\sqrt{3} = \frac{3000\sqrt{3}}{DE}$$

$$DE = 3000 \text{ m} \quad ...(2)$$

In right ΔBEC,

$$\tan 30^\circ = \frac{BC}{EC}$$

$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{DE + CD}$$

$$DE + CD = 3000 \times 3$$

$$3000 + 30x = 9000$$

$$30x = 6000$$

$$x = 200 \text{ m/s}$$

Hence, speed of plane is 200 m/s

$$= 200 \times \frac{18}{5} = 720 \text{ km/hr}$$

Hence, the speed of the aeroplane is 720 km/hr.

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PRACTICE

✍ The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changed to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr.

[Board Term-2 OD 2015]

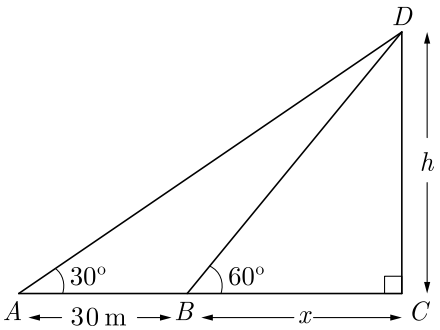
Ans : 720 km/h

65. The person standing on the bank of river observes that the angle of elevation of the top of a tree standing on opposite bank is 60°. When he moves 30 m away from the bank, he finds the angle of elevation to be 30°. Find the height of tree and width of the river.

Sol : [Board 2020 OD Basic]

Let CD be the tree of height h. Let A be the position of person after moving 30 m away from point B on bank of river. Let $BC = x$ be the width of the river.

As per given in question we have drawn figure below.



In right ΔDBC, $\frac{h}{x} = \tan 60^\circ$

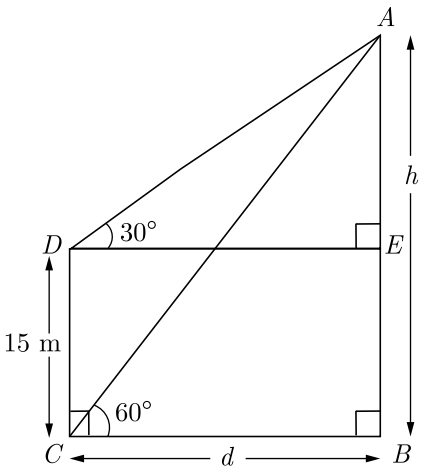
$$h = \sqrt{3}\, x \quad ...(1)$$

In right ΔADC,

$$\frac{h}{x+30} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\, h = x + 30 \quad ...(2)$$

Substituting the value of h from eq. (1) in eq.



Let AB and CD represent the tower of height h and building of height 15 m respectively and let the distance between the building and the tower be d.

In ΔABC, $\tan 60^\circ = \frac{h}{d}$

$$\sqrt{3}\, d = h \qquad ...(1)$$

In ΔADE, $\tan 30^\circ = \frac{h-15}{d}$

$$\frac{d}{\sqrt{3}} = h - 15 \qquad ...(2)$$

From (1) and (2) we have

$$\frac{h}{\sqrt{3} \times \sqrt{3}} = h - 15$$

$$h = 3h - 45$$

$$h = \frac{45}{2}$$

$$= 22.5 \text{ m}$$

Substituting value of h in (1), we get

$$d = \frac{45}{2\sqrt{3}} = \frac{45\sqrt{3}}{6}$$

$$= \frac{15\sqrt{3}}{2} \text{ m}$$

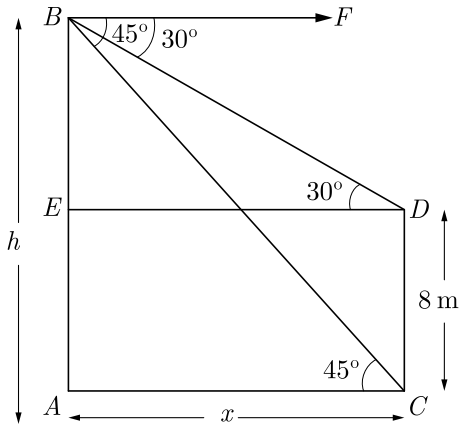
$$= 12.99 \text{ m (approx.)}$$

Height of tower is 22.5 m and distance between tower and the building is 12.99 m approximately.

63. The angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30° and 45°, respectively. Find the height of multi-storey building and distance between two buildings.

Sol : [Board Term-2 OD 2014]

As per given in question we have drawn figure below.



Here $AE = CD = 8$ m

$BE = AB - AE = (h - 8)$

and $AC = DE = x$

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$

In right angled ΔCAB we have

$$\tan 45^\circ = \frac{AB}{AC}$$

$$1 = \frac{h}{x} \Rightarrow x = h \qquad ...(1)$$

In right angled ΔEDB,

$$\tan 30^\circ = \frac{BE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{h-8}{x}$$

$$x = \sqrt{3}(h-8) \qquad ...(2)$$

From (1) and (2), we get

$$h = \sqrt{3}\, h - 8\sqrt{3}$$

$$8\sqrt{3} = \sqrt{3}\, h - h$$

$$h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 4\sqrt{3}(\sqrt{3}+1) = (12 + 4\sqrt{3}) \text{ m}$$

Since, $x = h$, $x = (12 + 4\sqrt{3})$

Distance $= (12 + 4\sqrt{3})$ m

$= 4(3 + \sqrt{3})$ m

Hence the height of multi storey building is $4(3+\sqrt{3})$ m.

PRACTICE

✎ The angle of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 45° respectively. Find the height of the tower

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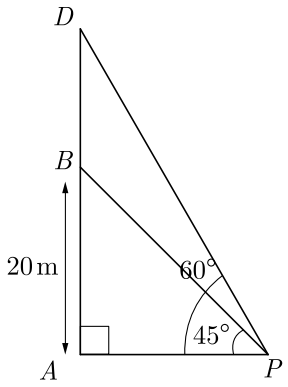

$$\text{In } \Delta PAB, \quad \tan 45^\circ = \frac{AB}{AP}$$

$$1 = \frac{20}{AP} \Rightarrow AP = 20\,\text{m}$$

$$\text{In } \Delta PAD, \quad \tan 60^\circ = \frac{AD}{AP}$$

$$= \frac{20 + BD}{20}$$

$$\sqrt{3} = \frac{20 + BD}{20}$$

$$20 + BD = 20\sqrt{3}$$

$$BD = 20\sqrt{3} - 20$$

$$= 20(\sqrt{3} - 1)$$

$$= 20(1.732 - 1)$$

$$= 20 \times 0.732$$

$$= 14.64\,\text{cm}.$$

Hence, height of the tower is 14.64 m.

PRACTICE

From a point P on the ground, the angles of elevation of the top of a 10 m tall building and a helicopter, hovering at some height vertically over the top of the building are 30° and 60° respectively. Find the height of the helicopter above the ground.

[Board Term-2 OD Compt. 2017]

Ans : 20 m

68. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Sol : [Board 2020 Delhi Standard]

Let AB be a building of height 7 m and CD be tower of height CD. From the given information we have drawn the figure as below.

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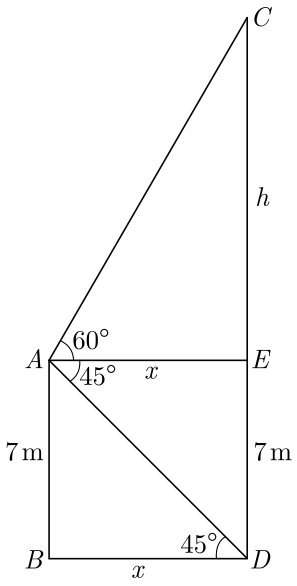


Now $CD = (7 + h)$

$$BD = AE = x$$

$$\text{In } \Delta ABD, \quad \tan 45^\circ = \frac{AB}{BD}$$

$$1 = \frac{7}{x} \Rightarrow x = 7\,\text{cm}$$

$$\text{In } \Delta CEA, \quad \tan 60^\circ = \frac{CE}{AE}$$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$$

Substituting the value of x, we get

$$h = 7\sqrt{3}$$

Now, $CD = CE + ED$

$$= (7 + 7\sqrt{3})\ \text{m}$$

$$= 7(1 + \sqrt{3})\ \text{m}$$

$$= 7(1 + 1.732)\ \text{m}$$

$$= 7 \times 2.732\ \text{m} = 19.124\ \text{m}$$

Hence height of tower is 19.12 m approximately.

PRACTICE

From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. (Use $\sqrt{3} = 1.732$)

[Board Term-2 Foreign 2013, 2017]

Ans : 19.124 m

69. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point P between them on the road, the angle of elevation of the top of a pole is

(2), we get

$$3x = x + 30$$
$$x = 15 \text{ m} \quad ...(3)$$

Thus $h = \sqrt{3} \times 15 = 15\sqrt{3}$
$= 15 \times 1.732 = 25.98$ m

Hence, height of tree is 25.98 m and width of river is 15 m.

PRACTICE

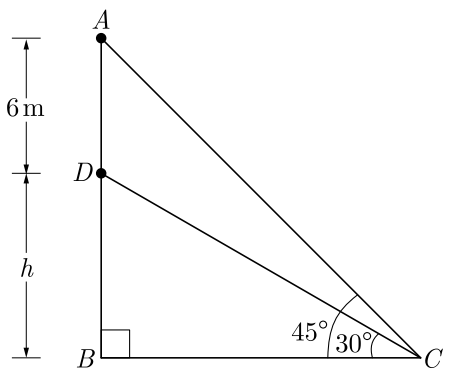
- A person standing on the bank of a river, observes that the angle of elevation of the top of the tree standing on the opposite bank is 60°. When he retreats 20 m from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the breadth of the river.
  [Board Term-2 OD 2012]
  Ans : 17.3 m and 10 m

- An observer finds the angle of elevation of the top of the tower from a certain point on the ground as 30°. If the observer moves 20 m, towards the base of the tower, the angle of elevation of the top increase by 15°, find the height of the tower.
  [Board Term-2 Delhi 2017]
  Ans : $10(\sqrt{3}+1)$ m

66. A vertical tower stands on horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the ground, angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take $\sqrt{3} = 1.73$)

Sol : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



Here AD is a flagstaff and BD is a tower.

In ΔABC $\tan 45^\circ = \dfrac{AB}{BC}$

$$1 = \frac{h+6}{BC}$$
$$BC = h + 6 \quad ...(1)$$

In ΔDBC, $\tan 30^\circ = \dfrac{DB}{BC}$ from (1)

$$\frac{1}{\sqrt{3}} = \frac{h}{h+6}$$
$$h\sqrt{3} = h + 6$$
$$h(\sqrt{3}-1) = 6$$
$$h = \frac{6}{\sqrt{3}-1}$$
$$= \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$
$$= \frac{6(\sqrt{3}+1)}{2}$$
$$= 3(\sqrt{3}+1)$$
$$= 3(1.73+1)$$
$$= 3 \times 2.73 = 8.19 \text{ m}$$

Thus height of tower is 8.19 m.

PRACTICE

- From a point P on the ground, the angle of elevation of the top of a tower is 30° and that of the top of the flagstaff is 45°. If height of flagstaff is 5 m, find the height of the tower. (Use $\sqrt{3} = 1.732$)
  [Board 2019 OD]
  Ans : 6.83 m

- The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff. [Use $\sqrt{3} = 1.73$]
  [Board Term-2 OD 2014]
  Ans : 87.6 m

67. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Sol : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below. Here AB is the building and BD is tower on building.

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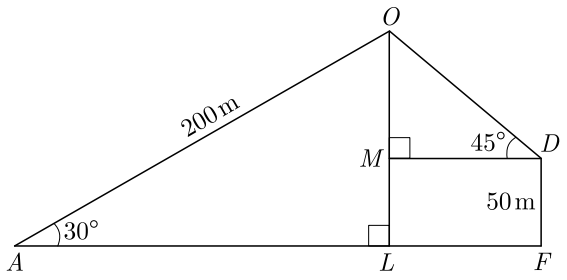


Let O be the position of the bird, A be the position for Amit, D be the position for Deepak and FD be the building at which Deepak is standing at height 50 m.

In ΔOLA, $\angle L = 90^\circ$

$$\sin 30^\circ = \frac{OL}{OA}$$

$$\frac{1}{2} = \frac{OL}{200}$$

$$\Rightarrow \quad OL = \frac{200}{2} = 100\text{ m}$$

$$OM = OL - LM$$

$$= OL - FD$$

$$= (100 - 50)\text{ m} = 50\text{ m}$$

In ΔOMD, $\angle M = 90^\circ$

$$\sin 45^\circ = \frac{OM}{OD}$$

$$\frac{1}{\sqrt{2}} = \frac{50}{OD}$$

$$OD = 50\sqrt{2}$$

$$= 50 \times 1.414$$

$$= 70.7\text{ m}$$

Thus, the distance of the bird from the Deepak is 70.7 m.

PRACTICE

A boy observes that the angle of elevation of a bird flying at a distance of 100 m is 30°. At the same distance from the boy, a girl finds the angle of elevation of the same bird from a building 20 m high is 45°. Find the distance of the bird from the girl.

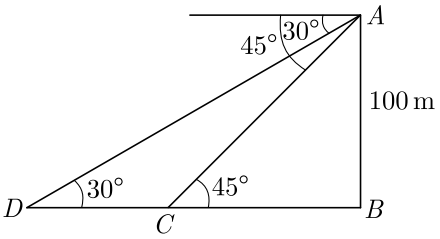
[Board Term-2 OD 2014]

Ans : $30\sqrt{2}$ m

72. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use $\sqrt{3} = 1.732$]

Sol : [Board 2018]

Let AB be the tower and ships are at points C and D. As per question statement we have shown digram below.



Now in ΔABC we have

$$\tan 45^\circ = \frac{AB}{AC}$$

$$\frac{AB}{AC} = 1 \Rightarrow AB = BC$$

Now in ΔABD we have

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$$

$$AB + CD = \sqrt{3}\,AB$$

$$CD = AB(\sqrt{3} - 1)$$

$$= 100 \times (1.732 - 1)$$

$$= 73.2\text{ m}$$

Distance between two ships is 73.2 m.

PRACTICE

From a top of a building 100 m high the angle of depression of two objects are on the same side observed to be 45° and 60°. Find the distance between the objects.

[Board Term-2 OD 2014]

Ans : $\frac{100}{3}(3 - \sqrt{3})$

73. From the top of tower, 100 m high, a man observes two cars on the opposite sides of the tower with the angles of depression 30° and 45° respectively. Find the distance between the cars. (Use $\sqrt{3} = 1.73$)

Sol : [Board Term-2 SQP 2016]

Let DC be tower of height 100 m. A and B be two car on the opposite side of tower. As per given in question we have drawn figure below.

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Car is at P at 30° and is at Q at 45° elevation.

Here $\angle AQB = 45^\circ$

Now, in right ΔABQ we have,

$$\tan 45^\circ = \frac{AB}{BQ}$$
$$1 = \frac{h}{BQ}$$
$$BQ = h$$

In right ΔAPB we have,

$$\tan 30^\circ = \frac{AB}{PB}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{x+h}$$
$$x+h = h\sqrt{3}$$
$$x = h(\sqrt{3}-1)$$

Thus, $\text{Speed} = \dfrac{h(\sqrt{3}-1)}{12}$ m/min

Time for remaining distance,

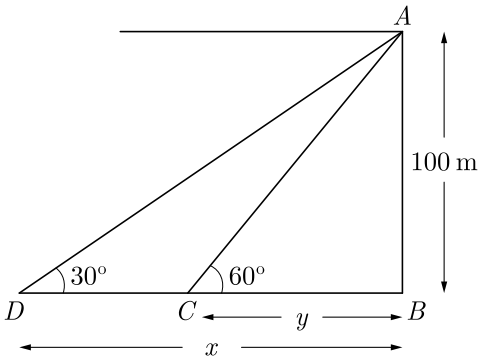
$$t = \frac{h}{\frac{h(\sqrt{3}-1)}{12}} = \frac{12}{(\sqrt{3}-1)}$$
$$= \frac{12(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{12(\sqrt{3}+1)}{3-1}$$
$$= \frac{12}{2}(\sqrt{3}+1) \quad = 6(\sqrt{3}+1)$$
$$t = 6 \times 2.73 = 16.38$$

Hence, time taken by car is 16.38 minutes.

78. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60°. Find the distance travelled by the ship during the period of observation. (Use $\sqrt{3} = 1.73$

Sol : [Board Term-2 OD 2016]

Let AB be the light house of height 100 m. Let C and D be the position of ship at elevation 60° and 30°. As per given in question we have drawn figure below.



In right ΔABC we have

$$\frac{AB}{BC} = \tan 60^\circ$$
$$\frac{100}{y} = \sqrt{3}$$
$$y = \frac{100}{\sqrt{3}}$$

In right ΔABD, we have

$$\frac{AB}{BD} = \tan 30^\circ$$
$$\frac{100}{x} = \frac{1}{\sqrt{3}}$$
$$x = 100\sqrt{3}$$

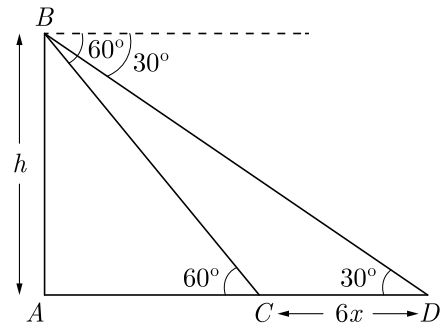
Distance CD travelled by ship,

$$x - y = 100\sqrt{3} - \frac{100}{\sqrt{3}} \text{ m}$$
$$= 100\left[\frac{3-1}{\sqrt{3}}\right] = \frac{100 \times 2}{\sqrt{3}}$$
$$= \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$$
$$= \frac{200 \times 1.73}{3} = \frac{3.46}{3} \text{ m}$$
$$= 115.33 \text{ m}$$

79. A straight highway leads to the foot of a tower. A man standing on its top observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. 6 seconds later, the angle of depression of the car becomes 60°. Find the time taken by the car to reach the foot of tower from this point.

Sol : [Board Term-2 Delhi Compt. 2017]

Let AB be the tower of height h. Let point C and D be location of car. As per given in question we have drawn figure below.



Let the speed of car be x.

Thus distance covered in 6 sec $= 6x$.

Hence $DC = 6x$

Let distance (remaining) CA covered in t sec.

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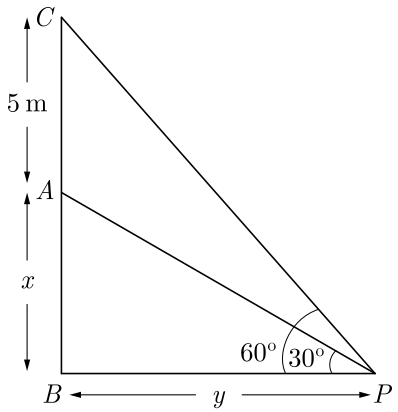
75. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 5 m. From a point on the ground the angles of elevation of top and bottom of the flagstaff are 60° and 30° respectively. Find the height of the tower and the distance of the point from the tower. (take $\sqrt{3} = 1.732$)

Sol : [Board Term-2 Foreign 2016]

Let AB be tower of height x and AC be flag staff of height 5 m. As per given in question we have drawn figure below.



In right ΔABP,

$$\frac{AB}{BP} = \tan 30^\circ$$

$$\frac{x}{y} = \frac{1}{\sqrt{3}}$$

$$y = \sqrt{3}\,x \qquad ...(1)$$

In right ΔCBP

$$\frac{x+5}{y} = \tan 60^\circ = \sqrt{3} \qquad ...(2)$$

Substituting the value of y from (1) we have

$$\frac{x+5}{\sqrt{3}\,x} = \sqrt{3}$$

$$x+5 = 3x \Rightarrow x = 2.5 \text{ m}$$

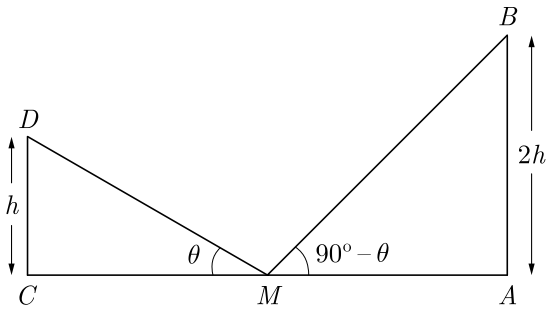
Height of tower is $= 2.5$ m

Distance of P from tower $= (2.5 \times 1.732)$ or 4.33 m.

76. Two post are k metre apart and the height of one is double that of the other. If from the mid-point of the line segment joining their feet, an observer finds the angles of elevation of their tops to be complementary, then find the height of the shorted post.

Sol : [Board Term-2 Foreign 2015]

Let AB and CD be the two posts such that $AB = 2CD$. Let M be the mid-point of CA. As per given in question we have drawn figure below.



Here $CA = k$, $\angle CMD = \theta$ and $\angle AMB = 90^\circ - \theta$

Clearly, $CM = MA = \frac{1}{2}k$

Let $CD = h$. then $AB = 2h$

Now, $$\frac{AB}{AM} = \tan(90^\circ - \theta)$$

$$\frac{2h}{\frac{k}{2}} = \cot\theta$$

$$\frac{4h}{k} = \cot\theta \qquad ...(1)$$

Also in right ΔCMD,

$$\frac{CD}{CM} = \tan\theta$$

$$\frac{h}{\frac{k}{2}} = \tan\theta$$

$$\frac{2h}{k} = \tan\theta \qquad ...(2)$$

Multiplying (1) and (2), we have

$$\frac{4h}{k} \times \frac{2h}{k} = \tan\theta \times \cot\theta = 1$$

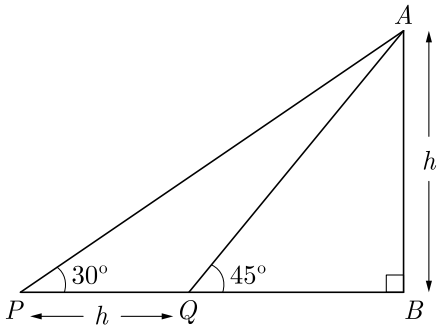
$$h^2 = \frac{k^2}{8}$$

$$h = \frac{k}{2\sqrt{2}} = \frac{k\sqrt{2}}{4}$$

77. A man on the top of a vertical tower observes a car moving at a uniform speed towards him. If it takes 12 min. for the angle of depression to change from 30° to 45°, how soon after this, the car will reach the tower ?

Sol : [Board Term-2 OD 2014]

Let AB be the tower of height h. As per given in question we have drawn figure below.



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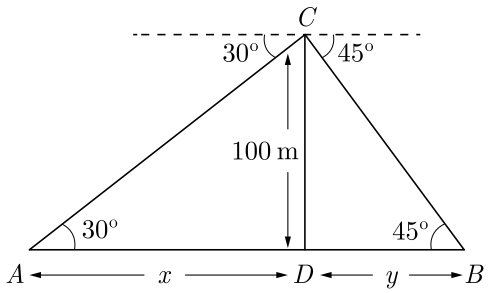


In right ΔADC,

$$\tan 30^\circ = \frac{CD}{AD}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{x}$$

$$x = 100\sqrt{3} \qquad ...(1)$$

In right ΔBDC,

$$\tan 45^\circ = \frac{CD}{DB}$$

$$1 = \frac{100}{y}$$

$$y = 100 \text{ m}$$

Distance between two cars

$$AB = AD + DB$$
$$= x + y$$
$$= (100\sqrt{3} + 100)$$
$$= (100 \times 1.73 + 100) \text{ m}$$
$$= (173 + 100) \text{ m}$$
$$= 273 \text{ m}$$

Hence, distance between two cars is 273 m.

PRACTICE

From the top of a 120 m high tower, a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of repression as 60° and 45°. Find the distance between two cars.

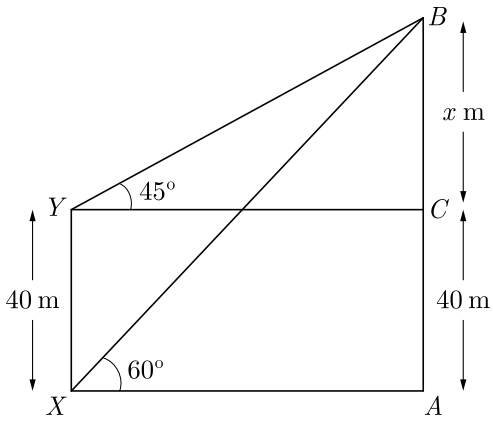
[Board Term-2 Delhi Compt. 2017]

Ans : 189.28 m

14. The angle of elevation of the top B of a tower AB from a point X on the ground is 60°. At point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Find the height of the tower AB and the distance XB.

Sol : [Board Term-2 OD 2016]

As per given in question we have drawn figure below.



In right ΔYCB, we have

$$\tan 45^\circ = \frac{BC}{YC}$$

$$1 = \frac{x}{YC}$$

$$YC = x = XA$$

In right ΔXAB we have

$$\tan 60^\circ = \frac{AB}{XA}$$

$$\sqrt{3} = \frac{x+40}{x}$$

$$\sqrt{3}\,x = x + 40$$

$$x\sqrt{3} - x = 40$$

$$x = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$
$$= 20(\sqrt{3}+1)$$
$$= 20\sqrt{3} + 20$$

Thus height of the tower,

$$AB = x + 40$$
$$= 20\sqrt{3} + 20 + 40$$
$$= 20\sqrt{3} + 60$$
$$= 20(\sqrt{3} + 3)$$

In right ΔXAB we have,

$$\sin 60^\circ = \frac{AB}{BX}$$

$$\frac{\sqrt{3}}{2} = \frac{AB}{BX}$$

$$BX = \frac{2AB}{\sqrt{3}}$$

$$= \frac{20 \times 2(\sqrt{3}+3)}{\sqrt{3}}$$

$$= 40(1 + \sqrt{3})$$

$$= 40 \times 2.73 = 109.20$$

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$$CA = tx$$

Now in right ΔADB,

$$AD = AC + CD = 6x + tx$$

$$\tan 30^\circ = \frac{h}{6x + tx}$$

$$\frac{h}{x} = \frac{6 + t}{\sqrt{3}} \quad ...(1)$$

In right ΔACB we have,

$$\tan 60^\circ = \frac{h}{tx}$$

$$\sqrt{3}\, t = \frac{h}{x} \quad ...(2)$$

From eqn. (1) and (2) we get

$$\sqrt{3}\, t = \frac{6 + t}{\sqrt{3}}$$

$$3t = 6 + t$$

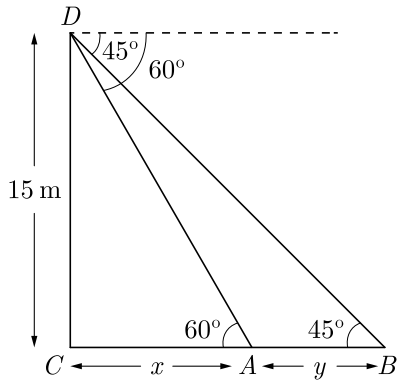
$$2t = 6 \Rightarrow t = 3$$

Hence, car takes 3 seconds.

80. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points.

Sol : [Board Term-2 OD 2017]

Let CD be the tower of height 15 m. Let A and B point on same side of tower As per given in question we have drawn figure below.



In right ΔDCA we have

$$\frac{DC}{CA} = \tan 60^\circ$$

$$\frac{15}{x} = \sqrt{3}$$

$$x = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

In right ΔDCB we have

$$\frac{DC}{CB} = \tan 45^\circ$$

$$\frac{15}{x + y} = 1$$

$$x + y = 15$$

$$5\sqrt{3} + y = 15$$

$$y = 15 - 5\sqrt{3}$$

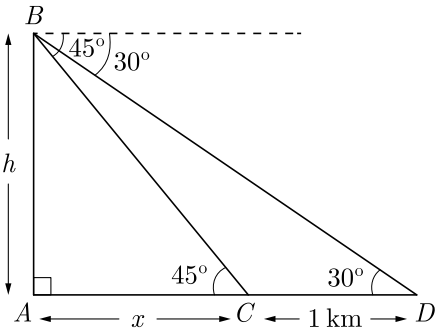
$$= 5(3 - \sqrt{3}) \text{ m}$$

Hence, the distance between points $= 5(3 - \sqrt{3})$m

81. From the top of a hill, the angle of depression of two consecutive kilometre stones due east are found to be 45° and 30° respectively. Find the height of the hill. [Use $\sqrt{3} = 1.73$]

Sol : [Board Term-2 OD 2016]

Let AB be the hill of height h. Angle of depression from point D and C are given 30° and 45° respectively. As per given in question we have drawn figure below.



In right ΔABC we have

$$\frac{AB}{AC} = \tan 45^\circ$$

$$\frac{h}{x} = 1 \Rightarrow h = x$$

In right ΔABD we have

$$\frac{AB}{AC + CD} = \tan 30^\circ$$

$$\frac{h}{x + 1000} = \frac{1}{\sqrt{3}}$$

$$h\sqrt{3} = h + 1000$$

$$h(\sqrt{3} - 1) = 1000$$

$$h = \frac{1000}{\sqrt{3} - 1}$$

$$= \frac{1000(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{1000(\sqrt{3} + 1)}{3 - 1}$$

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$$y = 80$$

In right DOC we have

$$\tan 30^\circ = \frac{80}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{x+y}$$

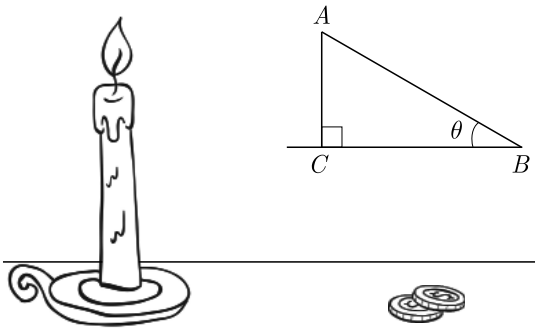
$$x+y = 80\sqrt{3}$$

$$x = 80\sqrt{3} - y = 80\sqrt{3} - 80$$

$$= 80(\sqrt{3}-1) = 58.4 \text{ m}.$$

Hence, speed of bird $= \frac{58.4}{2} = 29.2$ m

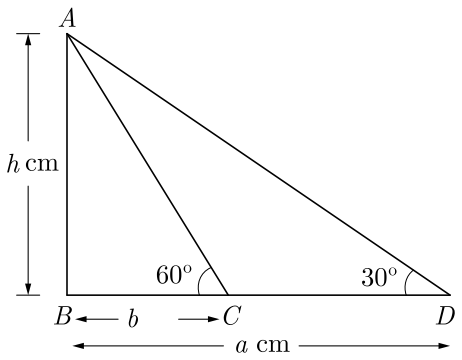
91. If the angles of elevation of the top of the candle from two coins distant a cm and b cm $(a > b)$ from its base and in the same straight line from it are 30° and 60°, then find the height of the candle.



If the angles of elevation of the top of the candle from two coins distant 'a' cm and 'b' cm $(a > b)$ from its base and in the same straight line from it are 30° and 60°, then find the height of the candle.

Sol : [Board SQP Standard 2021]

Let AB be the candle of height h. Let C and D be two points at distance b cm and a cm from the base of the candle. As per given condition we have shown the figure below.



Now $BC = b$ and $BD = a$

In ΔABC, $\tan 60^\circ = \frac{AB}{BC}$

$$\sqrt{3} = \frac{h}{b}$$

$$h = \sqrt{3}\, b \quad \ldots(1)$$

In ΔABD, $\tan 30^\circ = \frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{h}{a}$$

$$h = \frac{a}{\sqrt{3}} \quad \ldots(2)$$

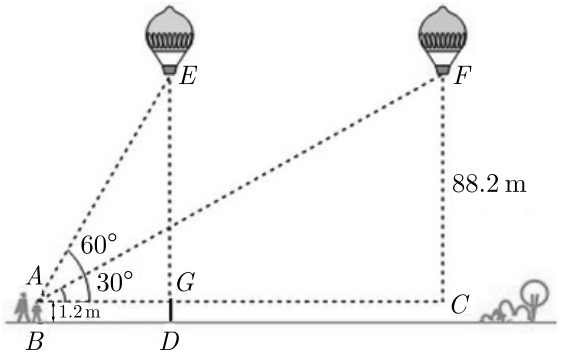
From equation (1) and (2), we have

$$h^2 = \sqrt{3}\, b \times \frac{a}{\sqrt{3}}$$

$$h^2 = ab \Rightarrow h = \sqrt{ab}$$

Hence, height of candle is $\sqrt{ab}$ cm.

92. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60° .After sometime, the angle of elevation reduces 30°.Find the distance travelled by the balloon during the interval.

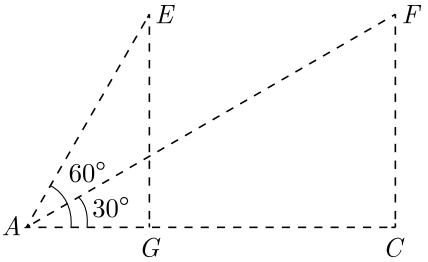


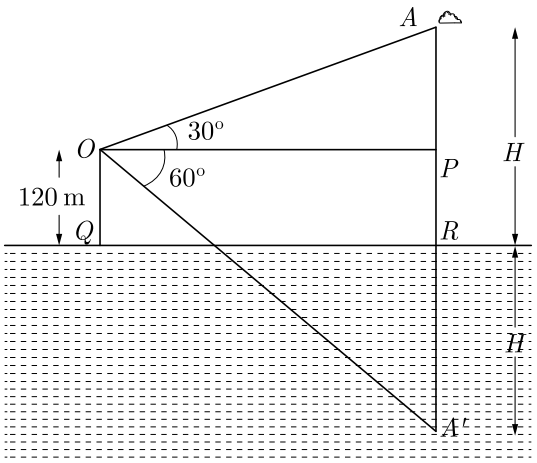
Sol : [Board SQP Basic 2021]

Let E be the initial position of the balloon and F be final position be F.

Height of balloon above the girl height

$$= 88.2 - 1.2 = 87 \text{ m}$$





Here A is cloud and A' is refection of cloud.

In right ΔAOP we have

$$\tan 30^\circ = \frac{PA}{OP}$$

$$\frac{1}{\sqrt{3}} = \frac{H-120}{OP}$$

$$OP = (H-120)\sqrt{3} \quad ...(1)$$

In right $\Delta OPA'$ we have

$$\tan 60^\circ = \frac{PA'}{OP}$$

$$\sqrt{3} = \frac{H+120}{OP}$$

$$OP = \frac{H+120}{\sqrt{3}} \quad ...(2)$$

From (1) and (2), we get

$$\frac{H+120}{\sqrt{3}} = \sqrt{3}(H-120)$$

$$H+120 = 3(H-120)$$

$$H+120 = 3H-360$$

$$2H = 480 \Rightarrow H = 240$$

Thus height of cloud is 240 m.

PRACTICE

An angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water is 60°. Find the height of the cloud from the surface of water.

[Board Term-2 Delhi 2017]

Ans : 120 m

At a point A, 20 metre above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A ?

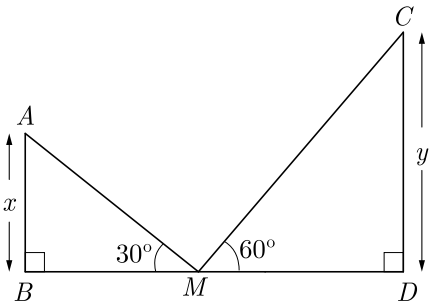
[Board Term-2 OD 2015]

Ans : 40 m

85. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x:y$.

Sol : [Board Term-2 OD 2015]

Let AB be the tower of height x and CD be the tower of height y. Angle of depressions of both tower at centre point M are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here M is the centre of the line joining their feet.

Let $BM = MD = z$

In right ΔABM we have,

$$\frac{x}{z} = \tan 30^\circ$$

$$x = z \times \frac{1}{\sqrt{3}}$$

In right ΔCDM we have,

$$\frac{y}{z} = \tan 60^\circ$$

$$y = z \times \sqrt{3}$$

From (1) and (2), we get

$$\frac{x}{y} = \frac{z \times \frac{1}{\sqrt{3}}}{z \times \sqrt{3}}$$

$$\frac{x}{y} = \frac{1}{3}$$

Thus $x:y = 1:3$

86. The angle of elevation of a jet fighter point A on ground is 60°. After flying 10 seconds, the angle changes to 30°. If the jet is flying at a speed of 648 km/hour, find the constant height at which the jet is flying.

Sol : [Board Term-2 Delhi 2012]

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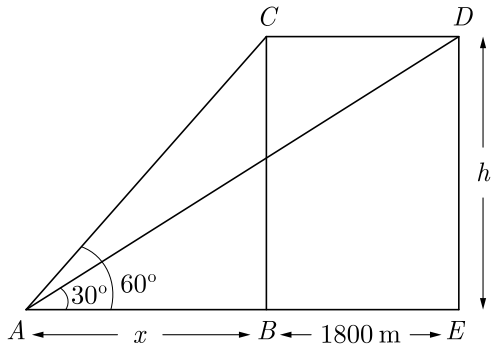
Let C and D are the point of location of jet at height h. Point B and E are foot print on ground of get at thee location. As per given in question we have drawn figure below.



In 3600 sec distance travelled by plane $= 648000$ m

In 10 sec distance travelled by plane

$$= \frac{648000}{3600} \times 10 = 1800 \text{ m}$$

In right ΔABC, we have

$$\frac{h}{x} = \tan 60^\circ = \sqrt{3}$$

$$h = x\sqrt{3} \qquad ...(1)$$

In right ΔADE we have

$$\frac{h}{x+1800} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$h = \frac{x+1800}{\sqrt{3}} \qquad ...(2)$$

From equations (1) and (2), we get

$$x\sqrt{3} = \frac{x+1800}{\sqrt{3}}$$

$$3x = x + 1800$$

$$2x = 1800$$

$$x = 900 \text{ m}$$

$$h = x\sqrt{3} = 900 \times 1.732 = 1558.5 \text{ m}$$

Thus height of jet is 1558.8 m.

87. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively. Find :

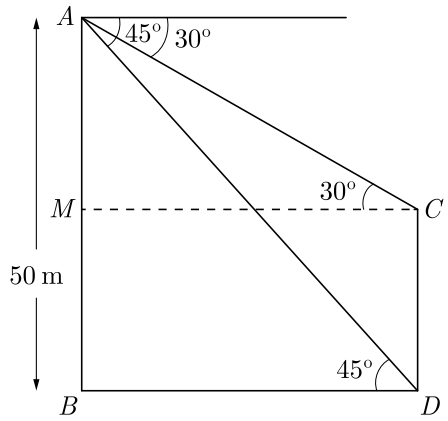
(1) How far the pole is from the bottom of the tower,

(2) The height of the pole. (Use $\sqrt{3} = 1.732$)

Sol : [Board Term-2 Foreign 2015]

Let AB be the tower of height 50 m and CD be the pole of height h. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively.

As per given in question we have drawn figure below.



In right ΔABD we have,

$$\tan 45^\circ = \frac{AB}{BD} = 1$$

$$1 = \frac{50}{x} \Rightarrow x = 50 \text{ m}$$

(1) Thus distance of pole from bottom of tower is 50 m.

Now in ΔAMC we have

$$\tan 30^\circ = \frac{AM}{MC} = \frac{AM}{x}$$

$$AM = \frac{50}{\sqrt{3}} \text{ or } 28.87 \text{ m.}$$

(2) Height pole $h = CD = BM$

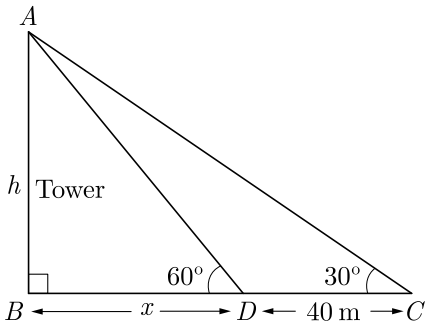
$$= 50 - 28.87 = 21.13 \text{ m.}$$

88. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30°, then when it is 60°. Find the height of the tower.

Sol : [Board Term-2 OD 2011]

Let AB be the tower of height h. Let BC be the shadow at 60° and BD be shadow at 30°.

As per given in question we have drawn figure below.



In right ΔABC we get,

NODIA

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x$$

In right ΔABD we have,

$$\tan 30^\circ = \frac{AB}{BC+40}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+40}$$

$$x+40 = \sqrt{3}\,h = \sqrt{3} \times \sqrt{3}\,x = 3x$$

$$40 = 2x \Rightarrow x = 20 \text{ m}$$

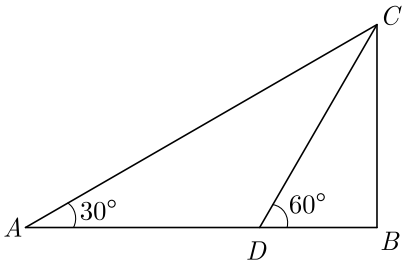
$$h = \sqrt{3} \times 20 = 20\sqrt{3} \text{ m}$$

Thus height of tower is $20\sqrt{3}$ m.

89. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$]

Sol : [Board 2019 Delhi]

As per given information in question we have drawn the figure below.



Here D is first position and A is position after 2 minutes.

Height of the light house,

$$BC = 100 \text{ m}$$

From ΔDBC, $\angle B = 90^\circ$

So, $$\tan 60^\circ = \frac{BC}{BD}$$

$$\sqrt{3} = \frac{100}{BD}$$

$$BD = \frac{100}{\sqrt{3}} \text{m}$$

Now, after time 2 minute boat is at A. New distance from light house is AB and angle is 30°.

From ΔABC, $\angle B = 90^\circ$

So, $$\tan 30^\circ = \frac{BC}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{AB}$$

$$AB = 100\sqrt{3}$$

Therefore, distance d travelled in 2 min,

$$AD = AB - DB = 100\sqrt{3} - \frac{100}{3}$$

$$= 173.2 - \frac{100}{3}\sqrt{3}$$

$$= 173.2 - 57.73$$

$$= 115.47 \text{ m}$$

Speed $$s = \frac{d}{t} = \frac{115.47 \text{ m}}{2 \text{ min}}$$

$$= 57.74 \text{ m/min}$$

Hence, going away from the light house with a speed of 57.74 m/min.

PRACTICE

A moving boat observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat.

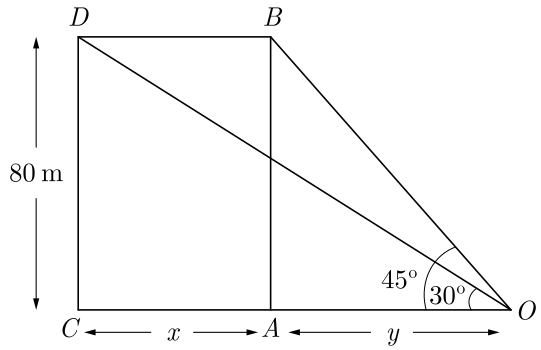
[Board Term-2 Delhi 2017]

Ans : $\frac{3}{2}(3-\sqrt{3})$ km/hr.

90. A bird sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take $\sqrt{3} = 1.732$)

Sol : [Board Term-2 Delhi 2016]

Let CD be the tree of height 80 m and bird is sitting at D. Point O on ground is reference point from where we observe bird. As per given in question we have drawn figure below.



In right AOB we have

$$\tan 45^\circ = \frac{80}{y}$$

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Now $\tan 45^\circ = \frac{h}{324}$

$1 = \frac{h}{324}$

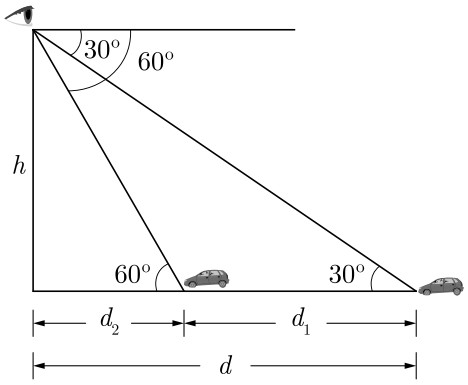
$h = 324$ m

Let d be the initial distance of car from Eiffel tower. At this point A the angle of depression of car from top of tower is 30°. After 15 minute car reaches at point B. At this point the angle of depression of car from top of tower is 60°. We draw a diagram of the situation as shown below.

Now $\tan 30^\circ = \frac{h}{d}$

$\frac{1}{\sqrt{3}} = \frac{324}{d}$

$d = 324\sqrt{3}$ m



Now $\tan 60^\circ = \frac{h}{d_2}$

$\sqrt{3} = \frac{324}{d_2}$

$d_2 = \frac{324}{\sqrt{3}} = \frac{324 \times \sqrt{3}}{3}$

$= 108\sqrt{3}$ m

Now $d_1 = d - d_2$

$= 324\sqrt{3} - 108\sqrt{3} = 216\sqrt{3}$

Here $d_2 = \frac{1}{2} d_1$. Thus time to cover d_2 is half of time to cover d_1 which is $\frac{15}{2} = 7.5$ min.

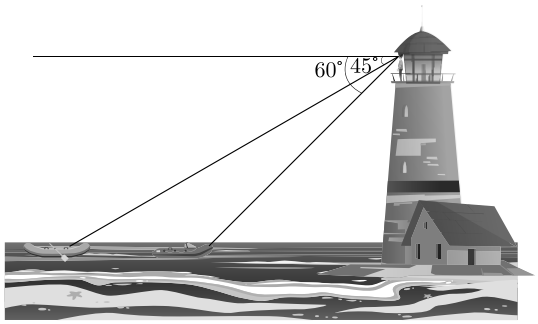
(i) Height of Eiffel tower is 324 m.

(ii) After 7.5 minute, the car will reach the base of the tower.

97. From the observation deck of a seaside building 200 m high, Jignesh sees two fishing boats in the distance. The angle of depression to the nearer boat is 60° while for the boat farther away the angle is 45°.

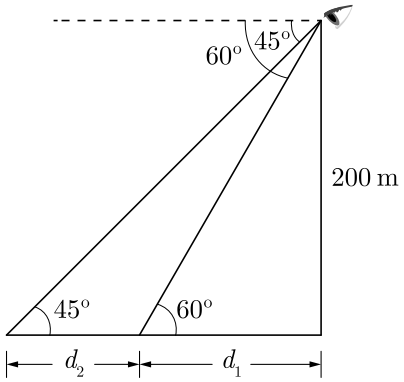
(i) How far out to sea is the nearer boat?

(ii) How far apart are the two boats?



Sol :

Let d_1 be the distance of nearer boat from sea and d_2 be the distance between two boat. We draw a diagram of the situation as shown below.



Now $\tan 60^\circ = \frac{150}{d_1}$

$\sqrt{3} = \frac{150}{d_1}$

$d_1 = \frac{150}{\sqrt{3}} = 50\sqrt{3}$

$= 50 \times 1.732 = 86.6$ m

Now $\tan 45^\circ = \frac{150}{d_1 + d_2}$

$1 = \frac{150}{d_1 + d_2}$

$d_1 + d_2 = 150$

Substituting value of d_1 we have

$86.6 + d_2 = 150$

$d_2 = 150 - 86.6 = 63.4$ m

(i) Thus distance of nearer boat from seaside is 86.6 m

(ii) Both boat are 63.4 m apart.

98. While doing some night fishing, Sarthak round a peninsula and a tall light house comes into view. Taking a sighting, Sarthak find the angle of elevation to the top of the lighthouse is 30°. If the lighthouse is known to be 25 m tall, how far

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In right triangle ΔEAG,

$$\tan 60^\circ = \frac{EG}{AG}$$

$$AG = \frac{87}{\sqrt{3}} = \frac{87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 29\sqrt{3}$$

In right triangle ΔFAC,

$$\tan 30^\circ = \frac{FC}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{87}{AC} \qquad (EC = FC)$$

$$AC = 87\sqrt{3}$$

Now, distance travelled by the balloon

$$GC = AC - AG$$

$$= 87\sqrt{3} - 29\sqrt{3}$$

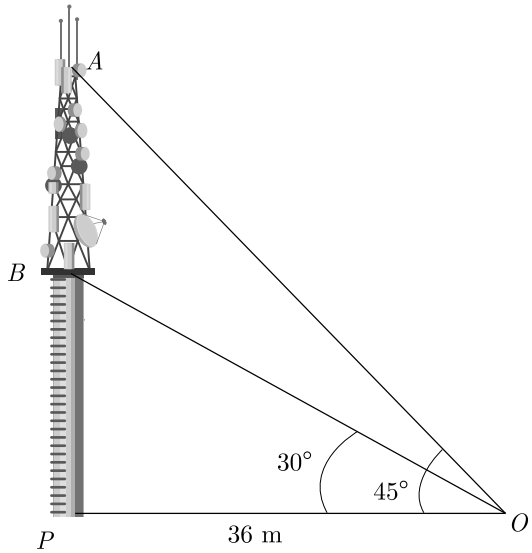
$$= \sqrt{3}(87 - 29)$$

Thus $GC = 58\sqrt{3}$ m

COMPETENCEY BASED QUESTIONS

93. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two sections A and B. Tower is supported by wires from a point O.

Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the section B is 30° and the angle of elevation of the top of section A is 45°.



Based on the above information, answer the following questions:

(i) Find the length of the wire from the point O to the top of section B.

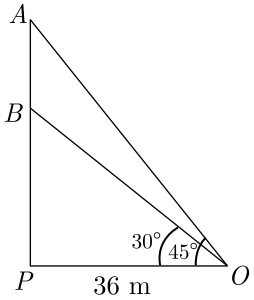
(ii) Find the distance AB.

(iii) Find the area of ΔOPB.

(iv) Find the height of the section A from the base of the tower.

Sol : [Board 2023 OD Standard]

We make the following diagram as per given information.



(i) Length of the wire from the point O to the top of section B

In ΔBPO $\cos 30^\circ = \frac{OP}{OB}$

$$\frac{\sqrt{3}}{2} = \frac{36}{OB}$$

$$OB = \frac{36 \times 2}{\sqrt{3}} = \frac{72}{\sqrt{3}}$$

$$= \frac{72}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{72\sqrt{3}}{3} = 24\sqrt{3} \text{ cm}$$

Thus, the length of wire from O to top of section $B = 24\sqrt{3}$ cm.

(ii) Distance AB

$$AB = AP - BP$$

In ΔBPO, $\tan 30^\circ = \frac{P}{B} = \frac{BP}{OP}$

$$\frac{1}{\sqrt{3}} = \frac{BP}{36}$$

$$BP = \frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{36\sqrt{3}}{3} = 12\sqrt{3} \text{ cm}$$

In ΔAPO, $\tan 45^\circ = \frac{AP}{PO}$

$$1 = \frac{AP}{36}$$

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$$AP = 36\text{ cm}$$

Distance $AB = 36 - 12\sqrt{3}$

$$= 36 - 20.78$$

$$= 15.22\text{ cm}$$

(iii) Area of ΔOPB

$$\text{Area of } \Delta OPB = \frac{1}{2} \times \text{Base} \times \text{height}$$

$$= \frac{1}{2} \times 36 \times 12\sqrt{3}$$

$$= 216\sqrt{3}\text{ cm}^2$$

$$= 374.12\text{ cm}^2$$

(iv) Height of the section A from the base of the tower

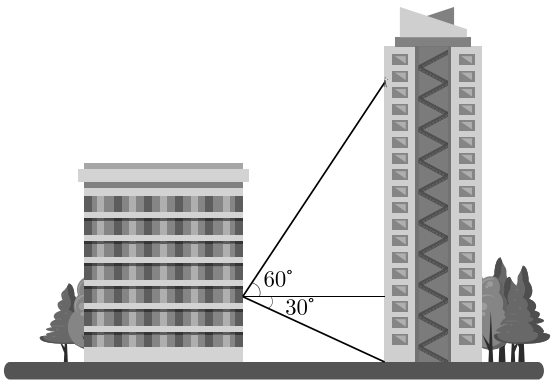
Height of Section A from base of tower is AP

In ΔAPO, $\tan 45^\circ = \frac{AP}{PO}$

$$1 = \frac{AP}{PO}$$

$$AP = 36\text{ cm}$$

94. From his hotel room window on the fourth floor, Ranjan notices some window washers high above him on the hotel across the street.



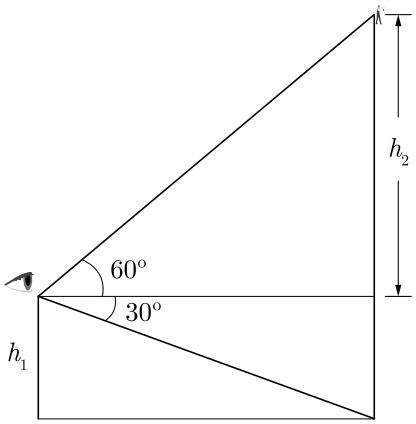
Curious as to their height above ground, he quickly estimates the buildings are 60 m apart, the angle of elevation to the workers is about 60°, and the angle of depression to the base of the hotel is about 30°.

(i) How high above ground is the window of Ranjan's hotel room?

(ii) How high above ground are the workers?

Sol :

Let h_1 be the height of Ranjan window from ground and h_2 be height of window washers from Ranjan. We draw a diagram of the situation as shown below.



Here $\tan 30^\circ = \frac{h_1}{60}$

$$\frac{1}{\sqrt{3}} = \frac{h_1}{60}$$

$$h_1 = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

$$= 20 \times 1.732 = 34.64\text{ m}$$

Now $\tan 60^\circ = \frac{h_2}{60}$

$$\sqrt{3} = \frac{h_2}{60}$$

$$h_2 = 60\sqrt{3}$$

$$= 60 \times 1.732 = 103.92\text{ m}$$

Height of workers from ground,

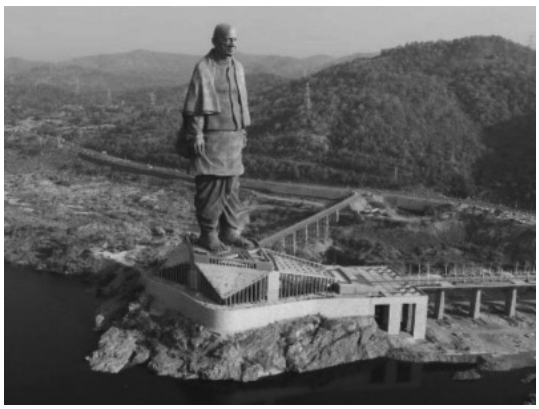
$$h_1 + h_2 = 34.64 + 103.92$$

$$= 138.56\text{ m}$$

(i) Window of Ranjan hotel is 34.64 meter above ground.

(ii) Workers are 138.64 meter above the ground.

95. Statue of Unity : It is a colossal statue of Indian statesman and independence activist Sardar Vallabh bhai Patel, who was the first Deputy Prime Minister and Home minister of independent India.



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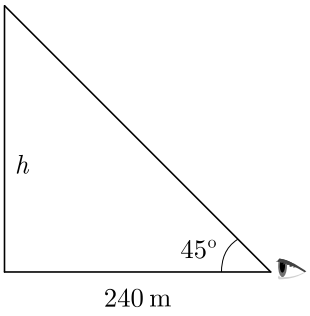
Patel was highly respected for his leadership in uniting the 562 princely states of India to form the single Union of India. It is located in the state of Gujarat and it is the world's tallest statue.

(i) For a person standing 240 m from the center of the base of the statue, the angle of elevation to the top of the statue is 45°. How tall is the statue?

(ii) A cop in helicopter near the top of the statue, notices a car wreck some distance from the statue. If the angle of depression from the cop's eyes to the wreck is 60°, how far away is the accident from the centre of base of the statue?

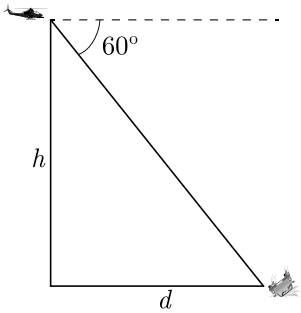
Sol :

Let h be the height of statue. We draw a diagram of the situation as shown below.



Now
$$\tan 45^\circ = \frac{h}{240}$$
$$1 = \frac{h}{240}$$
$$h = 240 \text{ m}$$

Let d be the distance of car wreck from he centre of base of the statue. We draw a diagram of the situation as shown below.



Now
$$\tan 30^\circ = \frac{d}{h}$$
$$\frac{1}{\sqrt{3}} = \frac{d}{240}$$
$$d = \frac{240}{\sqrt{3}} = 80\sqrt{3}$$
$$= 80 \times 1.732 = 138.56 \text{ m}$$

(i) The height of statue is 240 m.

(ii) Accident is 138.56 metre away from the centre of statue.

96. Eiffel Tower : The Eiffel Tower is a landmark and an early example of wrought-iron construction on a gigantic scale. The lower section consists of four immense arched legs set on masonry piers. The legs curve inward until they unite in a single tapered tower. Platforms, each with an observation deck, are at three levels; on the first is also a restaurant.

The tower, constructed of about 7000 tons of iron, has stairs and elevators. A meteorological station, a radio communications station, and a television transmission antenna, as well as a suite of rooms that were used by Eiffel are located near the top of the tower.

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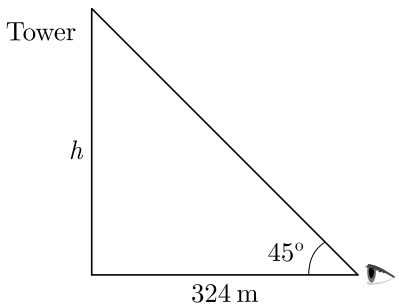
(i) For a person standing 324 m from the center of the base of the Eiffel Tower, the angle of elevation to the top of the tower is 45°. How tall is the Eiffel Tower?

(ii) A car is moving at uniform speed towards the Eiffel tower. It takes 15 minutes for the angle of depression from the top of tower to the car to change from 30° to 60°. After how much time after this, the car will reach the base of the tower?

Sol :

Let h be the height of Eiffel tower. We draw a diagram of the situation as shown below.



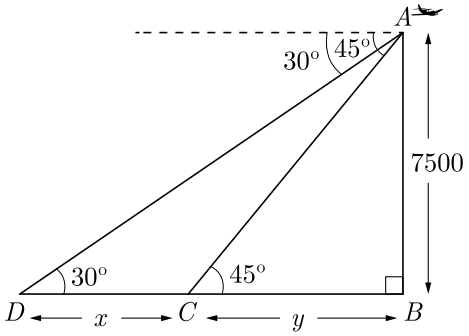
$$= 500(\sqrt{3}+1)$$
$$= 500(1.73+1)$$
$$= 500 \times 2.73 = 1365$$

Hence height of the hill is 1365 m.

82. The angle of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. if both the ships are in the same that one ship is exactly behind the other, find the distance between the ships.

Sol : [Board Term-2 Foreign 2017]

Let A, C and D be the position of aeroplane and two ship respectively. Aeroplane is flying at 7500 m height from point B. As per given in question we have drawn figure below.



In right ΔABC we have

$$\frac{AB}{BC} = \tan 45^\circ$$
$$\frac{7500}{y} = 1$$
$$y = 7500 \qquad ...(1)$$

In right ΔABD we have

$$\frac{AB}{BD} = \tan 30^\circ$$
$$\frac{7500}{x+y} = \frac{1}{\sqrt{3}}$$
$$x+y = 7500\sqrt{3} \qquad ...(2)$$

Substituting the value of y from (1) in (2) we have

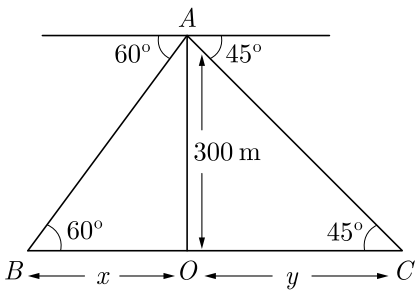
$$x+7500 = 7500\sqrt{3}$$
$$x = 7500\sqrt{3} - 7500$$
$$= 7500(\sqrt{3}-1)$$
$$= 7500(1.73-1)$$
$$= 7500 \times 0.73 = 5475 \text{ m}$$

Hence, the distance between two ships is 5475 m.

83. An aeroplane is flying at a height of 300 m above the ground. Flying at this height the angle of depression from the aeroplane of two points on both banks of a respectively. Find the width of the river. River in opposite direction are 45° and 60°.

Sol : [Board Term-2 OD 2017]

Let A be helicopter flying at a height of 300 m above the point O on ground. Let B and C be the bank of river. As per given in question we have drawn figure below.



Let BO be x and OC be y.

In right ΔAOC we have

$$\frac{AO}{OC} = \tan 45^\circ$$
$$\frac{300}{y} = 1 \Rightarrow y = 300$$

In right ΔAOB we have

$$\frac{AO}{BO} = \tan 60^\circ$$
$$\frac{300}{x} = \sqrt{3}$$
$$x\sqrt{3} = 300$$
$$x = \frac{300}{\sqrt{3}} = 100\sqrt{3}$$
$$BC = y + x$$
$$= 300 + 100\sqrt{3}$$
$$= 300 + 100 \times 1.732$$
$$= 473.2 \text{ m}$$

Hence, the width of river is 473.2 m.

84. The angle of elevation of a cloud from a point 120 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud.

Sol : [Board Term-2 OD 2012]

As per given in question we have drawn figure below.

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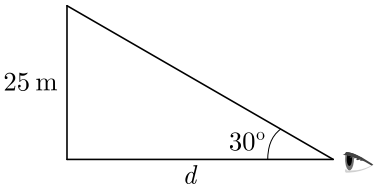
from the lighthouse is Sarthak ?

Sol :

Let d be distance between Sarthak and lighthouse. We draw a diagram of the situation as shown below.



Now $\tan 30^\circ = \frac{25}{d}$

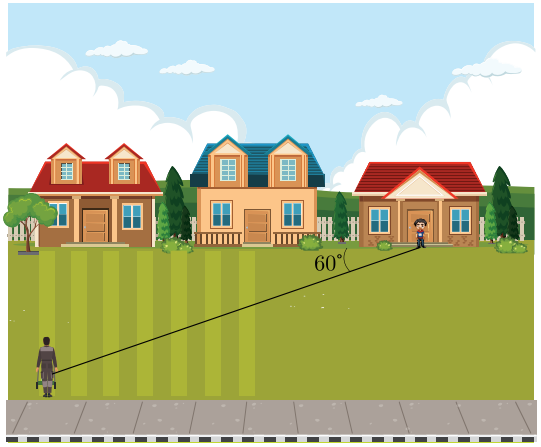
$$\frac{1}{\sqrt{3}} = \frac{25}{d}$$

$$d = \sqrt{3} \times 25$$

$$= 1.732 \times 25 = 43.3 \text{ m}$$

Thus Sarthak is 43.3 meter away from light house.

99. On a warm and lazy Saturday, Rishi is watching a county maintenance crew mow the park across the street. He notices the mower takes 16 sec to pass through 60° of rotation from one end of the park to the other.



If the corner of the park is 40 meter directly across the street from his house,

(i) How wide is the park?

(ii) How fast (in kmph) does the mower travel as it cuts the grass?

Sol :

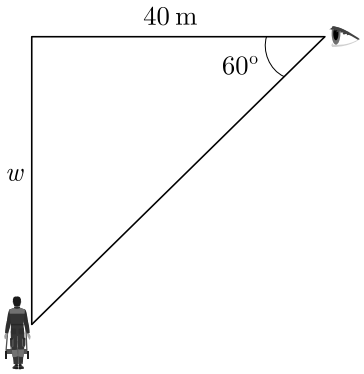
Let w be the width of park and s be the speed of mower. We draw a diagram of the situation as shown below.

(i) Now $\tan 60^\circ = \frac{w}{40}$

$$\sqrt{3} = \frac{w}{40}$$

$$w = \sqrt{3} \times 40 = 1.732 \times 40$$

$$= 69.28 \text{ m}$$



(ii) Speed of mower

$$s = \frac{d}{t} = \frac{w}{t} = \frac{69.28}{16}$$

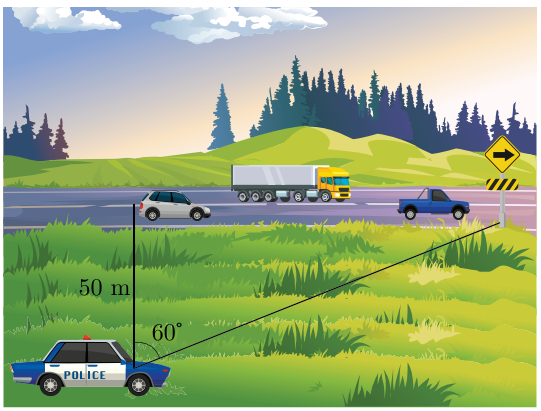
$$= 4.33 \text{ m/sec}$$

$$= 4.33 \times 3.6 = 15.6 \text{ kmph}$$

(i) Width of park is 69.28 meter

(ii) Mower travels by 15.6 kmph.

100. Speed Limit Enforcement : Rajendra works in traffic police and manage traffic on highway. His van is having radar detection equipment. He takes up a hidden position 50 meter from the highway. Using a sighting device he finds the angle between his position and a road sign in the distance is 60°.

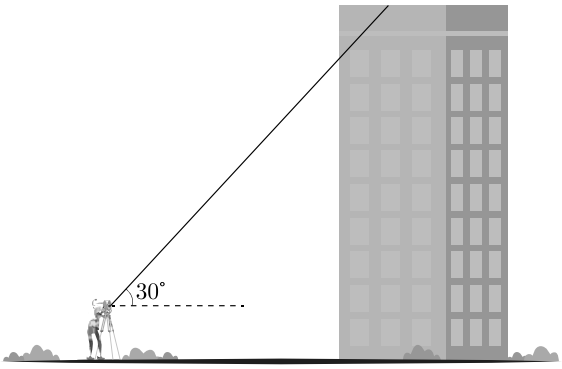


$$= \frac{20(\sqrt{3}+1)}{3-1}$$
$$= \frac{20(1.732+1)}{2}$$
$$= 10 \times 2.732 = 27.32 \text{ m}$$

(i) The height of space shuttle is 27.32 m.

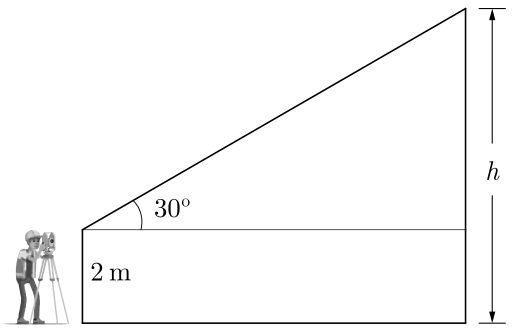
(ii) The distance of point A from space shuttle is 27.32 m because $h = d$

108. Height of a Building : A surveyor determines that the angle of elevation from a transit to the top of a building is 30°. The transit is positioned 2 meter above ground level and 30 meter from the building. Find the height of the building.



Sol :

Let h be the height of building from transit. We draw a diagram of the situation as shown below.



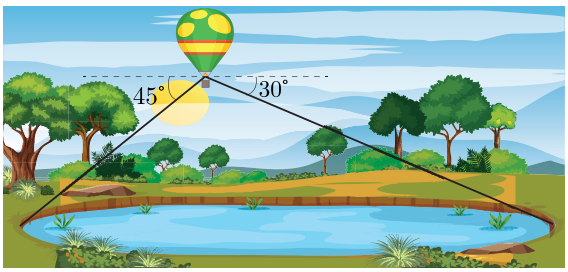
$$\tan 30^\circ = \frac{h}{30}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{30}$$
$$h = \frac{30}{\sqrt{3}}$$
$$= 10\sqrt{3}$$
$$= 10 \times 1.732 = 17.32 \text{ m}$$

Since transit is 2 meter high from ground, height of building from ground is $17.32 + 2 = 19.32$ m

109. Width of a Lake : The angle of depression to one side of a lake, measured from a balloon 300 meter above the lake as shown in the accompanying figure, is 45°. The angle of depression to the opposite side of the lake is 30°.

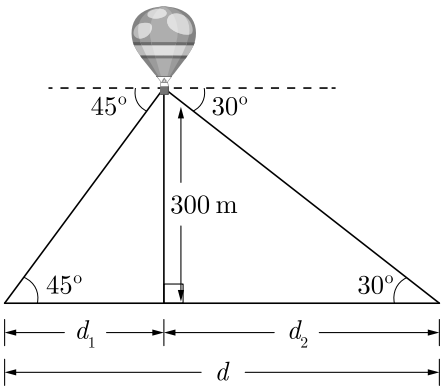
(i) Find the width of the lake.

(ii) Find the ground distance of balloon from sides of lake.



Sol :

Let w be the width of lake. We draw a diagram of the situation as shown below. Here d_1 and d_2 are the ground distance from balloon to the sides of lake.



Now $\tan 45^\circ = \frac{300}{d_1}$

$$1 = \frac{300}{d_1}$$
$$d_1 = 300 \text{ m}$$

Now $\tan 30^\circ = \frac{300}{d_2}$

$$\frac{1}{\sqrt{3}} = \frac{300}{d_2}$$
$$d_2 = 300\sqrt{3} \text{ m}$$
$$d_2 = 300 \times 1.732 = 519.6 \text{ m}$$
$$w = d_1 + d_2$$
$$= 300 + 519.6 = 819.6 \text{ m}$$

(i) Width of lake is 819.6 m.

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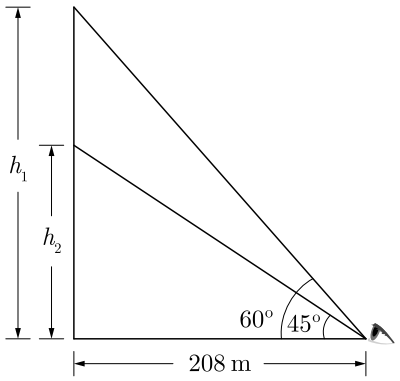
to the pinnacle of the tower is 60°. The angle of elevation to the restaurant from the same vantage point is 45°.

(i) How tall is the CN Tower?

(ii) How far below the pinnacle of the tower is the restaurant located?

Sol :

Let h_1 be the height of tower from ground and h_2 be height of restaurant from ground. We draw a diagram of the situation as shown below.



Now $\tan 60^\circ = \dfrac{h_1}{208}$

$\sqrt{3} = \dfrac{h_1}{208}$

$h_1 = 208 \times \sqrt{3}$

$= 208 \times 1.73 = 360$ m

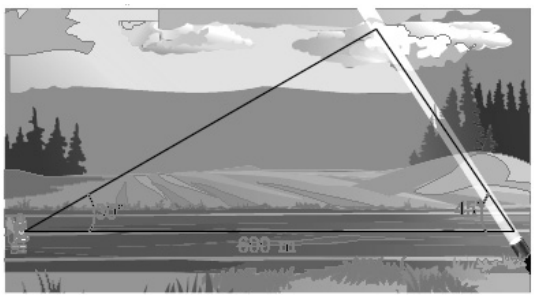
Now, $\tan 45^\circ = \dfrac{h_2}{208}$

$1 = \dfrac{h_2}{208}$ or $h_2 = 208$ m

(i) Height of CN Tower is 360 meter

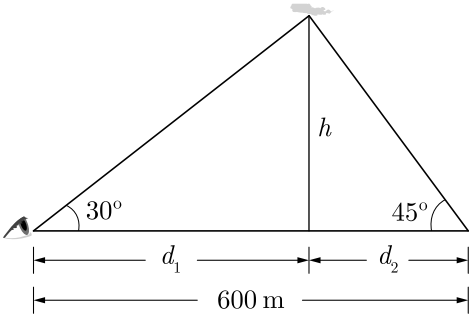
(ii) Restaurant is $360 - 208 = 152$ m below from top of tower.

103. Height of Cloud Cover : To measure the height of the cloud cover at an airport, a worker shines a spotlight upward at an angle 45° from the horizontal. An observer 600 m away measures the angle of elevation to the spot of light to be 30°. Find the height of the cloud cover.



Sol :

Let h be the height of cloud cover. We draw a diagram of the situation as shown below.



Now $\tan 30^\circ = \dfrac{h}{d_1}$

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{d_1} \Rightarrow d_1 = \sqrt{3}\,h$

Now $\tan 45^\circ = \dfrac{h}{d_2}$

$1 = \dfrac{h}{d_2} \Rightarrow d_2 = h$

Now $d_1 + d_2 = 600$

Substituting d_1 and d_2 we have

$\sqrt{3}\,h + h = 600$

$(\sqrt{3}+1)\,h = 600$

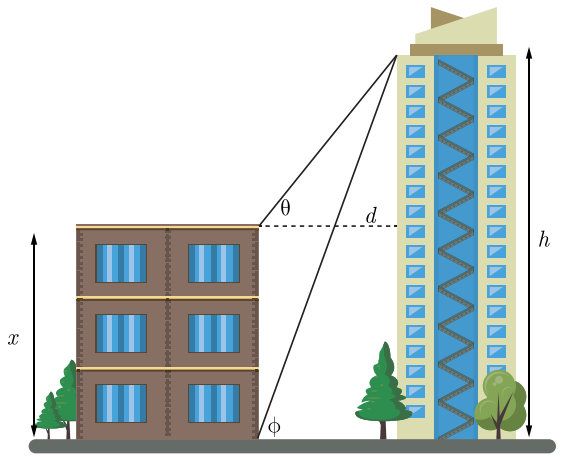
$h = \dfrac{600}{\sqrt{3}+1} = \dfrac{600(\sqrt{3}-1)}{3-1}$

$= 300(1.732 - 1)$

$= 300 \times 0.732 = 219.6$ m

Thus height of cloud cover is 219.6 m

104. Use the diagram given to derive a formula for the height h of the taller building in terms of the height x of the shorter building and the ratios for $\tan\theta$ and $\tan\phi$.



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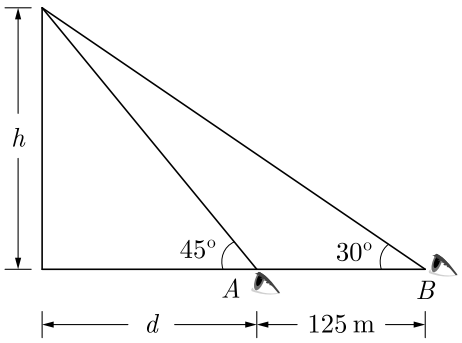


Now $\tan 45^\circ = \frac{h}{d}$

$1 = \frac{h}{d}$

$d = h$

Now $\tan 30^\circ = \frac{h}{d+125}$

$\frac{1}{\sqrt{3}} = \frac{h}{d+125}$

$d + 125 = \sqrt{3}\,h$

Substituting $d = h$ we have

$h + 125 = \sqrt{3}\,h$

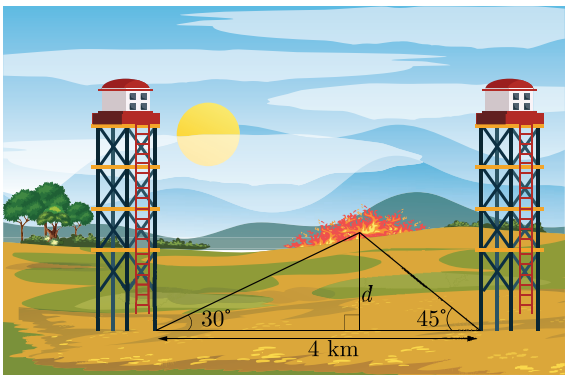
$125 = h(\sqrt{3}-1)$

$h = \frac{125}{\sqrt{3}-1} = \frac{125(\sqrt{3}+1)}{3-1}$

$= \frac{125(2.732)}{2} = 170.75$ m

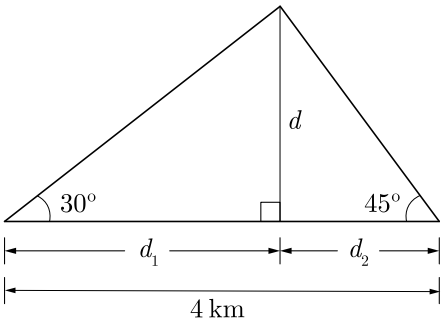
Height of Washington Monument is 170.75 m.

112. Fire towers : Two fire towers are 4 kilometres apart, where tower is due west of tower. A fire is spotted from the towers, and the angle of fire sight from tower is shown below. Find the distance of the fire from the line segment.



Sol :

Let d be the distance of the fire from the line segment. We draw a diagram of the situation as shown below.



Now $\tan 30^\circ = \frac{d}{d_1}$

$\frac{1}{\sqrt{3}} = \frac{d}{d_1} \Rightarrow d_1 = \sqrt{3}\,d$

Now $\tan 60^\circ = \frac{d}{d_2}$

$\sqrt{3} = \frac{d}{d_2} \Rightarrow d_2 = \frac{d}{\sqrt{3}}$

Now $d_1 + d_2 = 4$ km

Substituting d_1 and d_2 we have

$\sqrt{3}\,d + \frac{d}{\sqrt{3}} = 4$

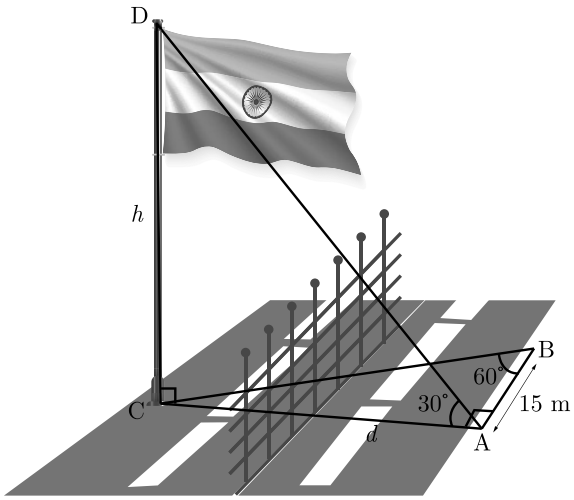
$\frac{3d}{\sqrt{3}} + \frac{d}{\sqrt{3}} = 4$

$\frac{4d}{\sqrt{3}} = 4$

$d = \frac{4\sqrt{3}}{4} = \sqrt{3} = 1.732$ km

The distance of the fire from the line segment is 1.732 km.

113. Flag Pole : Figure given below is a diagram that shows how Varsha estimates the height of a flagpole. She can't measure the distance between herself and the flagpole directly because there is a fence in the way.



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climbers and top of the northern rim respectively.

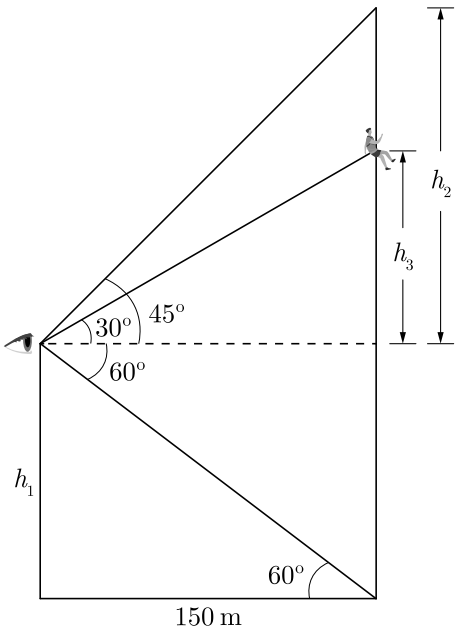
(i) How high is the southern rim of the canyon?

(ii) How high is the northern rim?

(iii) How much farther until the climber reaches the top?

Sol :

Let h_1 be the height of southern rim of the canyon from ground and h_2 be the height of the northern rim from southern rim. Let h_3 be the height of climbers on northern rim from southern rim. We draw a diagram of the situation as shown below.



For h_1, $\tan 60^\circ = \frac{h_1}{150}$

$\sqrt{3} = \frac{h_1}{150}$

$h_1 = 150 \times \sqrt{3}$

$= 150 \times 1.732 = 259.8$ m

For h_2, $\tan 45^\circ = \frac{h_2}{150}$

$1 = \frac{h_2}{150}$

$h_2 = 150$ m

For h_3, $\tan 30^\circ = \frac{h_3}{150}$

$\frac{1}{\sqrt{3}} = \frac{h_3}{150}$

$h_3 = \frac{150}{\sqrt{3}} = 50\sqrt{3}$

$= 50 \times 1.732 = 86.6$ m

$h_2 - h_3 = 150 - 86.6 = 63.4$ m

(i) $h_1 = 259.8$ metre is the height of the south rim.

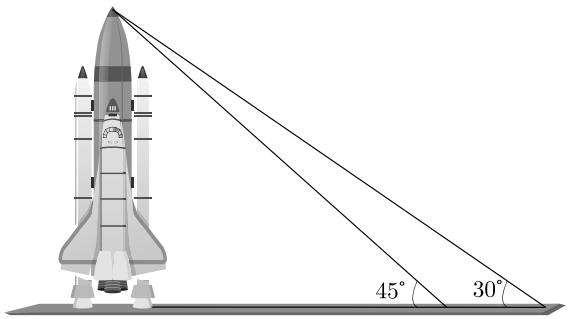
(ii) $h_1 + h_2 = 259.8 + 150 = 409.8$ metre is the height of the north rim.

(iii) Climbers have to go $h_2 - h_3 = 63.4$ m to the top.

107. An observer notes that the angle of elevation from point A to the top of a space shuttle is 45°. From a point 20 meters further from the space shuttle, the angle of elevation is 30°.

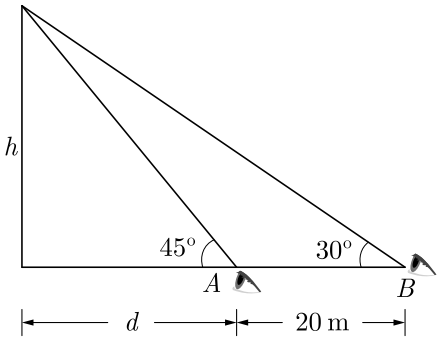
(i) Find the height of the space shuttle.

(ii) Find the distance of point A from space shuttle.



Sol :

Let h be the height of space shuttle and d be the distance from point A and space shuttle. We draw a diagram of the situation as shown below.



Now $\tan 45^\circ = \frac{h}{d}$

$1 = \frac{h}{d} \Rightarrow h = d$

Now $\tan 30^\circ = \frac{h}{d+20}$

Substituting $h = d$ in above equation,

$\frac{1}{\sqrt{3}} = \frac{h}{h+20}$

$h + 20 = \sqrt{3}\, h$

$20 = h(\sqrt{3} - 1)$

$h = \frac{20}{\sqrt{3} - 1}$

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He then uses a stop watch to determine how long it takes a vehicle to pass her location and reach the road sign. In quick succession—an 18-wheeler, a truck, and a car pass her position, with the time each takes to travel this distance noted. Find the speed of each vehicle in miles per hour if

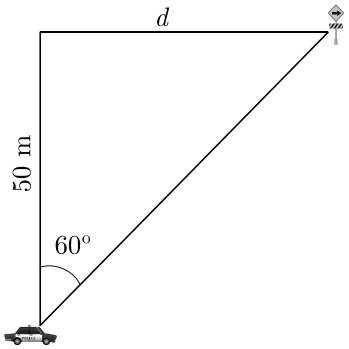
(i) the 18-wheeler takes 8 sec,

(ii) the truck takes 6 sec,

(iii) the car takes 4 sec.

Sol :

Let d be the distance between his hidden position on road and a road sign We draw a diagram of the situation as shown below.



Now $$\tan 60^\circ = \frac{d}{50}$$

$$\sqrt{3} = \frac{d}{50}$$

$$d = 50 \times \sqrt{3}$$

$$= 50 \times 1.732 = 86.6 \text{ m}$$

(i) Speed of 18-wheeler

$$s_1 = \frac{d}{t_1} = \frac{86.6}{8}$$

$$= 10.825 \text{ m/ sec}$$

$$= 10.825 \times \frac{3600}{1000} \text{ km/h}$$

$$= 38.97 \text{ km/h}$$

(ii) Speed of truck,

$$s_2 = \frac{d}{t_2} = \frac{86.6}{6}$$

$$= 14.43 \text{ m/ sec}$$

$$= 14.43 \times 3.6 \text{ km/h}$$

$$= 51.96 \text{ km/h}$$

(iii) Speed of car,

$$s_3 = \frac{d}{t_3} = \frac{86.6}{4}$$

$$= 21.65 \text{ m/ sec}$$

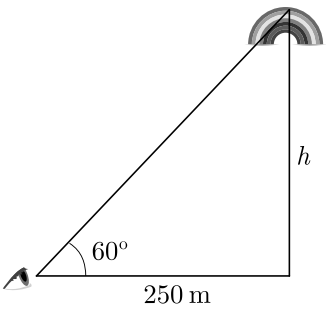
$$= 21.65 \times 3.6 \text{ km/h}$$

$$= 77.94 \text{ km/h}$$

101. Rainbow: While visiting the Mount Abu in Rajasthan, Taniya and Lavanya see a spectacularly vivid rainbow arching over the lake. Taniya speculates the rainbow is 250 m away, while Lavanya estimates the angle of elevation to the highest point of the rainbow is about 60°. What was the approximate height of the rainbow?

Sol :

Let h be height of rainbow. We draw a diagram of the situation as shown below.



$$\tan 60^\circ = \frac{h}{250}$$

$$\sqrt{3} = \frac{h}{250}$$

$$h = 250 \times \sqrt{3}$$

$$= 250 \times 1.173 = 432.5 \text{ m}$$

Thus height of rainbow is 432.5 meter.

102. CN Tower : The tallest free-standing tower in the world is the CN Tower in Toronto, Canada. The tower includes a rotating restaurant high above the ground.

From a distance of 208 meter the angle of elevation

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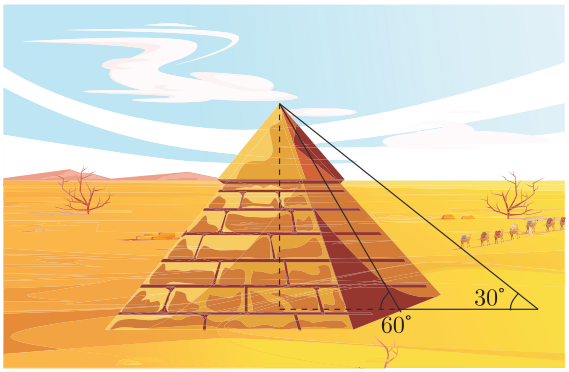
(ii) Ground distance of sides from balloon are 300 meter and 519.6 m.

110. Height of a Pyramid : The angle of elevation to the top of the Egyptian pyramid of Cheops is 30° measured from a point 50 meter from the base of the pyramid. The angle of elevation from the base of a face of the pyramid is 60°.

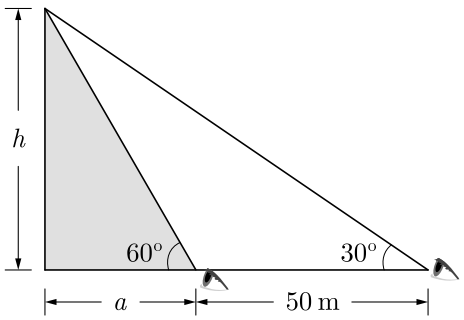


(i) Find the height of the Cheops pyramid.

(ii) Find the side of base of pyramid.

Sol :

Let h be the height of pyramid from centrer of base and $2a$ be the side of base of pyramid. We draw a diagram of the situation as shown below.



Now $\tan 60^\circ = \frac{h}{a}$

$\sqrt{3} = \frac{h}{a}$

$a = \frac{h}{\sqrt{3}}$

Now $\tan 30^\circ = \frac{h}{a+50}$

$\frac{1}{\sqrt{3}} = \frac{h}{a+50}$

$a+50 = \sqrt{3}\,h$

Substituting $a = \frac{h}{\sqrt{3}}$ we have

$\frac{h}{\sqrt{3}} + 50 = \sqrt{3}\,h$

$h + 50\sqrt{3} = 3h$

$50\sqrt{3} = 2h$

$h = 25\sqrt{3}$

$= 25 \times 1.732$

$= 43.3$ m

$2a = \frac{h}{\sqrt{3}} = \frac{25\sqrt{3}}{\sqrt{3}} = 25$ m

(i) Height of pyramid is 43.3 m.

(ii) Side of pyramid is $2x = 25 \times 2 = 50$ m.

111. Washington Monument : The Washington Monument is a large, tall, white obelisk near the west part of the National Mall in Washington, D.C. It was built to remember George Washington, who was the first President of the United States. It is the tallest stone structure in the world

From a point A on a line from the base of the Washington Monument, the angle of elevation to the top of the monument is 45°. From a point 125 m away from A and on the same line, the angle to the top is 30°. Find the height of the Washington Monument.

Sol :

Let h be the height of Washington Monument and d be distance of point A from the base of the Washington Monument. We draw a diagram of the situation as shown below.

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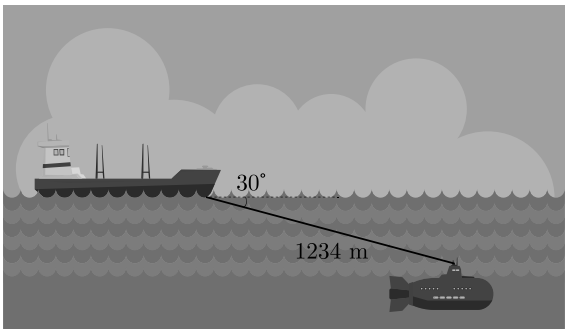
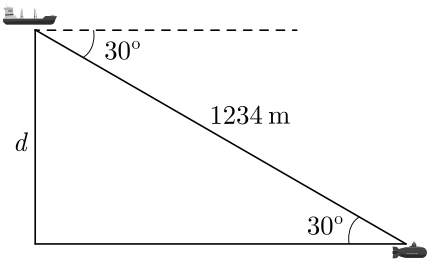


The sonar of a navy cruiser detects a sub marine that is 1234 meter from the cruiser. The angle between the water line and the submarine is 30°. How deep is the submarine?

Sol :

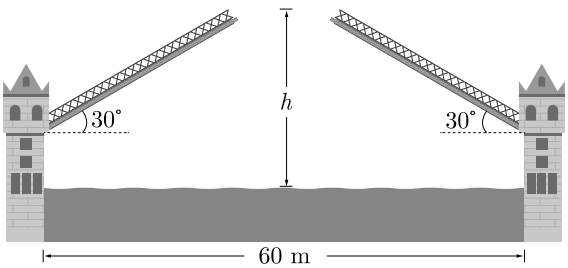
Let d be depth of submarine. We draw a diagram of the situation as shown below.



Now
$$\sin 30^\circ = \frac{d}{1234}$$
$$\frac{1}{2} = \frac{d}{1234}$$
$$d = \frac{1234}{2} = 617 \text{ m}$$

Thus submarine is 617 m below from the level of sea.

116. Drawbridge : A drawbridge is a bridge that can be moved in order to stop or allow passage across it. Modern drawbridges are often built across large, busy waterways. They can be lifted to allow large ships to pass or lowered to allow land vehicles or pedestrians to cross.



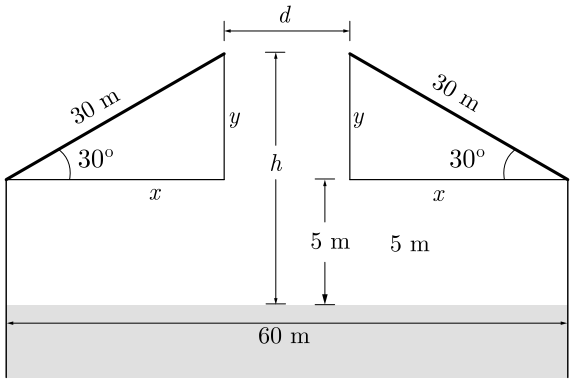
A drawbridge is 60 metre long when stretched across a river. As shown in the figure, the two sections of the bridge can be rotated upward through an angle of 30°.

(i) If the water level is 5 metre below the closed bridge, find the height h between the end of a section and the water level when the bridge is fully open.

(ii) How far apart are the ends of the two sections when the bridge is fully opened, as shown in the figure?

Sol :

It may be easily seen that length of each section of bridge is $\frac{60}{2} = 30$ m. We draw a diagram of the situation as shown below. Let h be height between the end of a section and the water level, when bridge is fully opened. Let d be distance between the end of a section, when bridge is fully opened.



Now
$$\sin 30^\circ = \frac{y}{30}$$
$$\frac{1}{2} = \frac{y}{30}$$
$$y = \frac{30}{2} = 15 \text{ m}$$
$$h = y + 5 = 15 + 5 = 20 \text{ m}$$

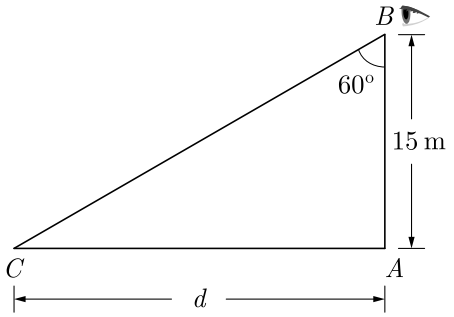
Now
$$\cos 30^\circ = \frac{x}{30}$$
$$\frac{\sqrt{3}}{2} = \frac{x}{30}$$
$$x = \frac{30\sqrt{3}}{2} = 15\sqrt{3} \text{ m}$$
$$d = 60 - x - x$$
$$= 60 - 15\sqrt{3} - 15\sqrt{3}$$
$$= 60 - 30\sqrt{3} = 30(2 - \sqrt{3})$$
$$= 30(2 - 1.732)$$
$$= 30 \times 0.268 = 8.04 \text{ m}$$

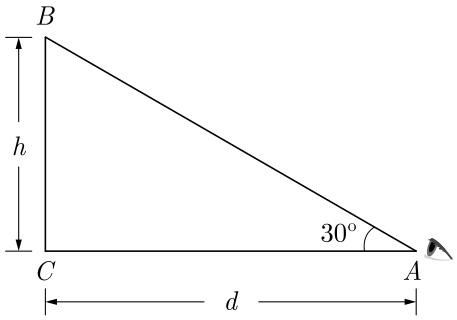
(i) When the bridge is fully open, height is 20 meter between the end of a section and the

So she stands at point A facing the pole and finds the angle of elevation from point A to the top of the pole to be 30°. Then she turns 90° and walks 15 metre to point B, where she measures the angle between her path and a line from B to the base of the pole. She finds that angle is 60°. Find the height of the pole.

Sol :

Let h be the height of the pole and d be the ground distance between point A and pole. We draw a diagram of the situation as shown below.





Now $\tan 60^\circ = \frac{d}{15}$

$\sqrt{3} = \frac{d}{15}$

$d = 15\sqrt{3}$

Now $\tan 30^\circ = \frac{h}{d}$

$\frac{1}{\sqrt{3}} = \frac{h}{15\sqrt{3}}$

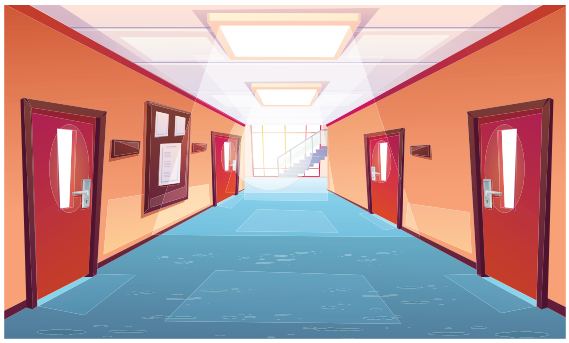
$h = 15 \text{ m}$

Thus height of pole is 15 m.

114. Height of a Door : From a point on the floor the angle of elevation to the top of a door is 30°, while the angle of elevation to the ceiling above the door is 60°. The ceiling is 6 metre above the floor.

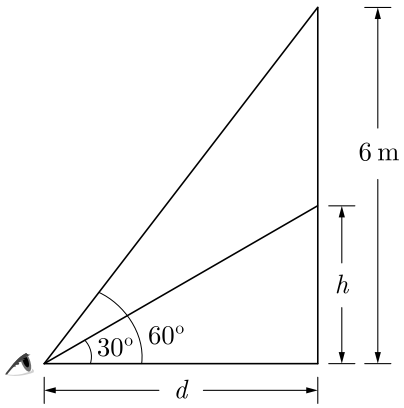
(i) What is the vertical dimension of the door ?

(ii) Find the distance of the point from door.



Sol :

Let h be the height of the door and d be the distance of door from the point. We draw a diagram of the situation as shown below.



Now $\tan 60^\circ = \frac{6}{d}$

$\sqrt{3} = \frac{6}{d}$

$d = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ m}$

$= 2 \times 1.732$

$= 3.564 \text{ m}$

Now $\tan 30^\circ = \frac{h}{d}$

$\frac{1}{\sqrt{3}} = \frac{h}{2\sqrt{3}}$

$h = 2 \text{ m}$

(i) The vertical dimension of the door is 2 meter.

(ii) The distance of the point from door is 3.564 m.

115. Sonar : It is a machine that uses underwater sound waves to find other objects in the sea. A sonar can work by sending out sound and listening for echoes (active sonar), like a radar, or by listening for sound made by the object it is trying to find.

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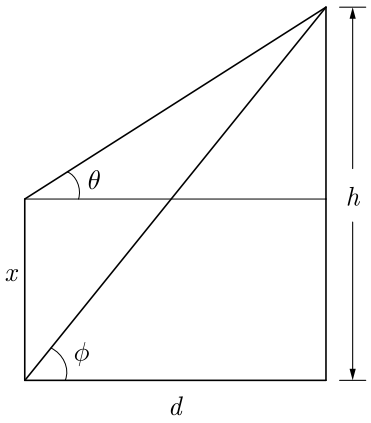
Sol :

We redraw the given diagram as shown below.



Now $\tan\phi = \frac{h}{d}$

$$d = \frac{h}{\tan\phi}$$

Now $\tan\theta = \frac{h-x}{d} = \frac{h-x}{\frac{h}{\tan\phi}}$

$$= \frac{(h-x)\tan\phi}{h}$$

$$h\tan\theta = h\tan\phi - x\tan\phi$$

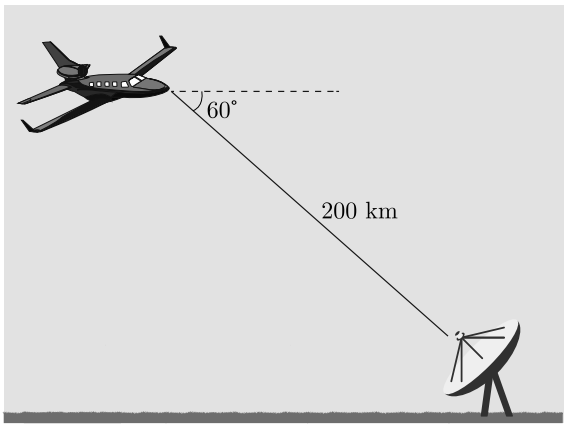
$$h\tan\theta - h\tan\phi = -x\tan\phi$$

$$h\tan\phi - h\tan\theta = x\tan\phi$$

$$h(\tan\phi - \tan\theta) = x\tan\phi$$

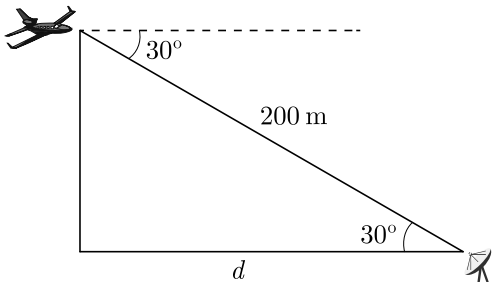
$$h = \frac{x\tan\phi}{\tan\phi - \tan\theta}$$

105. Distance Measuring Equipment : DME is standard avionic equipment on a commercial airplane. This equipment measures the distance from a plane to a radar station. If the distance from a plane to a radar station is 200 km and the angle of depression is 30°, find the ground distance from a point directly below the plane to the radar station.



Sol :

Let d be the distance from a point directly below the plane to the radar station. We draw a diagram of the situation as shown below.



$$\cos 30^\circ = \frac{d}{200}$$

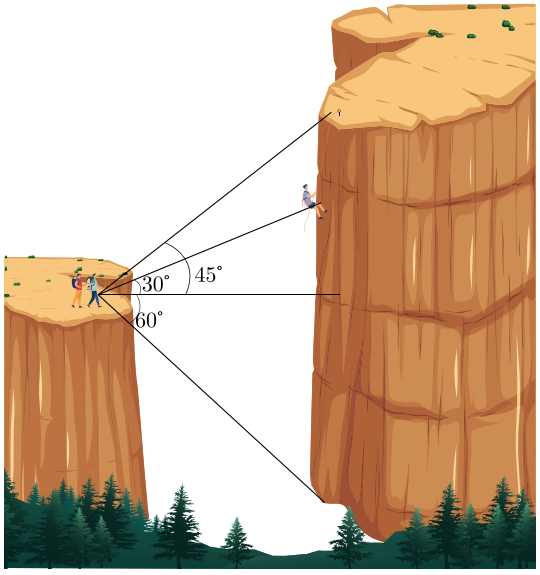
$$\frac{\sqrt{3}}{2} = \frac{d}{200}$$

$$d = 100\sqrt{3}$$

$$= 100 \times 1.732 = 173.2 \text{ km}$$

The ground distance from a point directly below the plane to the radar station is 173.2 meter.

106. Height of a Climber : Himalayan Trekking Club has just hiked to the south rim of a large canyon, when they spot a climber attempting to scale the taller northern face. Knowing the distance between the sheer walls of the northern and southern faces of the canyon is approximately 150 meter, they attempt to compute the distance remaining for the climbers to reach the top of the northern rim.



Using a homemade transit, they sight an angle of depression of 60° to the bottom of the north face, and angles of elevation of 30° and 45° to the

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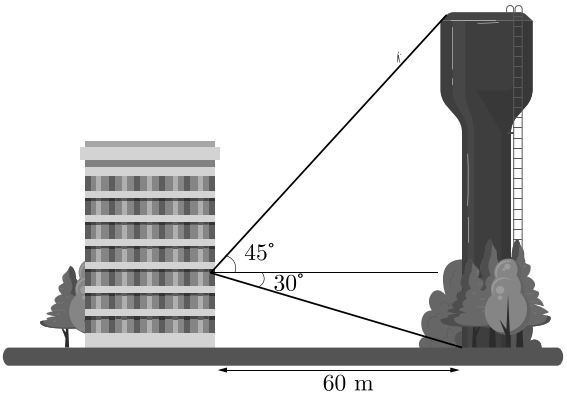
water level.

(ii) When the bridge is fully opened the ends of the two sections are 8.04 metre apart.

117. Water Tower : A water tower is a building that is used to hold and give out water. It is almost always built on a high place. It works because a pump gives water to the tower, and gravity makes the saved water go out to the places that need water. Those places are connected to the tower by pipes. A water tower is good when there is no power because it uses gravity to send out the water.



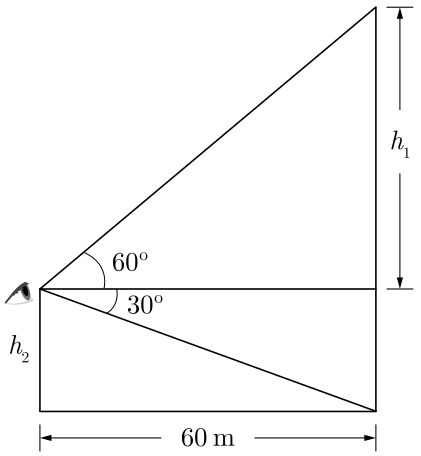
A water tower is located 60 meter from a building (see the figure). From a window in the building, an observer notes that the angle of elevation to the top of the tower is 60° and that the angle of depression to the bottom of the tower is 30°.

(i) How tall is the tower?

(ii) How high is the window?

Sol :

Let h_1 be the height of top of tower from window and h_2 be height of window from ground. We draw a diagram of the situation as shown below.



$$\text{Now} \quad \tan 60^\circ = \frac{h_1}{60}$$

$$\sqrt{3} = \frac{h_1}{60}$$

$$h_1 = 60 \times \sqrt{3}$$

$$= 60 \times 1.732 \text{ m}$$

$$= 103.56 \text{ m}$$

$$\text{Now} \quad \tan 30^\circ = \frac{h_2}{60}$$

$$\frac{1}{\sqrt{3}} = \frac{h_2}{60}$$

$$h_2 = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

$$= 20 \times 1.732 \text{ m}$$

$$= 34.64 \text{ m}$$

$$\text{Height of tower, } h = h_1 + h_2$$

$$= 103.56 + 34.64$$

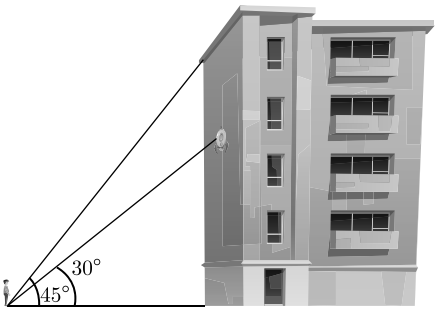
$$= 138.56 \text{ m}$$

(i) Tower is 138.56 m tall.

(ii) Height of the window is 103.56 meter.

118. Clinometer : A clinometer is a tool that is used to measure the angle of elevation, or angle from the ground, in a right - angled triangle. We can use a clinometer to measure the height of tall things that you can't possibly reach to the top of, flag poles, buildings, trees.

Ravish got a clinometer from school lab and started the measuring elevation angle in surrounding. He saw a building on which society logo is painted on wall of building.



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(i) What is the height of the section B ?

(ii) What is the height of the section A ?

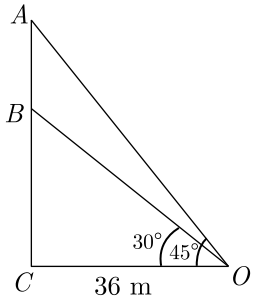
(iii) What is the length of the wire structure from the point O to the top of section A ?

(iv) What is the length of the wire structure from the point O to the top of section B ?

(v) What is the angle of depression from top of tower to point O ?

Sol :

(i) We make the following diagram as per given information.



$$\text{In } \Delta BCO \quad \tan 30^\circ = \frac{BC}{OC}$$

$$BC = OC\tan 30^\circ$$

$$BC = 36 \times \frac{1}{\sqrt{3}} = 12\sqrt{3} \text{ m}$$

(ii) In ΔACO,

$$\tan 45^\circ = \frac{AC}{OC} = 1$$

Thus $AC = OC = 36$ m

Now, $AB = AC - BC$

$$= 36 - 12\sqrt{3}$$

$$= 12(3 - \sqrt{3}) \text{ m}$$

(iii) In ΔACO,

$$\cos 45^\circ = \frac{OC}{OA}$$

$$\frac{1}{\sqrt{2}} = \frac{36}{OA}$$

$$OA = 36\sqrt{2} \text{ m}$$

(iv) In ΔBCO,

$$\cos 30^\circ = \frac{OC}{OB}$$

$$\frac{\sqrt{3}}{2} = \frac{36}{OB}$$

$$OB = \frac{72}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 24\sqrt{3} \text{ m}$$

(v) It is clear from figure that angle of elevation from point O to top of tower is 45°. This is equal to the angle of depression from top of tower to point O.

121. Navy Officer : Mr. Colin is tasked with planning a coup on the enemy at a certain date. Currently he is inspecting the area standing on top of the cliff. Agent Dev is on a chopper in the sky. When Mr. Colin looks down below the cliff towards the sea, he has Bhawani and Amar in boats positioned to get a good vantage point. Bhawani boat is behind the Amar boat.

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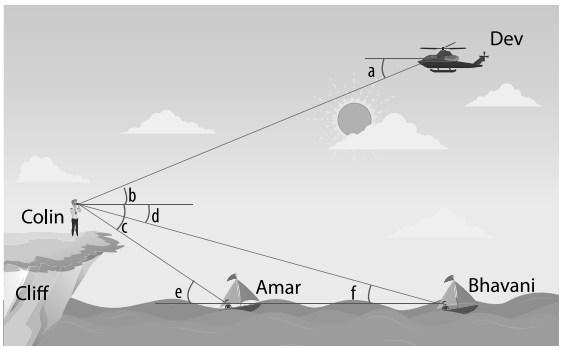

Following angle have been measured :

From Colin to Bhawani : 30°

From Dev to Colin : 60°

From Amar to Colin : 60°

(i) Which of the following is a pair of angle of elevation?

(a) $(\angle a, \angle e)$ (b) $(\angle b, \angle e)$

(c) $(\angle c, \angle d)$ (d) $(\angle a, \angle f)$

(ii) Which of the following is a pair of angle of depression?

(a) $(\angle a, \angle e)$ (b) $(\angle b, \angle e)$

(c) $(\angle c, \angle d)$ (d) $(\angle a, \angle f)$

(iii) If angle of elevation of Amar to Colin is 60° , what is the distance of Amar boat from the base of hill ?

(iv) If angle of depression of Colin to Bhawani is 30° , what is the distance of Amar boat from the Bhawani boat?

(v) If angle of depression of Dev to Colin is 60° , what is the height of Dev from base of hill ?

Sol :

(i) The angle of elevation of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (the line of sight). In our case clearly $(\angle b, \angle e, \angle f)$

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Lakshman is riding on a hot air balloon. After reaching at height x at point P, he spots a lorry parked at B on the ground at an angle of depression of 30°. The balloon rises further by 50 metres at point Q and now he spots the same lorry at an angle of depression of 45° and a car parked at C at an angle of depression of 30°.

(i) What is the relation between the height x of the balloon at point P and distance d between point A and B?

(ii) When balloon rises further 50 metres, then what is the relation between new height y and d?

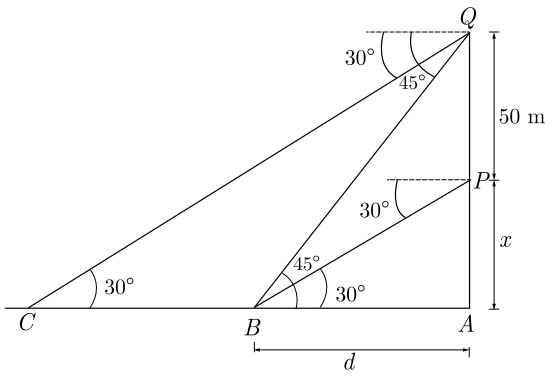
(iii) What is the new height of the balloon at point Q?

(iv) What is the distance AB on the ground ?

(v) What is the distance AC on the ground ?

Sol :

(i) We make the diagram as per given information.



In ΔAPB, $\tan 30^\circ = \dfrac{AP}{AB}$

$$\frac{1}{\sqrt{3}} = \frac{x}{d}$$

$$d = \sqrt{3}\,x \;\Rightarrow\; d^2 = 3x^2$$

(ii) In ΔBAQ,

$$\tan 45^\circ = \frac{AQ}{AB}$$

$$AB = AQ$$

$$d = y$$

(iii) From (i) and (ii) we have

$$d = \sqrt{3}\,x \text{ and } d = y$$

Since point Q is 50 m above point P, Thus

$$y = x + 50$$

Thus $$d = x + 50$$

Solving above equations we get

$$\sqrt{3}\,x = x + 50$$

$$x(\sqrt{3} - 1) = 50$$

$$x = \frac{50}{(\sqrt{3} - 1)} = 25(\sqrt{3} + 1)$$

$$y = x + 50 = 25(\sqrt{3} + 1) + 50$$

$$= 25\sqrt{3} + 25 + 50 = 25(\sqrt{3} + 3)$$

(iv) The distance AB on the ground is d and which is equal to

$$d = \sqrt{3}\,x$$

or $$d = y = 25(\sqrt{3} + 3)$$

(v) In ΔCAQ,

$$\tan 30^\circ = \frac{AQ}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{y}{AC} = \frac{25(\sqrt{3} + 3)}{AC}$$

$$AC = 25\sqrt{3}\,(\sqrt{3} + 3)$$

$$= 25(3 + 3\sqrt{3})$$

$$= 75(1 + \sqrt{3})$$

120. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure, including microwave dishes. They are among the tallest human-made structures. There are 2 main types: guyed and self-supporting structures.

On a similar concept, a radio station tower was built in two sections A and B. Tower is supported by wires from a point O. Distance between the base of the tower and point O is 36 m. From point O, the angle of elevation of the top of section B is 30° and the angle of elevation of the top of section A is 45°.

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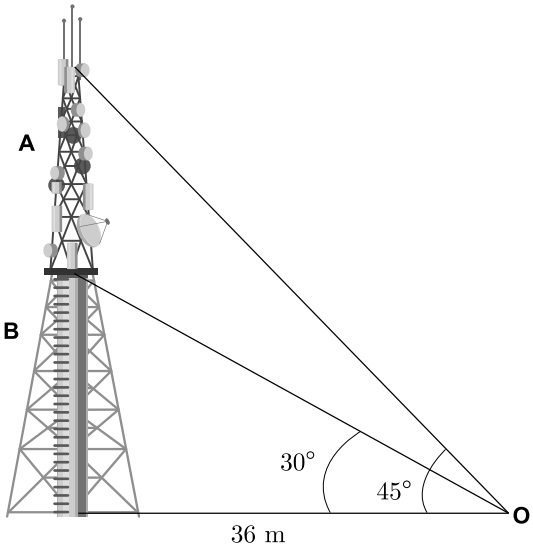


From a point P on the ground level, the angle of elevation of the roof of the building is 45°. The angle of elevation of the centre of logo is 30° from same point. The point P is at a distance of 24 m from the base of the building.

(i) What is the height of the building logo from ground ?

(ii) What is the height of the building from ground ?

(iii) What is the aerial distance of the point P from the top of the building ?

(iv) If the point of observation P is moved 9 m towards the base of the building, then the angle of elevation θ of the logo on building is given by

(a) $\tan\theta = \sqrt{3}$ (b) $\tan\theta = \dfrac{2}{\sqrt{3}}$

(c) $\tan\theta = \dfrac{1}{2}$ (d) $\tan\theta = \dfrac{8\sqrt{3}}{15}$

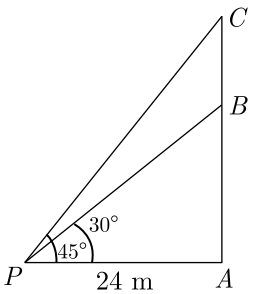
(v) In above case the angle of elevation ϕ of the top of building is given by

(a) $\tan\phi = 1.6$ (b) $\tan\phi = 1.5$

(c) $\tan\phi = 0.75$ (d) $\tan\phi = 0.8$

Sol :

(i) As per question statement we have shown the digram below.



The height of the building logo from ground is AB. Here C is top of building and AC is height of building.

$$\text{In } \Delta PAB, \quad \tan 30^\circ = \frac{AB}{PA}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{24}$$

$$AB = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 8\sqrt{3} \text{ m}$$

(ii) The height of the building from ground is AC .

$$\text{In } \Delta APC, \quad \tan 45^\circ = \frac{AC}{AP}$$

$$1 = \frac{AC}{24}$$

$$AC = 24 \text{ m}$$

(iii) In ΔAPC,

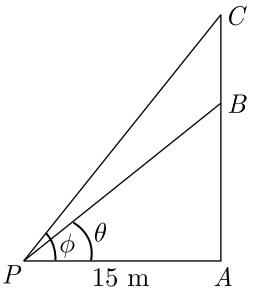
$$\cos 45^\circ = \frac{AP}{AC}$$

$$\frac{1}{\sqrt{2}} = \frac{24}{PC}$$

$$PC = 24\sqrt{2} \text{ m}$$

(iv) In this case we have made diagram as follows.

Now $AP = 24 - 9 = 15$ m



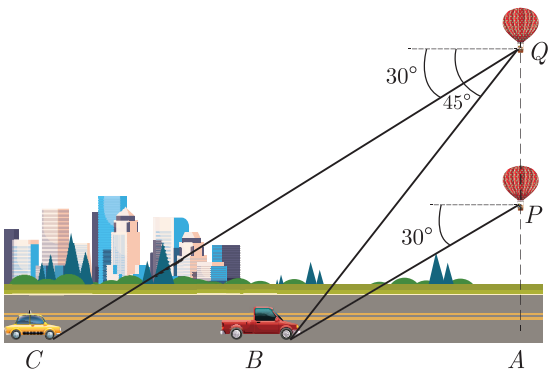
$$\tan\theta = \frac{AB}{AP} = \frac{8\sqrt{3}}{15}$$

Thus (d) is correct option.

(v) $\tan\phi = \dfrac{AC}{AP} = \dfrac{24}{15} = \dfrac{8}{5} = 1.6$

Thus (a) is correct option.

119. A hot air balloon is a type of aircraft. It is lifted by heating the air inside the balloon, usually with fire. Hot air weighs less than the same volume of cold air (it is less dense), which means that hot air will rise up or float when there is cold air around it, just like a bubble of air in a pot of water. The greater the difference between the hot and the cold, the greater the difference in density, and the stronger the balloon will pull up.



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are angle of depression.

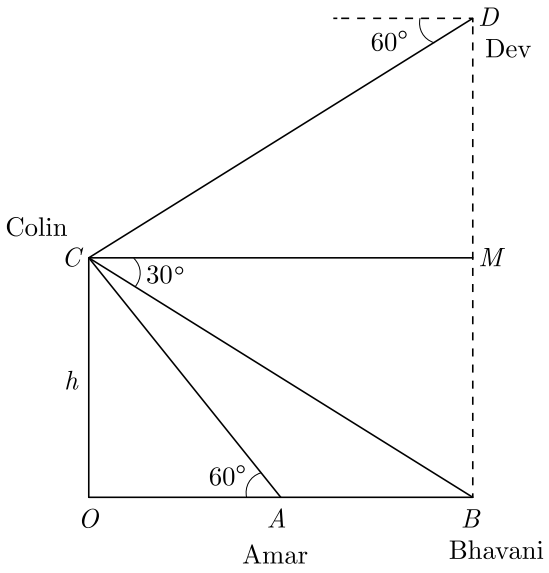
Thus (b) is correct option.

(ii) If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the angle of depression. In our case clearly $(\angle a, \angle c, \angle d)$ are angle of depression.

Thus (c) is correct option.

(iii) We make the figure as given below.

Here $\angle OAC = 60^\circ$ is angle of elevation.



$$\frac{h}{OA} = \tan 60^\circ = \sqrt{3}$$

$$OA = \frac{h}{\sqrt{3}}$$

(iv) Here $\angle OBC = \angle MCB = 30^\circ$

$$\frac{h}{OB} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$OB = \sqrt{3}\,h$$

$$AB = OB - OA$$

$$= \sqrt{3}\,h - \frac{h}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}(3h - h) = \frac{2h}{\sqrt{3}}$$

(v) Here $\angle DCM = 60^\circ$

Now, $\frac{DM}{CM} = \tan 60^\circ = \sqrt{3}$

$$DM = \sqrt{3}\,CM$$

But $CM = OB = \sqrt{3}\,h$

Thus $DM = \sqrt{3} \cdot \sqrt{3}\,h = 3h$

Height of Dev from Bhawani,

$$= DB = DM + MB$$

$$= 3h + h = 4h$$

122. When an eagle looks at a rat on the ground, eagle does not attack the rat at its initial position. It takes into account the speed of the rat and the direction in which rat is moving. After analysing the situation (how? it may be God's gift) eagle attacks the rat in such a way that it may successful in catching the rat.

Suppose an eagle sitting on the tree of height 90 m, observe a rat with angle of depression 45°, and moving away from the tree with some speed. Eagle start flying with 30° downward and catch rat in 10 second.

(i) What is the horizontal distance between tree and initial position of rat ?

(ii) What is the distance travelled by rat in 10 seconds?

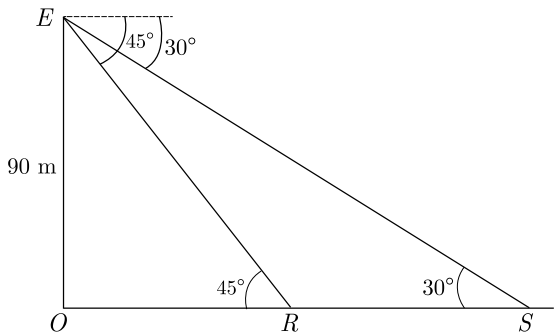
(iii) What is the distance travelled by eagle to catch rat?

(iv) What is the speed of rat ?

(v) What is the speed of eagle ?

Sol :

(i) As per information given in question we have made diagram below. Here E is initial position of eagle and R is initial position of rat. Eagle catch rat at S.



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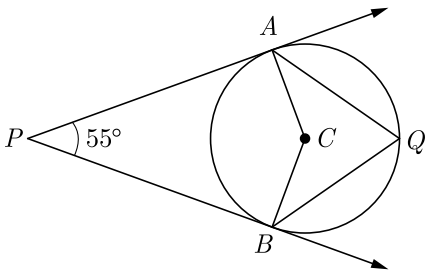
CHAPTER 10

Circle

OBJECTIVE QUESTIONS

1. In the given figure, PA and PB are tangents from external point P to a circle with centre C and Q is any point on the circle. Then the measure of $\angle AQB$ is



(a) $62\frac{1}{2}^\circ$ (b) 125°

(c) 55° (d) 90°

Sol : [Board 2023 OD Standard]

Sum of the angles between radii and between intersection point of tangent is always 180°.

Thus $\angle ACB + \angle APB = 180^\circ$

$\angle ACB + 55^\circ = 180^\circ$

$\angle ACB = 180^\circ - 55^\circ$

$= 125^\circ$

Angle subtended on the centre of a circle is twice of the angle subtended on circumference of circle

Thus $\angle AQB = \frac{1}{2}\angle ACB = \frac{1}{2} \times 125^\circ$

$= 62\frac{1}{2}^\circ$

Thus (a) is correct option.

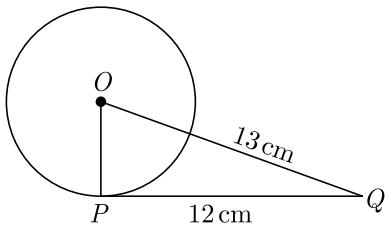
2. From an external point Q, the length of tangent to a circle is 12 cm and the distance of Q from the centre of circle is 13 cm. The radius of circle (in cm) is

(a) 10 (b) 5

(c) 12 (d) 7

Sol : [Board 2020 Delhi Basic]

Let O be the centre of the circle. As per given information we have drawn the figure below.



We have $OQ = 13$ cm

and $PQ = 12$ cm

Radius is perpendicular to the tangent at the point of contact.

Thus $OP \perp PQ$

In ΔOPQ, using Pythagoras theorem,

$OP^2 + PQ^2 = OQ^2$

$OP^2 + 12^2 = 13^2$

$OP^2 = 13^2 - 12^2$

$= 169 - 144 = 25$

Thus $OP = 5$ cm

Thus (b) is correct option.

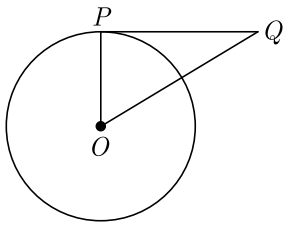
3. QP is a tangent to a circle with centre O at a point P on the circle. If ΔOPQ is isosceles, then $\angle OQR$ equals.

(a) 30° (b) 45°

(c) 60° (d) 90°

Sol : [Board 2020 Delhi Basic]

Let O be the centre of the circle. As per given information we have drawn the figure below.



We know that, the radius and tangent are perpendicular at their point of contact.

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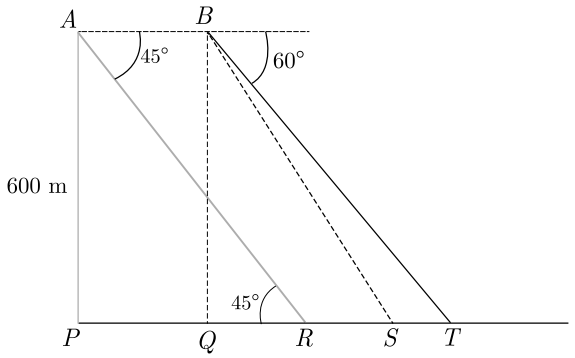
below.



In triangle $\triangle BQT$, $\angle QBT = 90^\circ - 60^\circ = 30^\circ$

$$\frac{BQ}{BT} = \cos 30^\circ$$

$$BT = \frac{BQ}{\cos 30^\circ} = \frac{600}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1200}{\sqrt{3}} = 400\sqrt{3} \text{ m}$$

Time taken to hit tank at T,

$$t_1 = \frac{BT}{s} = \frac{400\sqrt{3}}{\frac{500}{6}}$$

$$= \frac{4\sqrt{3} \times 6}{5} = \frac{24\sqrt{3}}{5} \text{ sec}$$

(iv) In figure this distance is given by QT

$$\frac{QT}{QB} = \tan 30^\circ$$

$$QT = QB \tan 30^\circ$$

$$= 600 \times \frac{1}{\sqrt{3}} = 200\sqrt{3}$$

(v) Speed of tank, $s_t = 90$ km/hour

$$= \frac{90000}{3600} \text{ m/sec}$$

$$= 25 \text{ m/sec}$$

Distance ST travelled by tank in $\frac{24\sqrt{3}}{5}$ sec

$$= 25 \times \frac{24\sqrt{3}}{5} = 120\sqrt{3} \text{ m}$$

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$$\frac{OE}{OR} = \tan 45^\circ = 1$$
$$OE = OR = 90\text{ m}$$

(ii) In triangle ΔEOS, $\angle OES = 90^\circ - 30^\circ = 60^\circ$

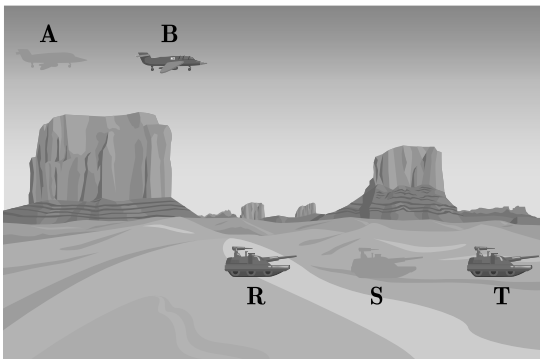
$$\frac{OS}{OE} = \tan 60^\circ$$
$$\frac{OS}{90} = \sqrt{3}$$
$$OS = 90\sqrt{3}\text{ m}$$
$$RS = OS - OR$$
$$= 90\sqrt{3} - 90 = 90(\sqrt{3}-1)$$

(iii) $$\frac{EO}{ES} = \cos 60^\circ$$
$$\frac{90}{ES} = \frac{1}{2}$$
$$ES = 90 \times 2 = 180\text{ m}$$

(iv) Speed of rat $s_r = \frac{RS}{t} = \frac{90(\sqrt{3}-1)}{10}$
$$= 9(\sqrt{3}-1)\text{ m/sec}$$

(v) Speed of Eagle,
$$s_e = \frac{ES}{t} = \frac{180}{10} = 18\text{ m/sec}$$
$$= \frac{18 \times 3600}{1000}\text{ km/h}$$
$$= 64.8\text{ km/h}$$

123. An air-to-surface missile (ASM) or air-to-ground missile (AGM or ATGM) is a missile designed to be launched from military aircraft and strike ground targets on land, at sea, or both. They are similar to guided glide bombs but to be deemed a missile,



A military fighter plane is flying at an altitude of 600 metres with the speed of 200 km/h. The pilot spots enemy tanks at point R on ground. After getting the permission from command centre to hit the target at R, pilot fires a missile. Fighter plane was at point A at the time of fire of missile. Missile moves to target at enemy tanks stationed at R at an angle of 45° at a speed of 300 km/h.

(i) What is the horizontal distance between fighter plane at A and tank at R ?

(ii) How much time will missile take to hit the target R ?

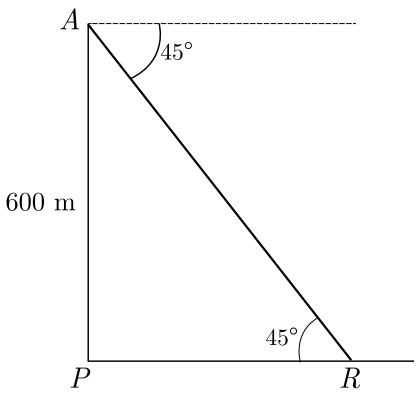
(iii) Another enemy tank at point S on ground moving with a speed of 90 km/h in straight line away from plane. Pilot fires another missile at an angle of 60° from its flight path position B at the instant when enemy's tank was at S and it hits this enemy tank at point T. How much time is taken by second missile to hit the enemy tank at point T ?

(iv) What is the horizontal distance between fighter plane at B and tank at T ?

(v) What is the distance of point T from S ?

Sol :

(i) We have shown this situation in following diagram



$$\frac{PR}{PA} = \tan 45^\circ = 1$$
$$\Rightarrow \quad PR = PA = 600\text{ m}$$

(ii) $$\frac{AP}{AR} = \cos 45^\circ$$
$$AR = \frac{AP}{\cos 45^\circ} = \frac{600}{\frac{1}{\sqrt{2}}} = 600\sqrt{2}\text{ m}$$

Speed of missile,
$$s = 300\text{ km/h} = \frac{300000}{3600}\text{ m/sec}$$
$$= \frac{500}{6}\text{ m/s}$$

Time taken to hit missile.
$$t = \frac{AR}{s} = \frac{600\sqrt{2}}{\frac{500}{6}} = \frac{36\sqrt{2}}{5}\text{ sec}$$

(iii) In this situation we have shown diagram

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Now, in isosceles triangle POQ we have

$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$

Equal sides subtend equal angles in isosceles triangle.

Thus $2\angle OQP + 90^\circ = 180^\circ$

$\angle OQP = 45^\circ$

Thus (b) is correct option.

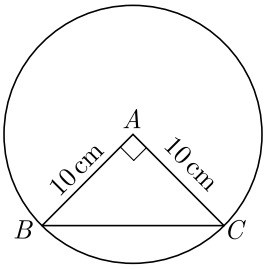
4. A chord of a circle of radius 10 cm, subtends a right angle at its centre. The length of the chord (in cm) is

(a) $\frac{5}{\sqrt{2}}$ (b) $5\sqrt{2}$

(c) $10\sqrt{2}$ (d) $10\sqrt{3}$

Sol : [Board 2020 OD Basic]

As per given information we have drawn the figure below.



Using Pythagoras theorem in ΔABC, we get

$BC^2 = AB^2 + AC^2 = 10^2 + 10^2$

$= 100 + 100 = 200$

$BC = 10\sqrt{2}$ cm

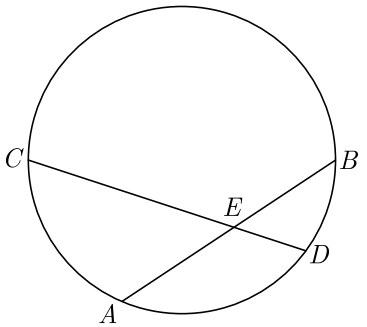
Thus (c) is correct option.

5. Two chords AB and CD of a circle intersect at E such that $AE = 2.4$ cm, $BE = 3.2$ cm and $CE = 1.6$ cm. The length of DE is

(a) 1.6 cm (b) 3.2 cm

(c) 4.8 cm (d) 6.4 cm

Sol :



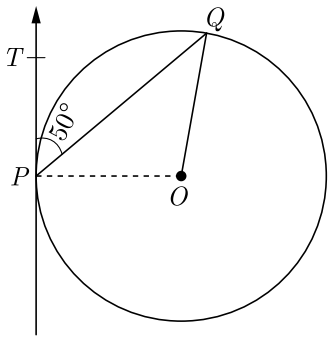
Applying the rule, $AE \times EB = CE \times ED$

$2.4 \times 3.2 = 1.6 \times ED$

$ED = 4.8$ cm

Thus (c) is correct option.

6. In figure, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50° with $PQ\angle POQ$ is



(a) 130° (b) 90°

(c) 100° (d) 75°

Sol : [Board 2020 OD Basic]

Due to angle between radius and tangent,

$\angle OPT = 90^\circ$

$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$

Also, $OP = OQ$ [Radii of a circle]

Since equal opposite sides have equal opposite angles,

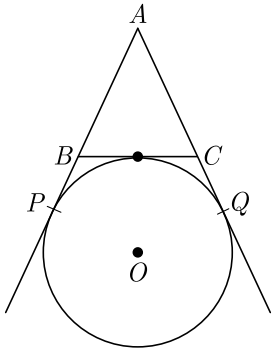
$\angle OPQ = \angle OQP = 40^\circ$

$\angle POQ = 180^\circ - \angle OPQ - \angle OQP$

$= 180^\circ - 40^\circ - 40^\circ = 100^\circ$

Thus (c) is correct option.

7. In figure, AP, AQ and BC are tangents of the circle with centre O. If $AB = 5$ cm, $AC = 6$ cm and $BC = 4$ cm, then the length of AP (in cm) is



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(a) 15 (b) 10

(c) 9 (d) 7.5

Sol : [Board 2020 Delhi Basic]

Due to tangents from external points,

$BP = BR, CR = CQ$, and $AP = AQ$

Perimeter of ΔABC,

$$\begin{aligned} AB + BC + AC &= AB + BR + RC + AC \\ 5 + 4 + 6 &= AB + BP + CQ + AC \\ 15 &= AP + AQ \\ 15 &= 2AP \end{aligned}$$

Thus $AP = \frac{15}{2} = 7.5$ cm

Thus (d) is correct option.

8. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is

(a) $3r$ (b) $6r$

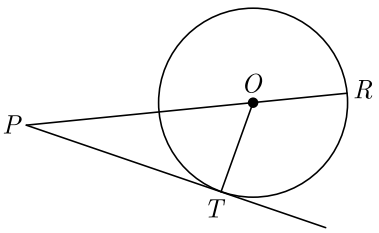
(c) $9r$ (d) $12r$

Sol :

Side of the regular hexagon inscribed in a circle of radius r is also r, the perimeter is $6r$.

Thus (b) is correct option.

9. In figure, on a circle of radius 7 cm, tangent PT is drawn from a point P such that $PT = 24$ cm. If O is the centre of the circle, then the length of PR is



(a) 30 cm (b) 28 cm

(c) 32 cm (d) 25 cm

Sol : [Board 2020 Delhi Basic]

Tangent at any point of a circle is perpendicular to the radius at the point of contact.

Thus $OT \perp PT$

Now in right-angled triangle PTO

$$\begin{aligned} OP^2 &= OT^2 + PT^2 \\ &= (7)^2 + (24)^2 \\ &= 49 + 576 = 625 \end{aligned}$$

Thus $OP = 25$ cm

Since $OR = OT$ because of radii of circle,

$$PR = OP + OR = 25 + 7 = 32 \text{ cm}$$

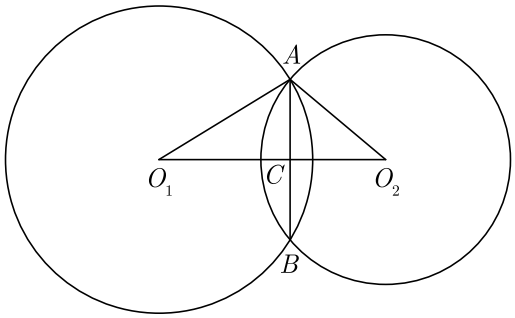
Thus (c) is correct option.

10. Two circles of radii 20 cm and 37 cm intersect in A and B. If O_1 and O_2 are their centres and $AB = 24$ cm, then the distance O_1O_2 is equal to

(a) 44 cm (b) 51 cm

(c) 40.5 cm (d) 45 cm

Sol :



Since C is the mid-point of AB,

$$AC = 12$$

$$AO_1 = 37$$

and $$AO_2 = 20$$

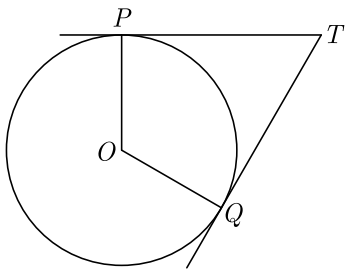
$$CO_1 = \sqrt{37^2 - 12^2} = 35$$

$$CO_2 = \sqrt{20^2 - 12^2} = 16$$

$$O_1O_2 = 35 + 16 = 51$$

Thus (b) is correct option.

11. In the adjoining figure, TP and TQ are the two tangents to a circle with centre O. If $\angle POQ = 110^\circ$, then $\angle PTQ$ is



(a) 60° (b) 70°

(c) 80° (d) 90°

Sol :

Here $OP \perp PT$ and $OQ \perp QT$,

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In quadrilateral $OPTQ$, we have

$$\angle POQ + \angle OPT + \angle PTQ + \angle OQT = 360^\circ$$
$$110^\circ + 90^\circ + \angle PTQ + 90^\circ = 360^\circ$$
$$\angle PTQ = 70^\circ$$

Thus (b) is correct option.

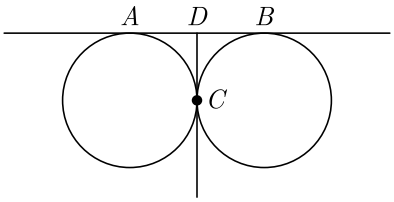
12. AB and CD are two common tangents to circles which touch each other at a point C. If D lies on AB such that $CD = 4$ cm then AB is

(a) 12 cm (b) 8 cm

(c) 4 cm (d) 6 cm

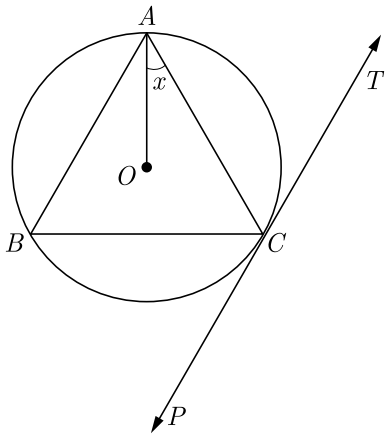
Sol :

$$AD = CD \text{ and } BD = CD$$
$$AB = AD + BD = CD + CD$$
$$= 2CD = 2 \times 4$$
$$= 8 \text{ cm}$$



Thus (b) is correct option.

13. In the adjoining figure, PT is a tangent at point C of the circle. O is the circumference of ΔABC. If $\angle ACP = 118^\circ$, then the measure of $\angle x$ is

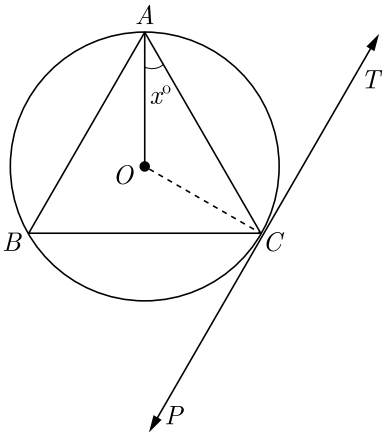


(a) 28° (b) 32°

(c) 42° (d) 38°

Sol :

We join OC as shown in the below figure. Here OC is the radius and PT is the tangent to circle at point C.



Thus $OC \perp PT$

$$\angle OCP = 90^\circ$$

Given, $\angle ACP = 118^\circ$

$$\angle ACO = \angle ACP - \angle OCP$$
$$= 118^\circ - 90^\circ = 28^\circ$$

Since O is the circumcentre, thus $OA = OC$ (radius)

$$\angle OAC = \angle ACO$$
$$x = 28^\circ$$

Thus (a) is correct option.

14. Two concentric circles of radii a and b where $a > b$,The length of a chord of the larger circle which touches the other circle is

(a) $\sqrt{a^2 + b^2}$ (b) $2\sqrt{a^2 + b^2}$

(c) $\sqrt{a^2 - b^2}$ (d) $2\sqrt{a^2 - b^2}$

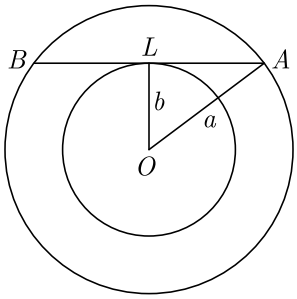
Sol :

In ΔOAL, $OA^2 = OL^2 + AL^2$

$$a^2 = OL^2 + b^2$$
$$OL = \sqrt{a^2 - b^2}$$

Length of chord,

$$2AL = 2\sqrt{a^2 - b^2}$$



Thus (d) is correct option.

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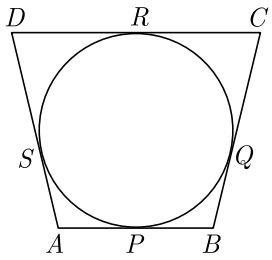
15. In the given figure, a circle touches all the four sides of quadrilateral $ABCD$ with $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm, then length of AD is



(a) 3 cm (b) 4 cm

(c) 5 cm (d) 6 cm

Sol :

Four sides of a quadrilateral $ABCD$ are tangent to a circle.

$$AB + CD = BC + AD$$
$$6 + 4 = 7 + AD$$
$$AD = 10 - 7 = 3 \text{ cm}$$

Thus (a) is correct option.

16. Two concentric circles are of radii 10 cm and 8 cm, then the length of the chord of the larger circle which touches the smaller circle is

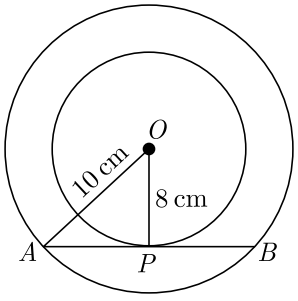
(a) 6 cm (b) 12 cm

(c) 18 cm (d) 9 cm

Sol :

Let O be the centre of the concentric circles of radii 10 cm and 8 cm, respectively. Let AB be a chord of the larger circle touching the smaller circles at P.

Then, $AP = PB$ and $OP \perp AB$

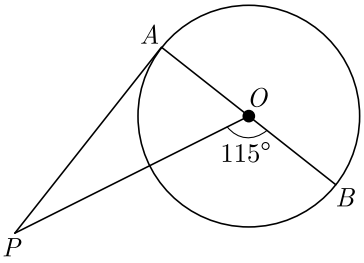


Applying Pythagoras theorem in ΔOPA, we have

$$OA^2 = OP^2 + AP^2$$
$$100 = 64 + AP^2$$
$$AP^2 = 100 - 64 = 36$$
$$AP = 6 \text{ cm}$$
$$AB = 2AP = 2 \times 6 = 12 \text{ cm}$$

Thus (b) is correct option.

17. In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115^\circ$, then perimeter of $\angle APO$ is



(a) 25° (b) 20°

(c) 30° (d) 65°

Sol :

Since tangent at a point to a circle is perpendicular to the radius,

$$\angle OAP = 90^\circ$$

Now, $\angle AOP + \angle BOP = 180^\circ$

$$\angle AOP + 115^\circ = 180^\circ$$
$$\angle AOP = (180^\circ - 115^\circ) = 65^\circ$$

From angle sum property of triangle we have

$$\angle OAP + \angle AOP + \angle APO = 180^\circ$$
$$90^\circ + 65^\circ + \angle APO = 180^\circ$$
$$155^\circ + \angle APO = 180^\circ$$
$$\angle APO = 180^\circ - 155^\circ = 25^\circ$$

Thus (a) is correct option.

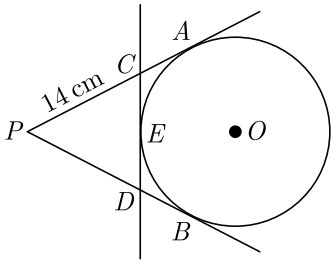
18. From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and $PA = 14$ cm. The perimeter of ΔPCD is

(a) 14 cm (b) 21 cm

(c) 28 cm (d) 35 cm

Sol :

As per information given in question we have drawn figure below.



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Here $PA = PB = 14\,\text{cm}$

Also, CD is tangent at point E on the circle.

So, CA and CE are tangent to the circle from point C.

Therefore, $CA = CE$,

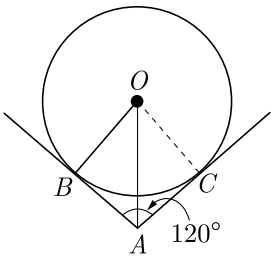
Similarly, $DB = DE$

Now, perimeter of ΔPCD,

$$\begin{aligned} PC + CD + PD &= PC + CE + ED + PD \\ &= PC + CA + PD + DB \\ &= PA + PB \\ &= 14 + 14 = 28\ \text{cm} \end{aligned}$$

Thus (c) is correct option.

19. In the given figure, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^\circ$, then OA is equal to that



(a) $2AB$ (b) $3AB$

(c) $4AB$ (d) $5AB$

Sol :

In ΔOAB and ΔOAC, we have,

$$\angle OBA = \angle OCA = 90^\circ$$

$$OA = OA \quad \text{[common]}$$

and $$OB = OC \quad \text{[radii of circle]}$$

So, by RHS congruence criterion,

$$\Delta OBA \cong \Delta OCA$$

$$\angle OAB = \angle OAC$$

$$= \frac{1}{2} \times 120^\circ$$

$$= 60^\circ$$

In ΔOBA, we have,

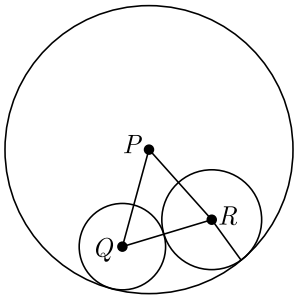
$$\cos 60^\circ = \frac{AB}{OA}$$

$$\frac{1}{2} = \frac{AB}{OA}$$

$$OA = 2AB$$

Thus (a) is correct option.

20. In the given figure, three circles with centres P, Q and R are drawn, such that the circles with centres Q and R touch each other externally and they touch the circle with centre P, internally. If $PQ = 10\,\text{cm}$, $PR = 8\,\text{cm}$ and $QR = 12\,\text{cm}$, then the diameter of the largest circle is:



(a) 30 cm

(b) 20 cm

(c) 10 cm

(d) None of these

Sol :

Let radii of the circles with centres P, Q and R are p, q and r, respectively.

Then, $PQ = p - q = 10$ ...(1)

$PR = p - r = 8$ ...(2)

and $QR = q + r = 12$ ...(3)

Adding equation (2) and (3), we get,

$$p + q = 20 \quad \text{...(4)}$$

Adding equation (1) and (4), we get,

$$2p = 30$$

Hence, diameter of the largest circle $2p = 30$.

Thus (a) is correct option.

21. If radii of two concentric circles are 4 cm and 5 cm, then the length of each of one circle which is tangent to the other circle, is

(a) 3 cm

(b) 6 cm

(c) 9 cm

(d) 1 cm

Sol :

Let C be the centre of two concentric circles C_1 and C_2, whose radii are $r_1 = 4\,\text{cm}$ and $r_2 = 5\,\text{cm}$. Now, we draw a chord $AB$of circle C_2, which touches C_1 at P.

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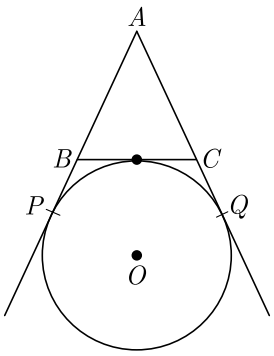
$$\angle POQ + \angle OPT + \angle PTQ + \angle OQT = 360^\circ$$
$$110^\circ + 90^\circ + \angle PTQ + 90^\circ = 360^\circ$$
$$\angle PTQ = 70^\circ$$

36. In figure, AP, AQ and BC are tangents of the circle with centre O. If $AB = 5$ cm, $AC = 6$ cm and $BC = 4$ cm, then what is the length of AP?



Sol : [Board 2020 Delhi Basic]

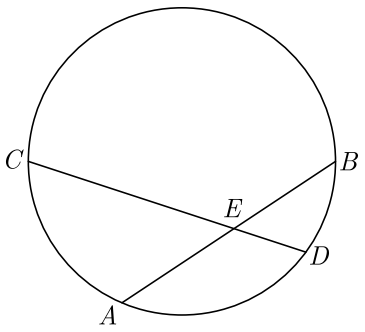
Due to tangents from external points,
$BP = BR, CR = CQ$, and $AP = AQ$
Perimeter of ΔABC,

$$AB + BC + AC = AB + BR + RC + AC$$
$$5 + 4 + 6 = AB + BP + CQ + AC$$
$$15 = AP + AQ$$
$$15 = 2AP$$

Thus $$AP = \frac{15}{2} = 7.5 \text{ cm}$$

37. Two chords AB and CD of a circle intersect at E such that $AE = 2.4$ cm, $BE = 3.2$ cm and $CE = 1.6$ cm. What is the length of DE?

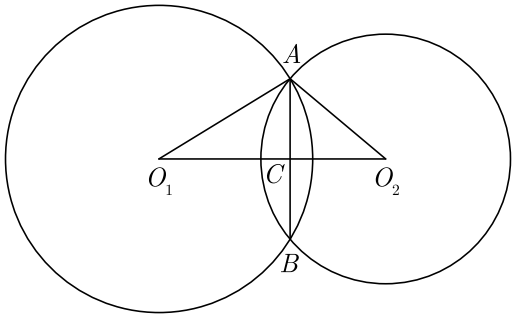
Sol : [Board Term-2 OD 2011]



Applying the rule, $AE \times EB = CE \times ED$
$$2.4 \times 3.2 = 1.6 \times ED$$
$$ED = 4.8 \text{ cm}$$

38. Two circles of radii 20 cm and 37 cm intersect in A and B. If O_1 and O_2 are their centres and $AB = 24$ cm, then find the distance O_1O_2.

Sol : [Board Term-2 Foreign 2016]

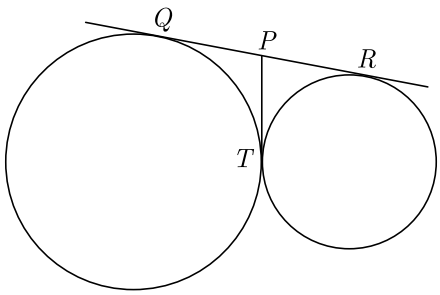


Since C is the mid-point of AB,

$$AC = 12$$
$$AO_1 = 37$$

and $$AO_2 = 20$$
$$CO_1 = \sqrt{37^2 - 12^2} = 35$$
$$CO_2 = \sqrt{20^2 - 12^2} = 16$$
$$O_1O_2 = 35 + 16 = 51$$

39. In the figure, QR is a common tangent to given circle which meet at T. Tangent at T meets QR at P. If $QP = 3.8$ cm, then find length of QR.



Sol : [Board Term-2 Delhi 2012, 2014]

Let us first consider large circle. Since length of tangents from external points are equal, we can write

$$QP = PT$$

Thus $$QP = PT = 3.8 \quad \ldots(1)$$

Now consider the small circle. For this circle we can also write using same logic,

$$PR = PT$$

But we have $PT = 3.8$ cm

Thus $$PR = PT = 3.8 \text{ cm}$$

NODIA

Sol : [Board 2024 OD Standard]

Both assertion (A) and reason (R) are true statements, but they are not related to each other. Therefore, the correct answer is (A) and (R) are true, but (R) is not the correct explanation of (A).

Thus (b) is correct option.

25. **Assertion :** If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent will be 4 cm.

Reason : $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

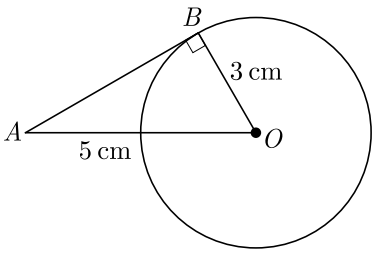
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :



$$\begin{aligned} OA^2 &= AB^2 + OB^2 \\ 5^2 &= AB^2 + 3^2 \\ AB &= \sqrt{25-9} \\ &= 4 \text{ cm} \end{aligned}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

26. **Assertion :** PA and PB are two tangents to a circle with centre O. Such that $\angle AOB = 110^\circ$, then $\angle APB = 90^\circ$.

Reason : The length of two tangents drawn from an external point are equal.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

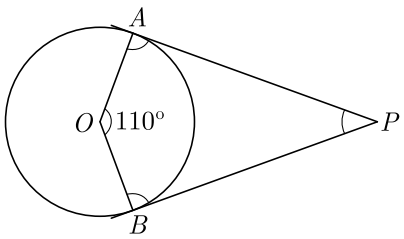
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol : (d) Assertion (A) is false but reason (R) is true.

As per information given in question we have drawn figure below.



Radius is perpendicular to the tangent at point of contact.

Thus, $OA \perp AP$ and $OB \perp PB$.

In quadrilateral, $OAPB$, we have

$$\begin{aligned} \angle OAP + \angle APB + \angle PBO + \angle AOB &= 360^\circ \\ 90^\circ + \angle APB + 90^\circ + 110^\circ &= 360^\circ \\ \angle APB &= 70^\circ \end{aligned}$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

27. **Assertion :** The two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre.

Reason : A parallelogram circumscribing a circle is a rhombus.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

From an external point the two tangents drawn subtend equal angles at the centre.

So assertion is true.

Also, a parallelogram circumscribing a circle is a rhombus, so reason is also true but R is not correct explanation of A.

Thus (b) is correct option.

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$$x = 28^\circ$$

43. If a circle can be inscribed in a parallelogram how will the parallelogram change?

Sol : [Board Term-2, 2014]

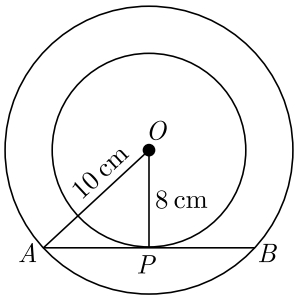
It changes into a rectangle or a square.

44. Two concentric circles are of radii 10 cm and 8 cm, then find the length of the chord of the larger circle which touches the smaller circle.

Sol : [Board Term-2 Delhi 2011]

Let O be the centre of the concentric circles of radii 10 cm and 8 cm, respectively. Let AB be a chord of the larger circle touching the smaller circles at P.

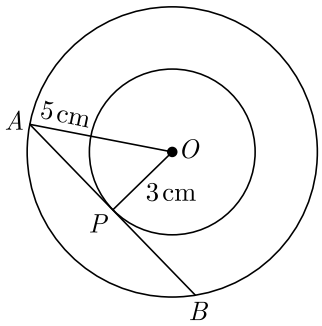
Then, $AP = PB$ and $OP \perp AB$



Applying Pythagoras theorem in ΔOPA, we have

$$OA^2 = OP^2 + AP^2$$
$$100 = 64 + AP^2$$
$$AP^2 = 100 - 64 = 36 \Rightarrow AP = 6 \text{ cm}$$
$$AB = 2AP = 2 \times 6 = 12 \text{ cm}$$

45. In given figure, the length PB = ... cm.



Sol : [Board 2020 OD Standard]

We have $AO = 5\,\text{cm}$

and $OP = 3\,\text{cm}$

Since AB is a tangent at P and OP is radius, we have

$$\angle APO = 90^\circ$$

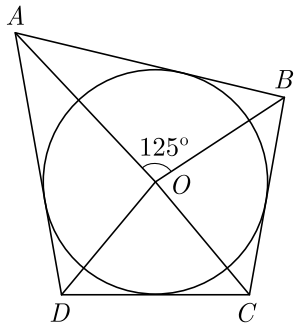
In right angled ΔOPA,

$$AP^2 = AO^2 - OP^2$$
$$= (5)^2 - (3)^2$$
$$= 25 - 9 = 16$$
$$AP = 4\,\text{cm}$$

Perpendicular from centre to chord bisect the chord.

Thus $AP = BP = 4\,\text{cm}$

46. In given figure, if $\angle AOB = 125^\circ$, then find the angle $\angle COD$?



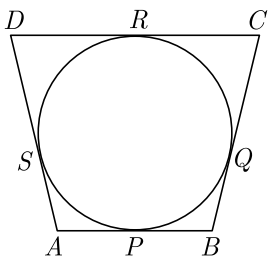
Sol : [Board Term-2 Delhi Compt. 2014]

We know that, a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

i.e. $\angle AOB + \angle COD = 180^\circ$

$$125^\circ + \angle COD = 180^\circ$$
$$\angle COD = 180^\circ - 125^\circ$$
$$= 55^\circ$$

47. In the given figure, a circle touches all the four sides of quadrilateral $ABCD$ with $AB = 6\,\text{cm}$, $BC = 7\,\text{cm}$ and $CD = 4\,\text{cm}$, then what is the length of AD ?



Sol : [Board Term-2 OD 2017]

Four sides of a quadrilateral $ABCD$ are tangent

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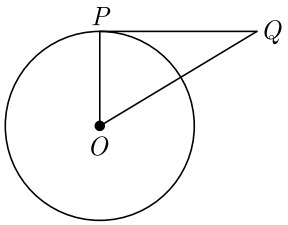


Equal sides subtend equal angles in isosceles triangle.

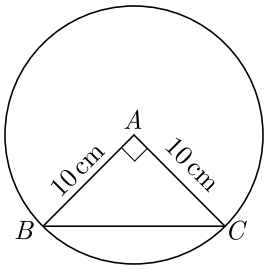
Thus $2\angle OQP + 90^\circ = 180^\circ$

$\angle OQP = 45^\circ$

32. A chord of a circle of radius 10 cm, subtends a right angle at its centre. What is the length of the chord?

Sol : [Board 2020 OD Basic]

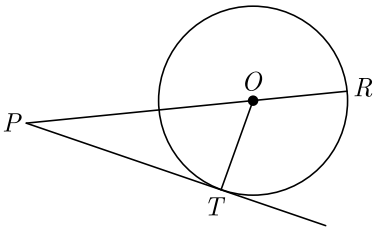
As per given information we have drawn the figure below.



Using Pythagoras theorem in ΔABC, we get

$$BC^2 = AB^2 + AC^2 = 10^2 + 10^2$$
$$= 100 + 100 = 200$$
$$BC = 10\sqrt{2} \text{ cm}$$

33. In figure, on a circle of radius 7 cm, tangent PT is drawn from a point P such that $PT = 24$ cm. If O is the centre of the circle, then what is the length of PR ?



Sol : [Board 2020 Delhi Basic]

Tangent at any point of a circle is perpendicular to the radius at the point of contact.

Thus $OT \perp PT$

Now in right-angled triangle PTO

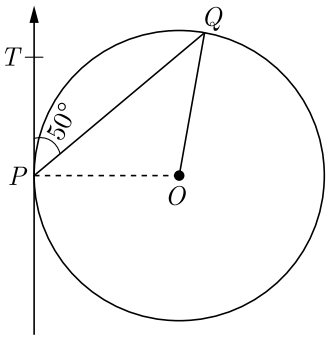
$$OP^2 = OT^2 + PT^2 = (7)^2 + (24)^2$$
$$= 49 + 576 = 625$$

Thus $OP = 25$ cm

Since $OR = OT$ because of radii of circle,

$$PR = OP + OR = 25 + 7 = 32 \text{ cm}$$

34. In figure, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50° with PQ. Find the angle $\angle POQ$.



Sol : [Board 2020 OD Basic]

Due to angle between radius and tangent,

$\angle OPT = 90^\circ$

$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$

Also, $OP = OQ$ [Radii of a circle]

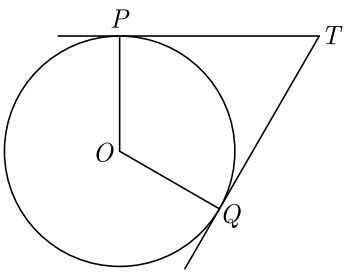
Since equal opposite sides have equal opposite angles,

$\angle OPQ = \angle OQP = 40^\circ$

$\angle POQ = 180^\circ - \angle OPQ - \angle OQP$

$= 180^\circ - 40^\circ - 40^\circ = 100^\circ$

35. In the adjoining figure, TP and TQ are the two tangents to a circle with centre O. If $\angle POQ = 110^\circ$, then find the angle $\angle PTQ$.



Sol : [Board Term-2 Compt. 2016]

Here $OP \perp PT$ and $OQ \perp QT$,

In quadrilateral $OPTQ$, we have

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28. **Assertion :** If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent will be 4 cm.

Reason : $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

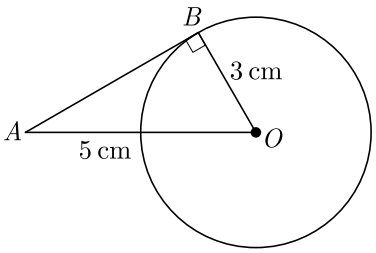
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :



$$OA^2 = AB^2 + OB^2$$
$$5^2 = AB^2 + 3^2$$
$$AB = \sqrt{25-9} = 4 \text{ cm}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

29. **Assertion :** The two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre.

Reason : A parallelogram circumscribing a circle is a rhombus.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

From an external point the two tangents drawn subtend equal angles at the centre.

So assertion is true.

Also, a parallelogram circumscribing a circle is a rhombus, so reason is also true but R is not correct explanation of A.

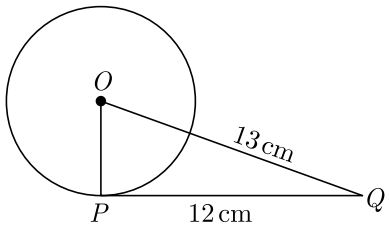
Thus (b) is correct option.

ONE MARK QUESTIONS

30. From an external point Q, the length of tangent to a circle is 12 cm and the distance of Q from the centre of circle is 13 cm. What is the radius of circle?

Sol : [Board 2020 Delhi Basic]

Let O be the centre of the circle. As per given information we have drawn the figure below.



We have $OQ = 13$ cm

and $PQ = 12$ cm

Radius is perpendicular to the tangent at the point of contact.

Thus $OP \perp PQ$

In ΔOPQ, using Pythagoras theorem,

$$OP^2 + PQ^2 = OQ^2$$
$$OP^2 + 12^2 = 13^2$$
$$OP^2 = 13^2 - 12^2$$
$$= 169 - 144 = 25$$

Thus $OP = 5$ cm

31. QP is a tangent to a circle with centre O at a point P on the circle. If ΔOPQ is isosceles, then find $\angle OQR$?

Sol : [Board 2020 Delhi Basic]

Let O be the centre of the circle. As per given information we have drawn the figure below.

We know that, the radius and tangent are perpendicular at their point of contact.

Now, in isosceles triangle POQ we have

$$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$$

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Now $QR = QP + PR$

$= 3.8 + 3.8 = 7.6$ cm.

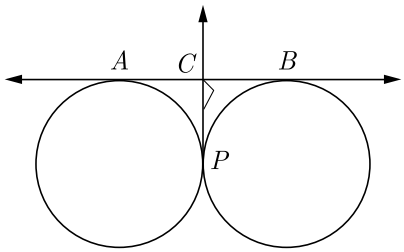
PRACTICE

- AB and CD are two common tangents to circles which touch each other at a point C. If D lies on AB such that $CD = 4$ cm then what is the length of AB.

[Board Term-2 OD 2015]

Ans : 8 cm

- In the given figure, if $BC = 4.5$ cm, find the length of AB.



[Board Term-2, 2012]

Ans : 9 cm

40. Two concentric circles of radii a and b where $a > b$,Find the length of a chord of the larger circle which touches the other circle.

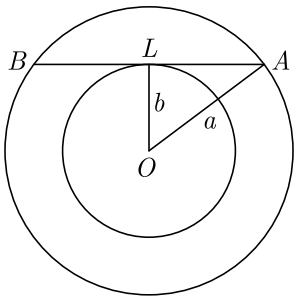
Sol : [Board Term-2 OD 2015]

In ΔOAL, $OA^2 = OL^2 + AL^2$

$a^2 = OL^2 + b^2$

$OL = \sqrt{a^2 - b^2}$

Length of chord, $2AL = 2\sqrt{a^2 - b^2}$



41. If the angle between two radii of a circle is 130°, then what is the angle between the tangents at the end points of radii at their point of intersection ?

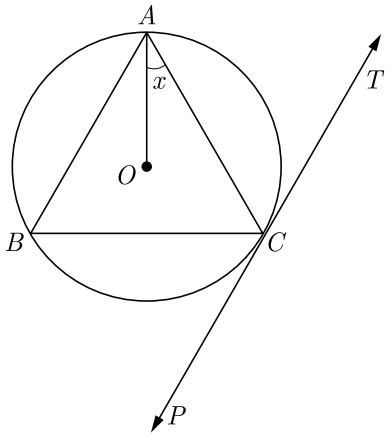
Sol : [Board Term-2 2012]

Sum of the angles between radii and between intersection point of tangent is always 180°.

Thus angle at the point of intersection of tangents

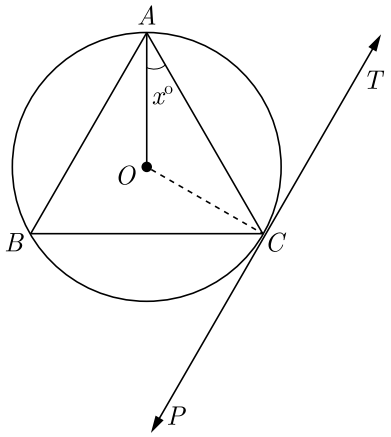
$= 180° - 130° = 50°$

42. In the adjoining figure, PT is a tangent at point C of the circle. O is the circumference of ΔABC. If $\angle ACP = 118°$, then find the angle $\angle x$?



Sol : [Board Term-2 Foreign 2012]

We join OC as shown in the below figure. Here OC is the radius and PT is the tangent to circle at point C.



Thus $OC \perp PT$

$\angle OCP = 90°$

Given, $\angle ACP = 118°$

$\angle ACO = \angle ACP - \angle OCP$

$= 118° - 90° = 28°$

$\angle ACO = 28°$

Since O is the circumcentre, thus $OA = OC$ (radius)

$\angle OAC = \angle ACO$

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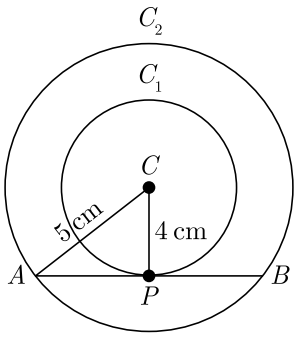


AB is tangent at P and CP is radius at P. Tangent at any point of circle is perpendicular to the radius through the point of contact.

Thus $CP \perp AB$

Now, in right triangle PAC

By Pythagoras theorem we have

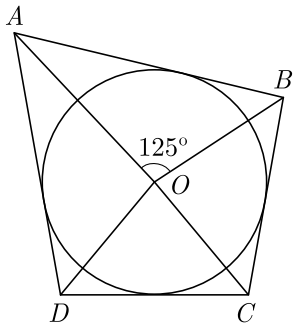
$$AP^2 = AC^2 - PC^2$$
$$= 5^2 - 4^2 = 25 - 16 = 9$$
$$AP = 3\,\text{cm}$$

So, length of chord,

$$AB = 2AP = 2 \times 3 = 6\,\text{cm}$$

Thus (b) is correct option.

22. In figure, if $\angle AOB = 125^\circ$, then $\angle COD$ is equal to



(a) 62.5° (b) 45°

(c) 35° (d) 55°

Sol :

We know that, a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

i.e. $\angle AOB + \angle COD = 180^\circ$

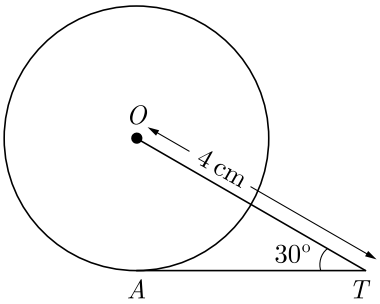
$125^\circ + \angle COD = 180^\circ$

$\angle COD = 180^\circ - 125^\circ$

$= 55^\circ$

Thus (d) is correct option.

23. In figure, AT is a tangent to the circle with centre O such that $OT = 4$ cm and $\angle OTA = 30^\circ$. Then, AT is equal to



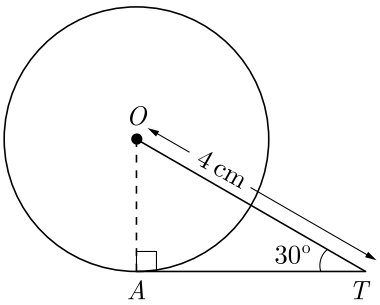
(a) 4 cm (b) 2 cm

(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

Sol :

First we joint OA. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\angle OAT = 90^\circ$ and $OT = 4$ cm (given)



In ΔOAT, $\cos 30^\circ = \dfrac{AT}{OT}$

$$\frac{AT}{4} = \frac{\sqrt{3}}{2}$$
$$AT = 2\sqrt{3}\,\text{cm}$$

Thus (c) is correct option.

24. **Assertion (A):** The tangents drawn at the end points of a diameter of a circle, are parallel.

Reason (R): Diameter of a circle is the longest chord.

(a) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A)

(b) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A)

(c) Assertion (A) is true but Reason (R) is false

(d) Assertion (A) is false but Reason (R) is true

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to a circle.

$$AB + CD = BC + AD$$
$$6 + 4 = 7 + AD$$
$$AD = 10 - 7 = 3 \text{ cm}$$

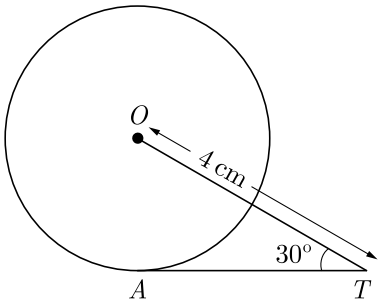
48. To draw a pair of tangents to a circle which are inclined to each other at an angle of 30°, it is required to draw tangents at end points of two radii of the circle, what will be the angle between them ?

Sol : [Board Term-2 2012]

Sum of the angles between radii and between intersection point of tangent is always 180°.

Angle between the radii$= 180^\circ - 30^\circ = 150^\circ$

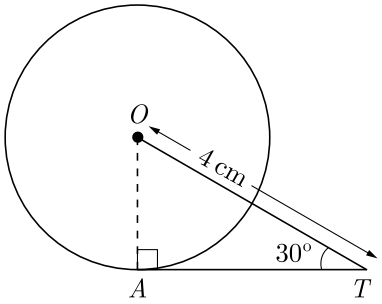
49. In figure, AT is a tangent to the circle with centre O such that $OT = 4$ cm and $\angle OTA = 30^\circ$. What is the length of AT?



Sol : [Board Term-2 Delhi 2013]

First we joint OA. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\angle OAT = 90^\circ$ and $OT = 4$ cm (given)



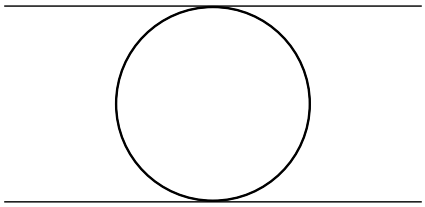
In ΔOAT, $\cos 30^\circ = \dfrac{AT}{OT}$

$$\frac{AT}{4} = \frac{\sqrt{3}}{2}$$
$$AT = 2\sqrt{3} \text{ cm}$$

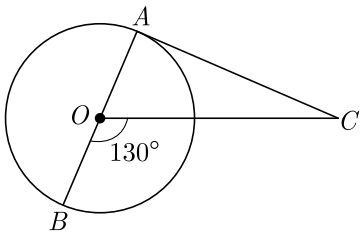
50. What is the maximum number of parallel tangents a circle can have on a diameter?

Sol : [Board Term-2 2012]

Tangent touches a circle on a distinct point. Only two parallel tangents can be drawn on the diameter of a circle. It has been shown in figure given below.



51. In the given figure, AOB is a diameter of the circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130^\circ$, the find $\angle ACO$.



Sol : [Board Term-2 Foreign 2016]

Here OA is radius and AC is tangent at A, since radius is always perpendicular to tangent, we have

$$\angle OAC = 90^\circ$$

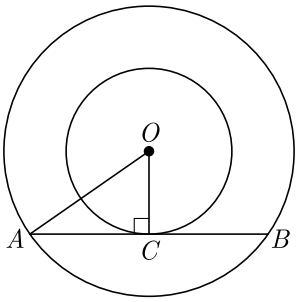
From exterior angle property,

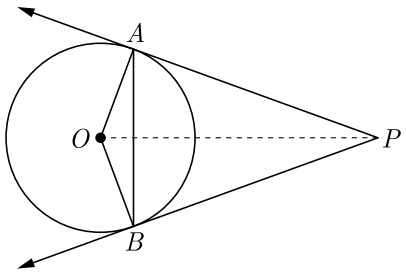
$$\angle BOC = OAC + \angle ACO$$
$$130^\circ = 90^\circ + \angle ACO$$
$$\angle ACO = 130^\circ - 90^\circ = 40^\circ$$

52. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle (in cm) which touches the smaller circle.

Sol : [Board 2020 OD Basic, Foreign 2014]

As per the given question we draw the figure as below.





Here angle $\angle AOB$ and $\angle APB$ are supplementary angle.

Thus $\angle AOB = 180^\circ - \angle APB$

$= 180^\circ - 70^\circ = 110^\circ$

OA and OB are radius of circle and equal in length, thus angle $\angle OAB$ and $\angle OBA$ are also equal. Thus in triangle ΔOAB we have

$\angle OBA + \angle OAB + \angle AOB = 180^\circ$

$\angle OAB + \angle OBA = 180^\circ - \angle AOB$

$2\angle OAB = 180^\circ - 110^\circ = 70^\circ$

$\angle OAB = 35^\circ$

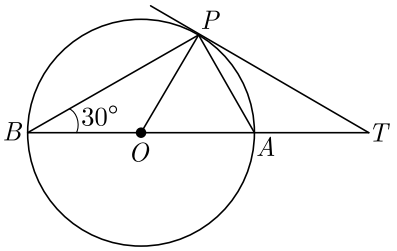
Since OA is radius and AP is tangent at A, $OA \perp AP$

$\angle OAP = 90^o$

Now $\angle PAB = \angle OAP - \angle OAB$

$= 90^o - 35^o = 55^o$

82. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If $\angle PBO = 30^o$, what is the measure of $\angle PTA$?



Sol : [Board Term-2, 2012]

Angle inscribed in a semicircle is always right angle.

$\angle BPA = 90^o$

Here OB and OP are radius of circle and equal in length, thus angle $\angle OBP$ and $\angle OPB$ are also equal.

Thus $\angle BPO = \angle PBO = 30^o$

Now $\angle POA = \angle OBP + \angle OPB$

$= 30^\circ + 30^\circ = 60^\circ$

Thus $\angle POT = \angle POA = 60^\circ$

Since OP is radius and PT is tangent at P, $OP \perp PT$

$\angle OPT = 90^o$

Now in right angle ΔOPT,

$\angle PTO = 180^\circ - (\angle OPT + \angle POT)$

Substituting $\angle OPT = 90^o$ and $\angle POT = 60^\circ$ we have

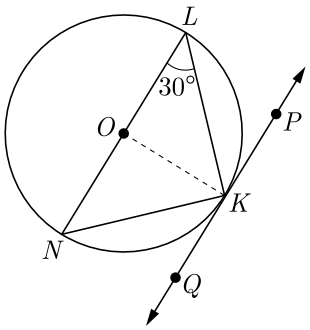
$\angle PTO = 180^\circ - (90^\circ + 60^\circ)$

$= 180^\circ - 150^\circ$

$= 30^o$

Thus $\angle PTA = \angle PTO = 30^o$

83. In figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and $\angle KLN = 30^o$, find $\angle PKL$.



Sol : [Board Term-2 OD Compt 2017]

Since OK and OL are radius of circle, thus

$OK = OL$

Angles opposite to equal sides are equal,

$\angle OKL = \angle OLK = 30^o$

Tangent is perpendicular to the end point of radius,

$\angle OKP = 90^o$ (Tangent)

Now $\angle PKL = \angle OKP - \angle OKL$

$= 90^o - 30^o = 60^o$

PRACTICE

In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is the tangent to the circle at the point A, find $\angle BAT$.

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Sol : [Board Term-2 Delhi 2012, 2014]

Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90^o$$

Due to symmetry we have

$$\angle AOP = \frac{\angle AOB}{2} = \frac{120^\circ}{2} = 60^\circ$$

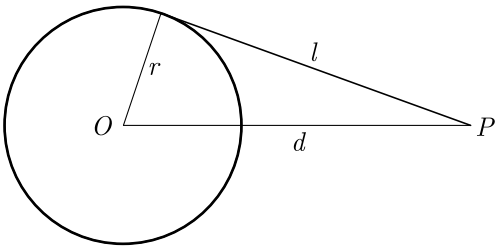
Now in right ΔAOP we have

$$\angle APO + \angle OAP + \angle AOP = 180^\circ$$
$$\angle APO + 90^\circ + 60^\circ = 180^\circ$$
$$\angle APO = 180^\circ - 150^\circ = 30^\circ.$$

58. What is the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm ?

Sol : [Board Term-2, 2012]

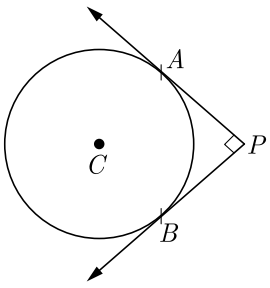
As per the given question we draw the figure as below.



Length of the tangent, $l = \sqrt{d^2 - r^2}$

$$= \sqrt{8^2 - 6^2}$$
$$= \sqrt{64 - 36}$$
$$= \sqrt{28} = 2\sqrt{3} \text{ cm.}$$

59. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If $PA \perp PB$, then find the length of each tangent.



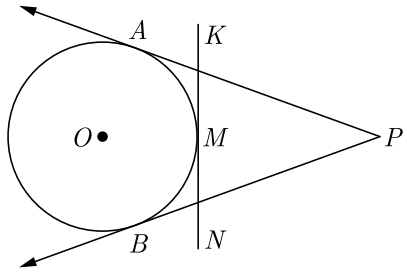
Sol : [Board Term-2, 2013]

Here tangent drawn on circle from external point P are at aright angle, $CAPB$ will be a square.

Thus $CA = AP = PB = BC = 4$ cm

Thus length of tangent is 4 cm.

60. PA and PB are tangents from point P to the circle with centre O as shown in figure. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that $KN = AK + BN$

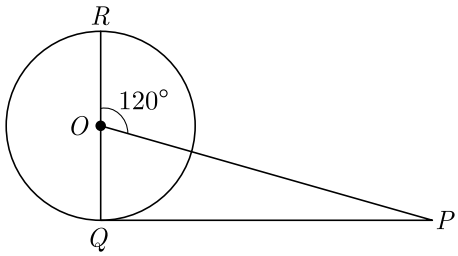


Sol : [Board Term-2, 2012]

Since length of tangents from an external point to a circle are equal,

$$PA = PB, KA = KM, NB = NM,$$
$$KA + NB = KM + NM$$
$$AK + BN = KN. \quad \text{Hence Proved}$$

61. PQ is a tangent drawn from an external point P to a circle with centre O, QOR is the diameter of the circle. If $\angle POR = 120^\circ$, What is the measure of $\angle OPQ$?



Sol : [Board Term-2 Foreign 2017]

Since PQ is a tangent to the circle, ΔOQP is right angle triangle

In ΔOQP because of exterior angle,

$$\angle POR = \angle OQP + \angle OPQ$$

Thus $\angle OPQ = \angle POR - \angle OQP$

$$= 120^\circ - 90^\circ$$
$$= 30^\circ$$

62. Two tangents making an angle of 60° between them are drawn to a circle of radius $\sqrt{3}$ cm, then find the length of each tangent.

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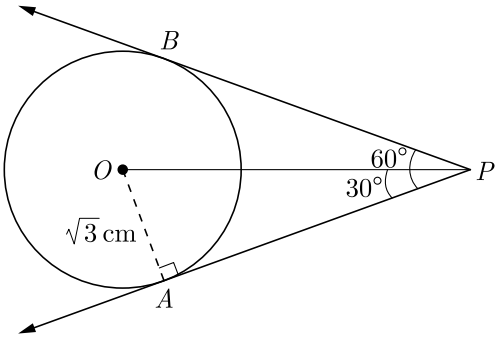
Sol : [Board, Term-2, 2013]

As per the given question we draw the figure as below.



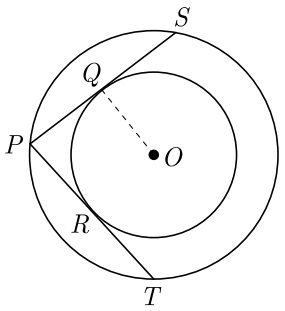
Since, $\tan\theta = \dfrac{OA}{AP}$

So. $\tan 30^\circ = \dfrac{OA}{AP}$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{AP}$$

$$AP = \sqrt{3} \times \sqrt{3} = 3 \text{ cm.}$$

63. In the figure there are two concentric circles with centre O. PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If $PR = 5$ cm find the length of PS.



Sol : [Board Term-2 Delhi Compt. 2017]

Since PQ and PR are tangent of the circle, $PQ = PR$

$$PQ = PR = 5 \text{ cm}$$

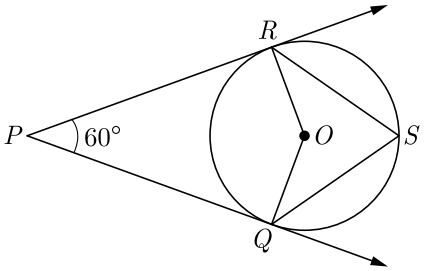
Since PS is chord of circle and point Q bisect it, thus

$$PQ = QS$$

$$PS = 2PQ$$

$$= 2 \times 5 = 10 \text{ cm}$$

64. In the given figure, find $\angle QSR$.



Sol : [Board Term-2, 2012]

Sum of the angles between radii and between intersection point of tangent is always 180°.

Thus $\angle ROQ + \angle RPQ = 180^\circ$

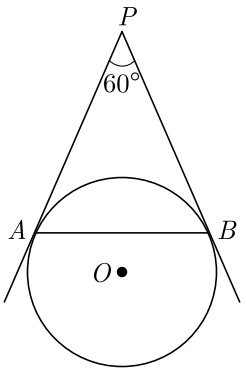
$$\angle ROQ = 180^\circ - 60^\circ = 120^\circ$$

We know that angle subtended on the centre of a circle is twice of the angle subtended on circumference of circle

Thus $\angle QSR = \frac{1}{2}\angle ROQ = \frac{1}{2} \times 120^\circ$

$$= 60^\circ$$

65. In figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.



Sol : [Board Term-2 Delhi 2016]

Since length of 2 tangents drawn from an external point to a circle are equal, we have

$$PA = PB$$

Thus $\angle PAB = \angle PBA = 60^\circ$

Hence ΔPAB is an equilateral triangle.

Therefore $AB = PA = 5$ cm.

66. In given figure, if AT is a tangent to the circle with centre O, such that $OT = 4$ cm and $\angle OTA = 30^\circ$, then find the length of AT .

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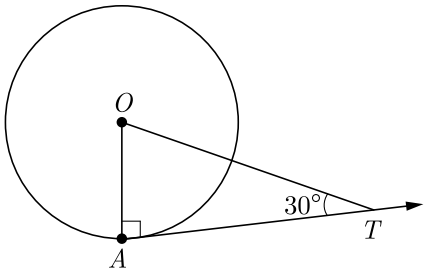


Sol : [Board Term-2, 2012]

Since AT is a tangent to the circle, ΔOAT is right angle triangle

Now $\cos 30^\circ = \frac{AT}{OT}$

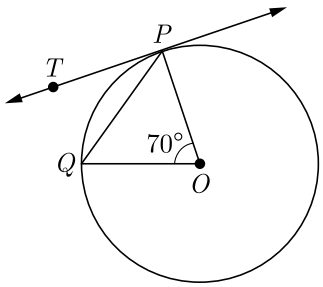
$AT = OT \cos 30^\circ$

$= 4 \times \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}$ cm.

Thus the length of AT is $2\sqrt{3}$ cm.

67. In figure, O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P. Find $\angle OPQ$ and $\angle TPQ$



Sol : [Board Term-2 OD 2017]

We have $\angle OPQ = \angle OQP$

$= \frac{180^\circ - 70^\circ}{2} = 55^\circ$

Thus $\angle TPQ = 90^\circ - 55^\circ = 35^\circ$

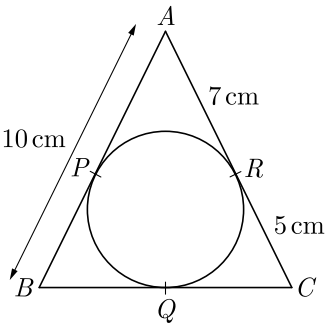
TWO MARKS QUESTIONS

68. A circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively. If $AB = 10$ cm $AR = 7$ cm and $CR = 5$ cm, then find the length of BC

Sol : [Board 2020 OD Basic]

As per given information we have drawn the figure below.

Here a circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively.



Since, tangents drawn to a circle from an external point are equal,

$AP = AR = 7$ cm

$CQ = CR = 5$ cm

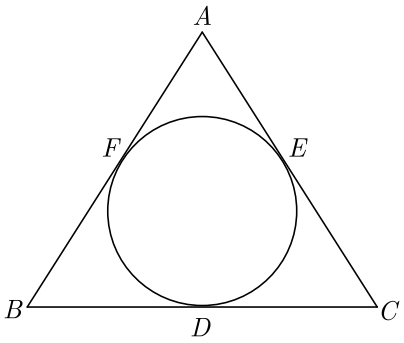
Now, $BP = (AB - AP) = 10 - 7 = 3$ cm

$BP = BQ = 3$ cm

$BC = BQ + QC = 3 + 5 = 8$ cm

PRACTICE

✍ A triangle ABC is drawn to circumscribe a circle. If $AB = 13$ cm, $BC = 14$ cm and $AE = 7$ cm, then find AC.



[Board Term-2 Delhi 2012]

Ans : 15 cm

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✍ In figure, ΔABC is circumscribing a circle, the length of BC is ..... cm.

Here AB is the chord of large circle which touch the smaller circle at point C. We can see easily that ΔAOC is right angled triangle.

Here, $AO = 5$ cm, $OC = 3$ cm

$$AC = \sqrt{AO^2 - OC^2} = \sqrt{5^2 - 3^2}$$
$$= \sqrt{25-9} = \sqrt{16} = 4 \text{ cm}$$

Length of chord, $AB = 8$ cm.

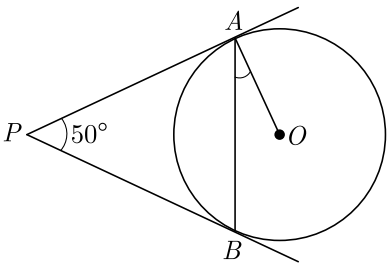
PRACTICE

If the radii of two concentric circle are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.

[Board Term-2 Delhi 2017]

Ans : 6 cm

53. In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$. Write the measure of $\angle OAB$.



Sol : [[Board Term-2 Delhi 2015]

We have $\angle APB = 50^\circ$

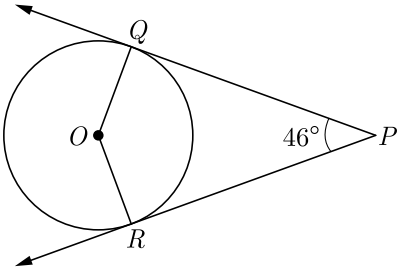
$$\angle PAB = \angle PBA = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90^\circ$$

Now $$\angle OAB = \angle OAP - \angle PAB$$
$$= 90^\circ - 65^\circ = 25^\circ$$

54. If PQ and PR are two tangents to a circle with centre O. If $\angle QPR = 46^\circ$ then find $\angle QOR$.



Sol : [Board Term-2 Delhi 2014]

We have $\angle QPR = 46^\circ$

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

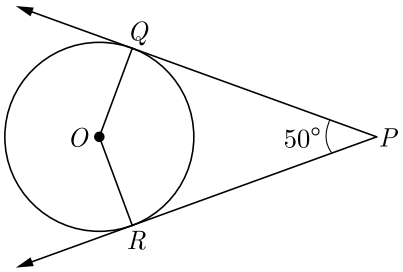
$$\angle QOR + \angle QPR = 180^\circ$$
$$\angle QOR + 46^\circ = 180^\circ$$
$$\angle QOR = 180^\circ - 46^\circ = 134^\circ$$

55. If a line intersects a circle in two distinct points, what is it called ?

Sol : [Board Term-2, 2012]

The line which intersects a circle in two distinct points is called secant.

56. In the given figure, PQ and PR are tangents to the circle with centre O such that $\angle QPR = 50^\circ$, Then find $\angle OQR$.



Sol : [Board Term-2 Delhi 2012, 2015]

We have $\angle QPR = \angle 50^\circ$ (Given)

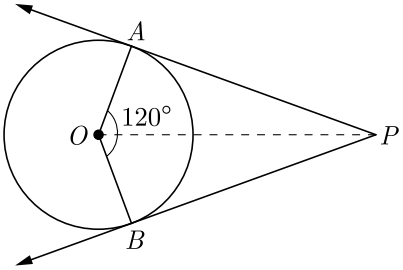
Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\angle QOR + \angle QPR = 180^\circ$$
$$\angle QOR = 180^\circ - \angle QPR$$
$$= 180^\circ - 50^\circ = 130^\circ$$

From ΔOQR we have

$$\angle OQR = \angle ORQ = \frac{180^\circ - 130^\circ}{2}$$
$$= \frac{50^\circ}{2} = 25^\circ$$

57. In the figure, PA and PB are tangents to a circle with centre O. If $\angle AOB = 120^\circ$, then find $\angle OPA$.



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$$\cos 60^\circ = \frac{PS}{OP}$$

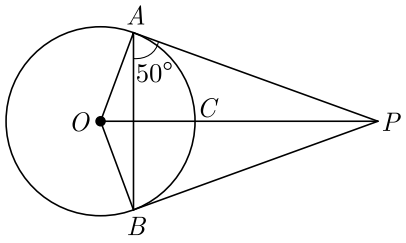
$$\frac{1}{2} = \frac{PS}{OP}$$

$$OP = 2PS \qquad \text{Hence proved.}$$

12. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50^o$, then find $\angle AOB$.

Sol : [Board Term-2 Delhi 2016]

As per the given question we draw the figure as below.



Since $PA \perp OA$, $\angle OAP = 90^\circ$

$$\angle OAB = \angle OAP - \angle BAP$$

$$= 90^o - 50^o = 40^o$$

Since OA and OB are radii, we have

$$\angle OAB = \angle OBA = 40^o$$

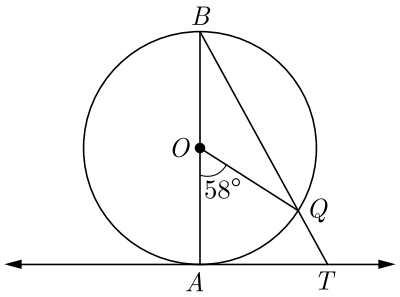
Now $$\angle AOB + \angle OAB + \angle OBA = 180^o$$

$$\angle AOB + 40^o + 40^o = 180^o$$

$$\angle AOB = 180^o - 80^o = 100^o$$

Hence $$\angle AOB = 100^o$$

13. In given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$.



Sol : [Board Term-2, 2015]

We have $\angle AOQ = 58^\circ$

Since angle $\angle ABQ$ and $\angle AOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle ABQ = \frac{1}{2}\angle AOQ$$

$$= \frac{1}{2} \times 58^\circ = 29^\circ$$

Here OA is perpendicular to TA because OA is radius and TA is tangent at A.

Thus $$\angle BAT = 90^\circ$$

$$\angle ABQ = \angle ABT$$

Now in ΔBAT,

$$\angle ATB = 90^\circ - \angle ABT$$

$$= 90^\circ - 29^\circ = 61^\circ$$

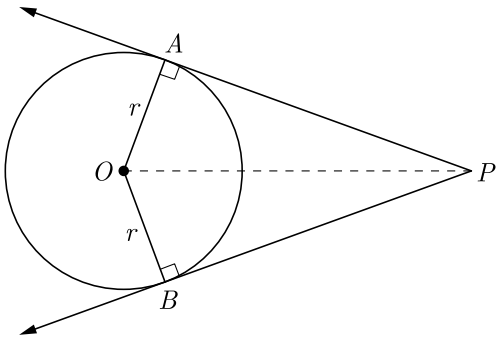
Thus $\angle ATQ = \angle ATB = 61^\circ$

14. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Sol : [Board 2020 OD Basic, 2018]

Consider a circle of radius r and centre at O as shown in figure below. Here we have drawn two tangent from P at A and B. We have to prove that

$$AP = PB$$



We join OA, OB and OP. In ΔPAO and ΔPBO , OP is common and $OA = OB$ radius of same circle.

Since radius is always perpendicular to tangent, at point of contact,

$$\angle OAP = \angle OBP = 90^\circ$$

Thus $$\Delta PAO \cong \Delta PBO.$$

and hence, $$AP = BP$$

Thus length of 2 tangents drawn from an external point to a circle are equal.

15. In the given figure PQ is chord of length 6 cm of the circle of radius 6 cm. TP and TQ are tangents to the circle at points P and Q respectively. Find $\angle PTQ$.

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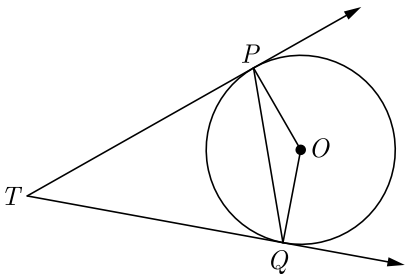


Sol : [Board Term-2 Delhi 2016]

We have $PQ = 6$ cm, $OP = OQ = 6$ cm

Since $PQ = OP = OQ$, triangle ΔPQO is an equilateral triangle.

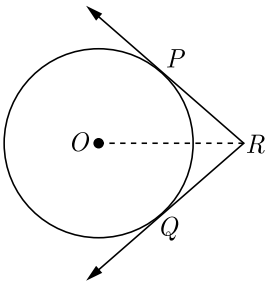
Thus $\angle POQ = 60^\circ$

Now we know that $\angle POQ$ and $\angle PTQ$ are supplementary angle,

$$\angle POQ + \angle PTQ = 180^\circ$$
$$\angle PTQ = 180^\circ - \angle POQ$$
$$= 180^\circ - 60^\circ = 120^\circ$$

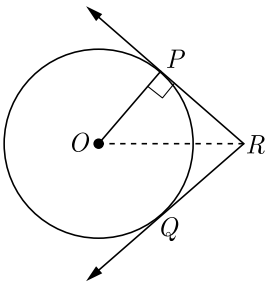
Thus $\angle PTQ = 120^\circ$

16. In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$.



Sol : [Board Term-2 OD 2015]

We redraw the given figure by joining O to P as shown below.



$$\angle PRO = \tfrac{1}{2}\angle PRQ$$
$$= \frac{120^\circ}{2} = 60^\circ$$

Here ΔOPR is right angle triangle, thus

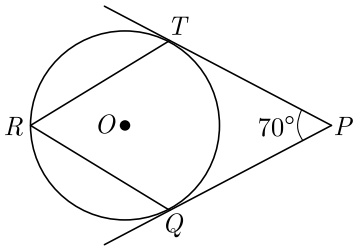
$$\angle POR = 90^\circ - \angle PRO$$
$$= 90^\circ - 60^\circ = 30^\circ$$

Now $$\frac{PR}{OR} = \sin 30^\circ = \frac{1}{2}$$
$$OR = 2PR$$
$$= PR + PR$$

Since $PR = QR$,

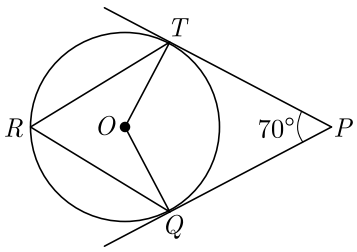
$$OR = PR + QR$$ Hence Proved

17. In figure, O is the centre of a circle. PT are tangents to the circle from an external point P. If $\angle TPQ = 70^\circ$, find $\angle TRQ$.



Sol : [Board Term-2 Foreign 2015]

We redraw the given figure by joining O to T and Q as shown below.



Here angle $\angle TOQ$ and $\angle TPQ$ are supplementary angle.

Thus $$\angle TOQ = 180^\circ - \angle TPQ$$
$$= 180^\circ - 70^\circ = 110^\circ$$

Since angle $\angle TRQ$ and $\angle TOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle TRQ = \frac{1}{2}\angle TOQ$$
$$= \frac{1}{2} \times 110^\circ = 55^\circ$$

18. Prove that tangents drawn at the ends of a chord of a circle make equal angles with the chord.

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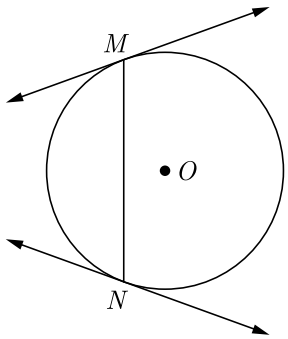
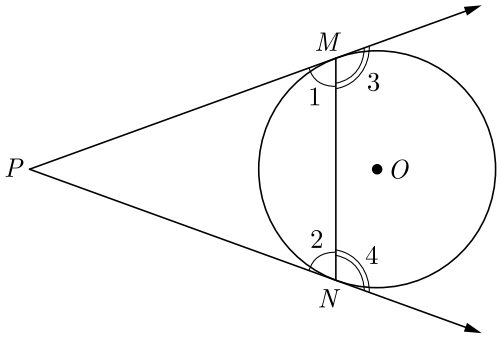


Sol : [Board Term-2 Delhi 2015]

We redraw the given figure by joining M and N to P as shown below.



Since length of tangents from an external point to a circle are equal,

$$PM = PN$$

Since angles opposite to equal sides are equal,

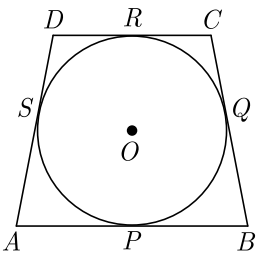
$$\angle 1 = \angle 2$$

Now using property of linear pair we have

$$180^\circ - \angle 1 = 180^\circ - \angle 2$$

$$\angle 3 = \angle 4$$ Hence Proved

79. In Figure a quadrilateral $ABCD$ is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD, and DA touch the circle at the points P, Q, R and S respectively. Prove that. $AB + CD = BC + DA$.



Sol : [Board Term-2 OD 2016]

Since length of tangents from an external point to a circle are equal,

At A, $AP = AS$ (1)

At B $BP = BQ$ (2)

At C $CR = CQ$ (3)

At D $DR = DS$ (4)

Adding eqn. (1), (2), (3), (4)

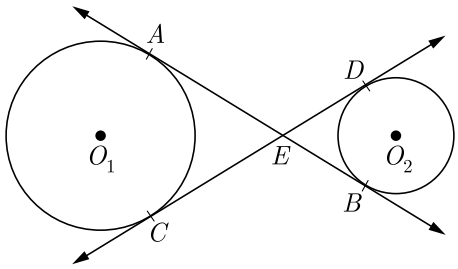
$$AP + BP + DR + CR = AS + DS + BQ + CQ$$

$$AP + BP + DR + RC = AS + SD + BQ + QC$$

$$AB + CD = AD + BC$$

Hence Proved

80. In Figure, common tangents AB and CD to the two circle with centres O_1 and O_2 intersect at E. Prove that $AB = CD$.



Sol : [Board Term-2 OD 2014]

Since EA and EC are tangents from point E to the circle with centre Q_1

$$EA = EC \quad ...(1)$$

and EB and ED are tangents from point E to the circle with centre O_2

$$EB = ED \quad (2)$$

Adding eq (1) and (2) we have

$$EA + BE = CE + ED$$

$$AB = CD$$ Hence Proved

81. Two tangents PA and PB are drawn from an external point P to a circle inclined to each other at an angle of 70°, then what is the value of $\angle PAB$?

Sol : [Board Term-2, 2012]

As per question we draw the given circle as shown below.

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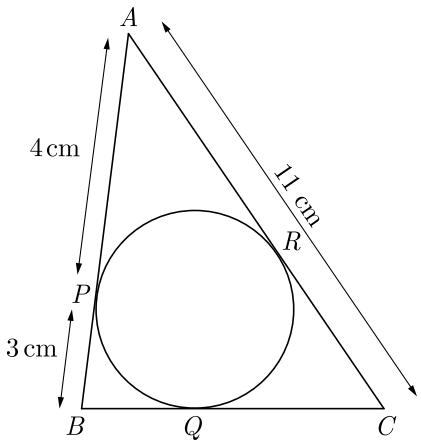


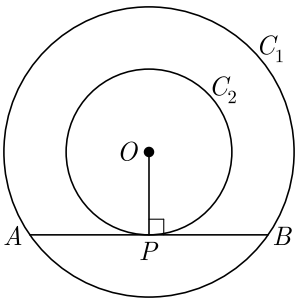
[Board 2020 Delhi Standard]

Ans : 10 cm

69. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle is bisected at the point of contact.

Sol : [Board Term-2, 2012]

As per the given question we draw the figure as below.

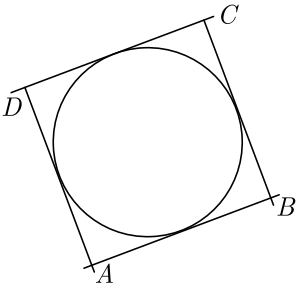


Since OP is radius and APB is tangent, $OP \perp AB$

Now for bigger circle, O is centre and AB is chord such that $OP \perp AB$.

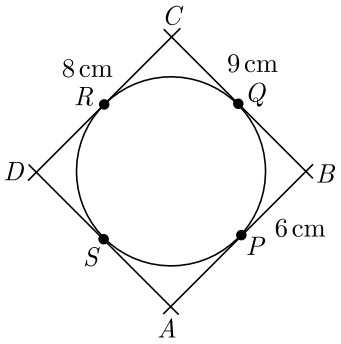
Thus OP bisects AB.

70. In figure, a circle touches all the four sides of a quadrilateral $ABCD$. If $AB = 6$ cm, $BC = 9$ cm and $CD = 8$ cm, then find the length of AD.



Sol : [Board 2020 Delhi Basic]

As per given information we have redrawn the figure below.



Tangents drawn from an external point to a circle are equal in length.

Thus $AP = AS$ and let it be x.

Similarly, $BP = BQ$, $CQ = CR$ and $RD = DS$

Now $BP = AB - AP = 6 - x$

$BP = BQ = 6 - x$

$CQ = BC - BQ = 9 - (6 - x) = 3 + x$

Now, $CQ = CR = 3 + x$

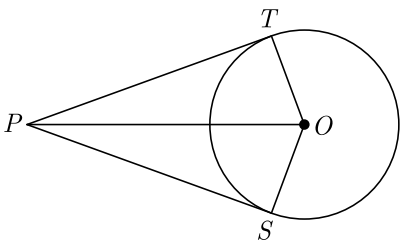
$RD = CD - CR = 8 - (3 + x) = 5 - x$

Now, $RD = DS = 5 - x$

$AD = AS + SD = x + 5 - x = 5$

Thus AD is 5 cm.

71. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle SPT = 120^\circ$, Prove that $OP = 2PS$.



Sol : [Board Term-2 Foreign 2016]

We have $\angle SPT = 120^\circ$

As OP bisects $\angle SPT$,

$$\angle OPS = \frac{120^\circ}{2} = 60^\circ$$

Since radius is always perpendicular to tangent,

$$\angle PTO = 90^\circ$$

Now in right triangle POS, we have

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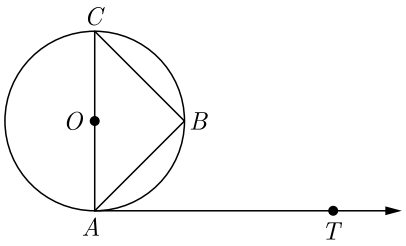
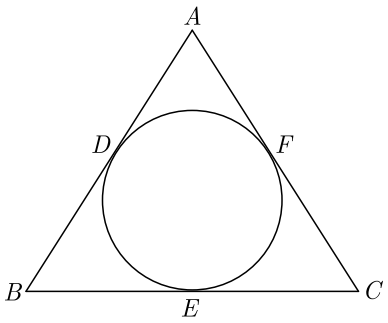


[Board Term-2 2012]

Ans : 50°

84. In the given figure, if $AB = AC$, prove that $BE = CE$.



Sol : [Board Term-2 OD 2017]

Since tangents from an external point to a circle are equal,

$$AD = AF \quad (1)$$

$$BD = BE \quad (2)$$

$$CE = CF \quad (3)$$

From $AB = AC$ we have

$$AD + DB = AF + FC$$

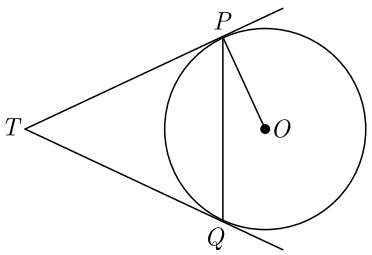
or $$DB = FC \quad (AD = AF)$$

From eq (2) and (3) we have

$$BE = EC$$ Hence Proved

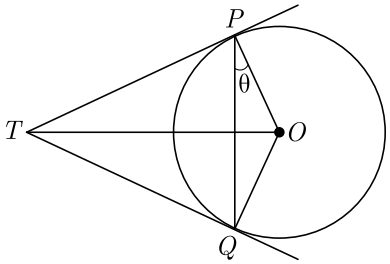
THREE MARKS QUESTIONS

85. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Sol : [Board 2023 OD Standard]

We redraw the given figure as shown below.



Let $\angle OPQ$ be θ, then

$$\angle TPQ = 90^\circ - \theta$$

Since, $TP = TQ$, due to opposite angles of equal sides we have

$$\angle TQP = 90^\circ - \theta$$

From angle sum property of a triangle we can write,

$$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$$

$$90^\circ - \theta + 90^\circ - \theta + \angle PTQ = 180^\circ$$

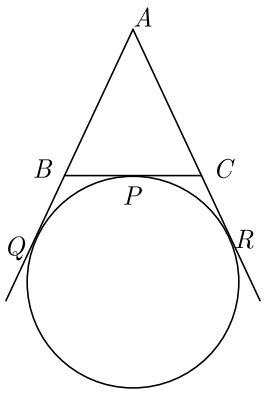
$$\angle PTQ = 180^\circ - 180^\circ + 2\theta$$

$$\angle PTQ = 2\theta$$

Hence, $$\angle PTQ = 2\angle OPQ$$

86. A circle touches the side BC of a ΔABC at a point P and touches AB and AC when produced at Q and R respectively. Show that

$$AQ = \tfrac{1}{2} \text{ (Perimeter of } \Delta ABC)$$



Sol : [Board 2023 OD Standard]

From the same external point, the tangent segments drawn to a circle are equal.

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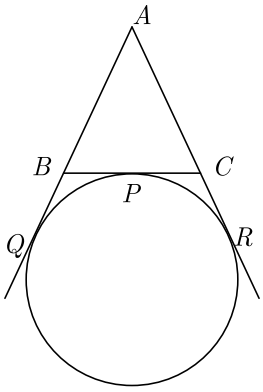


From the point A, $AQ = AR$ (1)

From the point B, $BQ = BP$ (2)

From the point C, $CP = CR$ (3)

Perimeter of ΔABC

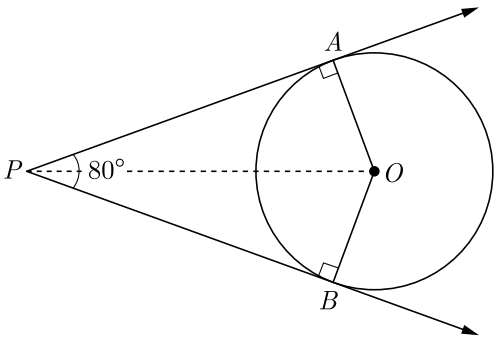
$$AB + BC + AC = AB + BP + PC + AC$$
$$= AB + BQ + CR + AC$$
Using (2) and (3)
$$= AQ + AR$$
$$= 2AQ \quad \text{From (1)}$$
$$AQ = \frac{1}{2} \times \text{Perimeter of } \Delta ABC.$$

Hence proved.

87. If tangents PA and PB drawn from an external point P to a circle with centre O are inclined to each other at an angle of 80°, then find $\angle POA$.

Sol : [Board 2020 Delhi Basic]

As per given information we have drawn the figure below.



Since PA and PB are the tangents, PO will be angle bisector of $\angle P$

Hence, $\angle APO = 40^\circ$

Now, in ΔAPO, $\angle PAO$ is 90° because this is angle between radius and tangent.

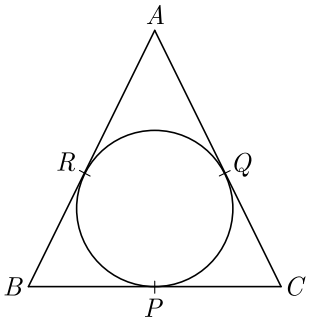
Now $\angle PAO + \angle APO + \angle POA = 180^\circ$

$$90^\circ + 40^\circ + \angle POA = 180^\circ$$
$$\angle POA = 50^\circ$$

88. An isosceles triangle ABC, with $AB = AC$, circumscribes a circle, touching BC at P, AC at Q and AB at R. Prove that the contact point P bisects BC.

Sol : [Board 2020 OD Basic]

As per given information we have drawn the figure below.



Since, the tangents drawn from externals points are equal,

$$AR = AQ$$
$$BR = BP$$
$$CP = CQ$$

Now we have, $AB = AC$

$$AR + BR = AQ + CQ$$
$$AR + BP = AQ + CP$$
$$AQ + BP = AQ + CP$$
$$BP = CP$$

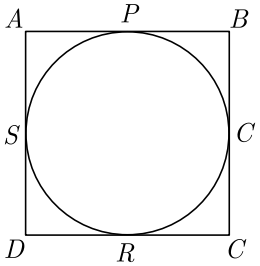
Hence, the point of contact P bisects BC.

89. Prove that the rectangle circumscribing a circle is a square.

Sol : [Board 2020 SQP Standard]

We have a rectangle $ABCD$ circumscribe a circle which touches the circle at P, Q, R, S. We have to prove that $ABCD$ is a square.

As per given information we have drawn the figure below.



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Since tangent drawn from an external point to a circle are equals,

$$AP = AS$$
$$PB = BQ$$
$$DR = DS$$
$$RC = QC$$

Adding all above equation we have

$$AP + PB + DR + RC = AS + SD + BQ + QC$$
$$AB + CD = AD + BC$$

Since $ABCD$ is rectangle, $AB = CD$ and $AD = BC$,

Thus $2AB = 2BC$

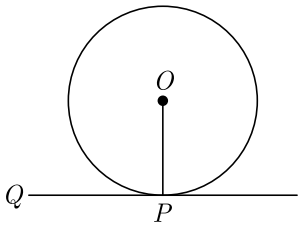
$$AB = BC$$

Since adjacent sides are equal are equal. So, $ABCD$ is a square.

90. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Sol : [Board 2020 Delhi Basic]

Given, a circle with centre O and tangent AB at P. We take a point Q on the tangent AB and join OQ meeting the circle at R.



To prove that $OP \perp AB$, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Clearly $OP = OR$ (radius)

$$OQ = OR + RQ$$
$$OQ > OR$$
$$OQ > OP$$

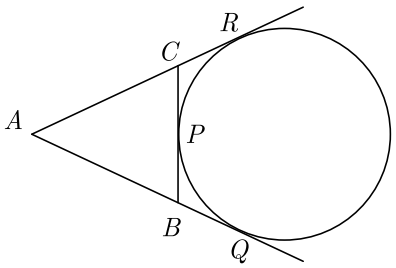
Thus OP is shorter than any other segment joining O to any other point of AB and shortest line is perpendicular.

Thus $OP \perp AB$ Hence Proved

91. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$

Sol : [Board 2020 OD Standard, 2016]

As per given information in question we have drawn the figure below,



From the same external point, the tangent segments drawn to a circle are equal.

From the point B, $BQ = BP$

From the point A, $AQ = AR$

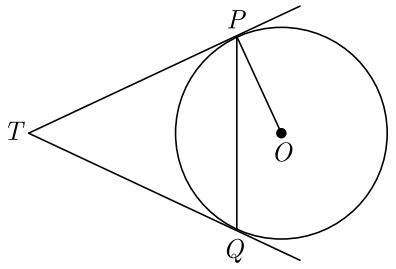
From the point C, $CP = CR$

Now

$$AB + BC + CA = (AQ - BQ) + (BP + PC) + (AR - CR)$$
$$= (AQ - BQ) + (BQ + CR) + (AQ - CR)$$
$$= 2AQ$$

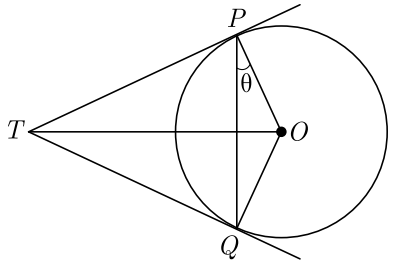
$$AQ = \frac{1}{2}(BC + CA + AB)$$ Hence proved.

92. In figure, two tangents TP and TQ are drawn to circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Sol : [Board 2020 Delhi Standard]

We redraw the given figure as shown below.



Let $\angle OPQ$ be θ, then

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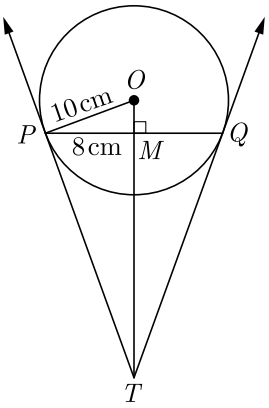
$$= \frac{256}{9} + 16$$

$$= \frac{400}{9}$$

$$TP = \frac{20}{3} \text{ cm}$$

PRACTICE

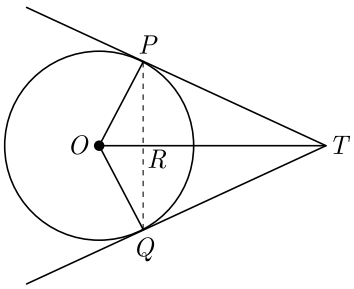
In figure, PQ, is a chord of length 16 cm, of a circle of radius 10 cm. the tangents at P and Q intersect at a point T. Find the length of TP.



[Board Term-2 Delhi 2014]

Ans : $\frac{40}{3}$ cm

In figure PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents drawn at P and Q intersect at T. Find the length of TP.



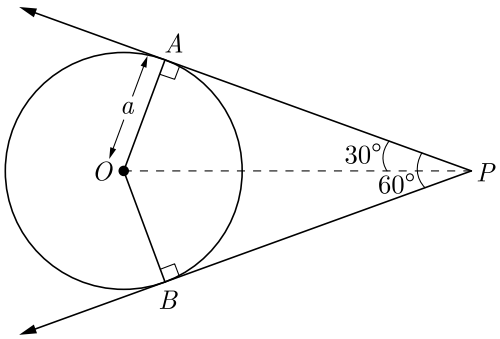
[Board Term-2 OD Compt 2017]

Ans : $\frac{20}{3}$ cm

104. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

Sol : [Board 2020 SQP STD]

As per the given question we draw the figure as below.



Tangents are always equally inclined to line joining the external point P to centre O.

$$\angle APO = \angle BPO = \frac{60^\circ}{2} = 30^\circ$$

Also radius is also perpendicular to tangent at point of contact.

In right ΔOAP we have,

$$\angle APO = 30^\circ$$

Now, $$\sin 30^\circ = \frac{OA}{OP}$$

Here OA is radius whose length is a, thus

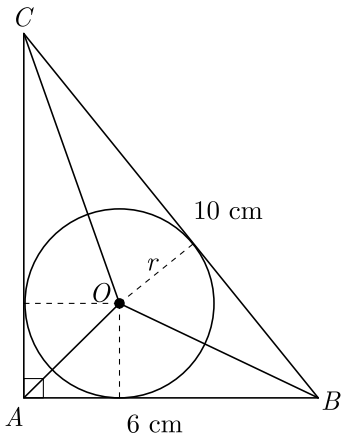
$$\frac{1}{2} = \frac{a}{OP}$$

or $$OP = 2a$$

105. A right triangle ABC, right angled at A is circumscribing a circle. If $AB = 6$ cm and $BC = 10$ cm, find the radius r of the circle.

Sol : [Board 2020 Delhi Basic]

As per question we draw figure shown below.



In triangle ΔABC,

$$AC = \sqrt{10^2 - 36^2} = 8 \text{ cm}$$

Area of triangle ΔABC,

$$\Delta ABC = \frac{1}{2} \times AB \times AC$$

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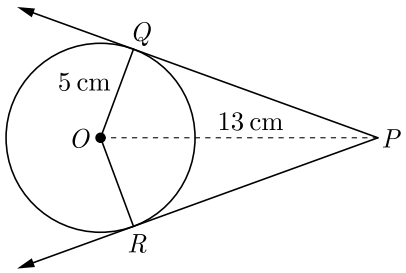


Here OQ is radius and QP is tangent at Q, since radius is always perpendicular to tangent at point of contact, $\Delta\, OQP$ is right angle triangle.

Now $PQ = \sqrt{OP^2 - OR^2}$

$= \sqrt{13^2 - 5^2} = \sqrt{169 - 25}$

$= \sqrt{144} = 12$ cm

Area of triangle $\Delta\, OQP$,

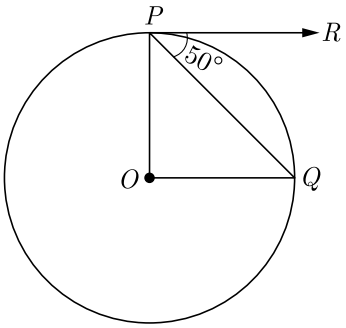
$\Delta = \frac{1}{2}(OQ)(QP)$

$= \frac{1}{2} \times 12 \times 5 = 30$

Area of quadrilateral $PQOR$,

$2 \times \Delta\, POQ = 2 \times 30 = 60$ cm^2

96. If O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, find $\angle POQ$.



Sol : [Board Term-2, 2012]

We have $\angle RPQ = 50^\circ$

Since $\angle OPQ + \angle QPR$ is right angle triangle,

$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$

Since, $OP = OQ$ because of radii of circle, we have

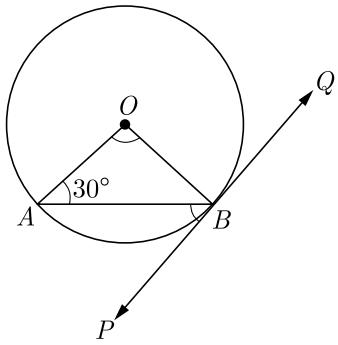
$\angle OPQ = \angle OQR = 40^\circ$

In ΔPOQ we have

$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$

$= 180^\circ - (40^\circ + 40^\circ) = 100^\circ$

97. In the figure, PQ is a tangent to a circle with centre O. If $\angle OAB = 30^\circ$, find $\angle ABP$ and $\angle AOB$.



Sol : [Board Term-2 Delhi 2014]

Here OB is radius and QT is tangent at B, $OB \perp PQ$

$\angle OBP = 90^\circ$

Here OA and OB are radius of circle and equal. Since angles opposite to equal sides are equal,

$\angle OAB = \angle OBA = 30^\circ$

Now $\angle AOB = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$

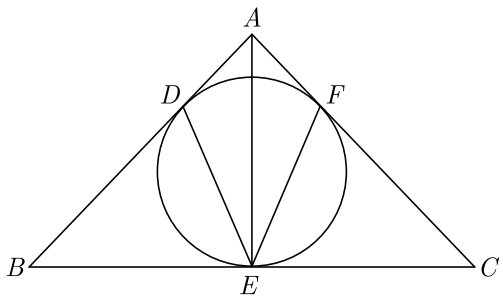
$\angle ABP = \angle OBP - \angle OBA$

$= 90^\circ - 30^\circ = 60^\circ$

98. In $\Delta ABD, AB = AC$. If the interior circle of ΔABC touches the sides AB, BC and CA at D, E and F respectively. Prove that E bisects BC.

Sol : [Board Term-2 Delhi 2014, 2012]

As per question we draw figure shown below.



Since length of tangents from an external point to a circle are equal,

At A, $AF = AD$ (1)

At B $BE = BD$ (2)

At C $CE = CF$ (3)

Now we have $AB = AC$

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$$\angle TPQ = 90^\circ - \theta$$

Since, $TP = TQ$, due to opposite angles of equal sides we have

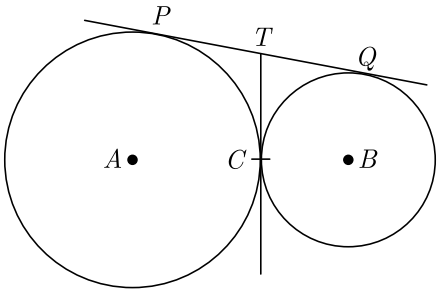
$$\angle TQP = 90^\circ - \theta$$

From angle sum property of a triangle we can write,

$$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$$
$$90^\circ - \theta + 90^\circ - \theta + \angle PTQ = 180^\circ$$
$$\angle PTQ = 180^\circ - 180^\circ + 2\theta$$
$$\angle PTQ = 2\theta$$

Hence, $\angle PTQ = 2\angle OPQ$

93. In given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



Sol : [Board 2020 OD Basic, 2020 Delhi Standard]

Here PT and TC are the tangents of circle A from extended point, thus

$$PT = TC$$

Here TQ and TC are the tangents of circle B from extended point, thus

$$QT = TC$$

Thus, $PT = QT$

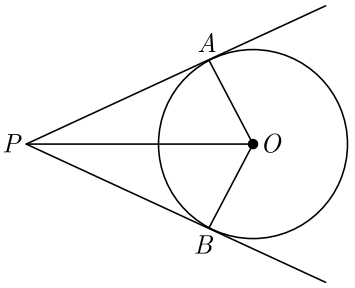
Now, $PQ = PT + TQ$

$$= PT + PT = 2PT$$

Thus $\frac{1}{2}PQ = PT$

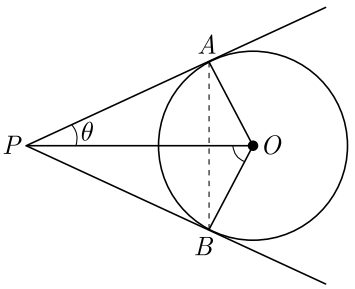
Hence, the common tangent to the circle at C, bisects the common tangents at P and Q.

94. In the given figure, OP is equal to the diameter of a circle with centre O and PA and PB are tangents. Prove that ABP is an equilateral triangle.



Sol : [Board Term-2, 2014]

We redraw the given figure by joining A to B as shown below.



Since OA is radius and PA is tangent at A, $OA \perp AP$. Now in right angle triangle ΔOAP, OP is equal to diameter of circle, thus

$$OP = 2OA$$
$$\frac{OA}{OP} = \frac{1}{2}$$
$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Since PO bisect the angle $\angle APB$,

Hence, $\angle APB = 2 \times 30^\circ = 60^o$

Now, in ΔAPB,

$$AP = AB$$
$$\angle PAB = \angle PBA$$
$$= \frac{180^\circ - 60^\circ}{2} = 60^o$$

Thus ΔAPB is an equilateral triangle.

95. From a point P, which is at a distant of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR are drawn to the circle, then the area of the quadrilateral $PQOR$ (in cm^2).

Sol : [Board Term-2, 2012]

As per the given question we draw the figure as below.

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Hence $\angle CAB = \angle ABF$

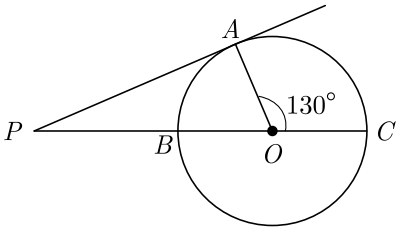
and $\angle ABE = \angle BAD$

Hence $\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles.

$CD \,||\, EF$ Hence Proved

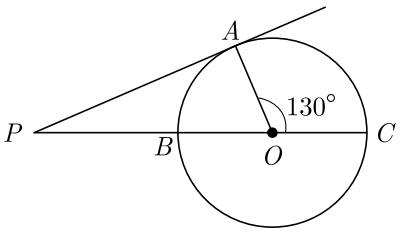
FIVE MARKS QUESTIONS

102. In the given figure, PA is a tangent to the circle drawn from the external point P and PBC is the secant to the circle with BC as diameter. If $\angle AOC = 130^\circ$, then find the measure of $\angle APB$, where O is the centre of the circle.



Sol : [Board 2023 OD Standard]

We have redrawn the given figure as below.



The tangent at a point to a circle is perpendicular to the radius passing through the point of contact.

Thus $\angle PAO = 90^\circ$

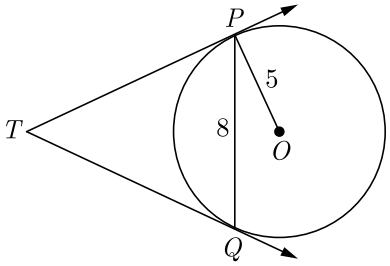
Now, due to linear pair,

$$\angle AOC + \angle AOB = 180^\circ$$
$$\angle AOP = 50^\circ$$

Sum of all angles of a triangle is 180°. Thus in ΔPAO we have

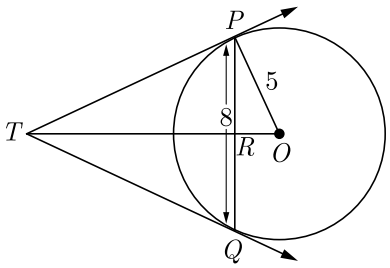
$$\angle APO + \angle PAO + \angle AOP = 180^\circ$$
$$\angle APO + 90^\circ + 50^\circ = 180^\circ$$
$$\angle APO + 140^\circ = 180^\circ$$
$$\angle APO = 180^\circ - 140^\circ$$
$$= 40^\circ$$

103. In Figure, PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.



Sol : [Board 2019 Delhi Standard]

We redraw the given figure as shown below. Here OT is perpendicular bisector of PQ,



Since, OT is perpendicular bisector of PQ,

, $PR = QR = 4$ cm

In right angle triangle ΔOTP and ΔPTR, we have

$$TP^2 = TR^2 + PR^2 \quad ...(1)$$

Also, $OT^2 = TP^2 + OP^2$

Substituting TP^2 from equation (1) we have

$$OT^2 = (TR^2 + PR^2) + OP^2$$
$$(TR + OR)^2 = TR^2 + PR^2 + OR^2$$

Now $OR^2 = OP^2 - PR^2$

$$= 5^2 - 4^2 = 3^2$$

Thus $OR = 3$ cm

Thus substituting $OR = 3$ cm we have

$$(TR + 3)^2 = TR^2 + 4^2 + 5^2$$
$$TR^2 + 9 + 6TR = TR^2 + 16 + 25$$
$$6TR = 32$$
$$TR = \frac{16}{3}$$

Now, from (1), $TP^2 = TR^2 + PR^2$

$$= \left(\frac{16}{3}\right)^2 + 4^2$$

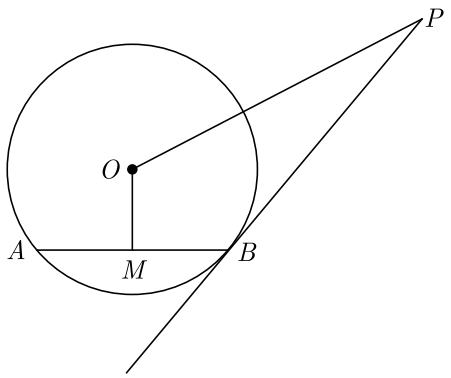
Since length of tangents from an external point to a circle are equal,

$$AN = AP, CN = CM \text{ and } BM = BP$$

Now taking $AN = AP$

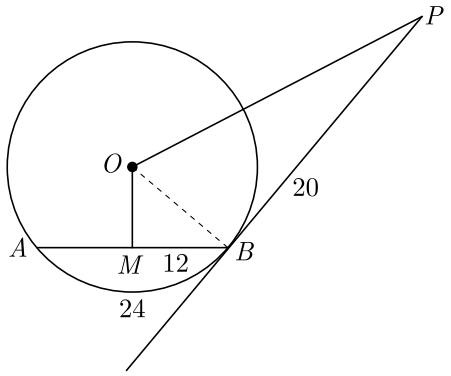
$$AC - CN = AB - BP$$
$$b - r = c - BM$$
$$b - r = c - (a - r)$$
$$b - r = c - a + r$$
$$2r = a + b - c$$
$$r = \frac{a + b - c}{2} \quad \text{Hence Proved.}$$

108. PB is a tangent to the circle with centre O to B. AB is a chord of length 24 cm at a distance of 5 cm from the centre. It the tangent is length 20 cm, find the length of PO.



Sol : [Board Term-2 Delhi 2015]

We redraw the given figure by joining O to B as shown below.



Here ΔOMB right angled triangle because AB is chord and OM is perpendicular on it.

In right angled triangle ΔOMB we have,

$$OB^2 = OM^2 + MB^2$$
$$= 5^2 + 12^2 = 13^2$$

Thus $OB = 13$

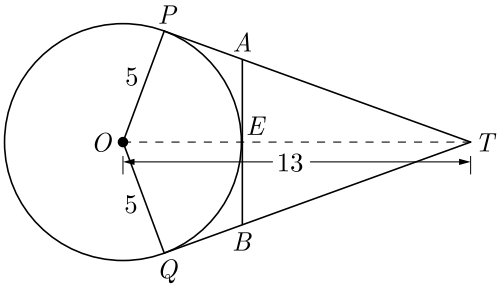
Here ΔOBP right angled triangle because PB is tangent on radius OB.

This in right angled triangle ΔOBP we have,

$$OP^2 = OB^2 + BP^2$$
$$= 13^2 + 20^2 = 569$$

Thus $OP = \sqrt{569} = 23.85$ cm

109. In figure O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Sol : [Board Term-2 Delhi 2016]

Here ΔOPT is right angled triangle because PT is tangent on radius OP.

Thus $PT = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = 12$ cm

and $TE = OT - OE = 13 - 5 = 8$ cm

Since length of tangents from an external point to a circle are equal,

Let $PA = AE = x$

Here ΔAET is right angled triangle because AB is tangent on radius OE.

In ΔAET, $TA^2 = TE^2 + EA^2$

$$(TP - PA)^2 = 8^2 + x^2$$
$$(12 - x)^2 = 64 + x^2$$
$$144 - 24x + x^2 = 64 + x^2$$
$$24x = 144 - 64 = 80$$

or, $x = 3.3$ cm.

Thus $AB = 2 \times x = 2 \times 3.3$

$= 6.6$ cm.

110. In figure, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm,

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$$= \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

Here we have joined AO, BO and CO.

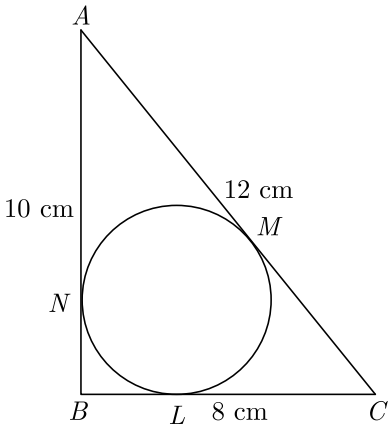
For area of triangle we have

$$\begin{aligned} \Delta ABC &= \Delta OBC + \Delta OCA + \Delta OAB \\ 24 &= \tfrac{1}{2} rBC + \tfrac{1}{2} rAC + \tfrac{1}{2} rAB \\ &= \tfrac{1}{2} r(BC + AC + AB) \\ &= \tfrac{1}{2} r(6 + 10 + 8) = 12r \end{aligned}$$

or $12r = 24$

Thus $r = 2$ cm.

106. In figure, a circle is inscribed in a ΔABC having sides $BC = 8$ cm, $AB = 10$ cm and $AC = 12$ cm. Find the length BL, CM and AN.



Sol : [Board 2019 Delhi]

Tangents from external a point on a circle are always equal in length.

Let x be length of BL, then we have

$$BL = x = BN$$

So, $LC = MC = (8 - x)$

and $AN = AM = (10 - x)$

Since, $AC = 12$

$$AM + MC = 12$$

$$(10 - x) + (8 - x) = 12$$

$$18 - 2x = 12 \Rightarrow x = 3$$

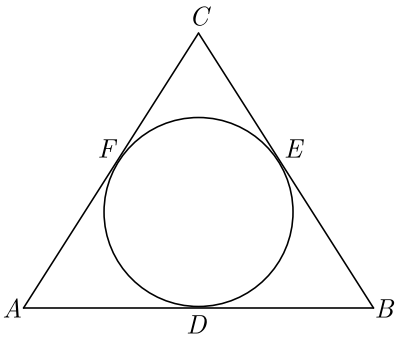
Hence, $BL = 3$ cm

$CM = 8 - 3 = 5$ cm

and $AN = 10 - 3 = 7$ cm

PRACTICE

In the given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



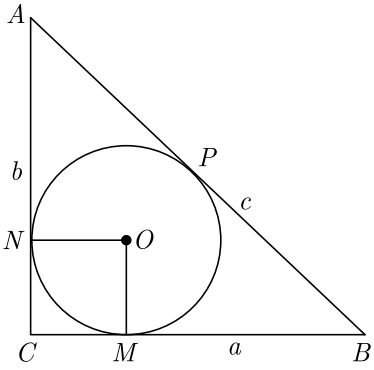
[Board Term-2 Delhi 2016, 2013, 2012]

Ans : 7 cm, 5 cm, 3 cm

107. a, b and c are the sides of a right triangle, where c is the hypotenuse. A circle, of radius r, touches the sides of the triangle. Prove that $r = \dfrac{a + b - c}{2}$.

Sol : [Board Term-2 Delhi 2016]

As per question we draw figure shown below.



Let the circle touches CB at M, CA at N and AB at P.

Now $OM \perp CB$ and $ON \perp AC$ because radius is always perpendicular to tangent

OM and ON are radius of circle, thus

$$OM = ON$$

CM and CN are tangent from C, thus

$$CM = CN$$

Therefore $OMCN$ is a square. Let

Let $OM = r = CM = CN = ON$

NODIA

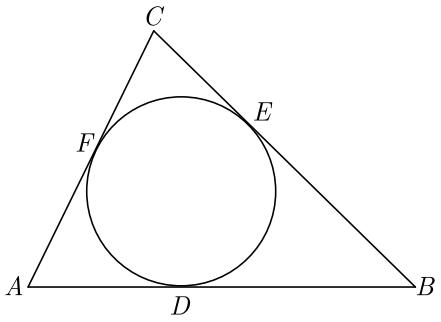
$$AD + DB = AF + FC$$
$$BD = FC \quad (AD = AF)$$
$$BE = EC \quad (BD = BE, CE = CF)$$

Thus E bisects BC.

99. A circle is inscribed in a ΔABC, with sides AC, AB and BC as 8 cm, 10 cm and 12 cm respectively. Find the length of AD, BE and CF.

Sol : [Board Term-2 Delhi 2013, 2012]

As per question we draw figure shown below.



We have $AC = 8$ cm

$AB = 10$ cm

and $BC = 12$ cm

Let AF be x. Since length of tangents from an external point to a circle are equal,

At A, $AF = AD = x$ (1)

At B $BE = BD = AB - AD = 10 - x$ (2)

At C $CE = CF = AC - AF = 8 - x$ (3)

Now $BC = BE + EC$

$12 = 10 - x + 8 - x$

$2x = 18 - 12 = 6$

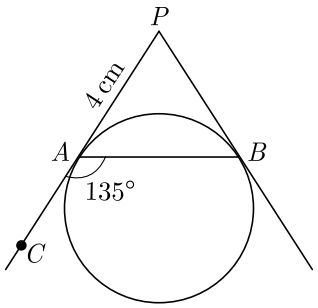
or $x = 3$

Now $AD = 3$ cm,

$BE = 10 - 3 = 7$ cm

and $CF = 8 - 3 = 5$

100. In the given figure, PA and PB are tangents to a circle from an external point P such that $PA = 4$ cm and $\angle BAC = 135^0$. Find the length of chord AB.



Sol : [Board Term-2 OD 2017]

Since length of tangents from an external point to a circle are equal,

$$PA = PB = 4 \text{ cm}$$

Here $\angle PAB$ and $\angle BAC$ are supplementary angles,

$$\angle PAB = 180^\circ - 135^\circ = 45^\circ$$

Angle $\angle ABP$ and $= \angle PAB = 45^\circ$ opposite angles of equal sides, thus

$$\angle ABP = \angle PAB = 45^\circ$$

In triangle ΔAPB we have

$$\angle APB = 180^\circ - \angle ABP - \angle BAP$$
$$= 180^\circ - 45^\circ - 45^\circ = 90^\circ$$

Thus ΔAPB is a isosceles right angled triangle

Now $AB^2 = AP^2 + BP^2 = 2AP^2$

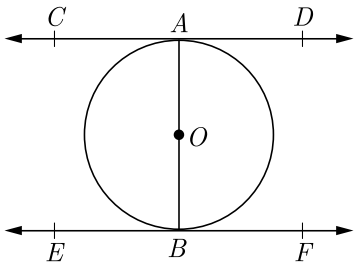
$= 2 \times 4^2 = 32$

Hence $AB = \sqrt{32} = 4\sqrt{2}$ cm

101. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Sol : [Board 2020 Delhi Basic, 2017, 2014]

Let AB be a diameter of a given circle and let CD and RF be the tangents drawn to the circle at A and B respectively as shown in figure below.



Here $AB \perp CD$ and $AB \perp EF$

Thus $\angle CAB = 90^o$ and $\angle ABF = 90^o$

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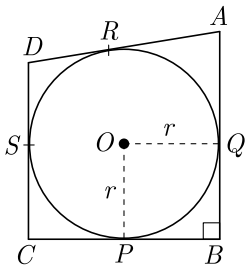
$\angle B = 90^\circ$ and $DS = 5$ cm, then find the radius of the circle (in cm).



Sol : [Board Term-2, 2013]

Since length of tangents from an external point to a circle are equal,

$$DR = DS = 5 \text{ cm}$$
$$AR = AQ$$
$$BQ = BP$$

Now $AR = AD - DR$

$= 23 - 5 = 18$ cm

$AQ = AR = 18$ cm

$QB = AB - AQ$

$= 29 - 18 = 11$ cm

$PB = QB = 11$

Now $\angle OQB = \angle OPB = 90^\circ$ because radius is always perpendicular to tangent.

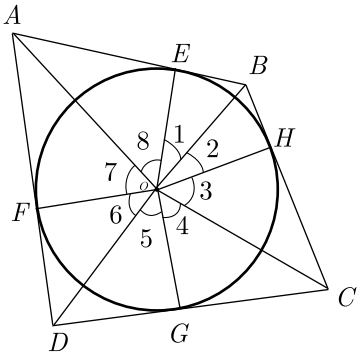
Thus $OP = OQ = PB = BQ$

So, $POQB$ is a square. Hence, $r = OP = PB = 11$ cm

111. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Sol : [Board Term-2 Foreign 2017, OD 2014]

A circle centre O is inscribed in a quadrilateral $ABCD$ as shown in figure given below.



Since OE and OF are radius of circle,

$$OE = OF$$

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\angle OEA = \angle OFA = 90^\circ$

Now in ΔAEO and ΔAFO,

$OE = OF$

$\angle OEA = \angle OFA = 90^\circ$

$OA = OA$ (Common side)

Thus $\Delta AEO \cong \Delta AFO$ (SAS congruency)

$\angle 7 = \angle 8$

Similarly, $\angle 1 = \angle 2$

$\angle 3 = \angle 4$

$\angle 5 = \angle 6$

Since angle around a point is 360°,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360^\circ$

$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180^\circ$

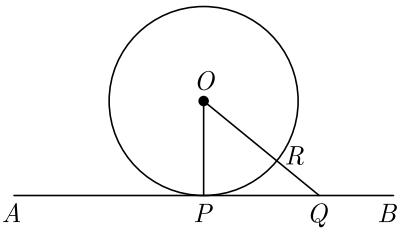
$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$

$\angle AOB + \angle COD = 180^\circ$ Hence Proved.

112. Prove that tangent drawn at any point of a circle perpendicular to the radius through the point contact.

Sol : [Board Term-2 OD 2016]

Consider a circle with centre O with tangent AB at point of contact P as shown in figure below



Let Q be point on AB and we join OQ. Suppose it touch the circle at R.

We $OP = OR$ (Radius)

Clearly $OQ > OR$

$OQ > OP$

Same will be the case with all other points on circle. Hence OP is the smallest line that connect

NODIA

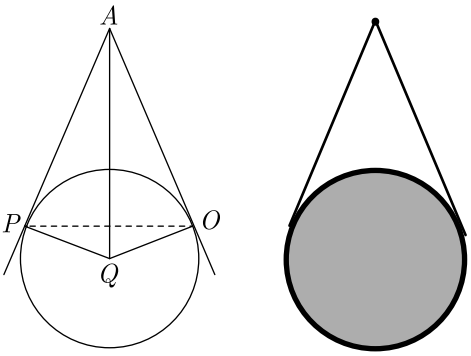
$$x = \left(\frac{\sqrt{274}-8}{2}\right)\text{m}$$

(iv) x and the radius r of circle

As $BQOR$ is square. So, $r = BR = 7 - x$]

$$\begin{aligned} r &= 7 - x \\ &= 7 - \left(\frac{\sqrt{274}-8}{2}\right) \\ &= \left(\frac{22-\sqrt{274}}{2}\right)\text{m} \end{aligned}$$

128. The picture given below shows a circular mirror hanging on the wall with a cord. The diagram represents the mirror as a circle with centre O . AP and AQ are tangents to the circle at P and Q respectively such that $AP = 30$ cm and $\angle PAQ = 60^\circ$.



Based on the above information, answer the following questions :

(i) Find the length PQ.

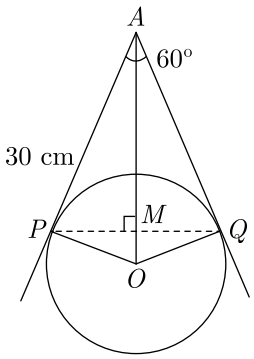
(ii) Find $\text{m}\angle POQ$.

(iii) Find the length OA.

(iv) Find the radius of the mirror.

Sol :

As per question we have redrawn the figure below.



(i) The length PQ,

Here $\angle PAQ = 60^\circ$

$$\angle PAM = 30^\circ$$

$$\text{In } \Delta PAM, \quad \sin 30^\circ = \frac{PM}{AP}$$

$$\frac{1}{2} = \frac{PM}{30}$$

$$2PM = 30 \text{ cm}$$

$$PQ = 2PM$$

$$= 30 \text{ cm}$$

(ii) Angle $\text{m}\angle POQ$

$$\angle POQ + \angle OQA + \angle QAP + \angle APO = 360^\circ$$

$$\angle POQ + 90^\circ + 60^\circ + 90^\circ = 360^\circ$$

$$\angle POQ + 240^\circ = 360^\circ$$

$$\text{m}\angle POQ = 120^\circ$$

(iii) The length OA

$$\text{In } \Delta OAP \quad \cos 30^\circ = \frac{AP}{OA}$$

$$\frac{\sqrt{3}}{2} = \frac{30}{OA}$$

$$OA = 20\sqrt{3} \text{ cm}$$

(iv) Radius of the mirror

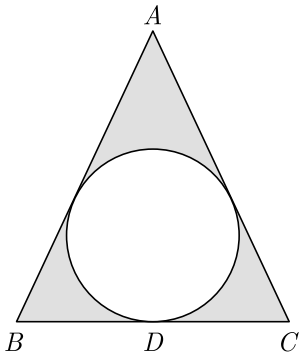
Radius or mirror $= OP$

$$\text{In } \Delta OAP \quad \tan 30^\circ = \frac{OP}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{OP}{30}$$

$$OP = 10\sqrt{3} \text{ cm}$$

129. A children's park is in the triangular shape as shown in Figure below. In the middle of the park, there is a circular region for younger children to play. It is fenced with three layers of wire. The radius of the circular region is 3 m.



Based on the above, answer the following questions:

(i) Find the perimeter (or circumference) of the circular region.

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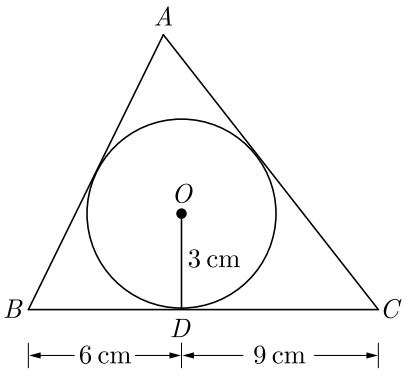
PRACTICE

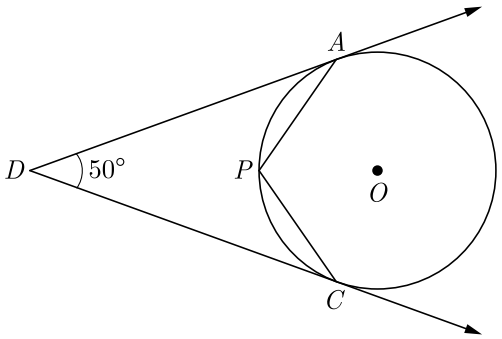
In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2, then find the lengths of sides AB and AC.



[Board Term-2 OD 2015]

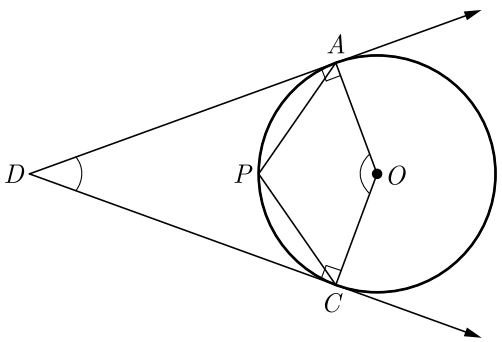
Ans : 9 and 12 cm

116. In the given figure, O is the centre of the circle. Determine $\angle APC$, if DA and DC are tangents and $\angle ADC = 50^\circ$.



Sol : [Board Term-2, 2015]

We redraw the given figure by joining A and C to O as shown below.



Since DA and DC are tangents from point D to the circle with centre O, and radius is always perpendicular to tangent, thus

$$\angle DAO = \angle DCO = 90^\circ$$

and

$$\angle ADC + \angle DAO + \angle DCO + \angle AOC = 360^\circ$$

$$50^\circ + 90^\circ + 90^\circ + \angle AOC = 360^\circ$$

$$230^\circ + \angle AOC = 360^\circ$$

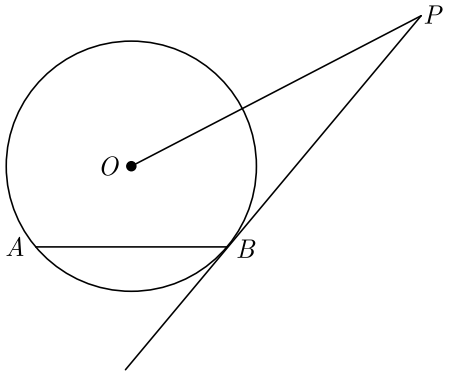
$$\angle AOC = 360^\circ - 230^\circ = 130^\circ$$

Now Reflex $\angle AOC = 360^\circ - 130^\circ = 230^\circ$

$$\angle APC = \frac{1}{2} \text{ reflex } \angle AOC$$

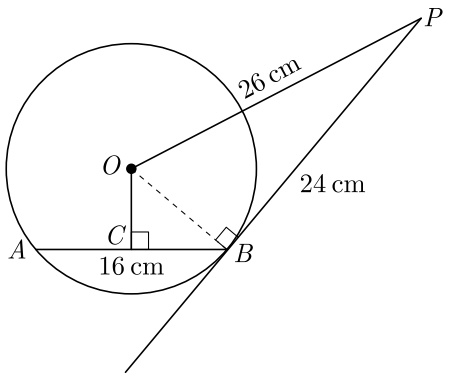
$$= \frac{1}{2} \times 230^\circ = 115^\circ$$

117. AB is a chord of circle with centre O. At B, a tangent PB is drawn such that its length is 24 cm. The distance of P from the centre is 26 cm. If the chord AB is 16 cm, find its distance from the centre.



Sol : [Board Term-2 Delhi 2014, 2012]

We redraw the given figure by joining O to B as shown below.



Here we have drawn perpendicular OC on chord AB. Thus Triangle ΔOCB is also right angled triangle, We have $PB = 24$ cm, $OP = 26$ cm.

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CHAPTER 11

Areas Related to Circles

OBJECTIVE QUESTIONS

1. The circumferences of two circles are in the ratio 4 : 5. What is the ratio of their radii ?

(a) 16 : 25 (b) 25 : 16

(c) $2 : \sqrt{5}$ (d) 4 : 5

Sol : [Board 2023 OD Standard]

Circumference of any circle is $2\pi r$.

$$\frac{2\pi r_1}{2\pi r_2} = \frac{4}{5}$$

$$\frac{r_1}{r_2} = \frac{4}{5}$$

Hence, ratio of their radii is 4 : 5.

Thus (d) is correct option.

2. The area of a circular ring formed by two concentric circles whose radii are 5.7 cm and 4.3 cm respectively is (Take $\pi = 3.1416$)

(a) 44 sq. cm. (b) 66 sq. cm.

(c) 22 sq. cm. (d) 33 sq. cm.

Sol : [Board 2022 Term-1 SQP STD]

Let the radii of the outer and inner circles be r_1 and r_2 respectively, we have

$$\text{Area} = \pi r_1^2 - \pi r_2^2 = \pi(r_1^2 - r_2^2)$$

$$= \pi (r_1 - r_2)(r_1 + r_2)$$

$$= \frac{22}{7} \times (5.7 - 4.3)(5.7 + 4.3)$$

$$= \frac{22}{7} \times 1.4 \times 10 \text{ sq. cm}$$

$$= 44 \text{ sq cm}$$

Thus (a) is correct option.

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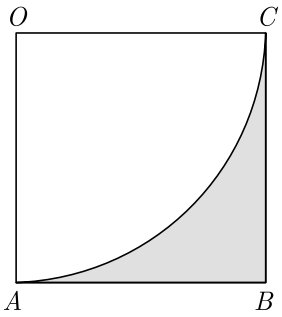
3. In the adjoining figure, $OABC$ is a square of side 7 cm. OAC is a quadrant of a circle with O as centre. The area of the shaded region is



(a) 10.5 cm^2 (b) 38.5 cm^2

(c) 49 cm^2 (d) 11.5 cm^2

Sol : [Board 2022 Term-1 SQP STD, Board Term-2 2017]

$$\text{Required area} = \left(r^2 - \frac{1}{4} \times \frac{22}{7} \times r^2\right) \text{cm}^2$$

$$= \left(7^2 - \frac{1}{4} \times \frac{22}{7} \times 7^2\right) \text{cm}^2$$

$$= (49 - 38.5) \text{cm}^2$$

Thus (a) is correct option.

4. A sector is cut from a circular sheet of radius 100 cm, the angle of the sector being 240°. If another circle of the area same as the sector is formed, then radius of the new circle is

(a) 79.5 cm (b) 81.5 cm

(c) 83.4 cm (d) 88.5 cm

Sol : [Board 2022 Term-1 SQP Basic]

$$\text{Area of sector} = \frac{240^\circ}{360^\circ} \times \pi(100)^2$$

$$= 20933 \text{ cm}^2$$

Let r be the radius of the new circle, then

$$20933 = \pi r^2$$

$$r = \sqrt{\frac{20933}{\pi}}$$

$$= 81.6 \text{ cm}$$

Thus (b) is correct option.

NODIA

$$\angle 1+\angle 2+\angle 3+\angle 4 = 180^\circ$$

From equation (1) and (2) we have

$$2\angle 2+2\angle 3 = 180^\circ$$

$$\angle 2+\angle 3 = 90^\circ$$

i.e., $\angle AOC+\angle BOC = 90^\circ$

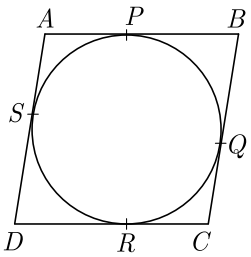
or $\angle AOB = 90^\circ$ Hence Proved

120. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol : [Board 2020 Delhi STD, 2013, 2014]

Let $ABCD$ be the parallelogram.

$$AB = CD, AD = BC \quad (1)$$



Since length of tangents from an external point to a circle are equal,

At A, $AP = AS$ (2)

At B $BP = BQ$ (3)

At C $CR = CQ$ (4)

At D $DR = DS$ (5)

Adding above 4 equation we have

$$AP+PB+CR+DR = AS+BQ+CQ+DS$$

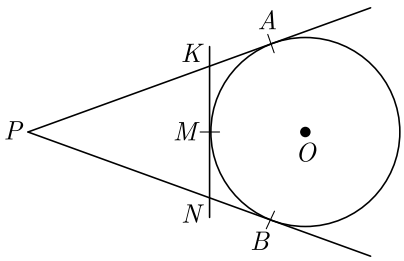
or, $AB+CD = AD+BC$

From (1) $2AB = 2AD$

or $AB = AD$

Thus $ABCD$ is a rhombus.

121. In given figure, PA and PB are tangents from a point P to the circle with centre O. At the point M, other tangent to the circle is drawn cutting PA and PB at K and N. Prove that the perimeter of $\Delta PNK = 2PB$.



Sol : [Board Term-2, 2012]

Since length of tangents from an external point to a circle are equal,

$$PA = PB$$

$$KM = KA$$

$$MN = BN$$

Now $KN = KM+MN$

$$= KA+BN$$

Now perimeter of ΔPNK

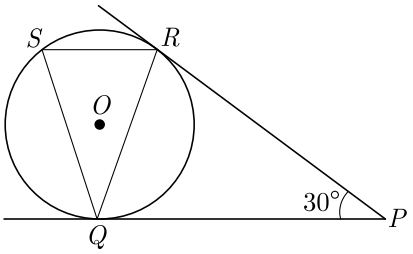
$$p = PN+KN+PK$$

$$= PN+BN+KA+PK$$

$$= PB+PA$$

$$= 2PB \quad (PA = PB)$$

122. In the figure, tangents PQ and PR are drawn from an external point P to a circle with centre O , such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.



Sol : [Board Term-2 Delhi 2015]

Since length of tangents from an external point to a circle are equal,

$$PR = PQ$$

Now $\angle PRQ = \angle PQR = \dfrac{180^\circ-30^\circ}{2}$

$$= \frac{150^\circ}{2} = 75^\circ$$

Since $SR \parallel QP$, $\angle SRQ$ and $\angle RQP$ are alternate angle,

$$\angle SRQ = \angle RQP = 75^\circ$$

Thus $SQ = RQ$

and $\angle RSQ = \angle SRQ = 75^\circ$

In triangle ΔAQR,

$$\angle SQR+\angle QSR+\angle QRS = 180^\circ$$

$$\angle SQR+75^\circ+75^\circ = 180^\circ$$

$$\angle SQR = 180^\circ-150^\circ$$

$$= 30^\circ$$

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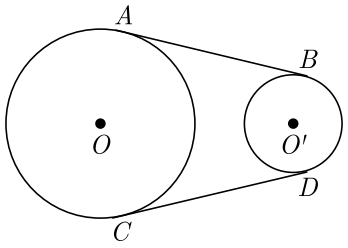
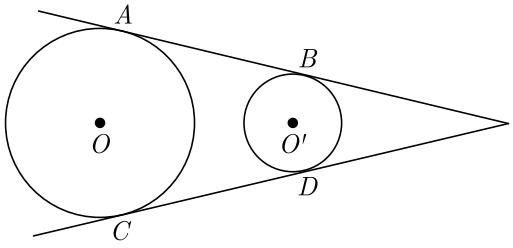
Thus $\angle SQR = 30^\circ$.

123. In the figure AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$



Sol : [Board Term-2 Delhi Compt. 2017]

We redraw the given figure by extending AB and BD which intersect at P as shown in figure below



Since length of tangents from an external point to a circle are equal,

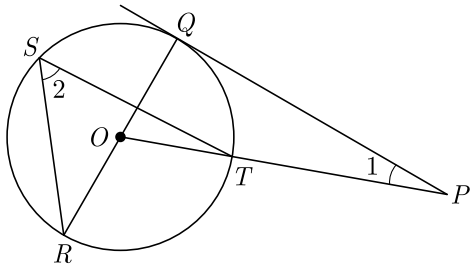
$$PA = PC$$

and $$PB = PD$$

Now, $$PA - PB = PC - PD$$

$$AB = CD$$ Hence Proved

124. In figure PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and $\angle QOR$ is a diameter. If $\angle POR = 130^\circ$ and S is a point on the circle, find $\angle 1 + \angle 2$.



Here $\angle OQP = 90^\circ$ because radius is always perpendicular to tangent at point of contact.

Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\angle 2 = \frac{1}{2}\angle TOR = \frac{1}{2}\angle POR$$

$$= \frac{1}{2} \times 130^\circ = 65^\circ$$

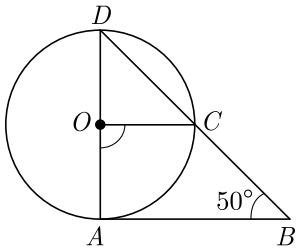
Now $$\angle POQ = 180^\circ - 130^\circ = 50^\circ$$

$$\angle 1 = 180^\circ - \angle OQP - \angle POQ$$

$$= 180^\circ - 90^\circ - 50^\circ = 40^\circ$$

Now $$\angle 2 + \angle 1 = 65^\circ + 40^\circ = 105^\circ$$

125. In the given figure, AD is a diameter of a circle with centre O and AB is a tangent at A. C is a point on the circle such that DC produced intersects the tangent at B and $\angle ABC = 50^\circ$. Find $\angle AOC$.



Sol : [Board Term-2 2015]

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Therefore $$\angle A = 90^\circ$$

Now in ΔDAB we have

$$\angle D + \angle A + \angle B = 180^\circ$$

$$\angle D + 90^\circ + 50^\circ = 180^\circ$$

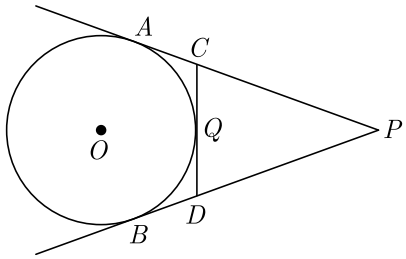
$$\angle D = 40^\circ$$

Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\angle AOC = 2\angle ADC = 2\angle D$$

$$= 2 \times 40^\circ = 80^\circ$$

126. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If $PA = 12$ cm, $QC = QD = 3$ cm, then find $PC + PD$.



NODIA

Sol : [Board Term-2 Delhi Compt. 2017]

Since length of tangents from an external point to a circle are equal,

$$CA = CQ = 3 \text{ cm}$$

$$DQ = DB = 3 \text{ cm}$$

and $$PB = PA = 12 \text{ cm}$$

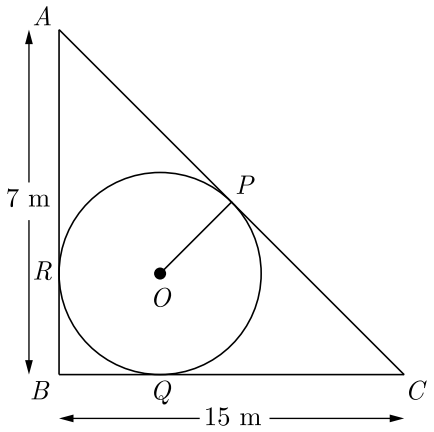
$$PA + PB = PC + CA + PD + DB$$

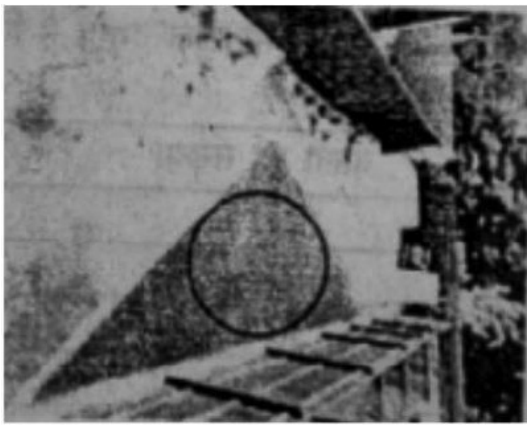
$$PC + PD = PA - CA + PB - DB$$

$$= 12 - 3 + 12 - 3 = 18 \text{ cm}$$

COMPETENCEY BASED QUESTIONS

121. A backyard is in the shape of a triangle ABC with right angle at B. $AB = 7$ m and $BC = 15$ m. A circular pit was dug inside it such that it touches the walls AC, BC and AB at P, Q and R respectively such that $AP = x$ m.





Based on the above information, answer the following questions:

(i) Find the length of AR in terms of x.

(ii) Write the type of quadrilateral $BQOR$.

(iii) Find the length PC in terms of x and hence find the value of x.

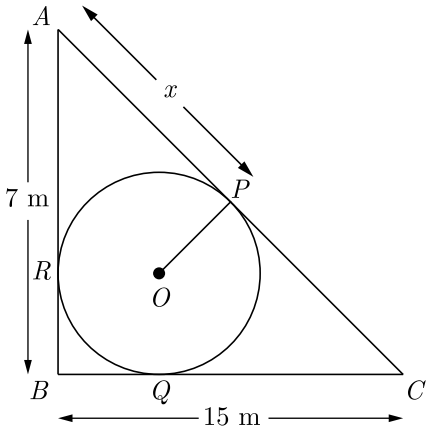
(iv) Find x and hence find the radius r of circle.

Sol : [Board 2024 OD Standard]

(i) Length of AR in terms of x.

Length of tangents from an external point to a circle are equal. Thus

$$AR = AP = x \text{ m}$$



(ii) Type of quadrilateral $BQOR$

Since Radius $\perp$ Tangent, we have $OR \perp AB$ and $OQ \perp BC$. Thus

$$\angle ORB = \angle OQB = \angle RBQ = 90^\circ$$

By angle sum property

$$\angle QOR = 360^\circ - 90^\circ - 90^\circ - 90^\circ = 90^\circ$$

Also, $OR = OQ$ Radii of circle

Thus $BQOR$ is a square

(iii) Length PC in terms of x

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{7^2 + 15^2}$$

$$= \sqrt{274} \text{ units}$$

$$PC = AC - AP$$

$$PC = (\sqrt{274} - x) \text{ units}$$

Now $$AR = AP = x$$

$$BR = BQ = 7 - x$$

$$CQ = PC = 15 - (7 - x)$$

$$= x + 8$$

$$AC = x + x + 8$$

$$= 2x + 8$$

$$\sqrt{274} = 2x + 8$$

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AB and smallest line is perpendicular.

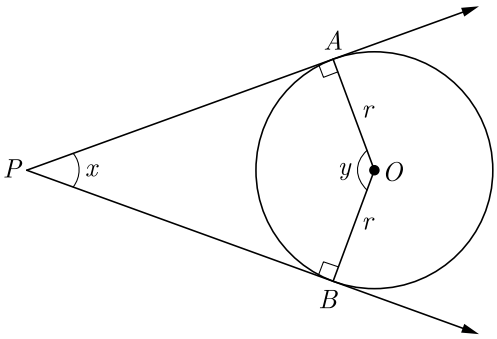
Thus $OP \perp AB$

or, $OP \perp PQ$ Hence Proved

113. Two tangents PA and PB are drawn from an external point P to a circle with centre O, such that $\angle APB = \angle x$ and $\angle AOB = y$. Prove that opposite angles are supplementary.

Sol : [Board Term-2, 2011]

As per question we draw figure shown below.



Now $OA \perp AP$ and $OB \perp BP$ because tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\angle A = \angle B = 90^\circ$

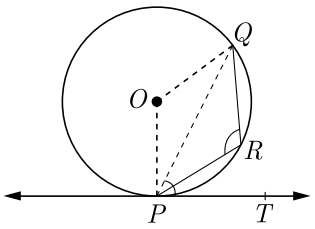
Since, $AOBP$ is a quadrilateral,

$$\angle A + \angle B + x + y = 360^\circ$$
$$90^\circ + 90^\circ + x + y = 360^\circ$$
$$180 + x + y = 360^\circ$$
$$x + y = 180^\circ$$

Therefore opposite angle are supplementary.

114. In figure, PQ is a chord of a circle O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$.

Sol : [Board Term-2 OD 2015, 2017]



We have $\angle QPT = 60^\circ$

Here $\angle OPT = 90^\circ$ because of tangent at radius.

Now $\angle OPQ = \angle OQP$

$= \angle OPT - \angle QTP$

$= 90^\circ - 60^\circ = 30^\circ$

$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$

$= 180^\circ - (30^\circ + 30^\circ)$

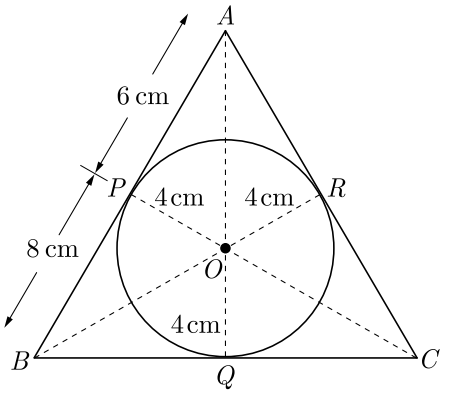
$= 180^\circ - 60^\circ = 120^\circ$

Now Reflex $\angle POQ = 360^\circ - 120^\circ = 240^\circ$

$\angle PRQ = \frac{1}{2}$ Reflex $\angle POQ$

$= \frac{1}{2} \times 240^\circ = 120^\circ$

115. In Figure the radius of incircle of ΔABC of area $84\,\text{cm}^2$ and the lengths of the segments AP and BP into which side AB is divided by the point of contact are 6 cm and 8 cm Find the lengths of the sides AC and BC.



Sol : [Board Term-2 Delhi 2012, 2014, OD Compt. 2017]

Since length of tangents from an external point to a circle are equal,

At A, $AP = AR = 6$ cm (1)

At B, $,BP = BQ = 8$ cm (2)

At C, $CR = CQ = x$ (3)

Perimeter of ΔABC,

$p = AP + PB + BQ + QC + CR + RA$

$= 6 + 8 + 8 + x + x + 6 = 28 + 2x$

Now area $\Delta ABC = \frac{1}{2}rp$

Here $r = 4$ is the radius of circle. Substituting all values we have

$84 = \frac{1}{2} \times 4 \times (28 + 2x)$

$84 = 56 + 4x$

$21 = 14 + x \Rightarrow x = 7$

Thus $AC = AR + RC = 6 + 7 = 13$ cm

$BC = BQ + QC = 8 + 7 = 15$ cm

NODIA

(ii) What is the total length of wire used?

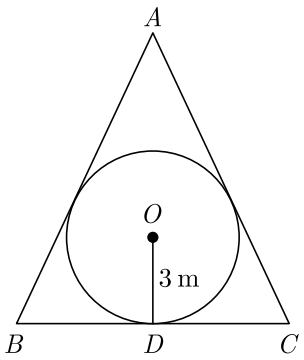
(iii) What is the area of the circular region?

(iv) If $BD = 6\text{ m}$, $DC = 9\text{ m}$ and $ar(\Delta ABC) = 54\text{ m}^2$, then find the length of sides AB and AC, respectively.

(v) Find the perimeter of ΔABC.

Sol : [Board 2021 Comp. Basic]

We have redrawn the figure as shown below.



(i) Perimeter of circular region $= 2\pi r$

$= 2\pi \times 3$

$= 6\pi\text{ m}$

(ii)Total length of wire$= 3 \times$ Circumference of circle

$= 3 \times 6\pi$

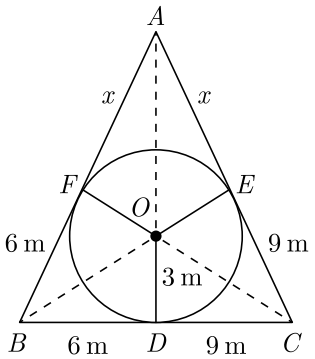
$= 18\pi\text{ m}$

(iii) Area of circular region, $A = \pi r^2$

$= \pi(3)^2$

$= 9\pi\text{ m}^2$

(iv) Now we have redrawn the figure as shown below.



Since tangents drawn from external points are of equal length. So,

$BF = BD = 6\text{ m}$

$CE = CD = 9\text{ m}$

Let $AF = AE = x\text{ m}$

Now,

$\text{ar}(\Delta ABC) = \text{ar}(\Delta AOB) + \text{ar}(\Delta BOC) + \text{ar}(\Delta AOC)$

$54 = \frac{1}{2}AB \times OF + \frac{1}{2}BC \times OD + \frac{1}{2}AC \times OE$

$54 = \frac{1}{2}(6+x) \times 3 + \frac{1}{2}(6+9) \times 3 + \frac{1}{2}(9+x) \times 3$

$54 = \frac{3}{2}[6+x+15+9+x]$

$36 = 2x + 30 \rightarrow x = 3$

So, $AB = x + 6 = 9\text{ m}$

and $AC = x + 9 = 12\text{ m}$

(v) Perimeter of ΔABC $s = AB + BC + AC$

$= 9 + 15 + 12$

$= 36\text{ m}$

****

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Triangle ΔOPB is right angled triangle because PB is tangent at radius OB and $\angle OPB = 90^\circ$.

In right angled ΔOPB, we have

$$OB = \sqrt{OP^2 - BP^2}$$
$$= \sqrt{26^2 - 24^2}$$
$$= \sqrt{676 - 576} = \sqrt{100} = 10 \text{ cm}$$

Since perpendicular drawn from the centre to a chord bisect it, we have

$$BC = \frac{1}{2}AB = \frac{16}{2} = 8 \text{ cm}$$

Now in ΔOBC, $OC^2 = OB^2 - BC^2$

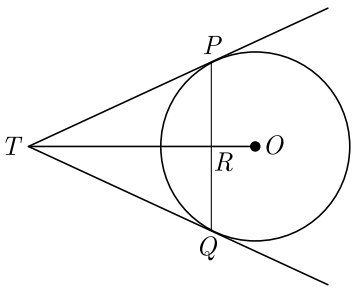
$$= 10^2 - 8^2 = 36$$
$$OC = 6 \text{ cm}$$

Thus distance of the chord from the centre is 6 cm.

118. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

Sol : [Board Term-2 Delhi 2015]

A circle with centre O. Tangents TP and TQ are drawn from a point T outside a circle as shown in figure below.



Since length of tangents from an external point to a circle are equal,

$$TP = TQ$$

Angle $\angle TPR$ and $\angle TQR$ are opposite angle of equal sides, thus

$$\angle TPR = \angle TQR$$

Now in ΔPTR and ΔQTR

$$TP = TQ$$
$$TR = TR \quad \text{(Common)}$$
$$\angle TPR = \angle TQR$$

Thus $\Delta PTR \cong \Delta QTR$

and $PR = QR$

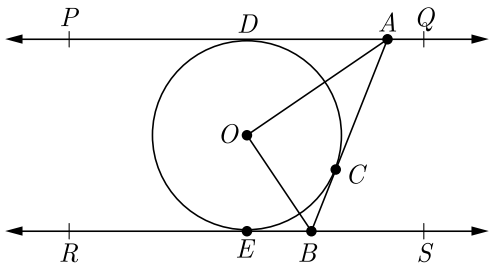
and $\angle PRT = \angle QRT$

But $\angle PRT + QRT = 180^\circ$ as PQ is line segment,

$$\angle PRT = \angle QRT = 90^\circ$$

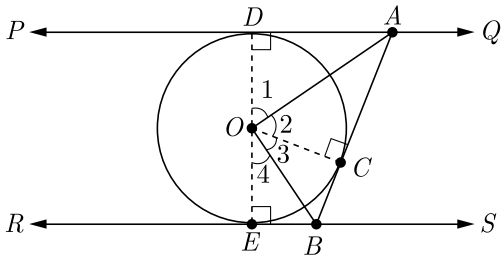
Therefore TR or OT is the right bisector of line segment PQ. Hence proved.

119. In Figure, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that $\angle AOB = 90^\circ$.



Sol : [Board 2019 OD STD, 2014, 2012]

We redraw the given figure as shown below.



In ΔDOA and ΔCOA, DA and AC are tangents drawn from common point,

Thus $DA = AC$

Due to angle between tangent and radius,

$$\angle ODA = \angle OCA = 90^\circ$$

Due to radius of circle,

$$OD = OC$$

By SAS symmetry we have

$$\Delta DOA \cong \Delta COA$$

Hence, by CPCT, $\angle 1 = \angle 2$

i.e., $\angle DOA = \angle COA$ ...(1)

Similarly, by SAS

$$\Delta BOC = \Delta BOE$$

and by CPCT $\angle 3 = \angle 4$

i.e., $\angle COB = \angle BOE$ ...(2)

Now, angles on a straight line,

5. If a circular grass lawn of 35 m in radius has a path 7 m wide running around it on the outside, then the area of the path is

(a) $1450\,m^2$ (b) $1576\,m^2$

(c) $1694\,m^2$ (d) $3368\,m^2$

Sol : [Board 2022 Term-1 SQP Basic]

Radius of outer concentric circle,

$$= (35+7)\,m = 42\,m.$$

Area of path $= \pi(42^2 - 35^2)\,m^2$

$$= \frac{22}{7}(42^2 - 35^2)\,m^2$$

Thus (c) is correct option.

6. If the area of a semi-circular field is 15400 sq m, then perimeter of the field is

(a) $160\sqrt{2}\,m$ (b) $260\sqrt{2}\,m$

(c) $360\sqrt{2}\,m$ (d) $460\sqrt{2}\,m$

Sol : [Board 2022 Term-1 STD]

Let the radius of the field be r.

Then, $\frac{\pi r^2}{2} = 15400$

$$\frac{1}{2} \times \frac{22}{7} \times r^2 = 15400$$

$$r^2 = 15400 \times 2 \times \frac{7}{22} = 9800$$

$$r = 70\sqrt{2}\,m$$

Thus, perimeter of the field

$$\pi r + 2r = \frac{22}{7} \times 70\sqrt{2} + 2 \times 70\sqrt{2}$$

$$= 220\sqrt{2} + 140\sqrt{2}$$

$$= \sqrt{2}(220+140)$$

$$= 360\sqrt{2}\,m$$

Thus (c) is correct option.

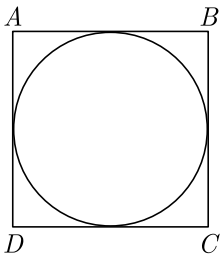
7. The area of the circle that can be inscribed in a square of side 6 cm is

(a) $36\pi\,cm^2$ (b) $18\pi\,cm^2$

(c) $12\pi\,cm^2$ (d) $9\pi\,cm^2$

Sol : [Board 2022 Term-1 STD]

Given, side of square $= 6\,cm$



Diameter of circle is equal to the side of square.

Diameter of a circle, $d = = 6\,cm$

Radius of a circle, $r = \frac{d}{2} = \frac{6}{2} = 3\,cm$

Area of circle, $\pi r^2 = \pi 3^2 = 9\pi\,cm^2$

Thus (d) is correct option.

8. The sum of the areas of two circle, which touch each other externally, is $153\,\pi$. If the sum of their radii is 15, then the ratio of the larger to the smaller radius is

(a) 4 : 1 (b) 2 : 1

(c) 3 : 1 (d) None of these

Sol : [Board 2022 Term-1 Basic]

Let the radii of the two circles be r_1 and r_2, then

$$r_1 + r_2 = 15 \quad ...(1)$$

and $$\pi r_1^2 + \pi r_2^2 = 153\,\pi \quad ...(2)$$

$$r_1^2 + r_2^2 = 153$$

$$r_1^2 + (15 - r_1)^2 = 153$$

$$r_1^2 + 225 - 30r_1 + r_1^2 = 153$$

$$2r_1^2 - 30r_1 + 72 = 0$$

$$r_1^2 - 15r_1 + 36 = 0$$

Solving, we get $r_1 = 12$ and $r_2 = 3$.

Thus required ratio is 12 : 3 or 4 : 1.

Thus (a) is correct option.

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9. A race track is in the form of a ring whose inner and outer circumference are 437 m and 503 m respectively. The area of the track is

(a) 66 sq. cm. (b) 4935 sq. cm.

(c) 9870 sq. cm (d) None of these

Sol : [Board 2022 Term-1 Basic]

We have $2\pi r_1 = 503 \Rightarrow r_1 = \frac{503}{2\pi}$

and $2\pi r_2 = 437 \Rightarrow r_2 = \frac{437}{2\pi}$

Area of ring

$$\pi(r_1^2 - r_2^2) = \pi(r_1 + r_2)(r_1 - r_2)$$

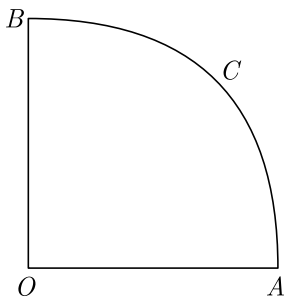
$$= \pi\left(\frac{503+437}{2\pi}\right)\left(\frac{503-437}{2\pi}\right)$$

$$= \frac{940}{2}\left(\frac{66}{2\pi}\right) = \frac{940}{2} \times \frac{66}{2} \times \frac{7}{22}$$

$= 235 \times 21 = 4935$ sq. cm.

Thus (b) is correct option.

10. In the given figure, $OACB$ is a quadrant of a circle of radius 7 cm. The perimeter of the quadrant is



(a) 11 cm (b) 18 cm

(c) 25 cm (d) 36 cm

Sol : [Board Term-2 2017]

$$\text{Perimeter} = \frac{1}{4} \times 2\pi r + 2r$$

$$= \left(\frac{1}{2} \times \frac{22}{7} \times 7 + 2 \times 7\right) \text{cm}$$

$$= 25 \text{ cm}$$

Thus (c) is correct option.

11. If the circumference of a circle increases from 4π to 8π, then its area is

(a) halved (b) doubled

(c) tripled (d) quadrupled

Sol : [Board Term-2 2017]

$$2\pi r = 4\pi \Rightarrow r = 2$$

$$\text{Area} = \pi(2)^2 = 4\pi$$

When, $2\pi r = 8\pi \Rightarrow r = 4$

$$\text{Area} = 16\pi$$

Thus area is quadrupled.

Thus (d) is correct option.

12. If the radius of a circle is diminished by 10%, then its area is diminished by

(a) 10% (b) 19%

(c) 36% (d) 20%

Sol : [Board Term-2 2013]

Let r be the radius of circle, then area $= \pi r^2$

When r is diminished by 10%

Then, $\text{area} = \pi\left(r - \frac{r}{10}\right)^2 = \pi r^2\left(\frac{81}{100}\right)$

Thus area is diminished by

$$\left(1 - \frac{81}{100}\right)\% = 19\%$$

Thus (b) is correct option.

13. If the perimeter of a semi-circular protractor is 36 cm, then its diameter is

(a) 10 cm (b) 14 cm

(c) 12 cm (d) 16 cm

Sol : [Board 2011]

$$\text{Perimeter} = \frac{2\pi r}{2} + 2r = \pi r + 2r$$

$$(\pi + 2)r = 36$$

$$\left(\frac{36}{7}\right) - r = 36 \Rightarrow r = 7 \text{ cm}$$

Hence, diameter $2r = 7 \times 2 = 14$ cm

Thus (b) is correct option.

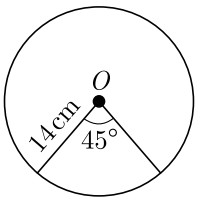
14. In a circle of radius 14 cm, an arc subtends an angle of 45° at the centre, then the area of the sector is

(a) 71 cm^2 (b) 76 cm^2

(c) 77 cm^2 (d) 154 cm^2

Sol : [Board Term-2 2016]

Given, $r = 14$ cm and $\theta = 45^\circ$



$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{1}{8} \times 22 \times 2 \times 14$$

$$= 77 \text{ cm}^2$$

Thus (c) is correct option.

15. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then

(a) $R_1 + R_2 = R$ (b) $R_1^2 + R_2^2 = R^2$

(c) $R_1 + R_2 < R$ (d) $R_1^2 + R_2^2 < R^2$

Sol : [Board 2009]

According to the given condition,

Area of circle = Area of first circle

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$+$ Area of second circle

$$\pi R^2 = \pi R_1^2 + \pi R_2^2$$
$$R^2 = R_1^2 + R_2^2$$

Thus (b) is correct option.

16. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then

(a) $R_1 + R_2 = R$ (b) $R_1 + R_2 > R$

(c) $R_1 + R_2 > R$ (d) $R_1 + R_2 < R$

Sol : [Board Term-2 2014]

According to the given condition,

$$2\pi R = 2\pi R_1 + 2\pi R_2$$
$$R = R_1 + R_2$$

Thus (a) is correct option.

17. If the circumference of a circle and the perimeter of a square are equal, then

(a) Area of the circle = Area of the square

(b) Area of the circle > Area of the square

(c) Area of the circle < Area of the square

(d) Nothing definite can be said about the relation between the areas of the circle and square

Sol : [Board 2008]

Let r and a be the radius of circle and side of square respectively.

$$2\pi r = 4a$$
$$\frac{22}{7} r = 2a$$
$$11\, r = 7a$$
$$r = \frac{7a}{11} \quad ...(1)$$

Now, area of circle, $A_1 = \pi r^2$

From equation (1), we get

$$A_1 = \pi\left(\frac{7a}{11}\right)^2 = \frac{22}{7} \times \frac{49a^2}{121}$$
$$A_1 = \frac{14a^2}{11} \quad ...(2)$$

and area of square, $A_2 = (a)^2$ ...(3)

From equations (2) and (3),

$$A_1 = \frac{14}{11} A_2$$
$$A_1 > A_2$$

Hence, Area of the circle >Area of the square.

Thus (b) is correct option.

18. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(a) 22:7 (b) 14:11

(c) 7:22 (d) 11:14

Sol : [Board 2010]

Let radius of circle be r and side of a square be a.

According to the given condition,

Perimeter of a circle = Perimeter of a square

$$2\pi r = 4a$$
$$a = \frac{\pi r}{2} \quad ...(1)$$

Now, $\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{(a)^2} = \frac{\pi r^2}{\left(\pi \frac{r}{2}\right)^2}$ [from Eq. (1)]

$$= \frac{\pi r^2}{\pi^2 \frac{r^2}{4}} = \frac{4}{\pi} = \frac{4}{\frac{22}{7}}$$
$$= \frac{28}{22} = \frac{14}{11}$$

Hence, the required ratio is 14:11.

Thus (b) is correct option.

19. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be

(a) 10 m (b) 15 m

(c) 20 m (d) 24 m

Sol : [Board Term-2 2015]

We have $\pi R^2 = \pi r_1^2 + \pi r_2^2$

$$R^2 = r_1^2 + r_2^2$$
$$= \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$$
$$= \left(\frac{16}{2}\right)^2 + \left(\frac{12}{2}\right)^2$$
$$= (8)^2 + (6)^2 = 100$$
$$R = \sqrt{100} = 10$$

Thus (a) is correct option.

20. The area of the square that can be inscribed in a circle of radius 8 cm is

(a) 256 cm^2 (b) 128 cm^2

(c) $64\sqrt{2}$ cm^2 (d) 64 cm^2

Sol : [Board Term-2 2014]

Radius of circle, $OC = 8\,\text{cm}$,

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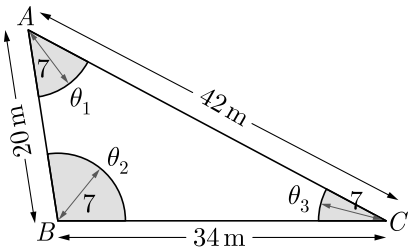
Hence, the area of the road is $770\ \text{m}^2$.

47. Three horses are tied each with 7 m long rope at three corners of a triangular field having sides 20 m, 34 m and 42 m. Find the area of the plot which can be grazed by the horses.

Sol : [Board 2020 Delhi Basic]

As per information given in question we have drawn the figure below.



Let $\angle A = \theta_1$, $\angle B = \theta_2$ and $\angle C = \theta_3$.

Now, area which can be grazed by the horses is the sum of the areas of three sectors with central angles θ_1, θ_2 and θ_3 each with radius $r = 7$ m.

$$\frac{\pi r^2\theta_1}{360^\circ} + \frac{\pi r^2\theta_2}{360^\circ} + \frac{\pi r^2\theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}(\theta_1 + \theta_2 + \theta_3) \quad ...(1)$$

From angle sum property of a triangle we have

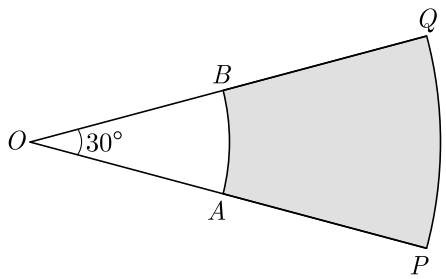
$$\theta_1 + \theta_2 + \theta_3 = 180^\circ$$

Substituting above in equation (1) we have

$$\begin{aligned}\frac{\pi r^2\theta_1}{360^\circ} + \frac{\pi r^2\theta_2}{360^\circ} + \frac{\pi r^2\theta_3}{360^\circ} &= \frac{\pi r^2}{360^\circ} \times 180^\circ \\ &= \frac{\pi r^2}{2} \\ &= \frac{22}{7} \times \frac{1}{2} \times (7)^2 \\ &= \frac{22}{7} \times \frac{1}{2} \times 7 \times 7 \\ &= 77\ \text{m}^2\end{aligned}$$

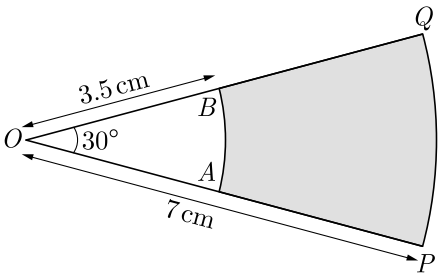
Hence, the area grazed by the horses is $77\ \text{m}^2$

48. In Figure, PQ and AB are two arcs of concentric circles of radii 7 cm and 3.5 cm respectively, with centre O. If $\angle POQ = 30^\circ$, then find the area of shaded region.



Sol : [Board 2020 OD Basic]

We redraw the given figure as below.



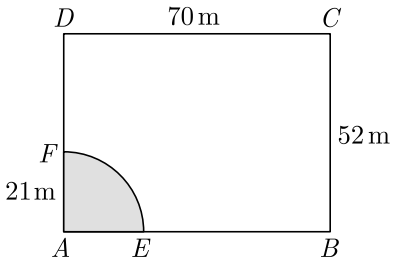
Area of shaded region

$$\begin{aligned}\pi[R^2 - r^2]\frac{\theta}{360^\circ} &= \frac{22}{7}[7^2 - (3.5)^2]\frac{30^\circ}{360^\circ} \\ &= \frac{22}{7}(7 + 3.5)(7 - 3.5) \times \frac{1}{12} \\ &= \frac{22}{7} \times 10.5 \times 3.5 \times \frac{1}{12} \\ &= 9.625\ \text{cm}^2\end{aligned}$$

49. A horse is tethered to one corner of a rectangular field of dimensions $70\ \text{m} \times 52\ \text{m}$, by a rope of length 21 m. How much area of the field can it graze?

Sol : [Board 2020 OD Basic]

As per information given in question we have drawn the figure below.



Length of the rope is 21 cm.

Shaded portion $AEFA$ indicates the area in which the horse can graze. Clearly it is the area of a quadrant of a circle of radius, $r = 21$ m.

Area of quadrant,

$$\begin{aligned}\frac{1}{4}\pi r^2 &= \frac{1}{4} \times \frac{22}{7} \times (21)^2 \\ &= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21 \\ &= 346.5\ \text{m}^2\end{aligned}$$

Hence, the graze area is $346.5\ \text{m}^2$

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Sol :

We have $C = 2 \times \frac{22}{7} \times r = 176$

$$r = \frac{176 \times 7}{2 \times 22} = 28 \text{ cm}$$

Both assertion and reason are correct. Also Reason is the correct explanation of the assertion.

Thus (a) is correct option.

25. **Assertion :** If the outer and inner diameter of a circular path is 10 m and 6 m then area of the path is $16\pi\, m^2$.

Reason : If R and r be the radius of outer and inner circular path, then area of path is $\pi(R^2 - r^2)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

$$\text{Area of the path} = \pi\left[\left(\frac{10}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right]$$
$$= \pi(25 - 9) = 16\,\pi$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

26. **Assertion :** In a circle of radius 6 cm, the angle of a sector 60°. Then the area of the sector is $18\frac{6}{7}$ cm^2.

Reason : Area of the circle with radius r is πr^2.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

$$\text{Area of the sector} = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6$$
$$= \frac{132}{7} = 18\frac{6}{7}\text{cm}^2.$$

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

ONE MARK QUESTIONS

27. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.

Sol : [Board 2020 Delhi Standard]

Angle subtended in 1 minutes $= 6^\circ$

Angle subtended in 35 minutes $= 6^\circ \times 35 = 210^\circ$

Area of the face of the clock by the minute hand, i.e. area of sector,

$$\frac{\pi r^2 \theta}{360^\circ} = \frac{22}{7} \times \frac{12 \times 12 \times 210^\circ}{360^\circ}$$
$$= \frac{22}{7} \times \frac{12 \times 12 \times 7 \times 30^\circ}{12 \times 30^\circ}$$
$$= 22 \times 12 = 264 \text{ cm}^2$$

PRACTICE

The length of the minute hand of clock is 14 cm. Find the area swept by the minute hand in 15 minutes.

[Board 2020 OD Basic]

Ans : 154 cm^2

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand from 9 a.m. to 9.35 a.m.

[Board Term-2 2012]

Ans : 259.33 cm2

28. The radius of a circle is 17.5 cm. find the area of the sector of the circle enclosed by two radii and an arc 44 cm in length.

Sol : [Board 2020 OD Basic]

Given, arc length $= 44$ cm

Radius of circle, $r = 17.5$ cm

So, $$\text{area of sector} = \frac{\text{arc length}}{2\pi r} \times \pi r^2$$
$$= \frac{\text{arc length} \times r}{2}$$
$$= \frac{44 \times 17.5}{2}$$
$$= 22 \times 17.5 = 385 \text{ sq. cm.}$$

NODIA

$$\text{Area}_{AFEA} = \frac{\theta}{360^\circ} \times \pi r^2$$
$$= \frac{60^\circ}{360^\circ} \times \pi (5)^2 = \frac{25}{6}\pi \text{ cm}^2$$

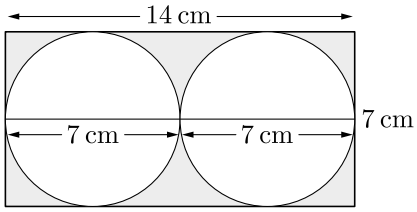
Thus total area of shaded region

$$\text{Area} = 3\left(\frac{25}{6}\pi\right) = \frac{25 \times 3.14}{2}$$
$$= 39.25 \text{ cm}^2$$

39. Two circular pieces of equal radii and maximum areas, touching each other are cut out from a rectangular cardboard of dimensions 14 cm $\times$ 7 cm. find the area of the remaining cardboard. (Use $\pi = \frac{22}{7}$)

Sol : [Board Term-2 Delhi 2013]

As per question the digram is shown below.



Area of the remaining cardboard

= Area of rectangular cardboard $-2 \times$ Area of circle

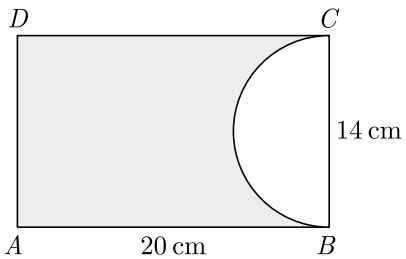
$$= 14 \times 7 - 2\pi\left(\frac{7}{2}\right)^2$$
$$= 14 \times 7 - 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$
$$= 98 - \frac{44}{7} \times \frac{49}{4} = 98 - 77 = 21$$

Hence, area of remaining card board is 21 cm^2

40. A paper is in the form of a rectangle $ABCD$ in which $AB = 20$ cm, $BC = 14$ cm. A semi-circular portion with BC as diameter is cut off. Find the area of the part. Use $\pi = \frac{22}{7}$.

Sol : [Board Term-2 2012, Foreign 2014]

As per question the digram is shown below.



Area of remaining part,

= Area of rectangle $-$ Area of semi-circle

$$= 20 \times 14 - \frac{1}{2}\pi 7^2$$
$$= 280 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$
$$= 280 - 77 = 203 \text{ cm}$$

Hence, area of remaining part is 203 cm.

41. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, find the circumference (in cm) of the circle.

Sol : [Board Term-2 Delhi 2012]

Let r be the radius of the circle.

Now, circumference $-$ radius $= 37$

$$2\pi r - r = 37$$
$$2 \times \frac{22}{7} r - r = 37$$
$$r\left(\frac{22-7}{7}\right) = 37$$
$$r \times \frac{37}{7} = 37$$
$$r = \frac{37 \times 7}{37} = 7 \text{ cm}$$

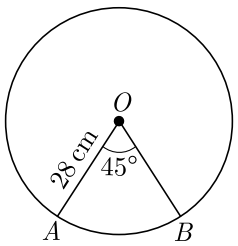
Circumference of the circle,

$$2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm.}$$

42. Find the area of the corresponding major sector of a circle of radius 28 cm and the central angle 45°.

Sol : [Board Term-2 2015]

As per question statement figure is shown below;



Area of major sector,

= area of circle $-$ area of minor sector

$$= \pi r^2\left(1 - \frac{\theta}{360^\circ}\right)$$
$$= \frac{22}{7} \times 28 \times 28\left(1 - \frac{45^\circ}{360^\circ}\right)$$
$$= \frac{22}{7} \times 28 \times 28 \times \frac{7}{8}$$
$$= 2156 \text{ cm}^2$$

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Radius of circle $r = 10$ cm, central angle $= 90^\circ$

Area of minor segment,

$$= \frac{1}{2} \times 10^2 \times \left[\frac{3.14 \times 90}{180} - \sin 90^\circ\right]$$

$$= \frac{1}{2} \times 100 \times [1.57 - 1] = 28.5 \text{ cm}^2$$

TWO MARKS QUESTIONS

36. The perimeter of a sector of a circle with radius 6.5 cm is 31 cm, then find the area of the sector.

Sol : [Board 2020 Delhi Basic]

Given, Radius $= 6.5$ cm

Let O be the centre of a circle with radius 6.5 cm and

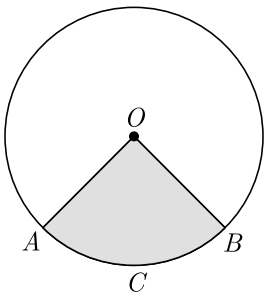
$OACBO$ be its sector with perimeter 31 cm.

Thus, we have

$$OA + OB + \overset{\frown}{ACB} = 31 \text{ cm}$$

$$6.5 + 6.5 + \overset{\frown}{ACB} = 31 \text{ cm}$$

$$\overset{\frown}{ACB} = 18 \text{ cm}$$



Now, area of sector $OACBO$

$$= \frac{1}{2} \times \text{radius} \times \overset{\frown}{ACB}$$

$$= \frac{1}{2} \times 6.5 \times 18 = 58.5 \text{ cm}^2$$

PRACTICE

The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

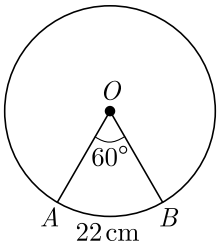
[Board 2020 Delhi Standard]

Ans : 15.6 cm^2

37. A piece of wire 22 cm long is bent into the form an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. [Use $\pi = \frac{22}{7}$]

Sol : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



NODIA

Here AB is an arc of a circle of radius r.

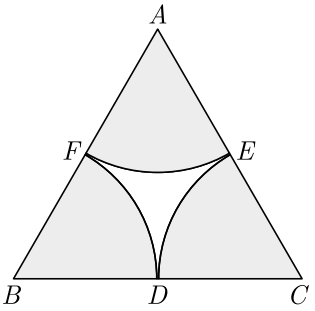
$$\text{Length of arc} = \frac{2\pi r\theta}{360^\circ}$$

$$22 = \frac{2 \times 22 \times r \times 60^\circ}{7 \times 360^\circ}$$

$$22 = \frac{22 \times r}{21} \Rightarrow r = 21$$

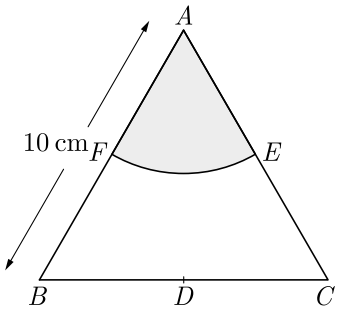
Hence, the radius of the circle is 21 cm.

38. In fig. arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm, to intersect the side BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region. (Use $\pi = 3.14$).



Sol : [Board Term-2 2011]

Figure given below shows the single sector.



Since ΔABC is an equilateral triangle

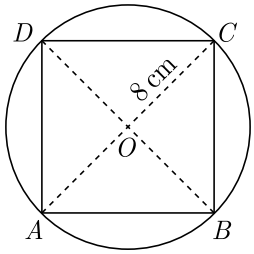
$$\angle A = \angle B = \angle C = 60^\circ$$

Here we have 3 sector and area of all three sector is equal.

Area of sector $AFEA$,

Diameter of the circle, $AC = 2 \times OC$

$= 2 \times 8 = 16$ cm



which is equal to the diagonal of a square.

Let side of square be x.

In right angled ΔABC,

$$AC^2 = AB^2 + BC^2$$
$$(16)^2 = x^2 + x^2$$
$$256 = 2x^2$$
$$x^2 = 128$$

Area of square, $x^2 = 128\ \text{cm}^2$

Alternate Method :

Radius of circle, $r = 8$ cm

Diameter of circle, $d = 2r = 2 \times 8 = 16$ cm

Since, square inscribed in circle.

Diagonal of square = Diameter of circle

Now, Area of square $= \dfrac{(\text{Diagonal})^2}{2}$

$= \dfrac{(16)^2}{2} = \dfrac{256}{2}$

$= 128\ \text{cm}^2$

Thus (b) is correct option.

21. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is

(a) 56 cm (b) 42 cm

(c) 28 cm (d) 16 cm

Sol : [Board Term-2 2013, OD 2015]

We have $2\pi r = 2\pi r_1 + 2\pi r_2$

$2\pi r = \pi d_1 + \pi d_2$

$2r = d_1 + d_2 = 36 + 20$

$2r = 56 \Rightarrow r = 28$ cm

Thus (c) is correct option.

22. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is

(a) 31 cm (b) 25 cm

(c) 62 cm (d) 50 cm

Sol : [Board Term-2 2012, Foreign 2014]

We have $\pi R^2 = \pi r_1^2 + \pi r_2^2$

$R^2 = r_1^2 + r_2^2$

$= 24^2 + 7^2 = 625$

$R = \sqrt{625} = 25$ cm

Diameter of a circle

$2R = 2 \times 25 = 50$ cm

Thus (d) is correct option.

23. **Assertion :** If a wire of length 22 cm is bent in the shape of a circle, then area of the circle so formed is $40\ \text{cm}^2$.

Reason : Circumference of the circle = length of the wire.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have $2\pi r = 22$

$r = 3.5$ cm

Area of the circle $= \dfrac{22}{7} \times 3.5 \times 3.5$

$= 38.5\ \text{cm}^2$

Assertion is not correct, but reason is true.

Thus (d) is correct option.

24. **Assertion :** If the circumference of a circle is 176 cm, then its radius is 28 cm.

Reason : Circumference $= 2\pi \times$ radius

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

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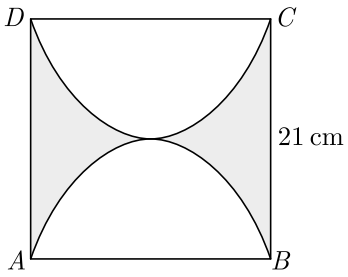
43. Find the perimeter of the shaded region if $ABCD$ is a square of side 21 cm and APB and CPD are semicircle. Use $\pi = \frac{22}{7}$.



Sol : [Board Term-2 SQP 2016]

It may be seen easily that perimeter of the shaded region include AD, BC and two semi circle arc.

Thus perimeter of the shaded region, $= AD + BC +$

$+$ lengths of the arcs of semi circles APB and CPD

$$= 21 + 21 + 2\left(\frac{22}{7} \times \frac{21}{2}\right) = 42 + 66 = 108 \text{ cm}.$$

44. The diameters of the front and rear wheels of a tractor are 80 cm and 200 cm respectively. Find the number of revolutions of rear wheel to cover the distance which the front wheel covers in 800 revolutions.

Sol : [Board Term-2 Delhi 2013]

Circumference of front wheel,

$$\pi d = \frac{22}{7} \times 80 = \frac{1760}{7} \text{ cm}$$

Distance covered by front wheel in 800 revolutions

$$= \frac{1760}{7} \times 800$$

Circumference of rear wheel

$$= \frac{22}{7} \times 200 = \frac{4400}{7} \text{ cm}$$

Revolutions made by rear wheel

$$= \frac{\frac{1760}{7} \times 800}{\frac{4400}{7}}$$

$$= \frac{1760 \times 800}{4400}$$

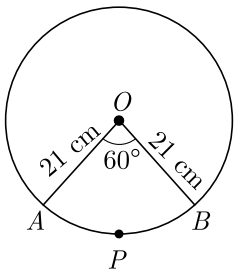
$$= 320 \text{ revolutions}$$

THREE MARKS QUESTIONS

45. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of the sector formed by the arc. Also, find the length of the arc.

Sol : [Board 2023 OD Standard]

As per question we have shown the diagram below



Area of sector APB,

$$\text{Area} = \frac{\theta}{360^\circ}\pi r^2$$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$$

$$= \frac{1}{6} \times 22 \times 3 \times 21$$

$$= 11 \times 21$$

$$= 231 \text{ cm}^2$$

Length of the arc APB,

$$l = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{60}{360^\circ} \times 2 \times \frac{22}{7} \times 21$$

$$= \frac{1}{6} \times 2 \times 22 \times 3$$

$$= 22 \text{ cm}$$

46. A road which is 7 m wide surrounds a circular park whose circumference is 88 m. Find the area of the road.

Sol : [Board 2020 Delhi Basic]

Let $w = 7$ m be the width of road.

Circumference of a circular park,

$$2\pi r = 88 \text{ m}$$

Inner radius of park,

$$r = \frac{88}{2\pi} = \frac{88 \times 7}{2 \times 22}$$

$$= 2 \times 7 = 14 \text{ m}$$

Outer radius of park including road width,

$$R = r + w$$

$$= 14 + 7 = 21 \text{ m}$$

Area of the road,

$$\pi(R^2 - r^2) = \pi(R + r)(R - r)$$

$$= \frac{22}{7}(21 + 14)(21 - 14)$$

$$= \frac{22}{7} \times 35 \times 7 = 770 \text{ m}^2$$

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29. Find the area of the sector of a circle of radius 6 cm whose central angle is 30°. (Take $\pi = 3.14$)

Sol : [Board 2020 OD Standard]

Radius, $r = 6$ cm

Central angle, $\theta = 30^\circ$

Area of the sector,

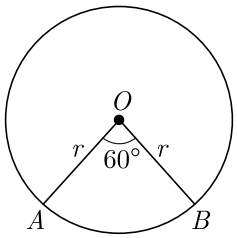
$$\frac{\pi r^2 \theta}{360^\circ} = \frac{3.14 \times 6 \times 6 \times 30^\circ}{360^\circ}$$

$$= 9.42 \text{ cm}^2$$

30. What is the perimeter of the sector with radius 10.5 cm and sector angle 60°.

Sol : [Board Term-2 2012]

As per question the digram is shown below.



Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^o}$$

$$= 10.5 \times 2 + 2 \times \frac{22}{7} \times \frac{10.5 \times 60}{360}$$

$$= 21 + 11 = 32 \text{ cm}$$

PRACTICE

✎ What is the perimeter of a sector of a circle whose central angle is 90° and radius is 7 cm? [Board Term-2 2012]

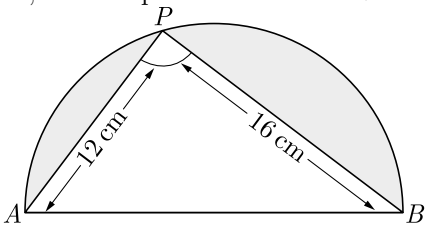
Ans : 25 cm

31. What is the name of a line which intersects a circle at two distinct points?

Sol : [Board Term-2 2012]

A line intersecting the circle at two distinct points is called a secant.

32. In the given figure, AB is the diameter where $AP = 12$ cm and $PB = 16$ cm. Taking the value of π as 3, find the perimeter of the shaded region.



Sol : [Board Term-2 2012]

From Pythagoras theorem we have

$$AB = \sqrt{(16)^2 + (12)^2}$$

$$= \sqrt{256 + 144}$$

$$= \sqrt{400} = 20 \text{ cm}$$

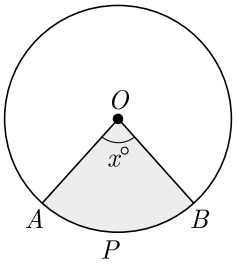
Radius of circle $= 10$ cm.

Perimeter of shaded region

$$\pi r + AP + PB = 3 \times 10 + 12 + 16$$

$$= 30 + 12 + 16 = 58 \text{ cm}$$

33. In given fig., O is the centre of a circle. If the area of the sector $OAPB$ is $\frac{5}{36}$ times the area of the circle, then find the value of x.



Sol : [Board Term-2 2012]

Area of the sector,

$$A_{\frown} = \frac{\pi r^2 \theta}{360^o}$$

Area of sector $OAPB$ is $\frac{5}{36}$ times the area of circle.

Thus $\pi r^2 \times \frac{x}{360} = \frac{5}{36}\pi r^2$

$$\frac{x}{360} = \frac{5}{36}$$

$$x = 50^o$$

34. If the radius of the circle is 6 cm and the length of an arc 12 cm. Find the area of the sector.

Sol : [Board Term-2 2014]

Area of the sector $= \frac{1}{2} \times$ (length of the corresponding arc) $\times$ radius

$$= \frac{1}{2} \times l \times r = \frac{1}{2} \times 12 \times 6$$

$$= 36 \text{ cm}^2$$

35. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find area of minor segment. ($\pi = 3.14$)

Sol : [Board Term-2 2012]

NODIA

50. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle. Use $\pi = \frac{22}{7}$.

Sol : [Board Term-2 2015]

Let radius of the circle be r.

Now as per question statement we have

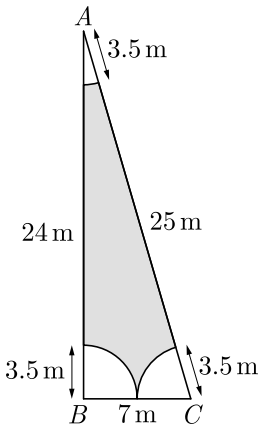
$$\begin{aligned}\text{Circumference} &= \text{Diameter}+16.8 \text{ cm}\\ 2\pi r &= 2r+16.8 \text{ cm}\\ 2\left(\frac{22}{7}\right)r &= 2r+16.8\\ \frac{44}{7}r &= 2r+16.8\\ 44r &= 14r+16.8\times 7\\ 30r &= 177.6\\ r &= \frac{117.6}{30} = 3.92\end{aligned}$$

Thus $r = 3.92$ cm.

51. Sides of a right triangular field are 25 m, 24 m and 7 m. At the three corners of the field, a cow, a buffalo and a horse are tied separately with ropes of 3.5 m each to graze in the field. Find the area of the field that cannot be grazed by these animals.

Sol : [Board 2020 SQP Standard]

As per information given in question we have drawn the figure below.



Let $\angle A = \theta_1$, $\angle B = \theta_2$ and $\angle C = \theta_3$.

Now, area which can be grazed by the animals is the sum of the areas of three sectors with central angles θ_1, θ_2 and θ_3 each with radius $r = 3.5$ m.

$$\frac{\pi r^2\theta_1}{360^\circ}+\frac{\pi r^2\theta_2}{360^\circ}+\frac{\pi r^2\theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}(\theta_1+\theta_2+\theta_3)...(1)$$

From angle sum property of a triangle we have

$$\theta_1+\theta_2+\theta_3 = 180^\circ$$

Substituting above in equation (1) we have

$$\begin{aligned}\frac{\pi r^2\theta_1}{360^\circ}+\frac{\pi r^2\theta_2}{360^\circ}+\frac{\pi r^2\theta_3}{360^\circ} &= \frac{\pi r^2}{360^\circ}\times 180^\circ\\ &= \frac{\pi r^2}{2}\\ &= \frac{22}{7}\times\frac{1}{2}\times(3.5)^2\\ &= 19.25\end{aligned}$$

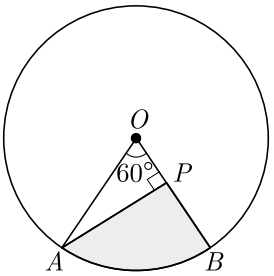
Hence, the area grazed by the horses is $19.25\,\text{m}^2$.

$$\begin{aligned}\text{Area of } \Delta ABC &= \frac{1}{2}\times AB\times BC\\ &= \frac{1}{2}\times 24\times 7 = 84\,\text{m}^2\end{aligned}$$

Area of the field that cannot be grazed by these animals = Area of triangle − Area of three sectors

$$= 84 - 1925 = 64.75\,\text{m}^2$$

52. In the given figure, AOB is a sector of angle 60° of a circle with centre O and radius 17 cm. If $AP \perp OB$ and $AP = 15$ cm, find the area of the shaded region.



Sol : [Board Term-2 2016]

Here $OA = 17$ cm $AP = 15$ cm and ΔOPA is right triangle

Using Pythagoras theorem, we have

$$OP = \sqrt{17^2-15^2} = 8 \text{ cm}$$

Area of the shaded region

= Area of the sector ΔOAB − Area of ΔOPA

$$\begin{aligned}&= \frac{60}{360}\times\pi r^2-\frac{1}{2}\times b\times h\\ &= \frac{60^\circ}{360^\circ}\times\frac{22}{7}\times 17\times 17-\frac{1}{2}\times 8\times 15\\ &= 151.38-60 = 91.38 \text{ cm}^2\end{aligned}$$

53. In the given figure, a chord AB of the circle with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment $AQBP$. Hence find the area of major segment $ALBQA$. (Use $\pi = 3.14$)

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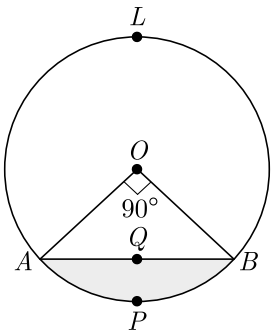


Sol : [Board Term-2 Foreign 2016]

Area of sector $OAPB$,

$$= \frac{90}{360}\pi(10)^2 = 25\pi \text{ cm}^2$$

Area of ΔAOB, $= \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$

Area of minor segment $AQBP$,

$$= (25\pi - 50) \text{ cm}^2$$

$$= 25 \times 3.14 - 50$$

$$= 78.5 - 50 = 28.5 \text{ cm}^2$$

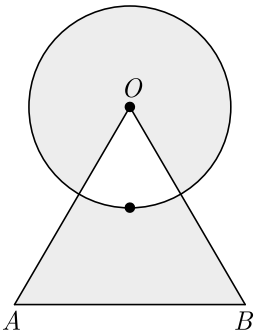
Also area of circle $= \pi(10)^2$

$$= 3.14 \times 100 = 314 \text{ cm}^2$$

Area of major segment $ALBQA = 314 - 28.5$

$$= 285.5 \text{ cm}^2$$

54. Find the area of shaded region shown in the given figure where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Sol : [Board Term-2 Foreign SQP 2016]

Since OAB is an an equilateral triangle,we have

$$\angle AOB = 60^\circ$$

Area of shaded region = Area of major sector + (Area of ΔAOB − Area of minor sector)

$$= \frac{300^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2 + \left(\frac{\sqrt{3}}{4}(12)^2 - \frac{60}{360} \times \frac{22}{7} \times 6^2\right)$$

$$= \frac{660}{7} + 36\sqrt{3} - \frac{132}{7} = 36\sqrt{3} + \frac{528}{7} \text{ cm}^2$$

55. Find the area of minor segment of a circle of radius 14 cm, when its centre angle is 60°. Also find the area of corresponding major segment. Use $\pi = \frac{22}{7}$.

Sol : [Board Term-2 OD 2015]

Here, $r = 14$ cm, $\theta = 60^\circ$

Area of minor segment,

$$\pi r^2 \frac{\theta}{360^\circ} - \frac{1}{2} r^2 \sin\theta$$

$$= \pi(14)^2 \frac{60^\circ}{360^\circ} - \frac{1}{2} \times (14)^2 \times \frac{\sqrt{3}}{2}$$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{60^\circ}{360^\circ} - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$$

$$= \left(\frac{308}{3} - 49\sqrt{3}\right) = 17.9 \text{ cm}^2 \text{ approx.}$$

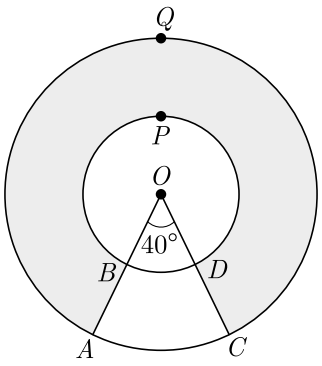
Area of major segment $= \pi r^2 - \left(\frac{308}{3} - 49\sqrt{3}\right)$

$$= \frac{22}{7} \times 14 \times 14 - \frac{308}{3} + 49\sqrt{3}$$

$$= \frac{1540}{3} + 49\sqrt{3} = 598.10$$

$$= 598 \text{ cm}^2 \text{ approx.}$$

56. In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40^\circ$. Use $\pi = \frac{22}{7}$.



Sol : [Board Term-2 OD 2016]

Radii of two concentric circle is 7 cm and 14 cm.

Angle $\angle AOC = 40^\circ$,

Angle $\angle AOC = 360^\circ - 40^\circ = 320^\circ$

Area of shaded region,

$$\frac{\theta}{360^\circ}\pi[R^2 - r^2] = \frac{320^\circ}{360^\circ} \times \frac{22}{7}[14^2 - 7^2]$$

$$= \frac{8}{9} \times 22 \times (14 \times 2 - 7)$$

$$= \frac{8}{9} \times 22 \times 21$$

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$$= \frac{8}{3} \times 22 \times 7$$

$$= \frac{8 \times 154}{3} \text{ cm}^2$$

Required area, $$= \frac{1232}{3} \text{ cm}^2$$

$$= 410.67 \text{ cm}^2$$

57. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of sector formed by the arc.

Sol : [Board Term-2 Delhi Compt. 2017]

We have $r = 21$ cm and $\theta = 60^\circ$

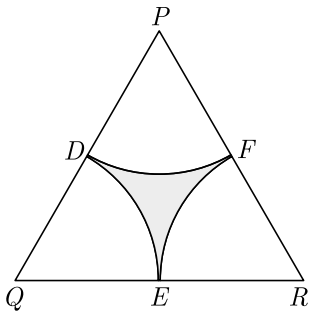
Area formed the sector $= \frac{\theta}{360} \times \pi r^2$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$$

$$= \frac{1}{6} \times 22 \times 3 \times 21$$

$$= 11 \times 21 = 231 \text{ cm}^2$$

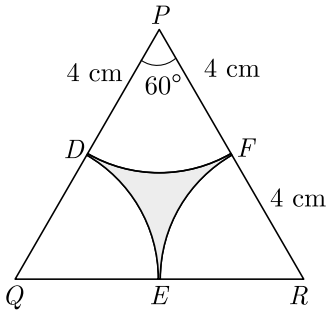
58. In the given figure, ΔPQR is an equilateral triangle of side 8 cm and D, E, F are centres of circular arcs, each of radius 4 cm. Find the area of shaded region. (Use $\pi = 3.14$) and $\sqrt{3} = 1.732$



Sol : [Board Term-2, 2012]

Here angle $\angle P = \angle Q = \angle R = 60^\circ$ because triangle is equilateral. side of triangle is 8 cm.

Consider circular section PDE. Radius of circular arc is 4 cm.



Area of sector PDF,

$$\frac{\theta}{360^\circ} \times \pi r^2 = \frac{60^\circ}{360^\circ} \times 3.14 \times 4 \times 4$$

$$= \frac{1}{6} \times 3.14 \times 16 = 8.373$$

Area of shaded region

$$= \text{Area of } \Delta PQR - 3(\text{area of sector})$$

$$= \frac{\sqrt{3}}{4}(8)^2 - 3 \times 8.373$$

$$= 16\sqrt{3} - 3 \times 8.373$$

$$= 16 \times 1.732 - 25.12$$

$$= 27.712 - 25.12 = 2.59 \text{ cm}^2$$

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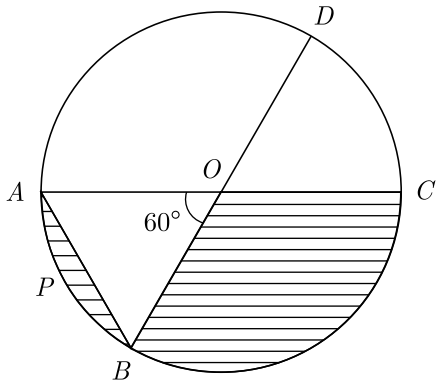
FIVE MARKS QUESTIONS

59. In the given figure, diameters AC and BD of the circle intersect at O. If $\angle AOB = 60^\circ$ and $OA = 10$ cm, then:



(i) find the length of the chord AB.

(ii) find the area of shaded region. (Take $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Sol : [Board 2024 OD Standard]

O is the centre of the circle because diameters AC

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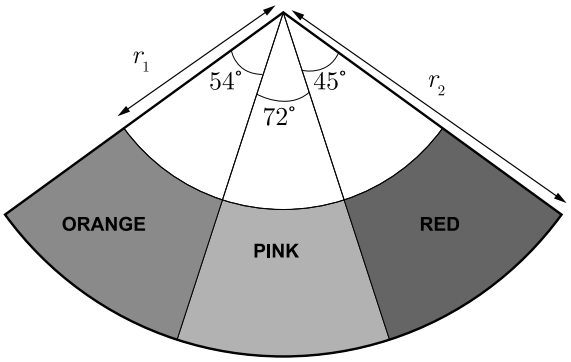
Lavanya hold a Japanease folding fan in her hand as shown in figure. It is shapded like a sector of a circle. The inner and outer raddi are 14 cm and 21 cm. The fan has three colour material.



(i) If the region containing the pink colour makes an angle of $\theta_2 = 72^\circ$ at the centre, then find the area of the region having pink colour.

(ii) If the region containing the orange colour makes an angle of $\theta_1 = 54^\circ$ at the centre, then find the area of the region having orange colour.

(iii) If the region containing the red colour makes an angle of $\theta_1 = 45^\circ$ at the centre, then find the perimeter of the region having red colour.

(iv) Find the area of the region having radius 14 cm.

Sol :

Here $r_1 = 14$ cm and $r_2 = 21$ cm.

(i) Area of region having pink colour,

$$A_p = \pi r_2^2 \frac{\theta_2}{360^\circ} - \pi r_1^2 \frac{\theta_2}{360^\circ}$$
$$= \frac{\pi\theta_2}{360^\circ}(r_2^2 - r_1^2)$$
$$= \frac{22}{7} \times \frac{72^\circ}{360^\circ}(21^2 - 14^2)$$
$$= \frac{22}{7} \times \frac{1}{5}(21+14)(21-14)$$
$$= \frac{22}{7} \times \frac{1}{5} \times 35 \times 7$$
$$= 22 \times 7 = 154 \text{ cm}^2$$

(ii) Area of region having orange colour,

$$A_o = \frac{\pi\theta_1}{360^\circ}(r_2^2 - r_1^2)$$
$$= \frac{22}{7} \times \frac{54^\circ}{360^\circ}(21^2 - 14^2)$$
$$= \frac{22}{7} \times \frac{3}{20}(21+14)(21-14)$$
$$= \frac{22}{7} \times \frac{3}{20} \times 35 \times 7$$
$$= \frac{22 \times 3 \times 7}{4} = 115.5 \text{ cm}^2$$

(iii) Perimeter of the region having red colour,

$$p_r = 2(r_2 - r_1) + 2\pi r_1 \frac{\theta_3}{360^\circ} + 2\pi r_2 \frac{\theta_3}{360^\circ}$$
$$= 2(r_2 - r_1) + 2\pi \frac{\theta_3}{360^\circ}(r_1 + r_2)$$
$$= 2(21-14) + 2 \times \frac{22}{7} \times \frac{45^\circ}{360^\circ}(21+14)$$
$$= 2 \times 7 + 2 \times \frac{22}{7} \times \frac{1}{8} \times 35$$
$$= 14 + \frac{11 \times 5}{2}$$
$$= 14 + 27.5 = 41.5 \text{ cm}$$

(iv) Area of the region having radius 14 cm

$$A_s = \pi r_1^2 \frac{(\theta_1 + \theta_2 + \theta_3)}{360^\circ}$$
$$= \frac{22}{7} \times (14)^2 \times \frac{(54^\circ + 72^\circ + 45^\circ)}{360^\circ}$$
$$= \frac{22}{7} \times 14 \times 14 \times \frac{171^\circ}{360^\circ}$$
$$= 22 \times 2 \times 14 \times \frac{19}{40}$$
$$= \frac{22 \times 7 \times 19}{10}$$
$$= 292.6 \text{ cm}^2$$

11. Pendulum Clock : It is a clock that uses a pendulum, a swinging weight, as its timekeeping element. From its invention in 1656 by Christiaan Huygens, the pendulum clock was the world's most precise timekeeper, accounting for its widespread use. Their greater accuracy allowed for the faster pace of life which was necessary for the Industrial Revolution. The home pendulum clock was replaced by less-expensive, synchronous, electric clocks in the 1930s and 40s. Pendulum clocks are now kept mostly for their decorative and antique value.

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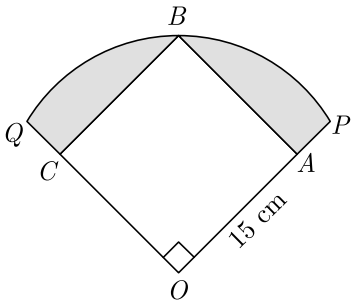
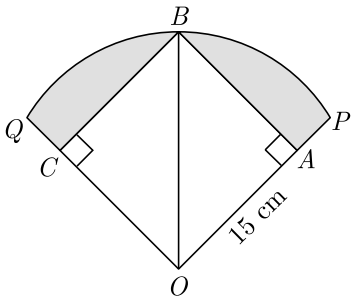
the shaded region. (Use $\pi = 3.14$).



Sol : [Board 2019 OD]

We have redrawn the figure given below.



Using Pythagoras theorem in ΔBAO,

$$OB^2 = OA^2 + AB^2 = 15^2 + 15^2$$
$$= 225 + 225 = 450$$
$$OB = \sqrt{450} = 15\sqrt{2}$$

Thus radius $OB = 15\sqrt{2}$ cm.

$$\text{Area of square} = (OA)^2 = (15)^2 = 225\ \text{cm}^2$$

Now, area of quadrant,

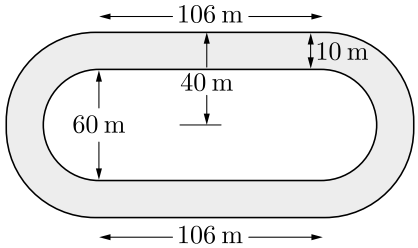
$$\frac{\pi r^2}{4} = \frac{1}{4} \times 3.14 \times (15\sqrt{2})^2$$
$$= \frac{1}{4} \times 3.14 \times 225 \times 2$$
$$= \frac{3.14 \times 225}{2}$$
$$= 353.25\ \text{cm}^2$$

Therefore, area of shaded region

= Area of quadrant $OPBQ$ − area of square $OABC$

$= 353.25 - 225 = 128.25\ \text{cm}^2$

63. Fig. depicts a racing track whose left and right ends are semi-circular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide everywhere, find the area of the track.



Sol : [Board Term-2 2011]

Width of the inner parallel lines = 60 m

Width of the outer lines $= 40 \times 2 = 80$ m

Radius of the inner semicircles $= \frac{60}{2} = 30$ m

Radius of the outer semicircles $= \frac{80}{2} = 40$ m

Area of inner rectangle $= 106 \times 60 = 3180\ \text{m}^2$

Area of outer rectangle $= 106 \times 80 = 4240\ \text{m}^2$.

Area of the inner semicircles

$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 30 \times 30 = \frac{19800}{7}\ \text{m}^2$$

Area of outer semicircles

$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 40 \times 40 = \frac{35200}{7}\ \text{m}^2$$

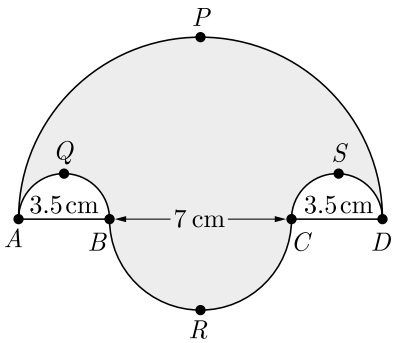
Area of racing track

= (area of outer rectangle + area of outer semicircles) − (area of inner rectangle + area of inner semicircles)

$$= 4240 + \frac{35200}{7} - \left(\frac{3180 + 19800}{7}\right)$$
$$= 1060 + \frac{15400}{7} = \frac{7420 + 15400}{7}$$
$$= \frac{22820}{7} = 3260\ \text{m}^2$$

Hence, area of track is 3260 m²

64. Find the area of the shaded region in Figure, $\widehat{APD}, \widehat{AQB}, \widehat{BRC}$ and $\widehat{CSD}$, are semi-circles of diameter 14 cm, 3.5 cm, 7 cm and 3.5 cm respectively. Use $\pi = \frac{22}{7}$.



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Sol : [Board Term-2 Foreign 2016]

Diameter of the largest semi circle $= 14$ cm

$$\text{Radius} = \frac{14}{2} = 7 \text{ cm}$$

Diameter of two equal unshaded semicircle $= 3.5$ cm

$$\text{Radius of each circle} = \frac{3.5}{2} \text{ cm}$$

Diameter of smaller shaded semi-circle $= 7$ cm

$$\text{Radius} = 3.5 \text{ cm}$$

Area of shaded portion

= area of largest semi-circle+

+ area of smaller shaded semicircle+

– area of two unshaded semicircles

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} - 2 \times \frac{1}{2} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$$

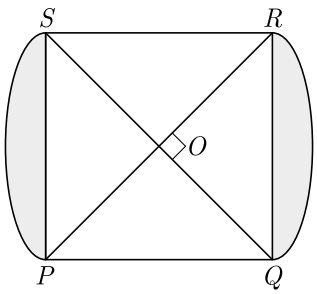
$$= \frac{1}{2} \times \frac{22}{7}\left[7^2 + \left(\frac{7}{2}\right)^2 - 2\left(\frac{7}{4}\right)^2\right] \text{ cm}^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2\left[1 + \frac{1}{4} - \frac{1}{8}\right]$$

$$= 11 \times 7\left[\frac{9}{8}\right]$$

$$= \frac{693}{8} \text{ sq. cm or } 86.625 \text{ cm}^2$$

65. In figure, $PQRS$ is square lawn with side $PQ = 42$ metre. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).



Sol : [Board Term-2 OD 2015]

Radius of circle with centre O is OR.

Let OR be x then using Pythagoras theorem we have

$$x^2 + x^2 = (42)^2 \Rightarrow x = 21\sqrt{2} \text{ m}$$

Area of segment of circle with centre angle 90°

$$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (21\sqrt{2})^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21 \times 2$$

$$= 11 \times 3 \times 21 = 693$$

Area of triangle ΔROQ,

$$= \frac{1}{2} \times (21\sqrt{2})^2 = 21 \times 21 = 441$$

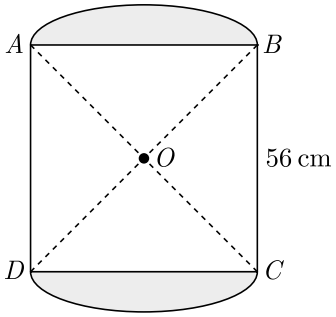
Area of the one side flower bed

$$= 693 - 441 = 252 \text{ m}^2$$

Area of flower bed of both

$$= 2 \times 252 = 504 \text{ m}^2$$

66. In fig., two circular flower beds have been shown on two sides of a square lawn $ABCD$ of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and flower beds.



Sol : [Board Term-2 2011]

Side of square $= 56$

Diagonal of square $= 56\sqrt{2}$

Radius of circle $= \frac{1}{2} \times 56\sqrt{2} = 28\sqrt{2}$

Total area = Area of sector OAB +

+ Area of sector ODC +

+ Area of ΔOAD +

+ Area of ΔOBC

$$= \frac{22}{7} \times (28\sqrt{2})^2 \times \frac{90^\circ}{360^\circ} + \frac{22}{7} \times (28\sqrt{2})^2 \times \frac{90^\circ}{360^\circ} + + \frac{1}{4} \times 56 \times 56 + \frac{1}{4} \times 56 \times 56$$

$$= \frac{1}{4} \times \frac{22}{7} \times (28\sqrt{2})^2 + \frac{1}{4} \times \frac{22}{7} \times (28\sqrt{2})^2 + + \frac{1}{4} \times 56 \times 56 + \frac{1}{4} \times 56 \times 56$$

$$= \frac{1}{4} \times 28 \times 56\left(\frac{22}{7} + \frac{22}{7} + 2 + 2\right) \text{ m}^2$$

$$= 7 \times 56\left(\frac{22 + 22 + 14 + 14}{7}\right) \text{ m}^2$$

$$= 56 \times 72 = 4032 \text{ m}^2.$$

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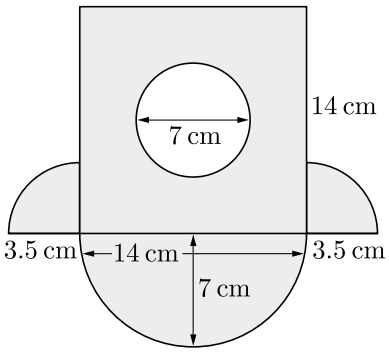
67. In fig., find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Sol : [Board Term-2 2011]

$$\text{Area of square} = (14)^2 = 196 \text{ cm}^2$$

$$\text{Area of internal circle} = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{ cm}^2$$

$$= \frac{77}{2} = 38.5 \text{ cm}^2$$

Area of semi-circle with 14 cm diameter

$$= \frac{1}{2} \times \frac{22}{7} \times 7^2 \text{ cm}^2$$

$$= 77 \text{ cm}^2$$

Area of two quarter circles of radius $\frac{7}{2}$ cm

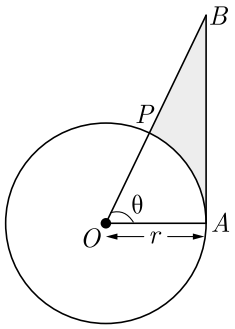
$$= 2 \times \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{77}{4} = 19.25 \text{ cm}^2$$

$$\text{Shaded area} = 196 - 38.5 + 77 + 19.25$$

$$= 292.25 - 38.5$$

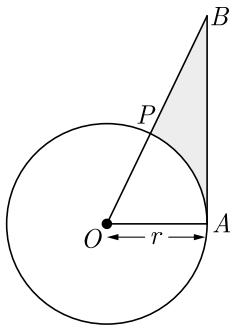
$$= 253.75 \text{ cm}^2.$$

68. The given figure shows a sector OAP of a circle with centre O, containing $\angle\theta$. AB is perpendicular to the radius OA and meets OP produced at B . Prove that the perimeter of shaded region is $r = [\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1]$.



Sol : [Board Term-2 OD 2015, 2016]

As per question statement we have redrawn this figure as given below.



Here OAP is sectors of circle with centre O, $\angle POA = \theta$ and $OA \perp AB$.

Perimeter of shaded region $= BP + AB + \overset{\frown}{AP}$ (1)

Now $\tan\theta = \frac{AB}{r} \Rightarrow r\tan\theta = AB$ ...(2)

$$\sec\theta = \frac{OB}{r} \Rightarrow r\sec\theta = OB$$

$OB - OP = BP \Rightarrow r\sec\theta - r = OP$ ...(3)

Length of arc AP,

$$\overset{\frown}{AP} = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{\theta}{360} \times 2\pi r = \frac{\theta\pi r}{180} \quad ...(4)$$

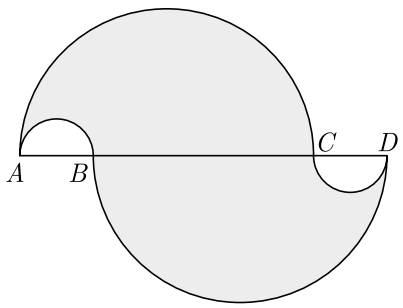
Substituting values from equation (2), (3), (4) in equation (1) we get perimeter of shaded region as

$$= r\tan\theta + r\sec\theta - r + \frac{\theta\pi r}{180}$$

$$= r\left[\tan\theta + \sec\theta + \frac{\theta\pi}{180} - 1\right]$$

Hence, Proved.

69. In fig., $AC = BD = 7$ cm and $AB = CD = 1.75$ cm. Semi-circles are drawn as shown in the figure. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Sol : [Board Term-2 2011]

Area of shaded region

$= 2$(Area of semi-circle of radius $\frac{7}{2}$ cm)

-2(Area of semi-circle of radius $\frac{7}{8}$ cm)

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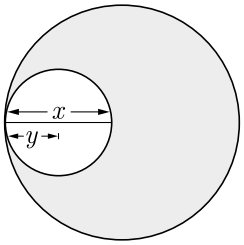
$$= 2\left[\frac{1}{2}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\right]-2\left[\frac{1}{2}\times\frac{22}{7}\times\left(\frac{7}{8}\right)^2\right]$$
$$= 2\times\frac{1}{2}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\left[1-\left(\frac{1}{4}\right)^2\right]$$
$$= \frac{77}{2}\left[1-\frac{1}{16}\right]=\frac{77}{2}\times\frac{15}{16}=\frac{1155}{32}\text{ cm}^2$$
$$= 36.09\text{ cm}^2$$

10. Two circles touch internally. The sum of their areas is 116π and the difference between their centres is 6 cm. Find the radii of the circles.

Sol : [Board Term-2 Foreign 2017]

Let the radius of larger circle be x and the radius of smaller circle be y. As per question statement we have shown diagram below.



Now $x-y = 6$ ...(1)

and $\pi x^2+\pi y^2 = 116\pi$

$\pi(x^2+y^2) = 116\pi$

$x^2+y^2 = 116$ ...(2)

From (1) and (2) we have

$x^2+(x-6)^2 = 116$

$x^2+x^2-12x+36 = 116$

$x^2-6x-40 = 0$

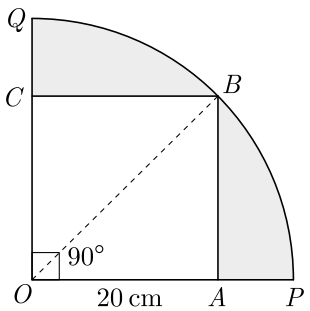
$x^2-10x+4x-40 = 0$

$x(x-10)+4(x+10) = 0$

$x = 10$, and $y = 10-6 = 4$

Hence, radii of the circles are 10 cm and 4 cm.

11. A square $OABC$ is inscribed in a quadrant $OPBQ$ of a circle. If $OA = 20$ cm, find the area of the shaded region. [Use $\pi = 3.14$]



Sol : [Board Term-2 Delhi 2014]

We have $OB = \sqrt{OA^2+AB^2}$

$= \sqrt{20^2+20^2} = \sqrt{800}$

Thus $OB = 20\sqrt{2}$ cm

Radius $r = 20\sqrt{2}$

Area of shaded region

= Area of sector $OQBPO$ − Area of square $OABC$

$= \pi r^2\frac{90^\circ}{360^\circ}-(20)^2$

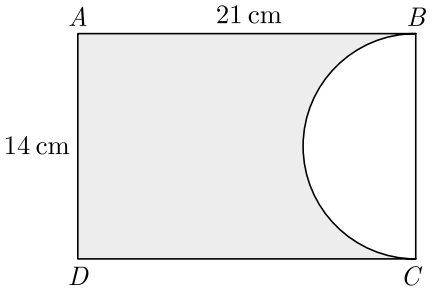
$= 3.14\times(20\sqrt{2})^2\times\frac{90^\circ}{360^\circ}-(20)^2$

$= 3.14\times200-400$

$= 628-400 = 228$

Required area is 228 cm^2.

12. In the given figure, $ABCD$ is a rectangle of dimensions 21 cm $\times$ 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



Sol : [Board Term-2 OD 2017]

Area of shaded region,

= Area of rectangle $ABCD$ − area of semicircle

$= 21\times14-\frac{\pi}{2}\times7^2$

$= 294-\frac{1}{2}\times\frac{22}{7}\times7\times7$

$= 294-77 = 217\text{ cm}^2$

Perimeter of shaded area

$= AB+AD+CD+\overset{\frown}{CB}$

$= 21+14+21+\frac{22}{7}\times7$

$= 21+14+21+22$

$= 78$ cm

Hence, area of shaded region is 217 cm^2 and perimeter is 78 cm.

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73. The long and short hands of a clock are 6 cm and 4 cm long respectively. Find the sum of distances travelled by their tips in 24 hours. (Use $\pi = 3.14$)

Sol : [Board Term-2 Foreign 2015]

Long hand makes 24 rounds in 24 hours and short hand makes 2 round in 24 hours. Distance travelled by tips of hands in one round is equal to the circumference of circle.

Radius of the circle formed by long hand = 6 cm. and radius of the circle formed by short hand = 4 cm.

Distance travelled by long hand in one round

$= \text{circumference of the circle } 2 \times 6 \times \pi$

Distance travelled by long hand in 24 rounds

$= 24 \times 12\pi = 288\pi$

Distance travelled by short hand in a round $= 2 \times 4\pi$

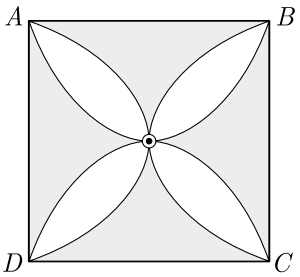
Distance travelled by short hand in 2 round

$= 2 \times 8\pi = 16\pi$

Sum of the distance $= 288\pi + 16\pi = 304\pi$

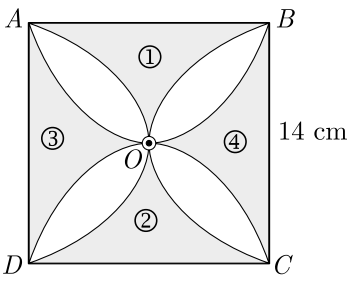
$= 304 \times 3.14 = 954.56 \text{ cm}$

74. In fig. $ABCD$ is a square of side 14 cm. Semi-circle are drawn with each side of square as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Sol : [Board Term-2 Delhi 2016, STD SQP 2021]

We have redrawn the given figure as shown below.



If we subtract area of two semicircle AOD and COB, from square $ABCD$ we will get area of part 1 and part 2.

Area of square $= 14 \times 14 = 196 \text{ cm}^2$

Radius of semicircle $= \frac{14}{2} = 7 \text{ cm}$

Area of semicircle $AOB + DOC$

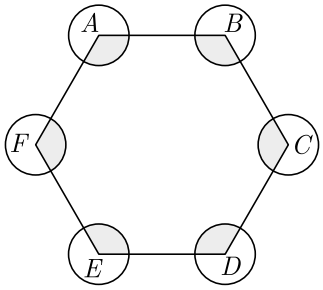
$= \frac{22}{7} \times 7^2 = 154 \text{ cm}^2$

So, area of each of two shaded part

$196 - 154 = 42 \text{ cm}^2$

Hence, area of four shaded parts is 84 cm^2.

75. In fig., $ABCDEF$ is any regular hexagon with different vertices A, B, C, D, E and F as the centres of circle with same radius r are drawn. Find the area of the shaded portion.



Sol : [Board Term-2 2011]

Let n be number of sides.

Now $n \times \text{each angle} = (n-2) \times 180^\circ$

$6 \times \text{each angle} = 4 \times 180^\circ$

$\text{each angle} = 120^\circ$

$\text{Area of a sector} = \pi r^2 \times \frac{120^\circ}{360^\circ}$

Area of 6 shaded regions $= 6\pi r^2 \times \frac{120^\circ}{360^\circ}$

$= 2\pi r^2$

COMPETENCEY BASED QUESTIONS

76. Traditional Japanese Fans : Japanese fans are made of paper on a bamboo frame, usually with a design painted on them. A Japanese Fan symbolises friendship, respect and good wishes and are given on special occasions, as well as to help cool you down in hot weather. The fan is an immediately recognizable icon of Japanese culture. Today they remain an important artistic medium and stylish fashion accessory.

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and BD intersect at O. OA and OB are the radii of circle.

$$OA = OB$$
$$\angle OAB = \angle OBA$$
$$\angle OAB = \angle OBA = \frac{180^\circ - 60^\circ}{2} = 60^\circ$$

Thus ΔOAB is an equilateral triangle.

Since ΔOAB is an equilateral triangle, we have

$$OA = OB = AB = 10 \text{ cm}$$

(i) Length of the chord $AB = OA = 10$ cm

(ii) Area of shaded region

Area of shaded region is the subtraction of area of equilateral ΔOAB from area of semicircle.

Area of ΔOAB $A_1 = \frac{\sqrt{3}}{4}(\text{side})^2$

$$= \frac{\sqrt{3}}{4}(10)^2$$
$$= 25\sqrt{3} \text{ cm}^2$$

Area of semicircle $A_2 = \frac{\pi(10)^2}{2}$

Area of shaded region,

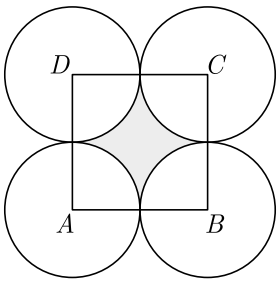
$$A = A_2 - A_1$$
$$= \frac{\pi(10)^2}{2} - 25\sqrt{3}$$
$$= 50\pi - 25\sqrt{3}$$
$$= 50(3.14) - 25(1.73)$$
$$= (157 - 43.25)$$
$$= 113.75 \text{ cm}^2$$

60. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circle is $\frac{24}{7}$ cm^2. Find the radius of each circle.

Sol : [Board Term-2 SQP 2017]

As per question statement the figure is shown below.

Let r be the radius of each circle.

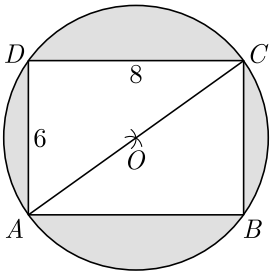


Area of square – Area of 4 sectors $= \frac{24}{7}$ cm^2

$$(2r)^2 - 4\left(\pi r^2 \times \frac{90^\circ}{360^\circ}\right) = \frac{24}{7}$$
$$4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$$
$$\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$$
$$6r^2 = 24$$
$$r^2 = 4 \Rightarrow r = \pm 2$$

Thus radius of each circle is 2 cm.

61. Find the area of the shaded region in Figure, if $ABCD$ is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take $\pi = 3.14$)



Sol : [Board 2019 Delhi]

In ΔABC, $\angle B = 90^\circ$

Using Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$= 8^2 + 6^2 = 64 + 36 = 100$$
$$AC = 10 \text{ cm}$$

Since, AC is the diameter of circle,

Radius of circle, $r = 5$ cm

Area of the shaded region

= (area of the circle) – (area of the rectangle)

$$= \pi r^2 - (AB \times BC)$$
$$= 3.14 \times 5^2 - (8 \times 6)$$
$$= 78.5 - 48 = 30.5 \text{ cm}^2$$

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62. In Figure, a square $OABC$ is inscribed in a quadrant $OPBQ$. If $OA = 15$ cm, find the area of

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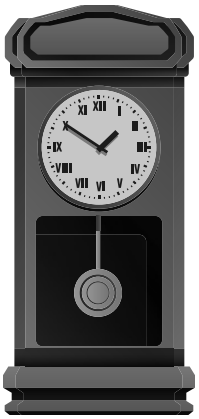

Dhriti bought a pendulum clock for her living room. the clock contains a small pendulam of lenght 45 cm. the minute hand and hour hand of the clock are 9 cm and 6 cm long respectively.

(i) Find the area swept by the minute hand in 14 minutes.

(ii) Find the angle described by hour hand in 10 minutes.

(iii) Find the distance covered by the tip of hour hand in 3.5 hours.

(iv) If the tip of pendulum covers a distance of 66 cm in complete oscillation, then find the angle described by pendulum at the centre.

Sol :

Let $r_m = 9$ cm, $r_h = 9$ cm and $r_p = 45$ cm

(i) Area swept by the minute hand in 60 minutes is πr^2, thus in 14 minutes.

$$= \pi r_m^2 \times \frac{14}{60}$$
$$= \frac{22}{7} \times 9 \times 9 \times \frac{14}{60}$$
$$= 22 \times 9 \times 9 \times \frac{1}{30}$$
$$= \frac{22 \times 9 \times 3}{10} = 59.4\,\text{cm}^2$$

(ii) Angle made by hour hand in 12 hours is 360° . Thus angle made in 10 minute.

$$\theta = \frac{360^\circ}{12} \times \frac{10}{60} = 5^\circ$$

(iii) Distance cover by tip of hour hand is $2\pi r$ in 12 hour.

Distance covered in 3.5 hours,

$$= 2\pi r_h \times \frac{3.5}{12}$$
$$= 2 \times \frac{22}{7} \times 6 \times \frac{3.5}{12}$$
$$= \frac{22}{2} = 11\,\text{cm}$$

(iv) If the tip of pendulum covers a distance of 66 cm in complete oscillation, distance covered in half oscillation is 33 cm.

If θ is the angle described by pendulum at the centre to cover distance 33 cm, then,

$$l = 2\pi r_p \times \frac{\theta}{360^\circ}$$
$$33 = 2 \times \frac{22}{7} \times 45 \times \frac{\theta}{360^\circ}$$
$$3 = \frac{4}{7} \times \frac{\theta}{8}$$
$$\theta = \frac{3 \times 7 \times 8}{4} = 42^\circ$$

78. UPHOLSTERY : Upholstery is the physical act of stuffing seats and furniture with webbing, padding, springs, foam, or cushions to furniture and covering it with upholstery fabric. The word upholstery comes from the Middle English word upholder, which referred to an artisan who help up their goods. A person who works with upholstery is an upholsterer.

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Eshika wants to cover the cushions of her papasan chair with a different fabric. If there are seven circular cushions that are the same size with a diameter of 32 cm, around a center cushion with a diameter of 46 cm, find the area of fabric that she will need to cover both sides of the cushions. Allow an extra 5 cm of fabric around each cushion.

Sol :

Radius of cloth for small cushions,

$$r_s = 16 + 5 = 21\,\text{cm}$$

Area, $A = \pi r_s^2$

$$= \frac{22}{7} \times 21^2$$

Curved width $\dfrac{2\pi r\theta}{360^\circ} = \dfrac{2\pi \times 4\sqrt{2} \times 90^\circ}{360^\circ}$

$= 2\pi\sqrt{2}$ m

(iii) Area of cross section

= Area of $AECD$ +

+ Area of section $ABCO$ − Area of triangle ACO

$= 8 \times 8 + \dfrac{\pi(4\sqrt{2})^2 \times 90^\circ}{360^\circ} - \dfrac{1}{2} \times 4 \times 8$

$= 64 + 8\pi - 16 = 48 + 8\pi = 8(6 + \pi)$

(iv) Curved surface area of roof

$= 2\pi\sqrt{2} \times 12 = 24\sqrt{2}\,\pi$ m^2

(v) Storage capacity of barn,

= Cross section area × Length

$= 8(6 + \pi) \times 12 = 96(6 + \pi)$ m^3

92. Speed of a River Current : The paddle wheel in the river functions like a wheel rolling on a road. To approximate the speed of the current of a river, a circular paddle wheel with radius 1.75 meter is lowered into the water. If the current causes the wheel to rotate at a speed of 10 revolutions per minute, what is the speed of the current?.

Sol :

The water rotate wheel with its current speed. Since wheel rotate 10 revolution in minute, water also flow equivalent 10 revolution distance in 1 minute.

Water flow in revolution of wheel, $= 2\pi r$

Water flow in 10 revolution of wheel,

$= 10 \times 2\pi r$

$= 10 \times 2 \times \dfrac{22}{7} \times 1.75$

$= 10 \times 2 \times \dfrac{22}{7} \times \dfrac{7}{4} = 110$ meter

Since the wheel makes 10 revolutions per minute a point on the circumference of the paddle wheel travels 110 meter. Hence, water flows 110 meter in one minute.

Water flow in 1 hour, $= 110 \times 60$ m

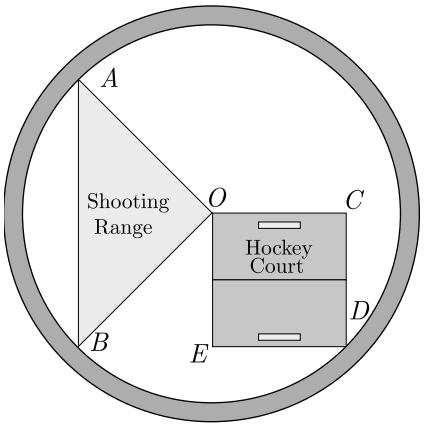
$= 6600$ m

$= 6.6$ km

Since water flow in 1 hour is 6.6 km, its current is 6.6 km/hour.

93. Jawaharlal Nehru Stadium is a multi-purpose sports stadium and a very popular sports stadium of Delhi. The stadium is a part of the Jawaharlal Nehru sports complex in central Delhi, which also houses the headquarters of the Sports Authority of India, the field arm of the Ministry of Youth Affairs and Sports, and the Indian Olympic Association . It has a capacity to seat 60,000 people. It is the third largest multi-purpose stadium in India. In 2010, the Jawaharlal Nehru Stadium was the main stadium for XIX Commonwealth Games; a major sporting event.

Jawaharlal Nehru stadium is conducting the annual sports competition soon. The curator of the stadium is tasked to figuring out the dimensions for carving out some areas allotted for a hockey court and a shooting range, as shown in the figure below.



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Sol :

Total area of 9 mini 3 inch cake

$$= 9\pi 3^2 \ = 81\pi$$

Total area of 1 large 9 inch cake

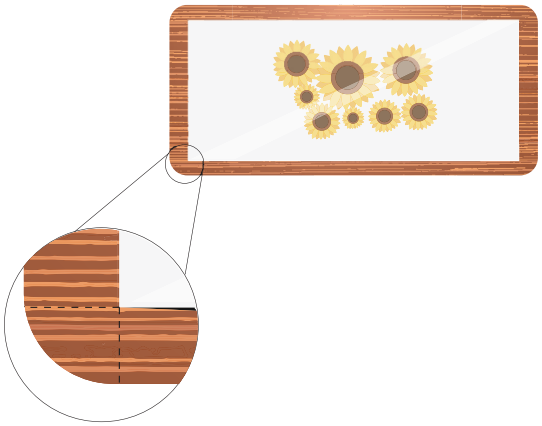
$$= \pi 9^2 \ = 81\pi$$

So when nine mini-cakes are compared to one large cake, the total area is equal. Nine minicakes are the same size as one 9-inch cake, but nine mini-cakes cost $9 \times 80 = 720$ Rs while the 9-inch cake is only Rs 645. The 9-inch cake gives more cake for the money.

Alternative :

The scale factor is $\frac{9}{3} = \frac{3}{1}$, so the ratio of areas between a large cake and a mini-cake is $(\frac{3}{1})^2 = \frac{9}{1}$. So when nine mini-cakes are compared to one large cake, the total area is equal. Nine minicakes are the same size as one 9-inch cake, but nine mini-cakes cost $9 \times 80 = 720$ Rs while the 9-inch cake is only Rs 645. The 9-inch cake gives more cake for the money.

81. FURNITURE : Varsha dining room table has hardwood around the outside. The outer dimension of table is 135 cm by 90 cm. Find the area of wood around the edge of the table. The table has wooden edge of 7 cm around it.



Sol :

The wooden edge of table can be separated into four rectangles and four corners. The four corners of the table form a circle with radius equal to width.

Width of wooden edge is 7 cm. Area of wood edge is equal to the sum of area of rectangles and area of circle.

$$A \ = 2l_1 w + 2l_2 w + 4 \times \tfrac{1}{4}\pi w^2$$

$$= 2 \times (90 - 14) \times 7 + 2 \times (135 - 14) \times 7 + \tfrac{22}{7} \times 7^2$$

$$= 2 \times 7 (76 + 121 + 11)$$

$$= 14 \times 208 \ = 2912 \ \text{cm}^2$$

82. Motion Detector : A motion sensor (or motion detector) is an electronic device that is designed to detect and measure movement. Motion sensors are used primarily in home and business security systems, but they can also be found in phones, paper towel dispensers, game consoles, and virtual reality systems.

A motion detector can detect movement up to 24 m away through an angle of $\theta = 70°$.

(i) What area can the motion detector monitor?

(ii) What angle is required to monitor 50% more area?

(iii) For $\theta = 91°$ what range is required for the detector to monitor 30% more area?

Sol :

(i) Let $r_1 = 24$ m and $\theta_1 = 70°$

Area monitor by motion detector,

$$A_1 \ = \pi r_1^2 \frac{\theta_1}{360°}$$

$$= \frac{22}{7} \times (24)^2 \times \frac{70}{360°}$$

$$= 22 \times 24 \times 24 \times \frac{1}{36}$$

$$= 22 \times 24 \times 2 \times \frac{1}{3}$$

$$= 22 \times 8 \times 2 \ = 352 \ \text{m}^2$$

(ii) Let θ_2 be angle to monitor 50% more area. Radius is equal to $r_1 = 24$ in this case.

$$A_2 \ = 1.5A_1$$

$$\pi r_1^2 \frac{\theta_2}{360°} \ = 1.5\pi r_1^2 \frac{\theta_1}{360°}$$

$$\theta_2 \ = 1.5\theta_1$$

$$= 70° \times 1.5 = 105°$$

(iii) Let r_3 be range required for the detector to monitor 30% more area where $\theta = 91°$.

$$A_3 \ = 1.3A_1$$

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$$\pi r_3^2 \frac{\theta}{360^\circ} = 1.3\pi r_1^2 \frac{\theta_1}{360^\circ}$$

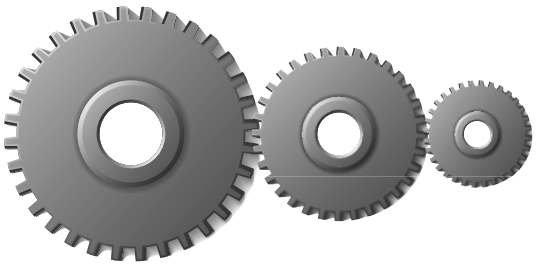
$$r_3^2\theta = 1.3 r_1^2 \theta_1$$

Substituting all values we have

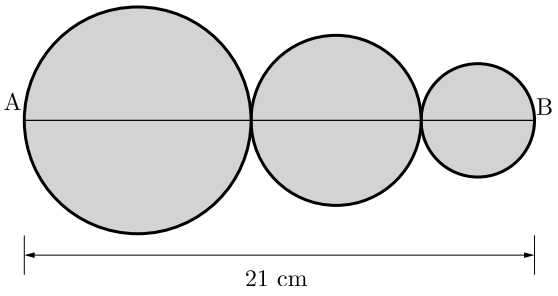
$$r_3^2 \times 91^\circ = 1.3 \times 24^2 \times 70^\circ$$

$$r_3^2 = 24^2 \Rightarrow r_3 = 24 \text{ m}$$

83. Gear : Gears are mechanical parts with cut teeth designed to fit with teeth on another part so as to transmit or receive force and motion. Gears are also sometimes called toothed wheels or cogged wheels or cogs. The cut teeth are also sometimes called cogs.

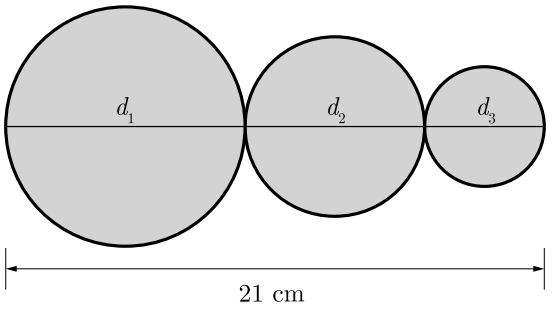


In following figure $AB = 21\,\text{cm}$. Find the perimeter of combined circles.



Sol :

We redraw the given circle as follows.



Perimeter of given figure will be sum of circumference of three circle.

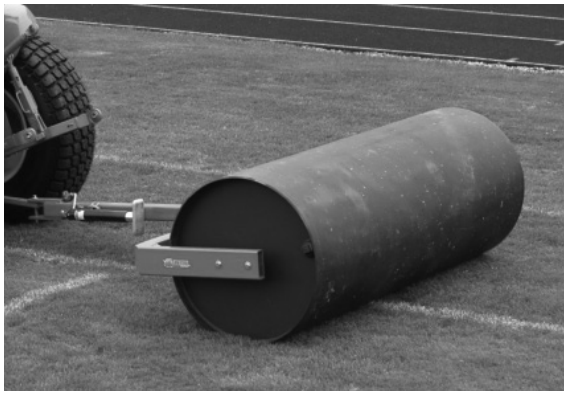
$$P = \pi d_1 + \pi d_2 + \pi d_3$$

$$= \pi(d_1 + d_2 + d_3)$$

$$= \frac{22}{7} \times 21$$

$$= 22 \times 3 = 66\,\text{cm}$$

84. Laying New Sod : Sod, also known as turf, is grass. When harvested into rolls it is held together by its roots and a thin layer of soil. Sod is typically used for lawns, golf courses, and sports stadiums around the world.



When new sod is laid, a heavy roller is used to press the sod down to ensure good contact with the ground beneath. The radius of the roller is 28 cm.

(i) Through what angle has the roller turned after being pulled across 11 meter of yard?

(ii) If it is turned through by 3 revolution and 45°, find the length, that the roller will press.

Sol :

(i) $$P = 2\pi r \frac{\theta}{360^\circ}$$

$$1100 = 2 \times \frac{22}{7} \times 28 \times \frac{\theta}{360^\circ}$$

$$100 = 2 \times 2 \times 4 \times \frac{\theta}{360^\circ}$$

$$100 = 4 \times \frac{\theta}{90^\circ}$$

$$\theta = \frac{100 \times 900}{4} = 2250^\circ$$

$$= 360^\circ \times 6 + 90^\circ$$

(ii) $$P = 2\pi r \frac{\theta}{360^\circ}$$

$$= 2 \times \frac{22}{7} \times 28 \times \frac{1125}{360^\circ}$$

$$= 2 \times 22 \times 4 \times \frac{125}{40}$$

$$= 22 \times 8 \times \frac{25}{8}$$

$$= 22 \times 25$$

$$= 550\,\text{cm} = 5.5\,\text{m}$$

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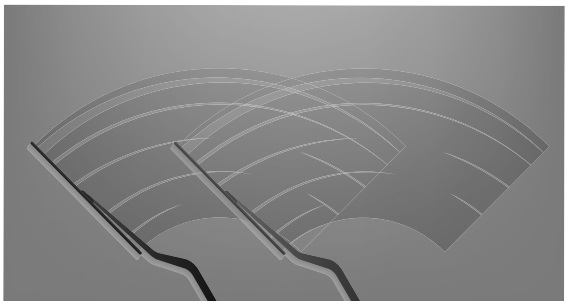
85. Windshield Wiper : Windshield wipers are a small part of your car, but they have a big impact on your driving and overall safety. They remove rain, snow, dirt, pollen, frost and other debris quickly and smoothly at the push of a button. The windshield wiper motor moves the windshield wiper arms across the windshield. The metal or hard plastic arms drag a thin rubber (or silicone) blade across the windshield to clear away water, giving you a better view of the road.



The armature for the rear windshield wiper has a length of 63 cm, with a rubber wiper blade that is 42 cm long. What area of my rear windshield is cleaned as the armature swings back-and-forth through an angle of 110° ?

Sol :

Area swept by full windshield wiper,

$$A_w = \pi(63)^2 \frac{110^\circ}{360^\circ}$$

Area swept by metal arm of windshield wiper,

$$A_m = \pi(42)^2 \frac{110^\circ}{360^\circ}$$

Area swept by rubber part of windshield wiper or area cleaned,

$$\begin{aligned} A_{\text{clean}} &= \pi\frac{110^\circ}{360^\circ}(63^2 - 42^2) \\ &= \frac{22}{7} \times \frac{11}{36} \times 3^2(21^2 - 14^2) \\ &= \frac{22}{7} \times \frac{11}{36} \times 9(21+14)(21-14) \\ &= \frac{22}{7} \times \frac{11}{4} \times 35 \times 7 \\ &= \frac{22 \times 11 \times 35}{4} \\ &= 2117.5\,\text{cm}^2 \end{aligned}$$

86. Electric Hoist : It is an electrically powered appliance that is used to lift, lower and even move heavy or awkward objects. They are mainly used to alleviate potential strain and injury on any person who needs to lift a heavy object or where the object is just much too heavy for a human to lift unaided.

An electric hoist is being used to lift a beam (see figure). The diameter of the drum on the hoist is 42 cm, and the beam must be raised 22 cm. Find the number of degrees through which the drum must rotate.

Sol :

Let drum must rotate by θ to raise beam by 22 cm.

Now $$\begin{aligned} h &= 2\pi r\frac{\theta}{360^\circ} \\ 22 &= 2 \times \frac{22}{7} \times \frac{42}{2} \times \frac{\theta}{360^\circ} \\ 1 &= 6 \times \frac{\theta}{360^\circ} = \frac{\theta}{60^\circ} \\ \theta &= 60^\circ \end{aligned}$$

87. Voltmeter : A voltmeter is an instrument used for measuring electric potential difference between two points in an electric circuit. When current carrying conductor placed in a magnetic field, a mechanical force acts on the conductor, if it is attached to a moving system, with the coil movement, the pointer moves over the scale.

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$$= 22 \times 3 \times 21 = 1386 \text{ cm}^2$$

Radius of cloth for large cushion,

$$r_l = 23 + 5 = 28 \text{ cm}$$

Area $A = \pi r_l^2$

$$= \frac{22}{7} \times 28^2$$

$$= 22 \times 4 \times 28 = 2464 \text{ cm}^2$$

We require 7 small cushions and 1 large cushions we have to multiply total area by 2 also because cloth will be used in both side.

Total Area $= 2(7 \times 1386 + 2464)$

$$= 24332 \text{ cm}^2$$

79. Swimming Pool : A swimming pool or simply pool is a structure designed to hold water to enable swimming or other leisure activities. Pools can be built into the ground (in-ground pools) or built above ground. In-ground pools are most commonly constructed from materials such as concrete, natural stone, metal, plastic, or fiberglass, and can be of a custom size and shape or built to a standardized size, the largest of which is the Olympic-size swimming pool.

The area of a circular pool is 616 square metre. The owner wants to replace the tiling at the edge of the pool.

(i) The edging is 25 cm wide, so she plans to use 6-inch square tiles to form a continuous inner edge. How many tiles will she need to purchase?

(ii) Once the square tiles are in place around the pool, there will be extra space between the tiles. What shape of tile will best fill this space? How many tiles of this shape should she purchase?

Sol :

Let r be radius of pool.

(i) Area of pool, $A = \pi r^2$

$$616 = \frac{22}{7} r^2$$

$$r^2 = \frac{616 \times 7}{22} = 28 \times 7 = 196$$

$$r = \sqrt{196} = 14$$

Circumference of pool,

$$C = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 14 = 176 \text{ m}$$

If tile 25 cm wide, we have to place 4 tile per meter. Therefore total $176 \times 4 = 704$ tiles for whole circumference.

(ii) The square tiles will touch along the inner edge of the border, but there will be gaps along the outer edge. The tiles used to fill the gaps should be triangles. There will be 704 gaps between the 704 square tiles, so 704 triangular tiles will be needed.

80. Bakery : A bakery is an establishment that produces and sells flour-based food baked in an oven such as bread, cookies, cakes, pastries, and pies. Some retail bakeries are also categorized as cafés, serving coffee and tea to customers who wish to consume the baked goods on the premises.

Tania runs a bakery shop and her bakery is very famous for tasty cake. She sells single-layer mini-cakes that are 3 inches in diameter for Rs 80 each. They also have a 9 inch cake for Rs 645. If both cakes are the same thickness, which option gives you more cake for the money, nine mini-cakes or one 9-inch cake? Explain.

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Now
$$\begin{aligned} A_2 &= A_1 \\ \pi r_2^2 \frac{\theta_2}{360^\circ} &= \pi r_1^2 \frac{\theta_1}{360^\circ} \\ r_2^2\theta_2 &= r_1^2\theta_1 \\ \theta_2 &= \left(\frac{r_1}{r_2}\right)^2 \theta_1 \\ \theta_2 &= \left(\frac{21}{28}\right)^2 \times 80 \\ &= \left(\frac{3}{4}\right)^2 \times 80 \\ &= \frac{9}{16} \times 80 = 45^\circ \end{aligned}$$

90. Pulley : A pulley is a wheel with a groove along its edge for holding a rope or cable. It is a simple machine that helps change the direction and point of application of a pulling force. Pulleys are used singly or in combination to transmit energy and motion.

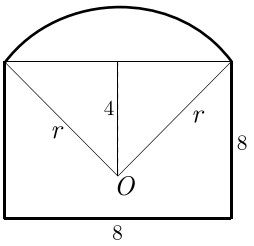
A boy is moving a box upward through the use of a pulley 21 cm in radius. If the pulley is rotated counterclockwise through an angle of 102°, find the height, that the weight will rise.

Sol :

Let weight rise by h, when pulley rotated through an angle of 102°,

$$\begin{aligned} h &= 2\pi r \frac{\theta}{360^\circ} \\ &= 2 \times \frac{22}{7} \times 21 \times \frac{102^\circ}{360^\circ} \\ &= 2 \times 22 \times 3 \times \frac{17}{60} \\ &= \frac{22 \times 17}{10} = 37.4 \text{ cm} \end{aligned}$$

91. Barn : A barn is an agricultural building usually on farms and used for various purposes. A barn refers to structures that house livestock, including cattle and horses, as well as equipment and fodder, and often grain.



Ramkaran want to build a barn at his farm. He has made a design for it which is shown above. Here roof is arc of a circle of radius r at centre O.

(i) What is the value of radius of arc ?

(ii) What is the curved width of roof ?

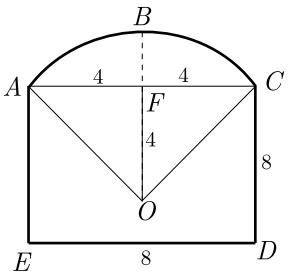
(iii) What is area of cross section of barn ?

(iv) If the length of the barn is 12 meters, what is the curved surface area of roof?

(v) What is the storage capacity of barn ?

Sol :

(i) We redraw the cross section of barn as shown below.



In right triangle ΔAFO,
$$\begin{aligned} AO &= \sqrt{AF^2 + FO^2} \\ &= \sqrt{4^2 + 4^2} = 4\sqrt{2} \text{ m} \end{aligned}$$

Thus $AO = 4\sqrt{2}$ which is also radius of curved arc.

(ii) In right angle triangle ΔAFO is also isosceles triangle

Thus, $\angle FAO = \angle FOA = 45^\circ$

Similarly, $\angle FOC = 45^\circ$

Thus $\angle AOC = \angle AOF + \angle FOC$
$= 45^\circ + 45^\circ = 90^\circ$

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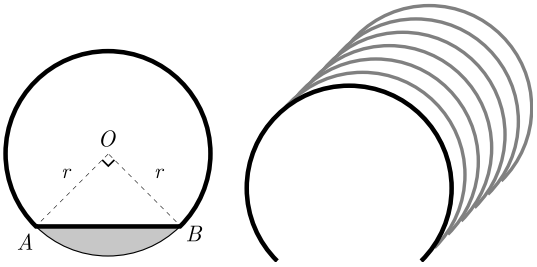
The space below AB is filled with rubble (debris from the demolition buildings).



Steel girders are erected above the tracks to strengthen the tunnel. The girders are erected at 6 m intervals along the length of the tunnel, with one at each end.

(i) What is the cross section area of tunnel before filling debris on ground plane ?

(ii) What is the area of cross section of tunnel after filling debris on ground plane?

(iii) What is the length of each girder ?

(iv) How many girders are erected ?

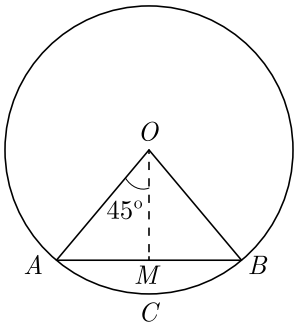
(v) If the weight of 1 meter girder is 25 kg, how much steel is required ?

Sol :

(i) Cross section area of tunnel before filling debris on ground plane,

$$\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ m}^2$$

(ii) The geometry of cross-section is shown below.



Triangle OAB is isosceles triangle having right angle at O.

Thus area of $\Delta AOB = \frac{r^2}{2}$

Area of circular section $OACB$

$$= \frac{\pi r^2 \theta}{360^\circ} = \frac{\pi r^2 90^\circ}{360^\circ} = \frac{\pi r^2}{4}$$

Area of cross section of tunnel,

$$= \text{Area of circle} - \text{Area } AMBC$$

$$= \text{Area of circle} - (\text{Area of } OACB - \text{Area of } \Delta OAB)$$

$$= \pi r^2 - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$$

$$= \frac{3\pi r^2}{4} + \frac{r^2}{2} = \frac{r^2}{4}(3\pi + 2)$$

$$= \frac{7 \times 7}{4}\left(\frac{3 \times 22}{7} + 2\right) = 140 \text{ m}^2$$

(iii) Length of each girder is length of curved part of cross-section,

$$= \frac{2\pi r(360^\circ - 90^\circ)}{360^\circ}$$

$$= \frac{2 \times \frac{22}{7} \times 7(360^\circ - 90^\circ)}{360^\circ}$$

$$= \frac{2 \times 22 \times 270^\circ}{360^\circ} = 33 \text{ m}$$

(iv) Since 1 girder is placed at every 6 m, total girder required,

$$= \frac{450}{6} + 1 = 75 + 1 = 76 \text{ girder}$$

(v) Total requirement of girder

$$= 76 \times 33 = 2508 \text{ meter}$$

Total weight of girder

$$= 2508 \times 25$$

$$= 62700 \text{ kg} = 627 \text{ quintal}$$

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The shapes of the hockey court and the shooting range are square and triangle respectively. Both of the courts have a common edge that touches the centre of stadium. The construction of the shooting range is such that the angle to centre is 90°. The radius of the stadium is 200 metres.

(i) What is the area allotted to shooting range ?

(ii) What is the area allotted to hockey court ?

(iii) If the team of the curators managing the stadium, likes to allot space for some more sports, how much area is available to them?

(iv) If the boundaries of the hockey court and shooting range are to be fenced, then what is the required length of the fence ?

(v) If the cost of fencing is Rs 6 per metre, what is the total cost of fencing ?

Sol :

(i) Here ΔAOB is a right-angled triangle in which AB is a hypotenuse.

Now $AO = OB$

$= \text{radius of circle} = 200\,\text{m}$

Thus area of ΔAOB,

$$= \frac{1}{2} \times OA \times OB$$

$$= \frac{1}{2} \times 200 \times 200$$

$$= 20{,}000\,\text{m}^2$$

(ii) Here $OCDE$ is a square whose diagonal is equal to the radius of the circle. Let a be side of square.

Now $a^2 + a^2 = (200)^2$

$2a^2 = 200 \times 200$

$a = \sqrt{100 \times 100 \times 2}$

$= 100\sqrt{2}\ \text{m}$

Area of square $a^2 = \left(100\sqrt{2}\right)^2 = 20{,}000\,\text{m}^2$

Area of hockey court is equal to area of shooting court.

(iii) Unoccupied area of stadium,

$=$Area of circle $-$ (Area of hockey court + area of shooting court)

$= \pi r^2 - (20{,}000 + 20{,}000)$

$= 40{,}000\pi - 40{,}000$

$= 40000\,(\pi - 1)$

$= 40{,}000 \times 2.14 = 85{,}600\ \text{m}^2$

(iv) Boundaries need to be fenced

$=$ perimeter of triangle + perimeter of square

For triangle, length of AB,

$AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{200^2 + 200^2} = 200\sqrt{2}\ \text{m}$

Perimeter of triangle,

$OA + OB + AB = 200 + 200 + 200\sqrt{2}$

$= 400 + 200\sqrt{2}$

Perimeter of square,

$4a = 4 \times 100\sqrt{2} = 400\sqrt{2}$

Boundary need to be fenced,

$= 400 + 200\sqrt{2} + 400\sqrt{2}$

$= 400 + 600\sqrt{2}$

$= 200\left(2 + 3\sqrt{2}\right)\ \text{m}$

(v) Cost of Fencing $=$ Length of Fence $\times$ Rate

$= 200\left(2 + 3\sqrt{2}\right) \times 6$

$= 1200\left(2 + 3\sqrt{2}\right)$

94. Tunnel : The tunnels are defined as the underground passages that are used for the transportation purposes. These permit the transmission of passengers and freights, or it may be for the transportation of utilities like water, sewage or gas etc. The tunnel engineering is one of the most interesting disciplines in engineering. The work is complex and difficult throughout its course, even though it is interesting.

Earth is excavated to make a road tunnel. The tunnel is a cylinder of radius 7 m and length 450 m.

A level surface is laid inside the tunnel to make road. Figure shows the circular cross - section of the tunnel. The level surface is represented by AB , the centre of the circle is O and $\angle AOB = 90^\circ$.

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The pointer on a voltmeter is 7 centimeters in length (see figure). Find the angle through which the pointer rotates when it moves 5.5 centimeters on the scale.

Sol :

Let the pointer rotates by θ when it moves 5.5 centimetres on the scale

Now $5.5 = 2\pi r\frac{\theta}{360^\circ}$

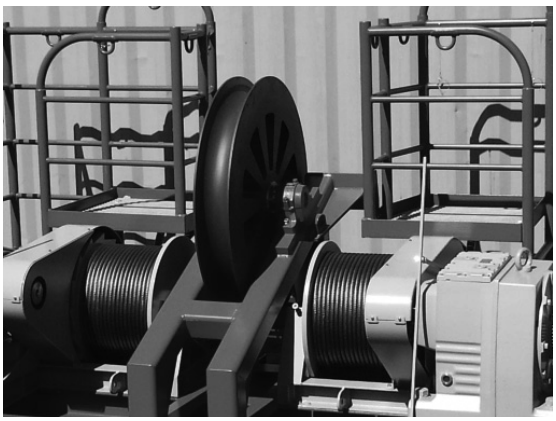
$5.5 = 2\times\frac{22}{7}\times 7\times\frac{\theta}{360^\circ}$

$11 = 4\times 22\times\frac{\theta}{360^\circ}$

$1 = 2\times\frac{\theta}{90^\circ}$

$\theta = \frac{90^\circ}{2} = 45^\circ$

88. Cable Winch: It is a device for hauling or lifting; made up of a rope, cable or chain wound around a horizontal rotating drum and turned by a crank or motor and typically mounted at the rear of a towing vehicle.



A large winch with a radius of 30 cm winds in 99 cm of cable.

(i) Through what angle has it turned?

(ii) If it is turned through an angle of 126°, find the length, that the winch will wind.

Sol :

(i) Let cable winch rotate by θ_1 to wind 99 cm.

Here $r = 30$ cm

Now $99 = 2\pi r\frac{\theta_1}{360^\circ}$

$99 = 2\times\frac{22}{7}\times 30\times\frac{\theta_1}{360^\circ}$

$9 = 2\times\frac{2}{7}\times\frac{\theta_1}{12}$

$9 = \frac{1}{7}\times\frac{\theta_1}{3}$

$\theta_1 = 9\times 7\times 3 = 189^\circ$

(ii) Let winch wind by l when it is turned through 126°.

$l = 2\pi r\frac{\theta_2}{360^\circ}$

$= 2\times\frac{22}{7}\times 30\times\frac{126^\circ}{360^\circ}$

$= 2\times 22\times\frac{18}{12}$

$= 2\times 22\times\frac{3}{2} = 66$ cm

89. Sprinkler : An irrigation sprinkler (also known as a water sprinkler or simply a sprinkler) is a device used to irrigate agricultural crops, lawns, landscapes, golf courses, and other areas. Irrigation sprinklers can be used for residential, industrial, and agricultural usage. It is useful on uneven land where sufficient water is not available as well as on sandy soil.

A water sprinkler is set to shoot a stream of water a distance of 21 m and rotate through an angle of 80°.

(i) What is the area of the lawn it waters?

(ii) For $r = 28$ m, what angle is required to water equal to previous area?

Sol :

(i) Let $r_1 = 21$ m and $\theta_1 = 80^\circ$

Area of the lawn watered,

$A_1 = \pi r_1^2\frac{\theta_1}{360^\circ}$

$= \frac{22}{7}\times 21^2\times\frac{80^\circ}{360^\circ}$

$= \frac{22}{7}\times 21\times 21\times\frac{2}{9}$

$= \frac{22}{7}\times 7\times 7\times 2$

$= 22\times 7\times 2 = 308\,\text{m}^2$

(ii) Let $r_2 = 28$ m and θ_2 be the angle for the same area.

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CHAPTER 12

Surface Areas and Volumes

CBSE has removed melting questions from exam 2023 but these question doesn't involve any extra concept. These questions are very useful to learn concept appearing in exam 2025. So we have included these question. However we have light marked these question, If you want you can skip these question.

OBJECTIVE QUESTIONS

1. The volume of the largest right circular cone that can be carved out from a solid cube of edge 2 cm is

(a) $\frac{4\pi}{3}$ cu cm (b) $\frac{5\pi}{3}$ cu cm

(c) $\frac{8\pi}{3}$ cu cm (d) $\frac{2\pi}{3}$ cu cm

Sol : [Board 2024 OD Standard]

From a 2 cm edge of cube we can carved out a right circular cone of height 2 cm and base diameter 2 cm.

Volume of such right circular cone,

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{2}{2}\right)^2 \times 2$$

$$= \frac{1}{3}\pi \times (1)^2 \times 2$$

$$= \frac{2\pi}{3} \text{ cu cm}$$

Thus (d) is correct option.

2. A solid sphere is cut into two hemispheres. The ratio of the surface areas of sphere to that of two hemispheres taken together, is

(a) 1 : 1 (b) 1 : 4

(c) 2 : 3 (d) 3 : 2

Sol : [Board 2024 OD Standard]

Surface area of a solid sphere

$$S_1 = 4\pi r^2$$

Surface area of a solid hemisphere

$$S_2 = 2\pi r^2 + \pi r^2 = 3\pi r^2$$

Surface area of a two solid hemisphere

$$2S_2 = 6\pi r^2$$

The ratio of the surface areas of sphere to that of two hemispheres taken together,

$$\frac{S_1}{2S_2} = \frac{4\pi r^2}{6\pi r^2} = \frac{2}{3}$$

Thus (c) is correct option.

3. The volume of a right circular cone whose area of the base is 156 cm^2 and the vertical height is 8 cm, is:

(a) 2496 cm^3 (b) 1248 cm^3

(c) 1664 cm^3 (d) 416 cm^3

Sol : [Board 2023 OD Standard]

Area of base $\pi r^2 = 156\,cm^2$

Volume of cone $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times 156 \times 8$$

$$= 416\,cm^3$$

Thus (d) is correct option.

4. If the radius of the sphere is increased by 100%, the volume of the corresponding sphere is increased by

(a) 200% (b) 500%

(c) 700% (d) 800%

Sol : [Board Term-2 Delhi 2017]

Let r be the original radius of sphere. If we increased radius by 100 %. it will be $2r$.

$$V_r = \frac{4}{3}\pi r^3$$

Now $V_{2r} = \frac{4}{3}\pi \times (2r)^3$

$$= \frac{4}{3}\pi \times 8r^2$$

Thus new volume is 8 times of original volume.

Hence when the radius is increased by 100%, the corresponding volume becomes 800% and thus

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increase is 700%.

Thus (c) is correct option.

5. A sphere is melted and half of the melted liquid is used to form 11 identical cubes, whereas the remaining half is used to form 7 identical smaller spheres. The ratio of the side of the cube to the radius of the new small sphere is

(a) $\left(\frac{4}{3}\right)^{1/3}$ (b) $\left(\frac{8}{3}\right)^{1/3}$

(c) $(3)^{1/3}$ (d) 2

Sol : [Board Term-2 OD 2016]

As per the given conditions,

$$11a^3 = 7 \times \frac{4}{3} \times \frac{22}{7} \times r^3$$

$$\frac{a}{r} = \left(\frac{8}{3}\right)^{1/3}$$

Thus (b) is correct option.

6. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is

(a) 2 : 1 (b) 1 : 2

(c) 1 : 3 (d) 3 : 1

Sol : [Board Term-2 OD 2013]

$$\pi rl = 2\pi rh$$

$$\frac{l}{h} = \frac{2}{1}$$

Thus (a) is correct option.

7. If the perimeter of one face of a cube is 20 cm, then its surface area is

(a) $120\ \text{cm}^2$ (b) $150\ \text{cm}^2$

(c) $125\ \text{cm}^2$ (d) $400\ \text{cm}^2$

Sol : [Board Term-2 Foreign 2014]

Edge of cube, $a = \frac{20}{4}\ \text{cm} = 5\ \text{cm}$

Surface area $6a^2 = 6 \times 5^2\ \text{cm}^2 = 150\ \text{cm}^2$

Thus (b) is correct option.

8. Ratio of lateral surface areas of two cylinders with equal height is

(a) 1 : 2

(b) $H : h$

(c) $R : r$

(d) None of these

Sol : [Board Term-2 Delhi 2017]

$$2\pi Rh : 2\pi rh = R : r$$

Thus (c) is correct option.

9. Volume of a spherical shell is given by

(a) $4\pi(R^2 - r^2)$ (b) $\pi(R^3 - r^3)$

(c) $4\pi(R^3 - r^3)$ (d) $\frac{4}{3}\pi(R^3 - r^3)$

Sol :

$$\text{Volume of spherical shell} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(R^3 - r^3)$$

Thus (d) is correct option.

10. Ratio of volumes of two cylinders with equal height is

(a) $H : h$ (b) $R : r$

(c) $R^2 : r^2$ (d) None of these

Sol : [Board Term-2 Foreign 2017]

$$\pi R^2 h : \pi r^2 h = R^2 : r^2$$

Thus (c) is correct option.

11. Ratio of volumes of two cones with same radii is

(a) $h_1 : h_2$ (b) $s_1 : s_2$

(c) $r_1 : r_2$ (d) None of these

Sol :

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$$

Since $\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_1^2 h_2 \quad (r_1 = r_2)$

$$h_1 : h_2$$

Thus (a) is correct option.

12. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. The length of the wire is

(a) 12 m (b) 18 m

(c) 36 m (d) 66 m

Sol : [Board Term-2 Foreign 2012]

Let the length of the wire be l. Since, metallic sphere is converted into a cylindrical shaped wire of length l,

Volume of the metal used in wire is equal to the volume of the sphere.

$$\pi r^2 l = \frac{4}{3}\pi R^3$$

$$\pi \times \left(\frac{2}{2} \times \frac{1}{10}\right)^2 \times l = \frac{4}{3} \times \pi \times \left(\frac{6}{2}\right)^3$$

$$\pi \times \frac{1}{100} \times h = \frac{4}{3} \times \pi \times 3^3$$

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$$\frac{l}{100} = 4 \times 3^2 = 36$$
$$l = 3600\,\text{cm} = 36\,\text{m}$$

Thus (c) is correct option.

13. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed, then the volume of remaining solid is

(a) $280\,\pi\text{cm}^3$ (b) $330\,\pi\text{cm}^3$

(c) $240\,\pi\text{cm}^3$ (d) $440\,\pi\,\text{cm}^3$

Sol : [Board Term-2 SQP 2016]

Volume of the remaining solid

$$= \text{Volume of the cylinder} - \text{Volume of the cone}$$
$$= \pi \times 6^2 \times 10 - \frac{1}{3} \times \pi \times 6^2 \times 10$$
$$= (360\pi - 120\pi) = 240\pi\,\text{cm}^3$$

Thus (c) is correct option.

14. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

(a) $4\pi r^2$ (b) $6\pi r^2$

(c) $3\pi r^2$ (d) $8\pi r^2$

Sol : [Board Term-2 OD 2011]

Because curved surface area of a hemisphere is $2\pi r^2$ and here, we join two solid hemispheres along their bases of radius r, from which we get a solid sphere.

Hence, the curved surface area of new solid $= 2\pi r^2 + 2\pi r^2 = 4\pi r^2$

Thus (a) is correct option.

15. A right circular cylinder of radius r and height h (where, $h > 2r$) just encloses a sphere of diameter

(a) r (b) $2r$

(c) h (d) $2h$

Sol : [Board Term-2 Delhi 2017]

Because the sphere encloses in the cylinder, therefore the diameter of sphere is equal to diameter of cylinder which is $2r$.

Thus (b) is correct option.

16. During conversion of a solid from one shape to another, the volume of the new shape will

(a) increase (b) decrease

(c) remain unaltered (d) be doubled

Sol : [Board Term-2 SQP 2015]

During conversion of a solid from one shape to another, the volume of the new shape will remain unaltered.

Thus (c) is correct option.

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A solid piece of iron in the form of a cuboid of dimensions $49\,\text{cm} \times 33\,\text{cm} \times 24\,\text{cm}$, is moulded to form a solid sphere. The radius of the sphere is

(a) 21 cm (b) 23 cm

(c) 25 cm (d) 19 cm

Sol : [Board Term-2 OD 2012]

Volume of the sphere = Volume of the cuboid

$$\frac{4}{3}\pi r^3 = 49 \times 33 \times 24 = 38808\,\text{cm}^3$$
$$4 \times \frac{22}{7} r^3 = 38808 \times 3$$
$$r^3 = \frac{38808 \times 3 \times 7}{4 \times 22}$$
$$= 441 \times 21$$
$$r^3 = 21 \times 21 \times 21$$
$$r = 21\,\text{cm}$$

Thus (a) is correct option.

18. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is

(a) 4 cm (b) 3 cm

(c) 2 cm (d) 6 cm

Sol : [Board Term-2 Delhi 2017]

Volume of the twelve solid sphere is equal to the volume of cylinder.

$$V_{12\,\text{sphere}} = V_{\text{cylinder}}$$
$$12 \times \frac{4}{3}\pi r^3 = \pi\left(\frac{2}{1}\right)^2 \times 16$$
$$16\pi r^3 = 16\pi$$
$$r^3 = 1 \Rightarrow r = 1\,\text{cm}$$

Diameter of each sphere, $d = 2r$

$$= 2 \times 1 = 2\,\text{cm}$$

Thus (c) is correct option.

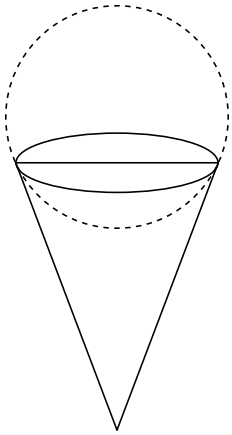
19. In a right circular cone, the cross-section made by a plane parallel to the base is a

(a) circle

(b) frustum of a cone

(c) sphere

(d) hemisphere



32. If two solid hemispheres of same base radius r are joined together along their bases, then find the curved surface area of this new solid.

Sol : [Board 2010]

Because curved surface area of a hemisphere is $2\pi r^2$ and here, we join two solid hemispheres along their bases of radius r, from which we get a solid sphere.

Hence, the curved surface area of new solid $= 2\pi r^2 + 2\pi r^2 = 4\pi r^2$

33. A solid piece of iron in the form of a cuboid of dimensions $49\,\text{cm} \times 33\,\text{cm} \times 24\,\text{cm}$, is moulded to form a solid sphere. What is the radius of the sphere ?

Sol : [Board 2006]

Volume of the sphere = Volume of the cuboid

$$\frac{4}{3}\pi r^3 = 49 \times 33 \times 24 = 38808\,\text{cm}^3$$

$$4 \times \frac{22}{7} r^3 = 38808 \times 3$$

$$r^3 = \frac{38808 \times 3 \times 7}{4 \times 22} = 441 \times 21$$

$$r^3 = 21 \times 21 \times 21$$

$$r = 21\,\text{cm}$$

***34.** Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. What is the diameter of each sphere?

Sol : [Board Term-2 Delhi 201]

Volume of the twelve solid sphere is equal to the volume of cylinder.

$$V_{12\,\text{sphere}} = V_{\text{cylinder}}$$

$$12 \times \frac{4}{3}\pi r^3 = \pi\left(\frac{2}{2}\right)^2 \times 16$$

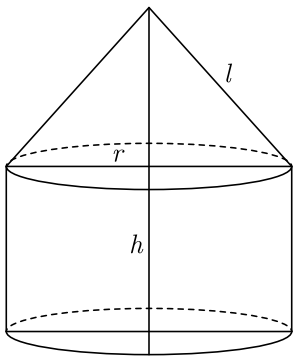
$$16\pi r^3 = 16\pi$$

$$r^3 = 1 \Rightarrow r = 1\,\text{cm}$$

Diameter of each sphere, $d = 2r$

$$= 2 \times 1 = 2\,\text{cm}$$

35. The total surface area of the given solid figure is ..... .



Sol : [Board 2020 SQP Standard]

Given figure is combination of right circular cone and cylinder.

Total surface area

= Area of base of cylinder+

+ Curved surface area of cylinder+

+ Curved surface area of cone

$$= \pi r^2 + 2\pi rh + \pi rl$$

$$= \pi r(r + 2h + l)$$

***36.** A solid metallic cuboid 24 cm × 11 cm × 7 cm is melted and recast into solid cones of base radius 3.5 cm and height 6 cm. Find the number of cones so formed.

Sol : [Board Term-2 Foreign 2011]

Let n be the number of cones formed.

Now, according to question,

Volume of n cones = Volume of cuboid

$$n \times \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6 = 24 \times 11 \times 7$$

$$n = \frac{24 \times 11 \times 7 \times 3 \times 7}{22 \times 3.5 \times 3.5 \times 6} = 24$$

Thus $n = 24$.

37. The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3. Find the ratio of its height to its diameter.

Sol : [Board Term-2 2014]

Curved Surface area of cylinder is $2\pi rh$ and volume of cylinder $\pi r^2 h$.

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$$= \sqrt{49 + 576}$$
$$= \sqrt{625}$$
$$= 25$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

ONE MARK QUESTIONS

24. If the radius of the sphere is increased by 100%, then how much volume of the corresponding sphere is increased ?

Sol : [Board Term-2 Delhi 2012]

Let r be the original radius of sphere. If we increased radius by 100 %. it will be $2r$.

$$V_r = \frac{4}{3}\pi r^3$$

Now $$V_{2r} = \frac{4}{3}\pi \times (2r)^3$$
$$= \frac{4}{3}\pi \times 8r^3$$

Thus new volume is 8 times of original volume.

Hence when the radius is increased by 100%, the corresponding volume becomes 800% and thus increase is 700%.

*25. A sphere is melted and half of the melted liquid is used to form 11 identical cubes, whereas the remaining half is used to form 7 identical smaller spheres. Find the ratio of the side of the cube to the radius of the new small sphere.

Sol : [Board Term-2 OD 2017]

As per the given conditions,

$$11a^3 = 7 \times \frac{4}{3} \times \frac{22}{7} \times r^3$$
$$\frac{a}{r} = \left(\frac{8}{3}\right)^{1/3}$$

26. Find the ratio of volumes of two cones with same radii.

Sol : [Board 2010]

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$$
$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_1^2 h_2 \qquad (r_1 = r_2)$$
$$h_1 : h_2$$

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27. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then what is the ratio of the slant height of the cone to the height of the cylinder?

Sol : [Board Term-2 Delhi 2013]

We have $$\pi r l = 2\pi r h$$
$$\frac{l}{h} = \frac{2}{1}$$

28. Find the ratio of lateral surface areas of two cylinders with equal height.

Sol : [Board Term-2 Foreign 2016]

$$2\pi Rh : 2\pi rh = R : r$$

29. Find the ratio of volumes of two cylinders with equal height.

Sol : [Board Term-2 Delhi 2014]

$$\pi R^2 h : \pi r^2 h = R^2 : r^2$$

*30. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. What is the length of the wire?

Sol : [Board 2009]

Let the length of the wire be l. Since, metallic sphere is converted into a cylindrical shaped wire of length l,

Volume of the metal used in wire is equal to the volume of the sphere.

$$\pi r^2 l = \frac{4}{3}\pi R^3$$
$$\pi \times \left(\frac{2}{2} \times \frac{1}{10}\right)^2 \times l = \frac{4}{3} \times \pi \times \left(\frac{6}{2}\right)^3$$
$$\pi \times \frac{1}{100} \times h = \frac{4}{3} \times \pi \times 3^3$$
$$\frac{l}{100} = 4 \times 3^2 = 36$$
$$l = 3600 \text{ cm}$$
$$= 36 \text{ m}$$

31. The diameter of hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, what portion of the ball will be outside the cone?

Sol : [Board 2007]

Though it is given that diameter of the cone is equal to the diameter of the spherical ball. But the ball will not fit into the cone because of its slant shape. Hence more than 50% of the portion of the ball will be outside the cone.

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Sol :

In a right circular cone, if any cut is made parallel to its base, the result would be the base of the cone, which in cross-section is a circle.

Thus (a) is correct option.

20. Volumes of two spheres are in the ratio 64 : 27. The the ratio of their surface areas is

(a) 3 : 4 (b) 4 : 3

(c) 9 : 16 (d) 16 : 9

Sol : [Board Term-2 OD 2011]

Let the radii of the two spheres are r_1 and r_2, respectively.

Given, ratio of their volumes,

$$V_1 : V_2 = 64 : 27$$

$$\frac{V_1}{V_2} = \frac{64}{27}$$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\frac{r_1^3}{r_2^3} = \frac{64}{27}$$

$$\frac{r_1}{r_2} = \frac{4}{3}$$

Now, ratio of their surface area,

$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Hence, the required ratio of their surface area is 16 :9.

Thus (d) is correct option.

21. **Assertion :** The slant height of the frustum of a cone is 5 cm and the difference between the radii of its two circular ends is 4 cm. Then the height of the frustum is 3 cm.

Reason : Slant height of the frustum of the cone is given by $l = \sqrt{(R-r)^2 + h^2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

We have, $l = 5\,\text{cm},\ R - r = 4\,\text{cm}$

$$5 = \sqrt{(4)^2 + h^2}$$

$$16 + h^2 = 25$$

$$h^2 = 25 - 16 = 9$$

$$h = 3\,\text{cm}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

22. **Assertion :** Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is $3872\,\text{cm}^2$

Reason : If r be the radius and h be the height of the cylinder, then total surface area $= (2\pi rh + 2\pi r^2)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Total surface area,

$$2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 14(30 + 14) = 88(44)$$

$$= 3872\,\text{cm}^2$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

23. **Assertion :** If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.

Reason : If r be the radius and h be the slant height of the cone, then slant height $= \sqrt{h^2 + r^2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Slant height $l = \sqrt{\left(\frac{14}{2}\right)^2 + (24)^2}$

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Now $\dfrac{\pi r^2 h}{2\pi r h} = \dfrac{924}{264}$

$\dfrac{r}{2} = \dfrac{7}{2} \Rightarrow r = 7$

Substituting $r = 7$ in $2\pi r h = 264$ we have

$2 \times \dfrac{22}{7} \times 7 \times h = 264$

$h = 6$ m

Now $\dfrac{h}{2r} = \dfrac{6}{14} = \dfrac{3}{6}$

Hence, $h : d = 3 : 7$

38. A rectangular sheet paper 40 cm $\times$ 22 cm is rolled to form a hollow cylinder of height 40 cm. Find the radius of the cylinder.

Sol : [Board Term-2 Foreign 2014]

Here, $h = 40$ cm, circumference $= 22$ cm

$2\pi r = 22$

$r = \dfrac{22 \times 7}{2 \times 22}$

$= \dfrac{7}{2} = 3.5$ cm

39. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.

Sol : [Board Term-2 Delhi 2014]

$V_{\text{cylinder}} : V_{\text{cone}} : V_{\text{hemisphere}} = \pi r^2 h : \dfrac{1}{3}\pi r^2 h : \dfrac{2}{3}\pi r^3$

$= \pi r^2 r : \dfrac{1}{3}\pi r^2 r : \dfrac{2}{3}\pi r^3 \qquad (h = r)$

$= 1 : \dfrac{1}{3} : \dfrac{2}{3}$

$= 3 : 1 : 2$

40. What is the volume of a right circular cylinder of base radius 7 cm and height 10 cm ? Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 2012]

We have $r = 7$ cm, $h = 10$ cm,

Volume of cylinder,

$\pi r^2 h = \dfrac{22}{7} \times (7)^2 \times 10$

$= 1540$ cm^3

41. What is the ratio of the total surface area of the solid hemisphere to the square of its radius.

Sol : [Board Term-2, 2012]

$\dfrac{\text{Total surface area of hemisphere}}{\text{Square of its radius}} = \dfrac{3\pi r^2}{r^2} = \dfrac{3\pi}{1}$

Thus required ratio is $3\pi : 1$.

42. The radius of sphere is r cm. It is divided into two equal parts. Find the whole surface of two parts.

Sol : [Board Term-2 2012]

Whole surface of each part

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

Total surface of two parts

$= 2 \times 3\pi r^2 = 6\pi r^2$

43. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3, find the ratio of their volumes.

Sol : [Board Term-2 OD 2012]

$\dfrac{\text{Volume of 1}^{\text{st}}\text{cylinder}}{\text{Volume of 2}^{\text{nd}}\text{cylinder}} = \dfrac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$

$= \left(\dfrac{r_1}{r_2}\right)^2 \times \dfrac{h_1}{h_2}$

$= \left(\dfrac{2}{3}\right)^2 \times \dfrac{5}{3}$

$= \dfrac{4}{9} \times \dfrac{5}{3} = \dfrac{20}{27}$

$= 20 : 27$

44. If the radius of the base of a right circular cylinder is halved, keeping the height same, find the ratio of the volume of the reduced cylinder to that of original cylinder.

Sol : [Board Term-2 2012]

$\dfrac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \dfrac{\pi \times \left(\frac{r}{2}\right)^2 h}{\pi r^2 h} = \dfrac{1}{4}$

$= 1 : 4$

45. Volume of two spheres are in the ratio 64 : 27, find the ratio of their surface areas.

Sol : [Board Term-2 Delhi 2012]

$\dfrac{\text{Volume of I}^{\text{st}}\text{sphere}}{\text{Volume of II}^{\text{nd}}} = \dfrac{64}{27}$

$\dfrac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \dfrac{64}{27}$

$\dfrac{r_1^3}{r_2^3} = \dfrac{4^3}{3^3}$

$\dfrac{r_1}{r_2} = \dfrac{4}{3}$

Ratio of their surface areas,

$\dfrac{2\pi r_1^2}{4\pi r_2^2} = \left(\dfrac{r_1}{r_2}\right)^2$

$= \left(\dfrac{4}{3}\right)^2 = \dfrac{16}{9}$

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46. Find the volume (in cm^3) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm.

Sol : [Board Term-2 OD 2012]

$$\text{Edge of the cube} = 4.2 \text{ cm.}$$

$$\text{Height of the cone} = 4.2 \text{ cm.}$$

$$\text{Radius of the cone} = \frac{4.2}{2} = 2.1 \text{ cm.}$$

Volume of the cone,

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 4.2$$
$$= 19.4 \text{ cm}^3$$

47. The circumference of the edge of a hemisphere bowl is 132 cm. When π is taken as $\frac{22}{7}$, find the capacity of the bowl in cm^3.

Sol : [Board Term-2 2012]

Let r be the radius of bowl, then circumference of bowl,

$$2\pi r = 132$$
$$r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Capacity i.e. volume of the bowl,

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$
$$= 19404 \text{ cm}^3$$

48. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

Sol : [Board Term-2 Delhi 2017]

Let radius of sphere be r.

Now Volume of sphere= S.A. of hemisphere

$$\frac{2}{3}\pi r^3 = 3\pi r^2$$
$$r = \frac{9}{2} \text{ units}$$

Diameter $$d = \frac{9}{2} \times 2 = 9 \text{ units}$$

***49.** Three solid metallic spherical balls of radii 3 cm, 4 cm and 5 cm are melted into a single spherical ball, find its radius.

Sol : [Board Term-2, 2014]

Let the radius of spherical ball be r.

Volume of spherical ball = Volume of three balls

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi[3^3 + 4^3 + 5^3]$$
$$r^3 = 27 + 64 + 125 = 216$$
$$r = 6 \text{ cm}$$

***50.** A solid sphere of radius r melted and recast into the shape of a solid cone of height r. Find the radius of the base of a cone.

Sol : [Board Term-2 Delhi 2012]

Let the radius of cone be R cm.

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 \times r$$
$$4r^3 = R^2 r$$
$$R^2 = 4r^2$$
$$R = 2r$$

51. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

Sol : [Board 2020 Delhi Standard]

Let h_1 and h_2 be height and r_1 and r_2 be radii of two cones.

Now $$\frac{h_1}{h_2} = \frac{1}{3} \text{ and } \frac{r_1}{r_2} = \frac{3}{1}$$

Ratio of their volumes,

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$
$$= \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right) = \left(\frac{3}{1}\right)^2 \left(\frac{1}{3}\right) = \frac{3}{1}$$

Hence, ratio of their volumes is 3 :1.

***52.** A solid metallic cone of radius 2 cm and height 8 cm is melted into a sphere. Find the radius of sphere.

Sol : [Board Term-2 2014]

Let R be the radius of sphere.

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$
$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (2)^2 \times 8$$
$$4R^3 = 4 \times 8$$
$$R^3 = 8 \Rightarrow R = 2 \text{ cm}$$
$$= 1 : 8$$

53. A glass cylinder with radius 10 cm has water to a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. Calculate the height by which water will rise in the cylinder. Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 2013]

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Let h be the height of water raised measured.

Volume of water displaced in cylinder $= \pi(10)^2 h$

Volume of cube,

$$\pi(10)^2 h = 8 \times 8 \times 8$$

$$h = \frac{8 \times 8 \times 8 \times 7}{22 \times 10 \times 10}$$

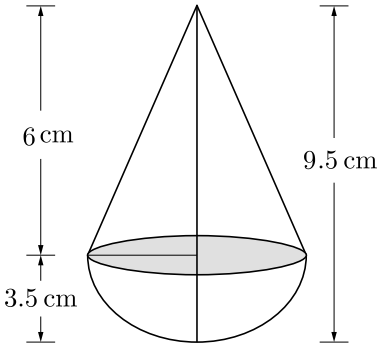
$$= 1.629 \text{ cm.}$$

TWO MARKS QUESTIONS

54. A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Sol : [Board Term-2 Delhi 2013]

As per question the figure is shown below. Here total volume of the toy is equal to the sum of volume of hemisphere and cone.



Volume of toy,

$$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times (6 + 2 \times 3.5)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times (6 + 7)$$

$$= \frac{1}{3} \times \frac{22}{2} \times 3.5 \times 13$$

$$= \frac{1}{3} \times 11 \times 3.5 \times 13$$

$$= \frac{500.5}{3} = 166.83 \text{ cm}^3 \qquad \text{(Approx)}$$

Hence, the volume of the solid is 166.83 cm^3.

55. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm?

Sol : [Board Term-2 OD 2017]

Let t be the time in which the level of the water in the tank will rise by 21 cm.

Length of water that flows in 1 hour is 15 km or 15000 m.

Radius of pipe is $\frac{14}{2} = 7$ cm or 0.07 m.

Volume of water in 1 hour,

$$= \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 15000 = 231 \text{ m}^3$$

Volume of water in time t,

$$= 231t \text{ m}^3$$

This volume of water is equal to the water flowed into the cuboidal pond which is 50 m long, 44 m wide and 0.21 m high.

Thus $231t = 50 \times 44 \times 0.21$

$$t = \frac{50 \times 44 \times 0.21}{231} = 2 \text{ Hours}$$

56. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

Sol : [Board Term-2 Delhi 2016]

Radius of conical heap $r = 12$ m

Height of heap, $h = 3.5$ m

Volume of rice,

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5 \text{ m}^3$$

$$= 528 \text{ m}^3$$

Slanted height,

$$l = \sqrt{12^2 + (3.5)^2} = 12.5 \text{ m}$$

Area of canvas cloth required,

$$\pi r l = \frac{22}{7} \times 12 \times 12.5$$

$$= 471.4 \text{ m}^2$$

57. Isha is 10 years old girl. On the result day, Isha and her father Suresh were very happy as she got first position in the class. While coming back to their home, Isha asked for a treat from her father as a reward for her success. They went to a juice shop and asked for two glasses of juice.

Aisha, a juice seller, was serving juice to her customers in two types of glasses.

Both the glasses had inner radius 3 cm. The height of both the glasses was 10 cm.

First Type : A glass with hemispherical raised bottom.

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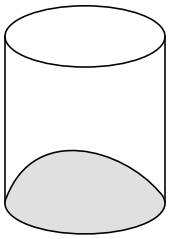
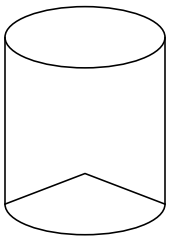


Second Type : A glass with conical raised bottom of height 1.5 cm.



Isha insisted to have the juice in first type of glass and her father decided to have the juice in second type of glass. Out of the two, Isha or her father Suresh, who got more quantity of juice to drink and by how much?

Sol : [Board 2020 SQP Standard]

Let H and h be the height of cylinder and height of cone. Let r be the common radius of cone and cylinder and hemisphere.

Capacity of first glass,

$$= \text{Volume of cylinder} - \text{Volume of hemisphere}$$
$$= \pi r^2 H - \frac{2}{3}\pi r^3$$
$$= \pi r^2\left[H - \frac{2}{3}r\right]$$
$$= \pi \times (3)^2\left[10 - \frac{2}{3}\times 3\right]$$
$$= 9\pi \times 8 = 72\pi \text{ cm}^2$$

Capacity of second glass,

$$= \text{Volume of cylinder} - \text{Volume of cone}$$
$$= \pi r^2 H - \frac{1}{3}\pi r^2 h$$
$$= \pi r^2\left[H - \frac{1}{3}h\right]$$
$$= \pi (3)^2\left[10 - \frac{1}{3}\times 1.5\right]$$
$$= 9\pi \times 9.5 = 85.5\pi \text{ cm}^2$$

Therefore Suresh got more juice of quantity,

$$= 85.5\pi - 72\pi \text{ cm}^2$$
$$= 13.5\pi \text{ cm}^3$$

58. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere.

Sol : [Board Term-2 2012]

Here diameter of sphere is equal to the radius of hemisphere which is 6 cm.

$$\text{Diameter of sphere} = \text{Radius of hemisphere}$$
$$= 6 \text{ cm}$$
$$\text{Radius of sphere} = 3 \text{ cm}$$

Volume, $V = \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times\frac{22}{7}\times 3^3 \text{ cm}^3.$$
$$= 113.14 \text{ cm}^3.$$

59. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 OD 2014]

Total surface area of hemisphere,

$$3\pi r^2 = 462 \text{ cm}^2$$
$$\frac{22r^2}{7} = \frac{462}{3}$$
$$r^2 = \frac{462\times 7}{22\times 3} = 49$$
$$r = 7 \text{ cm}.$$

Volume of hemisphere,

$$\frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{7}\times 7\times 7\times 7$$
$$= \frac{2156}{3}$$
$$= 718.67 \text{ cm}^3.$$

60. A solid metallic of dimensions 9m $\times$ 8m $\times$ 2 m is melted and recast into solid cubes of edge 2 m. Find the number of cubes so formed.

Sol : [Board Term-2 Foreign 2017]

$$\text{Volume of cuboid} = 9\times 8\times 2 \text{ cm}^3$$
$$\text{Volume of cube} = 2^3 \text{ cm}^3$$

Let number of recast cubes be n.

$$\text{Volume of } n \text{ cubes} = \text{Volume of cuboid}$$
$$n2^3 = 9\times 8\times 2$$
$$n\times 2\times 2\times 2 = 9\times 8\times 2$$
$$n = \frac{9\times 8\times 2}{2\times 2\times 2} = 18$$

Hence, number of cubes recast is 18.

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61. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs.25 per meter.

Sol : [Board Term-2 Foreign 2014, Delhi 2014]

We have radius $r = 7$ m and height $h = 24$ m

Slant height of tent,

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$$
$$= \sqrt{625} = 25 \text{ m}.$$

Curved surface area of cone,

$$\pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Curves surface area of tent will be required area of cloth. Let x meter of cloth is required

$$5x = 550$$

or, $$x = \frac{550}{5} = 110 \text{ m}.$$

Thus 110 m of cloth is required.

$$\text{Cost of cloth} = 25 \times 110$$
$$= \text{Rs.}2750.$$

62. A solid metallic cylinder of radius 3.5 cm and height 14 cm melted and recast into a number of small solid metallic ball, each of radius $\frac{7}{12}$ cm. Find the number of balls so formed.

Sol : [Board Term-2 2016]

Let the number of recasted balls be N.

Radius of cylinder $R = 3.5$ cm

Height of cylinder $h = 14$cm

Radius of recasted ball $r = \frac{7}{12}$

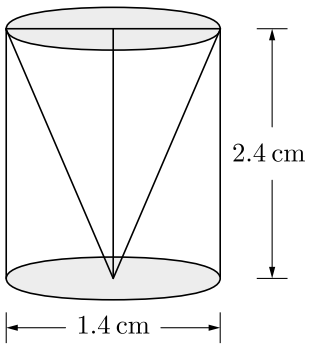
$$\text{Volume of balls} = \text{Volume of cylinder}$$
$$n\frac{4}{3}\pi r^3 = \pi R^2 h$$
$$n \times \frac{4}{3} \times \frac{7}{12} \times \frac{7}{12} \times \frac{7}{12} = 3.5 \times 3.5 \times 14$$
$$n = \frac{3.5 \times 3.5 \times 14 \times 3 \times 12 \times 12 \times 12}{4 \times 7 \times 7 \times 7}$$
$$= 0.5 \times 0.5 \times 2 \times 3 \times 3 \times 12 \times 12$$
$$= 648$$

Hence, number of recasted balls is 648.

63. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the volume of the remaining solid to the nearest cm^3. Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 2012]

As per question the figure is shown below.



Volume of remaining solid is difference of volume of cylinder and volume of cone.

$$\pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$
$$= \frac{2}{3} \times \frac{22}{7} \times (0.7)^2 \times 2.4$$
$$= 44 \times 0.1 \times 0.7 \times 0.8$$
$$= 4.4 \times .56 = 2.464 \text{ cm}^3.$$

PRACTICE

- From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed, then find the volume of remaining solid?
  [Board 2008]
  Ans : 240π cm^3

- From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.
  [Board 2020 OD Standard]
  Ans : 1056 cm^2

64. Find the number of plates, 1.5 cm in diameter and 0.2 cm thick, that can be fitted completely inside a right circular of height 10 cm and diameter 4.5 cm.

Sol : [Board Term-2 2014]

As per question we can arrange circular plate in right circular as follows. Here smaller circle is plate of 1.5 cm diameter and large circle is cylinder of 4.5 cm diameter.

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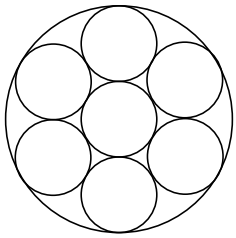


From figure it may be easily seen that 6 plate will be fitted in cylinder in one layer.

Height of six plate is 0.2 cm. Total height of cylinder is 10 cm. Thus layer of plate in cylinder is $\frac{10}{0.2} = 50$ layer. Thus total plate $50 \times 6 = 300$

65. Find the number of coins of 1.5 cm diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Sol : [Board Term-2 SQP 2016]

Volume of any cylinder shape is $\pi r^2 h$.

$$\text{Volume of coin} = \pi(0.75)^2 \times 0.2 \text{ cm}^3$$

$$\text{Volume of cylinder} = \pi(2.25)^2 \times 10 \text{ cm}^3$$

$$\text{No. of coins} = \frac{\text{Volume of cylinder}}{\text{Volume of coin}}$$

$$= \frac{\pi(2.25)^2 \times 10}{\pi(0.75)^2 \times 0.2}$$

$$= \frac{(3)^2 \times 10}{0.2} = 450$$

66. A cone of height 24 cm and radius of base 6 cm is made up of clay. If we reshape it into a sphere, find the radius of sphere.

Sol : [Board Term-2 2014]

$$\text{Volume of sphere} = \text{Volume of cone}$$

$$\frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 h$$

$$\frac{4}{3} \times r_1^3 = (6)^2 \times \frac{24}{3}$$

$$4r_1^3 = 36 \times 24$$

$$r_1^3 = 6^3 \Rightarrow r_1 = 6 \text{ cm}$$

Hence, radius of sphere is 6 cm.

67. A metallic sphere of total volume π is melted and recast into the shape of a right circular cylinder of radius 0.5 cm. What is the height of cylinder ?

Sol : [Board Term-2 2012]

$$\text{Volume of cylinder} = \text{Volume of sphere},$$

$$\pi r^2 h = \pi$$

where r and h are radius of base and height of cylinder

$$(0.5)^2 h = 1$$

$$0.25h = 1 \Rightarrow h = 4 \text{ cm}.$$

68. A metallic solid sphere of radius 4.2 cm is melted and recast into the shape of a solid cylinder of radius 6 cm. Find the height of the cylinder.

Sol : [Board Term-2, 2012]

$$\text{Volume of sphere} = \text{Volume of cylinder}$$

$$\frac{4\pi R^3}{3} = \pi r^2 h$$

$$\frac{4\pi}{3} \times (4.2)^3 = \pi 6^2 \times h$$

$$h = \frac{4 \times 4.2 \times 4.2 \times 4.2}{3 \times 6 \times 6}$$

Hence, height of cylinder is $h = 2.744$ cm.

69. A right circular cone of radius 3 cm, has a curved surface area of 47.1 cm^2. Find the volume of the cone. (Use $\pi = 3.14$)

Sol : [Board Term-2 Delhi 2016]

We have $r = 3, \pi rl = 47.1$

Thus $$l = \frac{47.1}{3 \times 3.14} = 5$$

$$h = \sqrt{5^2 - 3^2} = 4 \text{ cm}$$

Volume of cone,

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 3 \times 3 \times 4$$

$$= 37.68 \text{ cm}^3$$

70. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\pi = \frac{22}{7}$.

Sol : [Board Term-2 Delhi 2016]

We have $r + h = 37$ (1)

and $2\pi r(r + h) = 1628$ (2)

Thus $2\pi r \times 37 = 1628$

$$2\pi r = \frac{1628}{37} \Rightarrow r = 7 \text{ cm}$$

Substituting $r = 7$ in (1) we have

$$h = 30 \text{ cm}.$$

Here volume of cylinder

$$\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30$$

$$= 4620 \text{ cm}^3$$

NODIA

71. A glass is in the shape of a cylinder of radius 7 cm and height 10 cm. Find the volume of juice in litre required to fill 6 such glasses. Use $\pi = \frac{22}{7}$

Sol : [Board Term-2, 2015]

Radius of the glass $r = 7$ cm

Height of the glass $h = 10$ cm

Volume of 1 glass,

$$\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 10$$
$$= 1540 \text{ cm}^3$$

Volume of juice to fill 6 glasses,

$$6\pi r^2 h = 6 \times 1540$$
$$= 9240 \text{ cm}^3$$

$$\text{Volume in litre} = \frac{9240}{1000}$$
$$= 9.240 \text{ litre.}$$

72. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. (Use $\pi = \frac{22}{7}$)

Sol : [Board 2020 OD Standard]

Let r be the radius of base of cylinder and h be height.

Volume of a right circular cylinder $= 25\frac{1}{7}$ cm

$$\pi r^2 h = \frac{176}{7}$$
$$\frac{22}{7} \times h^2 \times h = \frac{176}{7}$$
$$h^3 = \frac{176}{22}$$
$$= 8 = 2^3.$$

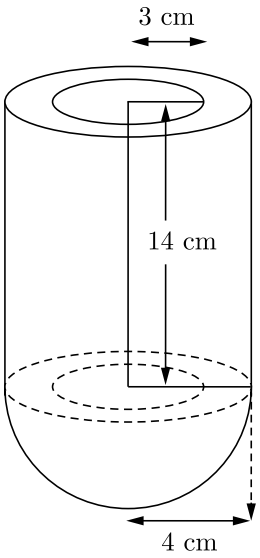
Hence, height of the cylinder $= 2$ cm.

THREE MARKS QUESTIONS

73. The inner and outer radii of a hollow cylinder surmounted on a hollow hemisphere of same radii are 3 cm and 4 cm respectively. If height of the cylinder is 14 cm, then find its total surface area (inner and outer).

Sol : [Board 2024 OD Standard]

As per question we have made the figure below. Total surface area of solid is equal to sum of the inner and outer curved surface areas of cylinder and hemisphere.



Total surface area of solid = Inner and outer curved surface areas of cylinder and hemisphere

$$S = 2\pi rh + 2\pi Rh + 2\pi r^2 + 2\pi R^2$$
$$= 2\pi h(r + R) + 2\pi(r^2 + R^2)$$

where, $r = 3$ cm, $R = 4$ cm and $h = 14$ cm

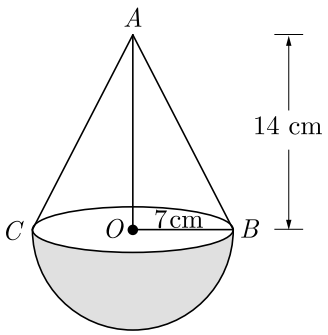
Thus $S = 2\pi h(r + R) + 2\pi(r^2 + R^2)$

$$= 2 \times \frac{22}{7} \times 14(3 + 4) + 2 \times \frac{22}{7}(9 + 16)$$
$$= 616 + \frac{1100}{7} = \frac{616 \times 7 + 1100}{7}$$
$$= \frac{5412}{7} \text{ cm}^2 = 773.14 \text{ cm}^2 \text{ (approx.)}$$

74. A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them being 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid.

Sol : [Board 2023 OD Standard]

As per given information in question we have drawn the figure below,



Here, radius $r = 7$ cm

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and height of a cone $= 14$ cm

Volume of the solid,

$= $ Volume of hemisphere + volume of a cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2(2r + h)$

$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times (2 \times 7 + 14)$

$= \frac{1}{3} \times \frac{22}{1} \times 7 \times (14 + 14)$

$= \frac{1}{3} \times 22 \times 7 \times 28$

$= \frac{4312}{3}$

$= 1437.33 \text{ cm}^3$ (approx)

75. From a solid cylinder whose height is 15 cm and the diameter is 16 cm, a conical cavity of the same height and same diameter is hollowed out, Find the total surface area of remaining solid. (Given your answer in terms of π).

Sol : [Board 2010]

Height of cylinder, $h = 15$ cm

Radius of cylinder, $r = \frac{16}{2} = 8$ cm

Radius of base of cone, $r = 8$ cm

Let slant height of cone be l, then we have

$l = \sqrt{r^2 + h^2} = \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225} = \sqrt{289}$

Thus $l = 17$ cm

TSA of reaming solid

$=$ Top area of cylinder +

+ CSA of cylinder + CSA of conical cavity

$= \pi r^2 + 2\pi rh + \pi rl$

$= \pi r(r + 2h + l)$

$= \pi \times 8(3 + 2 \times 15 + 17)$

$= \pi \times 8 \times 55 = 440\pi$

TSA of reaming solid is 440π.

PRACTICE

- From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of same height and same base radius is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)

  [Board Term-2 OD Compt. 2017]

  Ans : 603.43 cm^2

- From a solid cylinder of height 24 cm and diameter 14 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid.

  [Board Term-2 Delhi Compt. 2017]

  Ans : 1760 cm^2

- From a right circular cylinder of height 2.4 cm and radius 0.7 cm, a right circular cone of same radius is cut-out. Find the total surface area of the remaining solid.

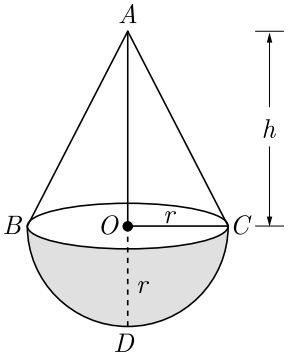
  [Board Term-2 OD 2017]

  Ans : 17.6 cm^2

76. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

Sol : [Board 2020 OD Standard]

Let ABC be a cone, which is mounted on a hemisphere.



We have $OC = OD = r$

Curved surface area of the hemispherical part

$= \frac{1}{2}(4\pi r^2) = 2\pi r^2$

Slant height of a cone, $l = \sqrt{r^2 + h^2}$

Curved surface area of a cone $= \pi rl = \pi r\sqrt{h^2 + r^2}$

Since curved surface areas of the hemispherical part and the conical part are equal,

$2\pi r^2 = \pi r\sqrt{h^2 + r^2}$

$2r = \sqrt{h^2 + r^2}$

Squaring both of the sides, we have

$4r^2 = h^2 + r^2$

$4r^2 - r^2 = h^2$

$3r^2 = h^2$

NODIA

$$\frac{r^2}{h^2} = \frac{1}{3}$$

$$\frac{r}{h} = \frac{1}{\sqrt{3}}$$

Hence, the ratio of the radius and the height is $1 : \sqrt{3}$

77. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

Sol : [Board 2020 OD Standard]

Let h and r be the height and radius of cylinder and cone.

Height, $h = 14$ cm

and radius, $r = 6$ cm

Volume of the remaining solid,

$$V_{\text{remain}} = V_{\text{cylinder}} - V_{\text{cone}}$$

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h$$

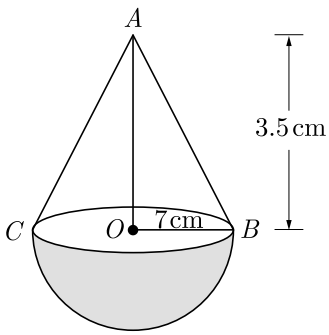
$$= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 14$$

$$= 1056\,\text{cm}^3$$

78. A solid is in the shape of a hemisphere surmounted by a cone. If the radius of hemisphere and base radius of cone is 7 cm and height of cone is 3.5 cm, find the volume of the solid. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Sol : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below,



Here, radius $r = 7$ cm

and height of a cone $= 3.5$ cm

Volume of the solid,

= Volume of hemisphere + volume of a cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times 7^3 + \frac{1}{3} \times \frac{22}{7} \times 7^3 \times 3.5$$

$$= \frac{1}{3}(2156 + 539)$$

$$= \frac{1}{3} \times 2695 = 898.33\,\text{cm}^3$$

79. A hemispherical bowl of internal diameter 36 cm contains liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer.

Sol : [Board Term-2 OD 2015]

Volume of the hemispherical bowl of internal diameter 36 cm will be equal to the 72 cylindrical bottles of diameter 6 cm.

$$\text{Volume of bowl} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\pi \times (18)^3\ \text{cm}^3$$

Volume of liquid in bowl is equal to the volume of bowl.

$$\text{Volume of liquid after wastage} = \frac{2}{3}\pi(18)^3 \times \frac{90}{100}\ \text{cm}^3$$

Volume of one bottle $= \pi r^2 h$

Volume of liquid in 72 bottles

$$= \pi \times (3)^2 \times h \times 72\ \text{cm}^2$$

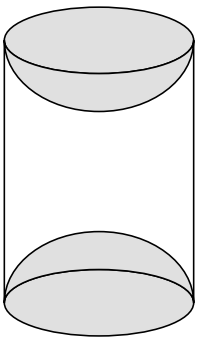
Volume of bottles= volume in liquid after wastage

$$\pi \times (3)^2 \times h \times 72 = \frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}$$

$$h = \frac{\frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}}{\pi \times (3)^2 \times 72}$$

Hence, the height of bottle $= 5.4$cm

80. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



Sol : [Board 2018]

Total surface Area of article

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= CSA of cylinder + CSA of 2 hemispheres

$$\text{CSA of cylinder} = 2\pi rh$$
$$= 2 \times \frac{22}{7} \times 3.5 \times 10$$
$$= 220\,\text{cm}^2$$

Curved surface area of two hemispherical scoops
$$= 2 \times 2\pi r^2$$
$$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$$
$$= 154\,\text{cm}^2$$

$$\text{Total surface area of article} = 220 + 154$$
$$= 374\,\text{cm}^2$$

81. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

Sol : [Board 2020 OD STD, 2019 Delhi, Delhi 2014, 2012]

Canal is the shape of cuboid where

$$\text{Breadth} = 6\,\text{m}$$
$$\text{Depth} = 1.5\,\text{m}$$
and $\text{speed of water} = 10\,\text{km/hr}$

Length of water moved in 60 minutes i.e. 1 hour
$$= 10\,\text{km}$$

Length of water moved in 30 minutes i.e. $\frac{1}{2}$ hours,
$$= \frac{1}{2} \times 10 = 5\,\text{km} = 5000\,\text{m}$$

Now, volume of water moved from canal in 30 minutes
$$= \text{Length} \times \text{Breadth} \times \text{Depth}$$
$$= 5000 \times 6 \times 1.5\,\text{m}^3$$

Volume of flowing water in canal
$$= \text{volume of water in area irrigated}$$
$$5000 \times 6 \times 1.5\text{m}^3 = \text{Area Irrigated} \times 8\,\text{cm}$$
$$5000 \times 6 \times 1.5\text{m}^3 = \text{Area Irrigated} \times \frac{8}{100}\,\text{m}$$
$$\text{Area Irrigated} = \frac{5000 \times 6 \times 1.5 \times 100}{8}\,\text{m}^2$$
$$= 5.625 \times 10^5\,\text{m}^2$$

PRACTICE

Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation?
[Board Term-2 Delhi 2016]

Ans : 162000 m^3

82. A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder in cubic metre. Use $\pi = \frac{22}{7}$.

Sol : [Board Term-2 Foreign 2015]

Volume of water in cylinder is equal to the volume of cylinder. Thus

Volume of water in cylinder = Volume of cylinder

$$\pi r^2 h = \pi(60)^2 \times 180$$
$$= 648000\pi\ \text{cm}^3$$

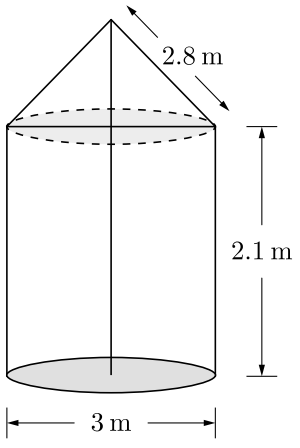
Water displaced on dropping cone is equal to the volume of solid cone, which is

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (30)^2 \times 60$$
$$= 18000\pi\ \text{cm}^3$$

Volume of water left in cylinder

$$= \text{Volume of cylinder} - \text{Volume of cone}$$
$$= 648000\pi - 18000\pi = 630000\pi\ \text{cm}^3$$
$$= \frac{630000 \times 22}{1000000 \times 7}\ \text{m}^3 = 1.98\ \text{m}^3$$

83. A tent is in the shape of cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs.500 per square meter. Use $\pi = \frac{22}{7}$.



Sol : [Board Term-2 OD 2016]

Area of canvas required will be surface area of tent.

Height of cylinder = 2.1 m

NODIA

Radius of cylinder = radius of cone $= \frac{3}{2}$ m

Slant height of cone $= 2.8$ m

Surface area of tent,

$= C.S.A$ of cone $+ C.S.A$ of cylinder.

$= \pi rl + 2\pi rh = \pi r(l + 2h)$

Thus $\pi r(l+2h) = \frac{22}{7} \times \frac{3}{2}(2.8 + 2 \times 2.1)$

$= \frac{33}{7} \times 7 = 33$ m^2

Total Cost $= 33 \times 500$

$= 16{,}500 Rs$

84. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weights, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$ cm and its depth $\frac{8}{9}$ cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

Sol : [Board Term-2 Foreign 2015]

Volume of cylinder,

$\pi r^2 h = \pi(3)^2 \times 5$

$= 45\pi$ cm^3

Volume of conical hole,

$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times \frac{8}{9} = \frac{2}{3}\pi$ cm^3

Metal left in cylinder $= 45\pi - \frac{2}{3}\pi = \frac{133\pi}{3}$

$\frac{\text{Volume of metal left}}{\text{Volume of metal taken out}} = \frac{\frac{133}{3}\pi}{\frac{2}{3}\pi} = 133 : 2.$

Hence required ratio is $133 : 2$

85. The rain water from 22m $\times$ 20 m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m. If the rain water collected from the roof fills $\frac{4th}{5}$ of cylindrical vessel then find the rainfall in cm.

Sol : [Board Term-2 Foreign 2015]

Let h be the rainfall.

Volume of water collected in cylindrical vessel,

$\frac{4}{5}\pi r^2 h = \frac{4}{5} \times \pi \times (1)^2 \times \left(\frac{7}{2}\right)$ m^3

$= \frac{44}{5}$ m^3

Rain water from roof $= 22 \times 20 \times h$ m^3

Now $22 \times 20 \times h = \frac{44}{5}$

$h = \frac{44}{5} \times \frac{1}{22 \times 20} = \frac{1}{50}$ m^3

$= \frac{1}{50} \times 100 = 2$ cm

86. A hollow cylindrical pipe is made up of copper. It is 21 dm long. The outer and inner diameters of the pipe are 10 cm and 6 cm respectively. Find the volume of copper used in making the pipe.

Sol : [Board Term-2, 2015]

Volume of copper used in making the pipe is equal to the difference of volume of external cylinder and volume of internal cylinder.

Height of cylindrical pipe,

$h = 21$ dm $= 210$ cm

External Radius, $R = \frac{10}{2} = 5$ cm

Internal Radius, $r = \frac{6}{2} = 3$ cm

Volume of copper used in making the pipe

= (Volume of External Cylinder)

$-$ (Volume of Internal Cylinder)

$= \pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - h^2)$

$= \frac{22}{7} \times 210 \times (5^2 - 3^2)$

$= \frac{22}{7} \times 210 \times (25 - 9)$

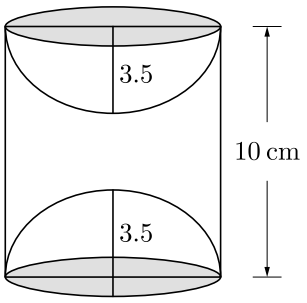
$= \frac{22}{7} \times 210 \times 16$

$= 10560$ cm^3.

87. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 Delhi 2013]

As per question the figure is shown below.



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Here radius of toy is equal to the radius of cylinder which is 3.5 cm.

Radius of toy = radius of cylinder = 3.5 cm

Vol. of toy = Vol. of cylinder $-2\times$ Vol. of hemisphere

$$= \pi r^2 h - 2\times\frac{2}{3}\pi r^3$$

$$= \pi r^2\left[h-\frac{4r}{3}\right]$$

$$= \frac{22}{7}\times(3.5)^2\left[10-\frac{4\times3.5}{3}\right]$$

$$= \frac{22}{7}\times3.5\times3.5\times\left[\frac{30-4\times3.5}{3}\right]$$

$$= \frac{22}{3}\times0.5\times3.5\times16 = 204.05 \text{ cm}^3.$$

88. The radii of two right circular cylinders are in the ratio of 2 : 3 and their height are in the ratio of 5 : 4. Calculate the ratio of their curved surface area and radio of their volumes.

Sol : [Board Term-2 2012]

Let the radii of two cylinders be $2r$ and $3r$ and their heights be $5h$ and $4h$ respectively.

Ratio of their curved surface areas,

$$= \frac{2\pi\times2r\times5h}{2\pi\times3r\times4h} = \frac{5}{6}$$

Thus their curved surface areas are in the ratio of 5 : 6.

Ratio of their volumes,

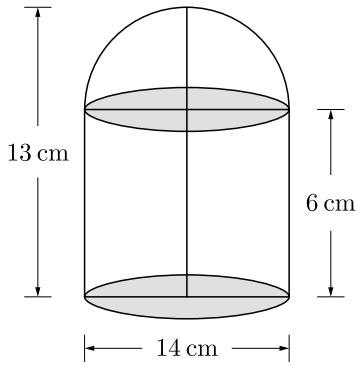
$$= \frac{\pi\times(2r)^2\times5h}{2\pi\times3r\times4h} = \frac{5\times4}{4\times9} = \frac{5}{9}$$

Hence, their volumes are in the ratio of 5 : 9 and their $C.SA$ are in the ratio of 5 : 6.

89. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel. Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 Delhi 2013]

As per question the figure is shown below.



Radius of hemisphere $r = \frac{14}{2} = 7$ cm

Height of cylinder $h = 13-7 = 6$ cm

Total slanted area of cylinder,

= S.A of hemisphere + S.A. of cylinder

$$= 2\pi r^2 + 2\pi rh = 2\pi r(r+h)$$

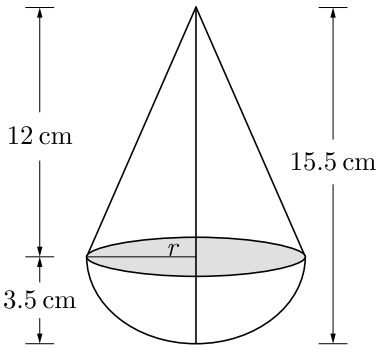
$$= \frac{2\times22\times7}{7}\times(7+6)$$

$$= 44\times13 = 572 \text{ cm}^2$$

90. A toy is in the form of a cone radius 3.5 cm mounted on a hemisphere of same radius. If the total height of the toy is 15.5 cm, find the total surface area of the toy. Use $\pi = \frac{22}{7}$

Sol : [Board OD 2020 Basic]

As per question the figure is shown below. Here total surface area of the toy is equal to the sum of surface area of hemisphere and curved surface area of cone.



Radius $r = \frac{7}{2} = 3.5$ cm

and height $h = 12$ cm

Slant height of cone,

$$l = \sqrt{r^2+h^2}$$

$$= \sqrt{3.5^2+12^2} = 12.5$$

Total surface area of the toy

= Surface area of hemisphere + + Curved surface area of cone

$$= 2\pi r^2 + \pi rl$$

$$= \pi r(2r+l)$$

$$= \frac{22}{7}\times3.5\times(2\times3.5+12.5)$$

$$= 11\times19.5$$

$$= 214.5 \text{ cm}^2$$

NODIA

PRACTICE

A toy is in the form of a cone surmounted on a hemisphere of common base of diameter 7 cm. If the height of the toy is 15.5 cm, find the total surface area of the toy. Use $\pi = \frac{22}{7}$

[Board Term-2 2012]

Ans : 214.5 cm^2

91. A metallic solid sphere of radius 10.5 cm melted and recasted into smaller solid cones each of radius 3.5 cm and height 3 cm. How may cones will be made ?

Sol : [Board Term-2 Delhi 2017]

Radius of given sphere $R = 10.5$ cm

Volume of sphere,

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(10.5)^3 \text{ cm}^3$$

Radius of one recasted cone,

$$r = 3.5\text{cm}$$

Height $h = 3$ cm

Volume $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3.5)^2 \times 3$

$$= \pi(3.5)^2 \text{ cm}^3$$

Let the number of recasted cones be n. Volume of sphere is equal to the n recasted cone.

$$n\pi(3.5)^2 = \frac{4}{3}\pi(10.5)^3$$

$$n = \frac{4}{3}\frac{(10.5)^3}{(3.5)^2}$$

$$= \frac{4}{3} \times 10.5 \times \left(\frac{10.5}{3.5}\right)^2$$

$$= \frac{4}{3} \times 10.5 \times (3)^3$$

$$= 4 \times 10.5 \times 3 = 126$$

Hence, number of recasted cones is 126.

PRACTICE

A solid metallic sphere of diameter 16 cm is melted and recasted into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed.

[Board Term-2 Delhi 2017]

Ans : 16 cones

92. A solid sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged into water, by how much will the level of water rise in the cylindrical vessel ?

Sol : [Board Term-2 SQP 2017]

Let h be the rise in level of water.

Radius of sphere $= 3$ cm.

Radius of cylinder $= \frac{12}{2} = 6$ cm

Volume of water displaced in cylinder will be equal to the volume of sphere.

$$\pi(6)^2 h = \frac{4\pi}{3}(3)^3$$

$$6 \times 6 \times h = \frac{4}{3} \times 3 \times 3 \times 3$$

$$6 \times 6 \times h = 4 \times 3 \times 3$$

$$h = \frac{4 \times 3 \times 3}{6 \times 6} = 1 \text{ cm}$$

Hence the water level rises is 1 cm.

93. A conical vessel, with base radius 5 cm height 24 cm, is full of water. This water emptied into a cylindrical vessel, of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 OD 2016]

Here radius and height of conical vessel are 5 cm and 24 cm.

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times 2.5 \times 24$$

When water is emptied into cylindrical vessel, water will rise in cylindrical vessel. Let rise in height be h.

Volume of water raised $= \pi r^2 h$. This volume is equal to the volume of cone.

Thus $\pi \times (10)^2 \times h = \frac{1}{3}\pi \times 25 \times 24$

$$100h = 25 \times 8$$

$$h = 2 \text{ cm}$$

94. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 m high embankment. Find the width of the embankment.

Sol : [Board Term-2 2012]

Depth of well, $d = 14$ m,

Radius, $r = 12$ m.

Volume of earth taken out,

NODIA

$$\pi r^2 h = \frac{22}{7} \times (2)^2 \times 14$$
$$= \frac{22}{7} \times 2 \times 2 \times 14$$
$$= 176 \text{ m}^3$$

Let r be the width of embankment. The radius of outer circle of embankment
$$= 2 + r$$
Area of upper surface of embankment
$$= \pi\left[(2+r)^2 - (2)^2\right]$$
Volume of embankment = Volume of earth taken out

$$\pi\left[(2+r)^2 - (2)^2\right] \times 0.4 = 176$$
$$\pi\left[4 + r^2 + 4r - 4\right] \times 0.4 = 176$$
$$\frac{0.4 \times 22}{7}(r^2 + 4r) = 176$$
$$r^2 + 4r = \frac{176 \times 7}{0.4 \times 22}$$
$$= 140$$
$$r^2 + 4r - 140 = 0$$
$$(r+14)(r-10) = 0 \Rightarrow r = 10$$

Hence width of embankment is 10 m.

95. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level into the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

Sol : [Board Term-2 OD 2016]

$$\text{Radius of sphere} = \frac{12}{2} = 6 \text{ cm}$$
$$\text{Volume} = \frac{4}{3}\pi \times 6^3 \text{ cm}^3$$

It is submerged into water, in cylindrical vessel, then water level rise by $3\frac{5}{9} = \frac{32}{9}$ cm. Volume of submerged sphere is equal to the volume of water rise in cylinder.

Volume submerged = Volume rise

Let r be radius of cylinder. Therefore

$$\pi \times r^2 \times \frac{32}{9} = \frac{4}{3}\pi \times 6^3 \text{ cm}$$
$$r^2 = \frac{216 \times 3 \times 4}{32}$$
$$= \frac{27 \times 3 \times 4}{4}$$
$$r^2 = 27 \times 3 = 81 \Rightarrow r = 9 \text{ cm}$$

Diameter $2r = 2 \times 9 = 18$ cm.

96. A well of diameter 4 m dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment.

Sol : [Board Term-2 Delhi 2016]

Radius of earth dug out $r = \frac{4}{2} = 2$ m

Depth of the earth $d = 21$,

Volume of earth $\pi r^2 d = \frac{22}{7} \times (2)^2 \times 21$
$$= 22 \times 4 \times 3$$
$$= 264 \text{ m}^2$$

Width of embankment $= 3$ m

Outer radius of ring $= 2 + 3 = 5$ m

Let the height of embankment be h.

Volume of embankment,

$$\pi(R^2 - r^2)h = 264$$
$$\frac{22}{7} \times (5^2 - 2^2) \times h = 264$$
$$\frac{22}{7} \times (25 - 4) \times h = 264$$
$$\frac{22}{7} \times 21 \times h = 264$$
$$22 \times 3 \times h = 264$$
$$h = \frac{264 \times 7}{22 \times 21} = 4$$

Height of embankment is 4 m.

PRACTICE

A well diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly around it to a width of 5 m. to form a embankment. Find the height of the embankment.
[Board Term-2 Foreign 2017]
Ans : 78.75 cm

97. A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice-cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cones.

Sol : [Board Term-2 Foreign 2016]

For cylindrical tub,

Radius $R = \frac{12}{2} = 6$ cm

Height $H = 15$ cm.

Downloaded from www.pdf.tube

Volume $\pi R^2 H = \pi(6)^2 \times 15 = 540\pi \text{ cm}^3$

Each child will get the ice-cream $\frac{540\pi}{10} \text{ cm}^3$

$= 54\pi \text{ cm}^3$

For cone, height $h = 2 \times d = 2 \times 2r = 4r$

Volume of cone,

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \times 4r = \frac{4}{3}\pi r^3$$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

Total volume of cone and hemisphere

$$= \frac{4}{3}\pi r^3 + \frac{2}{3}\pi r^3 = \frac{6}{3}\pi r^3 = 2\pi r^3$$

According to question,

$$2\pi r^3 = 54\pi$$

$$r^3 = 27 \Rightarrow r = 3$$

Hence diameter of conical part of ice-cream cones,

$$= 2r = 2 \times 3 = 6 \text{ cm.}$$

PRACTICE

A right cylindrical container of radius 6 cm and height 15 cm if full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone.

[Board 2019 OD]

Ans : 3 cm

98. A hemispherical tank, of diameter 3 m, is full of water. It is being emptied by a pipe at the rate of $3\frac{4}{7}$ litre per second. How much time will it take to make the tank half empty ? Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 Foreign 2016]

$$\text{Radius } r = \frac{3}{2} \text{ m}$$

Volume of hemispherical tank,

$$V = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi\left(\frac{3}{2}\right)^3 \text{ m}^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{27}{8} \text{ m}^3$$

$$= \frac{11}{7} \times \frac{9}{2} = \frac{99}{14} \text{ m}^3$$

Since $1 \text{ m}^3 = 1000$ litre, we have

$$V = \frac{99}{14} \times 1000 \text{ litre}$$

Volume of half of the hemisphere

$$\frac{V}{2} = \frac{1}{2} \times \frac{99}{14} \times 1000 \text{ Litres}$$

Let time taken for this volume to flow out be t. Then according to question,

$$3\frac{4}{7}t = \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$\frac{25t}{7} = \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$t = \frac{7}{25} \times \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$= 990 \text{ sec}$$

$$= 16 \text{ minutes } 30 \text{ sec.}$$

99. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 Delhi 2015]

Volume of single cone,

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h = \frac{\pi}{3} \times \left(\frac{3.5}{2}\right)^2 \times 3$$

Volume of recast sphere,

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

Volume of sphere is equal to the volume of 504 cones.

Thus $V_{\text{sphere}} = 504\, V_{\text{cone}}$

$$\frac{4\pi}{3} \times r^3 = 504 \times \frac{\pi}{3} \times \left(\frac{3.5}{2}\right)^2 \times 3$$

$$4r^3 = 504 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$r^3 = 126 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$= 7 \times 9 \times 2 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$= 3 \times 3 \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \times 3$$

$$r = 3 \times \frac{7}{2} = 10.5 \text{ cm}$$

Thus diameter is 21 cm.

Surface area $4\pi r^2 = 4 \times \frac{22}{7} \times 10.5 \times 10.5$

$$= 1386 \text{ cm}^2$$

100. The $\frac{3}{4}$th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

Sol : [Board Term-2 Delhi 2017]

Radius of conical vessel $= 5$ cm

NODIA

Weight of larger sphere, $W_2 = 7$ kg

Radius of smaller sphere, $r_1 = 3$ cm

Volume of smaller sphere, $V_1 = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(3)^3$

$= \frac{4}{3}\pi(27)$

$= 36\pi \text{ cm}^3$

Now weight of recanted metal sphere

$= (1+7)\text{kg} = 8\text{ kg}$

Since, 1 kg metal sphere occupies 36π cm^3 space.

Thus 8 kg metal sphere occupies $8 \times 36\pi$ cm^3 space.

Let R be the radius of new sphere, then volume of new 8 kg sphere is $\frac{4}{3}\pi R^3$.

Thus $\frac{4}{3}\pi R^3 = 36 \times 8\pi \text{ cm}^3$

$R^3 = 36 \times 2 \times 3$

$R^3 = 9 \times 4 \times 2 \times 3 = 3^3 \times 2^3$

$R = 2 \times 3 = 6$ cm

Diameter of new sphere

$2R = 2 \times 6 = 12$ cm

109. A hemispherical depression is cut from one face of a cubical block, such that diameter l of hemisphere is equal to the edge of cube. find the surface area of the remaining solid.

Sol : [Board Term-2 2014]

Let r be the radius of hemisphere.

Now $r = \frac{l}{2}$

Now, the required surface area

= Surface area of cubical block +

− Area of base of hemisphere +

+ Curved surface area of hemisphere.

$= 6(l)^2 - \pi r^2 + 2\pi r^2$

$= 6l^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2$

$= l^2\left(6 - \frac{\pi}{4} + \frac{2\pi}{4}\right)$

$= l^2\left(6 + \frac{\pi}{4}\right)$

$= l^2\left(6 + \frac{22}{7 \times 4}\right)$

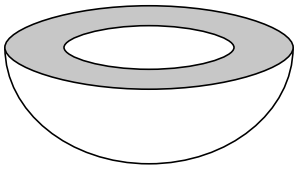
$= l^2\left(6 + \frac{11}{14}\right)$

$= \frac{95l^2}{14}$

110. The internal and external diameters of a hollow hemispherical vessel are 16 cm and 12 cm respectively. If the cost of painting 1 cm^2 of the surface area is Rs. 5.00, find the total cost of painting the vessel all over. (Use $\pi = 3.14$)

Sol : [Board Term-2 Delhi 2015]

As per question the figure is shown below.



Here $R = 8$ cm, $r = 6$ cm

Surface area $= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= \pi\left[2 \times 8^2 + 2 \times 6^2 + (8^2 - 6^2)\right]$

$= \pi\left[2 \times 64 + 2 \times 36 + (64 - 36)\right]$

$= \pi[128 + 72 + 28]$

$= 228 \times 3.14 = 715.92$ cm^2

Total cost $= 715.92 \times 5 = 3579.60$ Rs

111. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.

Sol : [Board Term-2 Delhi 2013]

Radius of pipe $r = \frac{2}{2} = 1$

Water flow rate $= 0.4$ m/s $= 40$ cm/s

Volume of water flowing through pipe in 1 sec.

$\pi r^2 h = \pi \times (1)^2 \times 40 = 40\pi$ cm^3

Volume of water flowing in 30 min $(30 \times 60 \text{ sec})$

$= 40\pi \times 30 \times 60 = 72000\pi$

Volume of water in cylindrical tank in 30 min,

Now $\pi R^2 H = \pi(40)^2 \times H$

$\pi(40)^2 \times H = 72000\pi$

$40 \times 40 \times H = 72000\pi$

Rise in water level

$H = \frac{72000}{40 \times 40} = 45$ cm.

Thus level of water in the tank is 45 cm.

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$$n \times \frac{1}{3}\pi r^2 h = \pi R^2 H$$

$$n \times \frac{1}{3} \times 3 \times 3 \times 9 = 6 \times 6 \times 15$$

$$n = \frac{6 \times 6 \times 15}{3 \times 9} = 20$$

Hence the number of toys is 20.

105. Find the volume of the largest solid right circular cone that can be cut out off a solid cube of side 14 cm.

Sol : [Board Term-2 2012]

The base of cone is the largest circle that can be inscribed in the face of the cube and the height will be equal to edge of the cube.

Radius of cone, $r = \frac{14}{2} = 7$ cm

Height of cone, $h = 14$ cm

Volume of cone, $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 14$$

$$= \frac{2156}{3} = 718.67.$$

106. A girl empties a cylindrical bucket, full of sand, of radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct upto one place of decimal.

Sol : [Board Term-2 Foreign 2014]

Here volume of cone is equal to the volume of cylinder.

Let r_1 and r_2 be the radii of the cylinder and cone respectively.

Volume of cone = Volume of Cylinder

$$\frac{1}{3}\pi r_2^2 h = \pi r_1 h^2$$

$$\frac{1}{3} \times r_2^2 \times 24 = 18 \times 18 \times 32$$

$$r_2^2 = \frac{3 \times 18 \times 18 \times 32}{24}$$

$$r_2^2 = 1296 \Rightarrow r_2 = 36 \text{ cm}$$

Radius of cone $r_2 = 36$ cm

Now, slant height of cone

$$l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 36^2}$$

$$= \sqrt{576 + 1296}$$

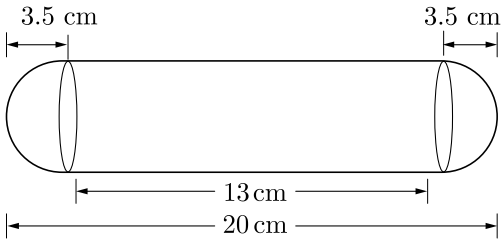
$$= 43.2 \text{ cm}.$$

FIVE MARKS QUESTIONS

107. A solid is in the form of a cylinder with hemispherical end. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid. (Use $\pi = \frac{22}{7}$)

Sol : [Board 2019 OD]

As per given information in question we have drawn the figure given below.



Height of the cylinder,

$$h = (20 - 7) \text{ cm} = 13 \text{ cm}$$

Radius of circular part,

$$r = \frac{7}{2} \text{ cm}$$

Volume of solid,

= Volume of cylinder + 2 × Volume of hemisphere

$$V = \pi r^2 h + 2 \times \left(\frac{2\pi}{3} r^3\right)$$

$$= \pi r^2 \left(h + \frac{4}{3} r\right)$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left[13 + \frac{4}{3} \times \frac{7}{2}\right]$$

$$= \frac{77}{2}\left(\frac{53}{3}\right) \text{cm}^3$$

$$= 680.2 \text{ cm}^3$$

PRACTICE

A medicine capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends, the length of the entire capsule is 15 mm and the diameter of the capsule is 5 mm. Find the Volume of the capsule.

[Board Term-2 OD 2012]

Ans : 242.26 mm^3

108. The weight of two spheres of same metal are 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere.

Sol : [Board 2019 OD]

Weight of smaller sphere, $W_1 = 1$ kg

NODIA

$$= 5.4 \times 1.8 \times 25000 \text{ m}^3$$

$$= 54 \times 18 \times 250 \text{ m}^3 = 243000 \text{ m}^3$$

(ii) Water flow in 40 minutes,

$$= 243000 \times \frac{40}{60} \text{ m}^3$$

$$= 162000 \text{ m}^3$$

Let A be the irrigated area then volume of water in irrigated area is equal to the water flow.

Thus $A \times 0.1 = 162000$

$$A = 1620000 \text{ m}^3$$

(iii) Length of water that flows from circular pipe in 1 sec is 7 m or 700 cm.

Radius of pipe is $\frac{4}{2} = 2$ cm.

Thus volume of water in 1 second,

$$= \pi \times (2)^2 \times 700 \text{ cm}^3$$

Volume of water in 30 minutes,

$$= \pi \times (2)^2 \times 700 \times 60 \times 30 \text{ cm}^3$$

Let h be height of water in tank. Since base of tank is 2 m, radius of tank is 1 m i.e. 100 cm.

Volume of water in the tank,

$$\pi 100^2 \times h = \pi \times 4 \times 700 \times 60 \times 30$$

$$h = \frac{700 \times 60 \times 30 \times 4}{100 \times 100} = 504 \text{ cm}$$

Hence, water level increased is 504 cm or 5.04 m.

124. Raju is a design engineer at a large pharmacy company. He has to decide the shape of medicine and for this he has to chose between tablet or captab.

a. Tablet : This is circular tablet.

b. Captab : This is long tablet having semicircular at both end.

The volume of medicine is 924 mm^2.

(i) If Raju decide to make tablet of diameter 14 mm, what is the thickness of tablet ?

(ii) If Raju decide to make captab of dimension 7 by 23.5 mm, what is the thickness of captab ?

Sol :

(i) Surface area of tablet,

$$S_t = \pi\left(\frac{d}{2}\right)^2 = \frac{22}{7} \times \left(\frac{14}{2}\right)^2$$

$$= \frac{22}{7} \times 7^2$$

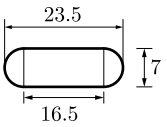
$$= 22 \times 7 = 154 \text{ mm}^2$$

Let t_t be thickness of tablet, then volume of tablet must be 924 mm^3.

Thus, $924 = 154 \times t_t$

$$t_t = \frac{924}{154} = 6 \text{ mm}$$

(ii) Now we make the digram of captab as follows



Surface area of cabtab

$$S_c = \pi\left(\frac{7}{2}\right)^2 + 16.5 \times 7$$

Let t_c be thickness of cabtab, then volume of captab must be 924 mm^2.

$$924 = \left[\pi\left(\frac{7}{2}\right)^2 + 16.5 \times 7\right] t_c$$

$$924 = \left[\frac{22}{7} \times \frac{7^2}{2^2} + 16.5 \times 7\right] t_c$$

$$924 = \left[11 \times \frac{7}{2} + 16.5 \times 7\right] t_c$$

$$924 = \frac{11 \times 7}{2}[1 + 3] t_c$$

$$924 = 77 \times 2\, t_c$$

$$t_c = \frac{924}{77 \times 2} = 6 \text{ mm}$$

125. Silo : A silo is a structure for storing bulk materials. Silos are used in agriculture to store grain or fermented feed known as silage. Silos are commonly used for bulk storage of grain, coal, cement, carbon black, woodchips, food products and sawdust.

A silo is in the shape of cylinder surmounted by a conical top. The height and diameter of cylindrical part are 40 feet and 42 feet respectively and the

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PRACTICE

✍ Water is flowing at 7 m/s through a circular pipe of internal diameter of 4 cm into a cylindrical tank, the radius of whose base is 40 cm. Find the increase in water level in 30 minutes.

[Board Term-2 2012]

Ans : 31.5 m.

✍ Water is flowing at the rate of 0.7 m/sec through a circular pipe whose internal diameter is 2 cm into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the level of water in half hour.

[Board Term-2 SQP 2016]

Ans : 78.75 cm

112. Water is flowing at the rate of 15 km/hr through a cylindrical pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time the level of water in pond rise by 21 cm ?

Sol : [Board Term-2 Delhi 2012]

Radius of pipe, $r = \frac{14}{2} = 7$ cm

Cross section area of pipe,

$$\pi r^2 = \frac{22}{7} \times \left(\frac{7}{100}\right)^2$$

Speed of water flowing through the pipe

$$= 15 \text{ km/hr} = 15000 \text{ m/hr}$$

In an hour length of water = 15000 m

Volume of water flowing from pipe in 1 hr,

$$\pi r^2 h = \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 15000 \text{ m}^3$$

Let t be time taken to fill the tank. Now total volume of water flowing in time t,

$$\pi r^2 ht = \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 15000t$$

Volume of water flown= Volume of water in tank

$$\pi r^2 ht = l \times b \times y$$

$$\frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 1500t = 50 \times 44 \times \frac{21}{100}$$

$$\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000t = 50 \times 44 \times \frac{21}{100}$$

$$\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000t = 50 \times 44 \times \frac{21}{100}$$

$$22 \times 7 \times 150t = 50 \times 44 \times 21$$

$$t = \frac{50 \times 44 \times 21}{22 \times 150 \times 7} = 2$$

Hence, time taken to fill the tank is 2 hours.

PRACTICE

✍ Water is flowing at the rate of 5 km/hour through a pipe of diameter 14 cm into a rectangular tank of dimensions 50 m × 44 m. Find the time in which the level of water in the tank will rise by 7 cm.

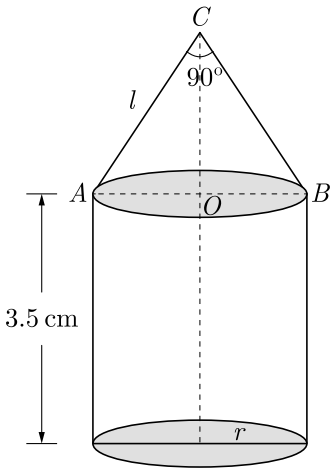
[Board Term-2 Delhi Compt. 2017]

Ans : 2 hours

113. A toy is in the form of a cylinder of diameter $2\sqrt{2}$ m and height 3.5 m surmounted by a cone whose vertical angle is 90°. Find total surface area of the toy.

Sol : [Board Term-2 2012]

As per question the figure is shown below.



Here $\angle C = 90^\circ$ and $AC = BC = l$

Thus $AB^2 = AC^2 + BC^2$

$= l^2 + l^2 = 2l^2$

Now $(2\sqrt{2})^2 = 2l^2$

Thus $l = 2$ and $r = \sqrt{2}$ m

Slant height of conical portion, $l = 2$ m

Total surface area of toy

$$2\pi rh + \pi r^2 + \pi rl = \pi r[7 + \sqrt{2} + 2] \text{ m}^2$$

$$= \pi\sqrt{2}[9 + \sqrt{2}] \text{ m}^2$$

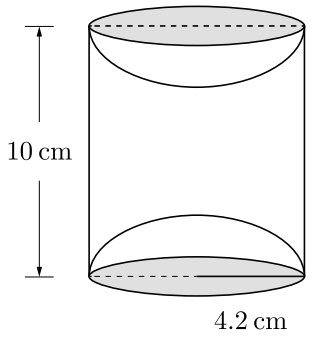
$$= \pi[2 + 9\sqrt{2}] \text{ m}^2$$

114. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. Use $\pi = \frac{22}{7}$

NODIA

Sol : [Board Term-2 OD 2015]

As per question the figure is shown below.



Volume of cylinder,

$$\pi R^2 H = \pi(4.2)^2 \times 10 \text{ cm}^3$$

Volume of metal scooped out ,

$$= 2 \times \text{Volume of hemisphere}$$

$$= 2 \times \frac{2}{3} \times \pi r^3 = \frac{4}{3}\pi r^3$$

$$= \frac{4\pi}{3}(4.2)^3$$

Volume of rest of cylinder,

$$= \pi(4.2)^2 \times 10 - \frac{4\pi}{3}(4.2)^3 \text{ cm}^3$$

$$= \pi(4.2)^2\left(10 - \frac{4}{3} \times 4.2\right) \text{ cm}^3$$

$$= \pi(4.2)^2(10 - 5.6) \text{ cm}^3$$

$$= \pi(4.2)^2 \times 4.4 \text{ cm}^3$$

Now from rest volume a wire of thickness 1.4 cm i.e radius 0.7 cm is formed. Let l be length of wire. Volume of wire and rest cylinder will be equal.

Volume of wire,

$$\pi r^2 l = \pi(4.2)^2 \times 4.4 \text{ cm}^3$$

$$\pi(0.7)^2 l = \pi(4.2)^2 \times 4.4 \text{ cm}^3$$

$$l = \frac{4.2 \times 4.2 \times 4.4}{07 \times 0.7} \text{ cm}^3$$

$$= 6 \times 6 \times 4.4 = 158.4 \text{ cm}$$

115. A milk tanker cylindrical in shape having diameter 2 m and length 4.2 m supplies milk to the two booths in the ratio of 3 : 2. One of the milk booths has cuboidal vessel having base area 3.96 sq. m. and the other has a cylindrical vessel having radius 1 m. Find the level of milk in each of the vessels. Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 Delhi 2012]

Radius of milk tanker $R = \frac{2}{2} = 1$ m

Length of mil tanker L 4.2 m

Volume of milk tanker,

$$\pi R^2 L = \frac{22}{7} \times 1 \times 4.2 = 13.2 \text{ m}^3$$

Supply of milk to booth I,

$$= 13.2 \times \frac{3}{5} = 2.64 \times 3 = 7.92 \text{ m}^3$$

Supply of milk to booth II,

$$= 13.2 \times \frac{2}{5} = 2.64 \times 2 = 5.28 \text{ m}^3$$

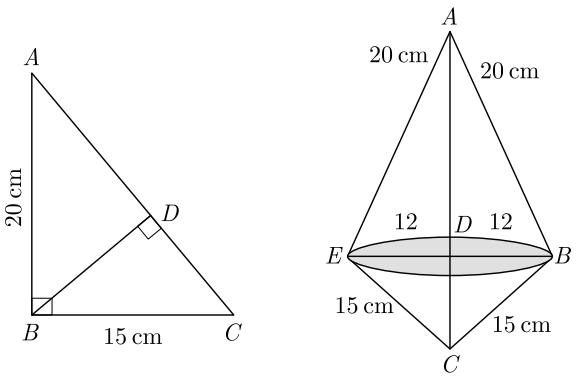
Height in 1st vessel $= \frac{7.92}{3.96} = 2$ m

Height in 2nd vessel $= \frac{5.28}{\frac{22}{7} \times 1} = \frac{5.28 \times 7}{22} = 1.68$ m

116. A right triangle whose sides are 20 cm and 15 cm is made to revolve about its hypotenuse. Find the volume and the surface area of the double cone so formed. (Use $\pi = 3.14$)

Sol : [Board Term-2 2012]

As per question the figure is shown below.



We have $AC^2 = 20^2 + 15^2 = 625$

$$AC = 25 \text{ cm}$$

$$\text{area}(\Delta ABC) = \text{area}(\Delta ABC)$$

$$\frac{1}{2} \times AC \times BD = \frac{1}{2} \times BC \times AB$$

$$25 \times BD = 15 \times 20 = 300$$

$$BD = 12 \text{ cm}$$

Volume of double cone,

= Volume of upper cone + Volume of lower cone

$$= \frac{1}{3}\pi(BD)^2 \times AD + \frac{1}{3}\pi(BD)^2 \times CD$$

$$= \frac{1}{3}\pi(BD)^2(AD + CD)$$

$$= \frac{1}{3}\pi(BD)^2(AC)$$

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$$= \frac{1}{3} \times 3.14 \times (12)^2 \times 25$$

$$= \frac{1}{3} \times 3.14 \times 144 \times 25 = 3768 \text{ cm}^2$$

Surface area= CSA of upper cone + CSA of lower cone

$$= \pi(12)(20) + \pi(12)(15)$$
$$= 12\pi\{20 + 15\}$$
$$= 12 \times 3.14 \times 35 = 1318.8 \text{ cm}^2$$

PRACTICE

A right angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the longest side. Find the surface area of figure obtained. Use $\pi = \frac{22}{7}$

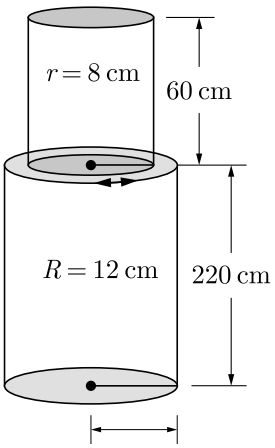
[Board Term-2 2012]

Ans : 52.8 cm^2.

117. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pipe, given that 1 cm3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

Sol : [Board 2019 OD, 2012]

As per question the figure is shown below.



Radius of lower cylinder, $R = 12$ cm

Height of lower cylinder, $H = 220$ cm

Radius of upper cylinder, $r = 8$ cm

Height of upper cylinder, $h = 60$ cm

Volume of solid iron pole,

$\pi R^2 H + \pi r^2 h$

$$= 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$$
$$= 111532.8 \text{ cm}^3$$

Mass of pole $= 111532.8 \times 8$ g

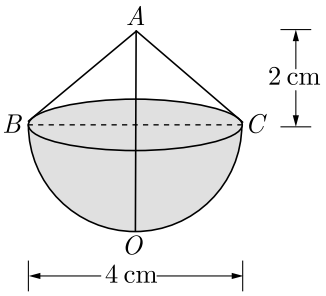
$= 892262.4$ g

$= 892.2624$ kg.

118. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volume of the cylinder and toy. (Use $\pi = 3.14$)

Sol : [Board Term-2 2012]

Let BOC is a hemisphere and ABC is a cone. As per question the figure is shown below.



Radius of hemisphere is equal to the radius of cone which is $\frac{4}{2} = 2$ cm.

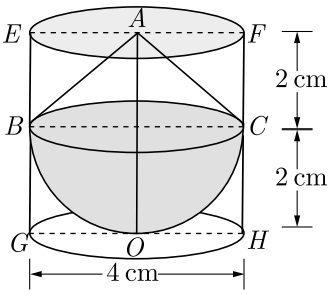
Height of cone, $h = 2$ cm

Volume of toy $= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi r^2(2r + h) = \frac{1}{3} \times 3.14 \times 2 \times 2(2 \times 2 + 2)$

$$= \frac{1}{3} \times 3.14 \times 4 \times 6$$
$$= 25.12 \text{ cm}^3$$

Let right circular cylinder $EFGH$ circumscribe the given solid toy.



Radius of cylinder $= 2$ cm

Height of cylinder $= 4$ cm

Volume of right circular cylinder

$\pi r^2 h = 3.14 \times (2)^2 \times 4 \text{ cm}^3$

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Height of conical vessel $= 24$ cm

Volume of this vessel $= \frac{\pi}{3} \times (5)^2 \times 24$

$= 200\pi$ cm^3

Internal radius of cylindrical vessel$= 10$

Let the h be the height of emptied water.

Volume of water in cylinder,

$$\pi r^2 h = \frac{3}{4} \times \text{Volume of cone}$$

$$\pi \times 10 \times 10 \times h = \frac{3}{4} \times 200\pi$$

$$100h = 150$$

$$h = 1.5 \text{ cm}$$

Hence the height of water is 1.5 cm.

101. Rampal decided to donate canvas for 10 tents conical in shape with base diameter 14 m and height 24 m to a centre for handicapped person's welfare. If the cost of 2 m wide canvas is Rs. 40 per meter, find the amount by which Rampal helped the money.

Sol : [Board Term-2 OD Compt. 2017]

Radius of tent $r = \frac{14}{2} = 7$ m

Height $h = 24$ m

Slant height $l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 24^2}$

$= \sqrt{49 + 576} = 25$ m

Surface area of the tent,

$$\pi rl = \pi rl$$

$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Surface area of 10 tents,

$$= 550 \times 10 = 5500$$

$$\text{Total cost} = 5500 \times \frac{40}{2} = 110000$$

Hence, Rampal helped the centre of 110000 Rs.

102. A cone of maximum size is curved out from a cube edge 14 cm. Find the surface area of remaining solid after the cone is curved out.

Sol : [Board Term-2 SQP 2017]

If a cone of maximum size is curved out from a cube edge a, diameter and height of cone will be a

Side of cube $a = 14$ cm.

If cone of maximum size is curved out,

Radius of cone $r = \frac{14}{2} = 7$ cm

Height of cone $h = 7$ cm

Slant height $l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 14^2}$

$= \sqrt{49 + 196}$

$= \sqrt{245}$

$= 15.65$ cm.

Total surface area = Surface area cube

+ curved Surface area of cone

− Circular area of base of cone

$= 6a^2 + \pi rl - \pi r^2$

$= 6 \times 14 \times 14 + \frac{22}{7} \times 7 \times 15.65 - \frac{22}{7} \times 7 \times 7$

$= 1176 + [22(15.65 - 7)]$

$= 1176 \times 22 \times 8.65$

$= 223792.8$ cm^2

103. The ratio of the volumes of two spheres is $8 : 27$. If r and R are the radii of sphere respectively, then find the $(R - r) : r$.

Sol : [Board Term-2 2012]

Ratio of volumes

$$\frac{\text{Volume of 1}^{\text{st}}\text{sphere}}{\text{Volume of 2}^{\text{nd}}\text{sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{8}{27}$$

$$\frac{r^3}{R^3} = \frac{8}{27}$$

$$\frac{r}{R} = \frac{2}{3}$$

$$\frac{r}{R - r} = \frac{2}{3 - 2} = \frac{2}{1}$$

$$\frac{R - r}{r} = \frac{1}{2}$$

104. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of a cone of radius 3 cm and height 9 cm. Find the number of toys formed so.

Sol : [Board Term-2 OD Compt. 2017]

Height of cylinder, $H = 15$ cm

Radius of cylinder, $R = \frac{12}{2} = 6$ cm

Radius of cone $r = 3$ cm

Height $h = 9$ cm

Let the number of toys recast be n.

Volume of n conical toys= Volume of cylinder

NODIA

$$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 3.5$$

$$= 11 \times 3 = 33 \text{ m}^3$$

$$l = \sqrt{3^2 + 3.5} = 4.609 \text{ m}$$

Canvas required to cover the heap,

$$\pi rl = \frac{22}{7} \times 3 \times 4.609$$

$$= 43.45 \text{ m}^2.$$

COMPETENCEY BASED QUESTIONS

122. Well Embankment : Well embankment is a raised wall that is built around the well. These are often constructed using soils obtained from a digging well. It provide protection to person from felling into the well.

A well of diameter 6 m is dug 14 m deep. $\frac{1}{15}$ of the earth taken out is spread evenly all around the well to form a embankment.

(i) Find the volume of the earth taken out.

(ii) If the height of embankment is 1.2 m, what is the width of the embankment ?

Sol :

(i) Depth of well, $d = 14$ m,

Radius, $r = \frac{6}{2} = 3$ m.

Volume of earth taken out,

$$\pi r^2 h = \frac{22}{7} \times (3)^2 \times 14 = 396 \text{ m}^3$$

(ii) Let w be the width of embankment. The radius of outer circle of embankment

$$= 3 + w$$

Area of upper surface of embankment

$$= \pi[(3+w)^2 - (3)^2]$$

Volume of embankment = $\frac{1}{15}$ of volume of earth taken out

$$\pi[(3+w)^2 - (3)^2] \times 1.2 = \frac{1}{15} \times 396$$

$$\pi(9 + w^2 + 6w - 9) \times 1.2 = \frac{1}{15} \times 396$$

$$\frac{22}{7}(w^2 + 6w) \times 1.2 = \frac{1}{15} \times 396$$

$$w^2 + 6w = \frac{396 \times 7}{15 \times 1.2 \times 22} = 7$$

$$w^2 + 6w - 7 = 0$$

$$(w+7)(w-1) = 0 \Rightarrow w = 1$$

Hence width of embankment is 1 m.

123. Irrigation Canals : Irrigation canals are the main waterways that bring irrigation water from a water source to the areas to be irrigated. The water is taken either from the river, tank or reservoirs. The canals can be constructed either by means of concrete, stone, brick or any sort of flexible membrane which solves the durability issues like seepage and erosion.

One such canal shown above is of width 5.4 m wide and depth 1.8 m deep through which water is flowing with a speed of 25 km/hour.

(i) How much water is flowing through the canal in 1 hour.

(ii) How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ?

(iii) At some distance from canal, a framer is having a large cylindrical tank the radius of whose base is 2 m. Suppose the farmer connects this tank to canal by a circular pipe of internal diameter of 4 cm for irrigation his field. If water is flowing at 7 m/s through a circular pipe, find the increase in water level in 30 minutes.

Sol :

(i) Water flow in 1 hour,

= Area of cross-section × Speed of water

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$= 50.24 \text{ cm}^3$

Difference of two volume

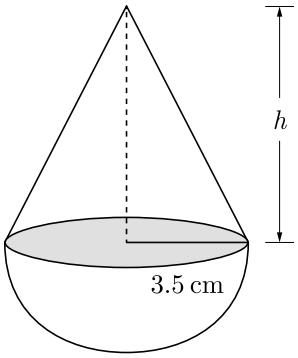
$=$ Volume of cylinder $-$ Volume of toy

$= 50.24 - 25.12 = 25.12 \text{ cm}^3$.

119. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also find the cost of painting the hemisphere part of the toy at the rate of Rs. 10 per cm^2. Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 Delhi 2015]

As per question the figure is shown below.



Radius of hemisphere is equal to the radius of cone which is 3.5 cm.

$$\text{Volume of toy} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$166\frac{5}{6} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$\frac{1001}{6} = \frac{\pi r^2}{3}(2r + h)$$

$$1001 = 2\pi r^2(2r + h)$$

$$1001 = 2 \times \frac{22}{7} \times (3.5)^2(2 \times 3.5 + h)$$

$$1001 = 22 \times 3.5 \times (7 + h)$$

$$91 = 2 \times 3.5 \times (7 + h)$$

$$13 = 7 + h \Rightarrow h = 6$$

Height of the toy $= 6 + 3.5 = 9.5$ cm.

CSA of hemisphere,

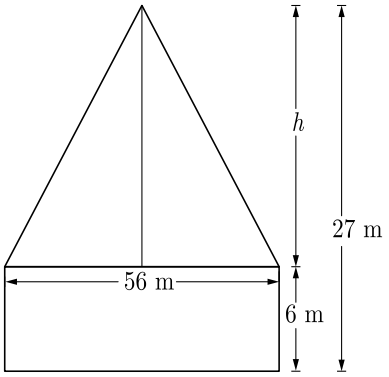
$$2\pi r^2 = 2 \times \frac{22}{7} \times 3.5 \times 3.5 = 77 \text{ cm}^2$$

$$\text{Cost of painting} = 10 \times 77 = 770 \text{ Rs}$$

120. A circus tent is in the shape of a cylinder surmounted by a conical top of same diameter. If their common diameter is 56 m, the height of cylindrical part is 6 m and the total height of the tent above the ground is 27 m, find the area of canvas used in the tent.

Sol : [Board Term-2 Delhi Compt. 2017]

As per question the figure is shown below.



Total height of tent $H_{\text{Total}} = 27$ m

Height of cylindrical part $h = 6$ m

Height of conical part $H = 27 - 6 = 21$ m

Radius of cone $R = \frac{56}{2} = 28$ m

Radius of cylinder $R = \frac{56}{2} = 28$ m

Slant height of cone

$$L = \sqrt{R^2 + H^2}$$

$$= \sqrt{28^2 + 21^2}$$

$$= \sqrt{784 + 441} = \sqrt{1225}$$

$$= 35 \text{ m}$$

Area of canvas used,

$$2\pi rh + \pi rl = \pi r(2h + l)$$

$$= \frac{22}{7} \times 28(2 \times 6 + 35)$$

$$= 22 \times 4 \times 47$$

$$= 4136 \text{ m}^2$$

121. A heap of wheat is in the form of cone of diameter 6 m and height 3.5 m. Find its volume . How much canvas cloth is required to just cover the heap ? Use $\pi = \frac{22}{7}$

Sol : [Board Term-2 Delhi 2016]

Radius of cone, $r = \frac{6}{2} = 3$ cm

Height of cone, $h = 3.5$ cm

Volume of wheat in the form of cone

$$V = \frac{1}{3}\pi r^2 h$$

NODIA

slant height of conical part is 29 feet.

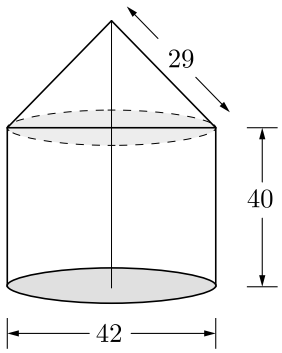
(i) How much metal sheet is required to make this silo ?

(ii) Find the cost of metal sheet needed to make the silo if the metal sheet is available at the rate of Rs. 250 per square feet.

(iii) What is the storage capacity of silo ?

Sol :

The structure of silo will be as given below.



Area of metal sheet required will be surface area of silo.

Height of cylinder part,

$$h_{cy} = 40 \text{ feet}$$

Diameter of cylindrical part is 42 feet i.e radius is 21 feet. Since radius of cylinder part is equal to the radius of conical part, for both part

$$r = 21 \text{ feet}$$

Slant height of conical part,

$$l = 29 \text{ feet}$$

Height of conical part,

$$h_{co} = \sqrt{29^2 - 21^2} = 20 \text{ feet}$$

Surface area of silo,

$$\begin{aligned} S_{silo} &= C.S.A \text{ of cone} + C.S.A \text{ of cylinder.} \\ &= \pi rl + 2\pi r h_{cy} \\ &= \pi r(l + 2h_{cy}) \\ &= \frac{22}{7} \times 21(29 + 2 \times 40) \\ &= 22 \times 3 \times 109 \\ &= 7194 \text{ feet}^2 \end{aligned}$$

(ii) $\text{Total Cost} = 7194 \times 125$

$$= 899250 \text{ Rs}$$

(iii) Storage capacity of silo is the volume of silo.

$$\begin{aligned} \text{Volume, } V &= \frac{1}{3}\pi r^2 h_{co} + \pi r^2 h_{cy} \\ &= \pi r^2\left(\frac{1}{3}h_{co} + h_{cy}\right) \\ &= \frac{22}{7} \times 21^2\left(\frac{20}{3} + 40\right) \\ &= \frac{22}{7} \times 21^2 \times \frac{140}{3} \\ &= 22 \times 21 \times 140 \\ &= 61600 \text{ feet}^3 \end{aligned}$$

126. Rainfall is one of the most commonly shared experiences on Earth. Rainfall is also necessary, since it provides water to plants and ultimately fills rivers. Because rainfall is both a needed resources and a threat, it is important to better understand this natural phenomenon. The most common rainfall measurement is the total rainfall depth during a given period, expressed in millimetres (mm). For instance, we might want to know how many millimetres of rain fell over the course of 1 h, 1 day, 1 month, or 1 year.

The rain water from 22m $\times$ 20m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m.

(i) If the rain water collected from the roof fills $\frac{4th}{5}$ of cylindrical vessel then find the rainfall in cm.

(ii) If rainfall is the 1.5 cm find the hight of water collected in cylindrical vessel.

Sol :

(i) Height of cylindrical vessel is 3.5 m and radius is $\frac{2}{2} = 1$ m.

Volume of water collected in cylindrical vessel,

$$\begin{aligned} V &= \frac{4}{5}\pi r^2 h_{cy} \\ &= \frac{4}{5} \times \frac{22}{7} \times (1)^2 \times \left(\frac{7}{2}\right) \text{ m}^3 \\ &= \frac{44}{5} \text{ m}^3 \end{aligned}$$

Let h be the rainfall.

Rain water from roof $= 22 \times 20 \times h \text{ m}^3$

Now $22 \times 20 \times h = \frac{44}{5}$

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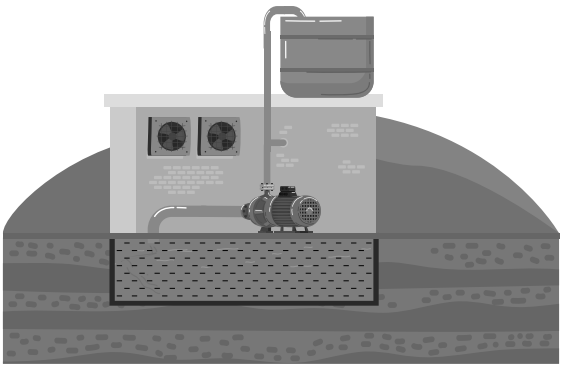
Cost of box of 100 toys,

$$= 4356 \times 100 = 435600 \text{ paise}$$

$$= 4356 \text{ Rs}$$

136. Underground water tank is popular in India. It is usually used for large water tank storage and can be built cheaply using cement-like materials. Underground water tanks are typically chosen by people who want to save space. The water in the underground tank is not affected by extreme weather conditions. The underground tanks maintain cool temperatures in both winter and summer. Electric pump is used to move water from the underground tank to overhead tank.



Ramesh has build recently his house and installed a underground tank and overhead tank. Dimensions of tanks are as follows :

Underground Tank : Base 2 m × 2 m and Height 1.1 m.

Overhead tank : Radius 50 cm and Height 175 cm

(i) What is the capacity of the underground tank?

(ii) What is the ratio of the capacity of the underground tank to the capacity of the overhead tank?

(iii) If curved part of overhead tank need to be painted to save it from corrosion, how much area need to be painted?

(iv) If water is filled in the overhead tank at the rate of 11 litre per minute, the tank will be completely filled in how much time?

(v) If the amount of water in the underground tank, at an instant, is 2400 litres, find then the water level in the underground tank at that instant.

Sol :

(i) Volume of underground tank,

$$lbh = 2 \times 2 \times 1.1 = 4.4 \text{ m}^3$$

Since 1 m^3 is equal to 1000 litres,

$$4.4 \text{ m}^3 = 4.4 \times 1000 = 4400 \text{ litres}$$

(ii) Radius of overhead is 50 cm i.e. $\frac{1}{2}$ meter and height is 175 cm i.e $1.75 = \frac{7}{4}$ metre.

Thus volume of overhead tank,

$$\pi r^2 h_{cy} = \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{7}{4} = \frac{11}{8} \text{ m}^3$$

$$\frac{\text{Capacity of sump}}{\text{Capacity of Overhead tank}} = \frac{lbh}{\pi r^2 h_{cy}}$$

$$= \frac{4.4}{\frac{11}{8}} = 3.2$$

(iii) C.S.A. of cylindrical tank

$$2\pi r h_{cy} = 2 \times \frac{22}{7} \times \frac{1}{2} \times \frac{7}{4} = 5.5 \text{ m}^2$$

(iv) Volume of water in cylindrical tank is $\frac{11}{8}$ m^3.

$$\frac{11}{8} \text{ m}^3 = \frac{11}{8} \times 1000 \text{ litres}$$

Thus time taken to fill tank,

$$= \frac{11}{8} \times 1000 \times \frac{1}{11} = 125 \text{ minutes}$$

(v) Volume of water in underground tank

$$= 2400 \text{ litres} = 2.4 \text{ m}^3$$

Then, $V = lbh_1$

$$2 \times 2 \times h_1 = 2.4$$

$$h_1 = \frac{2.4}{2 \times 2} = 0.6 \text{ m} = 60 \text{ cm}$$

137. Atal Tunnel : Atal Tunnel (also known as Rohtang Tunnel) is a highway tunnel built under the Rohtang Pass in the eastern Pir Panjal range of the Himalayas on the Leh-Manali Highway in Himachal Pradesh. At a length of 9.02 km, it is the longest tunnel above 10,000 feet (3,048 m) in the world and is named after former Prime Minister of India, Atal Bihari Vajpayee. The tunnel reduces the travel time and overall distance between Manali and Keylong on the way to Leh. Moreover, the tunnel bypasses most of the sites that were prone to road blockades, avalanches, and traffic snarls.

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128. Advertising columns are cylindrical outdoor sidewalk structures with a characteristic style that are used for advertising and other purposes. They are common throughout Germany including its capital Berlin, where the first 100 columns were installed in 1855. Advertising columns are typically used to display advertisements in the form of posters, traditionally chiefly theatre, cinema, nightclub, and concert announcements. Some are motorized and rotate very slowly.

Rajesh has been given the task of designing a advertising column for a client. It consist of a cylindrical part surmounted by hemisphere part on top. The base diameter of column is 7 feet and height of cylindrical part is 11 feet.

(i) Find the surface area of cylindrical part of column?

(ii) Find the total surface area of advertising column?

(iii) If cost of construction of column is Rs 75 per sq feet, find the total cost of a column.

(iv) Find the volume of advertising column ?

Sol :

Radius of both cylindrical part and hemispherical part is same and equal to $r = \frac{7}{2} = 3.5$ feet

(i)Surface area of cylindrical part of column,

$$T_{cy} = 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 3.5 \times 11 = 242 \text{ feet}^2$$

(ii) Total surface area of column

= SA of Cylindrical part + SA of hemisphere

$$= 242 + 2\pi r^2$$

$$= 242 + 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 242 + 22 \times \frac{7}{2}$$

$$= 242 + 11 \times 7$$

$$= 242 + 77 = 319 \text{ feet}^2$$

(iii) Total cost of column,

$$C = 319 \times 75 = 23925 \text{ Rs}$$

(iv) Volume of column,

= Volume of cylinder + Volume of hemisphere

$$= \pi r^2 h + \frac{2\pi r^3}{3}$$

$$= \pi r^2 \left(h + \frac{2}{3} r\right)$$

$$= \frac{22}{7} \times 3.5 \times 3.5 \left(11 + \frac{2 \times 3.5}{3}\right)$$

$$= 11 \times 3.5 \left(\frac{33 + 2 \times 3.5}{3}\right)$$

$$= 11 \times 3.5 \times \frac{40}{3}$$

$$= 513.33 \text{ feet}^3$$

129. Amul Dairy : Over the years, Amul, one of the most beloved brands of our country, has become the taste of India, just as its tagline claims. Every Indian millennial has grown up listening to the jingles of its many dairy products, and the Amul girl, the brand's mascot in the polka-dotted dress, has become a nostalgia-evoking symbol. Amul has truly come a long way since its founding in 1946.

A milk tanker of Amul diary, cylindrical in shape having diameter 2 m and length 4.2 m supplies milk to the two booths in the ratio of 3 : 2. One of the milk booths has cuboidal vessel having base area 3.96 sq. m. and the other has a cylindrical vessel having radius 1 m. Use $\pi = \frac{22}{7}$.

(i) What is the volume of milk tanker ?

(ii) Find the level of milk in cuboidal vessel ?

(iii) How much milk is supplied to cylindrical vessel ?

(iv) Find the level of milk in cylindrical vessel ?

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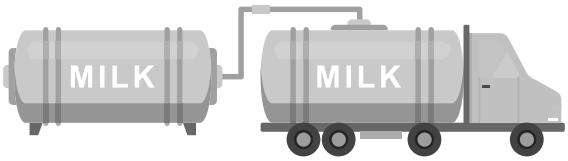

Sol :

(i) Radius of milk tanker $R = \frac{2}{2} = 1$ m

Length of milk tanker $L = 4.2$ m

Volume of milk tanker,

$$V = \pi R^2 L$$

$$= \frac{22}{7} \times \left(\frac{2}{2}\right)^2 \times 4.2 = 13.2 \text{ m}^3$$

(ii) Supply of milk to booth I,

$$= 13.2 \times \frac{3}{5} = 2.64 \times 3 = 7.92 \text{ m}^3$$

Height in 1st vessel $= \frac{7.92}{3.96} = 2$ m

(iii) Supply of milk to booth II,

$$= 13.2 \times \frac{2}{5} = 2.64 \times 2 = 5.28 \text{ m}^3$$

Area of base of cylindrical vessel $= \pi(1)^2 = \frac{22}{7}$ m^2

Height in 2nd vessel $= \frac{5.28}{\frac{22}{7}} = \frac{5.28 \times 7}{22} = 1.68$ m

130. Suppose a sugar cone is 10 centimeters deep and has a diameter of 4 centimeters. A spherical scoop of ice cream with a diameter of 4 centimeters rests on the top of the cone.

(i) If all the ice cream melts into the cone, will the cone overflow? Explain.

(ii) If the cone does not overflow, what percent of the cone will be filled?

Sol :

Radius is common for both cone and spherical scoop which is $r = \frac{4}{2} = 2$ cm.

For the cone, height is $h = 10$ cm, thus

Volume, $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(2)^2(10)$$

$$= 41.9 \text{ cm}^3$$

For the ice cream,

Volume, $V = \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi(2)^3$$

$$= 33.5 \text{ cm}^3$$

Since the volume of the ice cream is less than the volume of the cone, the cone will not overflow.

Now $\frac{\text{volume of ice cream}}{\text{volume of cone}} = \frac{33.5}{41.9}$

$$= 0.80$$

(i) Since the volume of the ice cream is less than the volume of the cone, the cone will not overflow.

(ii) The cone will be about 80% filled.

131. Earth, the third planet from the sun, is the fifth-largest planet in the solar system The radius of Earth at the equator is 3,963 miles (6,378 kilometers), according to NASA's Goddard Space Flight Center in Greenbelt, Maryland. However, Earth is not quite a sphere. The planet's rotation causes it to bulge at the equator. Earth's polar radius is 3,950 miles (6,356 km) — a difference of 13 miles (22 km).

The diameter of Earth is d_1 miles from the North Pole to the South Pole and d_2 miles from opposite points at the equator.

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(i) Find the surface area of Earth using each measure.

(ii) About 75% of Earth's surface is covered by water. Find the surface area of water on Earth, using the mean of the two diameters.

(iii) If the atmosphere of Earth extends to about 100 miles above the surface, find the surface area of the atmosphere surrounding Earth. Use the mean of the two diameters.

(iv) Find the volume of the atmosphere surrounding Earth using the mean of the two diameters..

Sol :

(i) Pole to pole: $T = \pi d^2 = \pi d_1^2$

Equator: $T = \pi d_2^2$

(ii) Mean diameter,

$$d = \tfrac{1}{2}(d_1 + d_2)$$

75% of Earth's surface,

$$= 0.75(\pi d^2)$$
$$= 0.75\pi d^2$$
$$= \frac{3}{4} \times \frac{22}{7} \times \frac{1}{4}(d_1 + d_2)^2$$
$$= \frac{33}{56}(d_1 + d_2)^2$$

(iii) If the atmosphere of Earth extends to about 100 miles above the surface , the diameter of the atmosphere is 200 miles longer than the mean value i.e. $200 + \frac{1}{2}(d_1 + d_2)$.

$$T = \pi d^2$$
$$= \pi[200 + \tfrac{1}{2}(d_1 + d_2)]^2$$
$$= \tfrac{\pi}{4}(400 + d_1 + d_2)^2$$

(iv) Volume of sphere,

$$V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$$

Volume of atmosphere surrounding,

$$V_{\text{earth}} = \frac{1}{6}\pi\left(200 + \frac{d_1 + d_2}{2}\right)^3 - \frac{1}{6}\pi\left(\frac{d_1 + d_2}{2}\right)^3$$
$$= \frac{1}{48}\pi[(200 + d_1 + d_2)^3 - (d_1 + d_2)^3]$$

132. Mother's Day is a celebration honouring the mother of the family or individual, as well as motherhood, maternal bonds, and the influence of mothers in society. It is celebrated on different days in many parts of the world, most commonly in the month of May.

India today did a survey among children for the amount of time spend with mothers. The result of survey is shown in figure. Suppose the bubble in the graphic is a sphere with a radius of 21 cm.

INDIA TODAY SURVEY
The world needs our mothers.
What kids 8-12 say they feel about the amount of time they spend with their mothers.
Wish they spend less time together
10%
30%
Wish they spend more time together
60%
Just right

(i) What is the volume of the bubble?

(ii) What is the volume of the portion of the bubble in which the kids had just the right amount of time with their mother?

(ii) What is the surface area of the portion of the bubble in which the kids had wish for more amount of time with their mother?

(iv) What is the area of the two-dimensional sector of the circle in which the kids wish they could spend less time with their mother?

Sol :

(i) Radius of bubble is 21 cm, thus volume.

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(21^3) = \frac{4}{3} \times \frac{22}{7} \times (21^3)$$
$$= 4 \times 22 \times 21 \times 21$$
$$= 38808 \text{ cm}^3$$

The volume is 38808 cm^2.

(ii) total volume × "just right" percentage

$$= 38808 \times 0.6$$
$$= 23284.8 \text{ cm}^2$$

The "just right" volume is approximately 23284.8 cm^3.

(iii) Surface area, $T = 4\pi r^2$

$$= 4\pi(21^2)$$
$$= 4 \times \tfrac{22}{7}(21)^2$$
$$= 4 \times 22 \times 3 \times 21$$
$$= 5544 \text{ cm}^2$$

total area × "wish for more" percentage

$$= 5544 \times 0.3$$

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$= 1663.2 \text{ cm}^2$

The "wish for more" surface area is approximately 1163 mm^2.

(iv) Area of 2 dimensional circle,

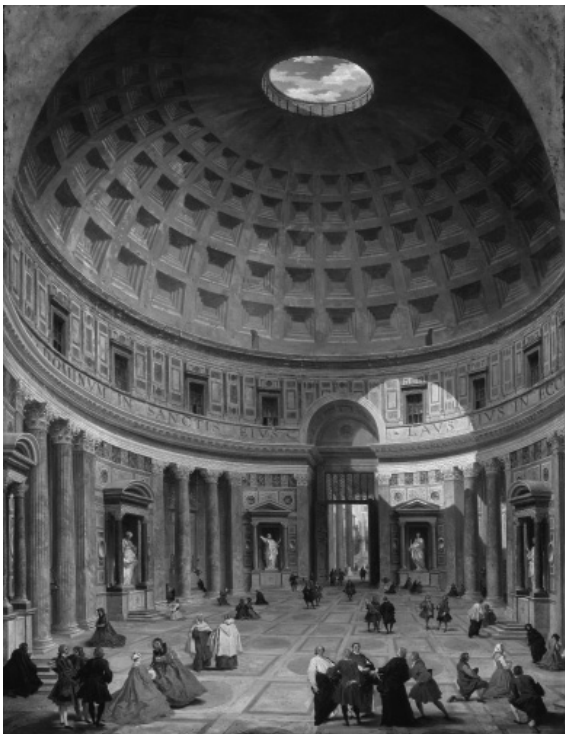
$$A = \pi r^2$$
$$= \tfrac{22}{7} \times (21^2) = 22 \times 3 \times 21$$
$$= 1386 \text{ cm}^2$$

total area $\times$ "wish for less" percentage

$$= 1386 \times 0.1$$
$$= 138.6 \text{ cm}^2$$

The area of the "wish for less" sector is approximately 138.6 mm^2.

133. Pantheon : The Pantheon is the best-preserved building from ancient Rome. Its magnificent dome is a lasting testimony to the genius of Roman architects and as the building stands virtually intact it offers a unique opportunity for the modern visitor to step back 2,000 years and experience the glory that was Rome.



The Pantheon in Rome is able to contain a perfect sphere. The building is a cylinder 42 meter diameter with a hemispherical domed roof. The total height is 42 meter. Find the volume of the interior of the Pantheon.

Sol :

Radius of cylindrical part and hemispherical part is $\frac{42}{2} = 21$ meter. Since total height of Pantheon is 42 metre, height of cylinder is 21 meter.

Volume = volume of cylinder + volume of hemisphere

$$V = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$
$$= \pi(21^2)(21) + \tfrac{2}{3} \times \pi(21^3)$$
$$= \pi(21^3)(1 + \tfrac{2}{3})$$
$$= \pi(21^3) \times \tfrac{5}{3}$$
$$= \frac{22}{7} \times (21^3) \times \frac{5}{3}$$
$$= 22 \times 21 \times 21 \times 5 = 48510 \text{ m}^3$$

The volume is approximately 48510 m^3.

134. DK Jain runs a company that makes ball bearings. The bearings are shipped in boxes that are then loaded onto trucks. Each bearing has a diameter of 18 mm.

(i) Each box can hold $3888\pi \text{ cm}^3$ of ball bearings. How many ball bearings can a box hold?

(ii) Each ball bearing has a mass of 4 gm. Determine the mass of each box.

(iii) The maximum mass a truck can carry is 11000 kg. What is the maximum number of boxes that can be loaded into a truck?

Sol :

(i) Volume of a ball bearing

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi\left(\frac{18}{2}\right)^3$$
$$= \frac{4 \times \pi \times 9 \times 9 \times 9}{3}$$
$$= 972\pi \text{ mm}^3$$

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Since box can hold $3888\pi\ \text{cm}^3$ or $3888000\pi\ \text{mm}^2$, ball bearing in box

$$= \frac{3888000\pi}{972\pi}$$
$$= 4000 \text{ ball bearing}$$

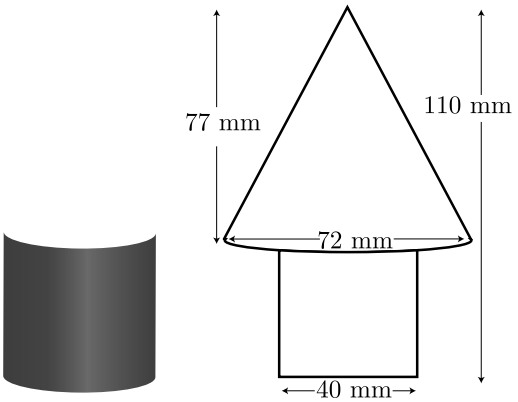
(ii) Mass of box

$$= 4000 \times 4$$
$$= 16000\ \text{gm} = 16\ \text{kg}$$

(iii) Maximum number of box loaded in truck,

$$= \frac{12000}{16} = 750 \text{ box}$$

135. In a toys manufacturing company, wooden parts are assembled and painted to prepare a toy. For the wood processing activity center, the wood is taken out of storage to be sawed, after which it undergoes rough polishing, then is cut, drilled and has holes punched in it. It is then fine polished using sandpaper. For the retail packaging and delivery activity center, the polished wood sub-parts are assembled together, then decorated using paint.



One specific toy is in the shape of a cone mounted on a cylinder. The total height of the toy is 110 mm and the height of its conical part is 77 mm. The diameters of the base of the conical part is 72 mm and that of the cylindrical part is 40 mm.

(i) If its cylindrical part is to be painted red, what is the surface area need to be painted ?

(ii) If its conical part is to be painted blue, what is the surface area need to be painted ?

(iii) How much of the wood have been used in making the toy ?

(iv) If the cost of painting the toy is 2 paise for $8\pi\ \text{mm}^2$, then what is the cost of painting of a box of 100 toys?

(v) If the toy manufacturer company charge 3 paise for $32\pi\ \text{mm}^3$ of wood, what is the price of a box of 100 toys?

Sol :

(i) Radius of cylindrical part, $r_{cy} = \frac{40}{2} = 20$ mm

Height of cylindrical part,

$$h_{cy} = 110 - 77 = 33 \text{ mm}$$

$$\text{C.S.A. of cylinder} = 2\pi r_{cy} h_{cy} + \pi r_{cy}^2$$
$$= \pi r_{cy}(2h_{cy} + r_{cy})$$
$$= 20\pi(2 \times 33 + 20)$$
$$= 1720\pi \text{ mm}^2$$

(ii) Radius of conical part is $\frac{72}{2} = 36$.

C.S.A. of conical part,

$$\pi r_{co} l_{co} + \pi r_{co}^2 - \pi r_{cy}^2$$
$$= \pi r_{co}\sqrt{h_{co}^2 + r_{co}^2} + \pi(r_{co}^2 - r_{cy}^2)$$
$$= \pi[r_{co}\sqrt{h_{co}^2 + r_{co}^2} + (r_{co}^2 - r_{cy}^2)]$$
$$= \pi[36\sqrt{77^2 + 36^2} + (36^2 - 20^2)]$$
$$= \pi[36 \times 85 + (36^2 - 20^2)]$$
$$= \pi(3060 + 1296 - 400)$$
$$= 3956\pi\ \text{mm}^2$$

(iii) Volume of toy = Volume of cone + Volume of cylinder

$$= \frac{1}{3}\pi r_{co}^2 h_{co} + \pi r_{cy}^2 h_{cy}$$
$$= \pi\left(\frac{1}{3} \times 36 \times 36 \times 77 + 20 \times 20 \times 33\right)$$
$$= 48\pi(9 \times 77 + 5 \times 5 \times 11)$$
$$= 46464\pi\ \text{mm}^3$$

(iv) Surface area = S.A. of cone + S.A. of cylinder

$$= 3956\pi + 1720\pi = 5676 \text{ mm}^2$$

Cost of painting is 2 paise for $8\pi\ \text{mm}^2$, thus cost of painting of 1 mm^2 is $\frac{2}{8\pi}$ paise.

Hence cost of painting of a toy of $3956\pi\ \text{mm}^2$,

$$= \frac{2}{8\pi} \times 5676\pi = 1419 \text{ paise}$$

Cost of painting of 100 toys,

$$= 1419 \times 100 = 141900 \text{ paise}$$
$$= 1419 \text{ Rs}$$

(v) Cost of toy is 3 paise for $32\pi\ \text{mm}^3$ of wood, thus cost of wood of 1 mm^3 is $\frac{3}{32\pi}$ paise.

Hence cost of a toy of $46464\pi\ \text{mm}^3$,

$$= \frac{3}{32\pi} \times 46464\pi = 4356 \text{ paise}$$

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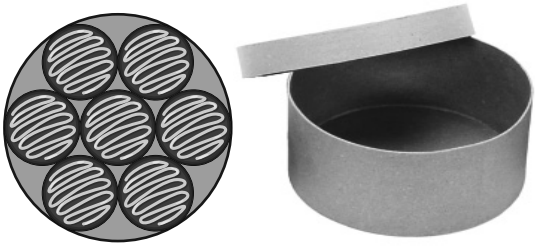
required to make one biscuit is 18 cu cm. Before it is cooked, the mixture is rolled into a sphere. After the biscuit is cooked, the biscuit becomes a cylinder of radius 3 cm and height 0.7 cm. The increase in volume is due to air being trapped in the biscuit. Biscuits are packed in a cylindrical card box of height 14 cm. The arrangement of biscuits is shown below.



(i) What is the volume of the biscuits after it is cooked ?

(ii) What is the volume of air trapped, while cooking the biscuit ?

(iii) How many biscuits will be there in a box ?

(iv) How much space is vacant in box after biscuits are packed ?

(v) If weight of 7 biscuits is 50 grams, what will be the weight of box of biscuits?

Sol :

(i) Volume of the biscuit,

$$= \pi r^2 h = \frac{22}{7} \times 3^2 \times 0.7 = 19.8 \text{ cu cm}$$

(ii) Volume of air trap

$$= \text{Volume of biscuit} - \text{Volume of sphere}$$

$$= 19.8 - 18 = 1.8 \text{ cu cm}$$

(iii) In a layer, 7 biscuits are arranged whose height is 0.7 cm.

Thus total layer in box,

$$= \frac{14}{0.7} = 20 \text{ layer}$$

Total biscuits in box $= 20 \times 7 = 140$ biscuits

(iv) From figure it is clear that radius of box is 3 times of biscuit radius i.e. $3 \times 3 = 9$.

$$\text{Volume of box} = \pi R^2 H$$

$$= \frac{22}{7} \times 9 \times 9 \times 14$$

$$= 22 \times 9 \times 9 \times 2$$

$$= 3564 \text{ cm}^3$$

$$\text{Volume of biscuits} = \pi r^2 h \times 140$$

$$= 19.8 \times 140 = 2772 \text{ cm}^3$$

$$\text{Vacant volume} = 3564 - 2772 = 792 \text{ cm}^3$$

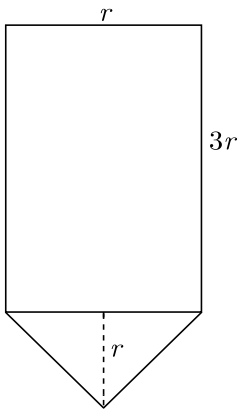
(v) Weight of 7 biscuits $= 50$ grams

$$\text{Weight of 140 biscuits} = \frac{50}{7} \times 140$$

$$= 1000 \text{ grams} = 1 \text{ kg}$$

139. Conical Tank : The advantages of cone bottom tanks are found in nearly every industry, especially where getting every last drop from the tank is important. This type of tank has excellent geometry for draining, especially with high solids content slurries as these cone tanks provide a better full-drain solution. The conical tank eliminates many of the problems that flat base tanks have as the base of the tank is sloped towards the centre giving the greatest possible full-drain system in vertical tank design.

Rajesh has been given the task of designing a conical bottom tank for his client. Height of conical part is equal to its radius. Length of cylindrical part is the 3 times of its radius. Tank is closed from top. The cross section of conical tank is given below.



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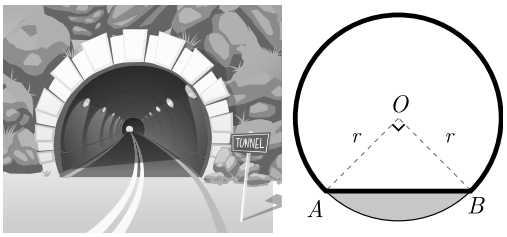
Earth is excavated to make a railway tunnel. The tunnel is a cylinder of radius 7 m and length 450 m. A level surface is laid inside the tunnel to carry the railway lines. Figure given below shows the circular cross - section of the tunnel. The level surface is represented by AB, the centre of the circle is O and $\angle AOB = 90°$. The space below AB is filled with rubble (debris from the demolition buildings).



(i) How much volume of earth is removed to make the tunnel ?

(ii) If the cost of excavation of 1 cubic meter is Rs 250, what is the total cost of excavation?

(iii) A coating is to be done on the surface of inner curved part of tunnel. What is the area of tunnel to be being coated ?

(iv) Costing of coating is Rs 30 per m^2. What is the total cost of coating ?

(v) How much volume of debris is required to fill the ground surface of tunnel ?

Sol :

(i) Cross-section area of tunnel to be excavated $= \pi r^2$

Volume of earth to be removed,

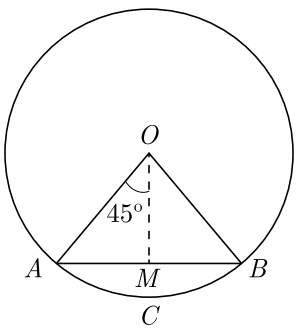
$$\pi r^2 l = \frac{22}{7} \times 7 \times 7 \times 450$$

$$= 69300\ m^3$$

(ii) Total cost of excavation

$$= 69300 \times 250 = ₹\,1732500$$

(iii) The geometry of cross-section is shown below.



Triangle OAB is isosceles triangle having right angle at O.

Length of curved part of cross-section,

$$= \frac{2\pi r(360° - 90°)}{360°}$$

$$= \frac{2 \times \frac{22}{7} \times 7(360° - 90°)}{360°}$$

$$= \frac{2 \times 22 \times 270°}{360°} = 33\ m$$

Total curved surface area of tunnel

= Length of curved part of cross-section $\times$ Length of tunnel

$= 33 \times 450 = 14850\ m^2$

(iv) Cost of coating on curved part,

$$= 14850 \times 30$$

$$= ₹\,445500$$

(v) Cross-section area of debris part of tunnel

$$= \text{Area of } OACB - \text{Area of } \Delta OAB$$

$$= \frac{\pi r^2}{4} - \frac{r^2}{2}$$

$$= \frac{\frac{22}{7} \times 7 \times 7}{4} - \frac{7 \times 7}{2}$$

$$= \frac{11 \times 7}{2} - \frac{7 \times 7}{2}$$

$$= \frac{4 \times 7}{2} = 14\ m^2$$

Volume of debris required

$$= 14 \times 500 = 7000\ \ m^3$$

138. A bakery is an establishment that produces and sells flour-based food baked in an oven such as bread, cookies, cakes, pastries, and pies. Some retail bakeries are also categorized as cafés, serving coffee and tea to customers who wish to consume the baked goods on the premises.

Tania runs a bakery shop and her bakery is very famous for tasty biscuits. The amount of mixture

$$h = \frac{44}{5} \times \frac{1}{22 \times 20} = \frac{1}{50} \text{ m}$$
$$= \frac{1}{50} \times 100 = 2 \text{ cm}$$

(ii) Since rainfall is 1.5 cm, volume of water collected from roof

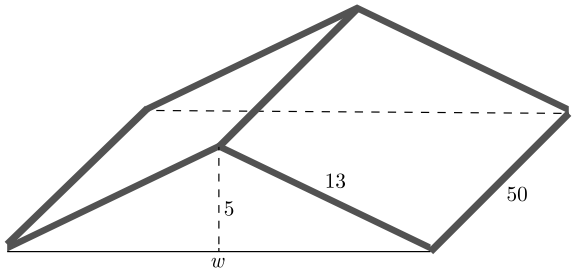
$$V = 22 \times 20 \times h$$
$$= 22 \times 20 \times 0.015$$

This water is collected in cylinder. Let h_1 be the height of water in cylinder. Thus volume of water in cylinder is $\pi r^2 h_1$ which is equal to the water collected from roof.

$$\pi r^2 h_1 = 22 \times 20 \times 0.015$$
$$\frac{22}{7} \times (1)^2 \times h_1 = 22 \times 20 \times 0.015$$
$$h_1 = 7 \times 20 \times 0.015 = 2.1 \text{ m}$$

127. Rain Water Harvesting : Water harvesting is the activity of direct collection of rainwater, rather than allowing it to run off. The rainwater collected can be stored for direct use or can be recharged into the groundwater.

Rajkumar has installed rain water harvesting system in his house. His has a roof with dimensions shown below. He knows that the average rainfall in his suburb is 77 cm per year. Rajkumar would like to install a underground cylindrical rain water tank to hold the water that runs off the roof.



Can you help the Rajkumar answer the following questions

(i) On average what volume of water will fall on the roof each year ?

(ii) How many litres of water does the tank need to hold?

(iii) If the tank has base diameter 7 m, how deep it need to be?

(iv) What is the inner surface area of the tank ?

Sol :

Length of roof, $l = 50$ m

(i) For the rainfall calculation base area of roof will be taken. Let w be the width of base.

Then, $\frac{w}{2} = \sqrt{13^2 - 5^2}$
$= 12$
$w = 24 \text{ m}$

Area of base,

$$S = w \times l$$
$$= 24 \times 50$$
$$= 1200 \text{ m}^2$$

Since rainfall is 77 cm, the volume of water collected in a year,

$$V = 1200 \times 0.77$$
$$= 924 \text{ m}^3$$

(ii) Volume in litre,

$$= 924 \times 1000$$
$$= 924000 \text{ litre}$$

Tank need to hold 924000 litre.

(iii) Volume of tank must be 924 m^3. Let h be height of tank.

Thus, $\pi r^2 h = 924$

$$\frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times h = 924$$
$$h = \frac{924 \times 7 \times 2 \times 2}{22 \times 7 \times 7}$$
$$= 24 \text{ m}$$

(iv) Surface area of tank,

$$S = 2\pi r h$$
$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 24$$
$$= 22 \times 24$$
$$= 528 \text{ m}^2$$

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(i) If radius of cylindrical part is taken as 3 meter, what is the volume of above conical tank ?

(ii) What is the area of metal sheet used to make this conical tank ? Assume that tank is covered from top.

(iii) What is the ratio of volume of cylindrical part to the volume of conical part?

(iv) The cost of metal sheet is Rs 2000 per square meter and fabrication cost is 1000 per square meter. What is the total cost of tank ?

(v) A oil is to be filled in the tank. The density of oil is 1050 kg per cubic meter. What is the weight of oil filled in tank ?

Sol :

(i) Length of cylindrical part is three times of radius of conical part and height of conical part is equal to its radius.

If we assume r be the common radius of cylindrical part and conical part, height of conical part will be r and length of cylindrical part will be $3r$.

Volume of conical tank = Volume of cylindrical part + Volume of conical part

$$= \pi r^2 l + \frac{1}{3}\pi r^2 h$$

$$= \pi r^2 \cdot 3r + \frac{1}{3}\pi r^2 \cdot r$$

$$= 3\pi r^3 + \frac{1}{3}\pi r^3 = \frac{10}{3}\pi r^3$$

$$= \frac{10}{3}\pi (3)^3 = 90\,\pi \text{ m}^3$$

(ii) Surface area of tank,

= SA of top + CSA of cylinder + CSA of cone

$$= \pi r^2 + 2\pi rl + \pi r\sqrt{h^2 + r^2}$$

$$= \pi r^2 + 2\pi r \cdot 3r + \pi r\sqrt{r^2 + r^2}$$

$$= \pi r^2 + 6\pi r^2 + \sqrt{2}\,\pi r^2$$

$$= (1 + 6 + \sqrt{2})\,\pi r^2$$

$$= (7 + \sqrt{2})\,\pi (3)^2$$

$$= 9(7 + \sqrt{2})\,\pi \text{ m}^2$$

(iii) Volume of cylindrical part,

$$= \pi r^2 l = \pi r^2 \cdot 3r = 3\pi r^3$$

Volume of conical part,

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 r = \frac{1}{3}\pi r^3$$

Ratio of volume of cylindrical part to conical part

$$= \frac{3\pi r^3}{\frac{1}{3}\pi r^3} = 9$$

(iv) Surface area of sheet used $= 9(7 + \sqrt{2})\,\pi \text{ m}^2$

Total cost = Cost of sheet + Fabrication cost

$$= 2000 + 1000$$

$$= 3000 \text{ per sq. meter}$$

Total cost of tank $= 9(7 + \sqrt{2})\,\pi \times 3000$

$$= 27000(7 + \sqrt{2})\,\pi$$

(v) Volume of tank i.e. volume of oil $= 90\,\pi \text{ m}^3$

Density of oil = 1050 kg per cubic meter

Weight of oil $= 90\pi \times 1050$

$$= 90 \times \frac{22}{7} \times 1050$$

$$= 90 \times 22 \times 150$$

$$= 297000 \text{ kg} = 297 \text{ Tonne}$$

140. Boiler : The boiler is essentially a closed vessel inside which water is stored. Fuel is burnt in a furnace and hot gasses are produced. These hot gasses come in contact with water vessel where the heat of these hot gases transfer to the water and consequently steam is produced in the boiler. Then this steam is piped to the turbine of thermal power plant.

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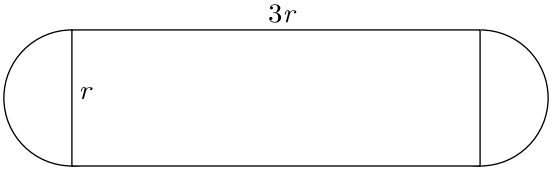
Rajesh has been given the task of designing a boiler for NTPC. Boiler consist of a cylindrical part in middle and two hemispherical part at its both end. The cross section of boiler is given below. Length of cylindrical part is the 3 times of radius of hemispherical part.



(a) 0-20 (b) 20-30
(c) 30-40 (d) 50-60

Sol : [Board Term-1 Foreign 2015]

Let us first construct the following frequency distribution table.

Marks	Number of Students
0-10	3
10-20	9
20-30	16
30-40	29
40-50	18
50-60	5

Since, the maximum frequency is 29 and the class corresponding to this frequency is 30-40. So, the modal class is 30-40.
Thus (c) is correct option.

22. If X, M and Z are denoting mean, median and mode of a data and $X:M=9:8$, then the ratio $M:Z$ is
(a) 3 : 4 (b) 4 : 9
(c) 4 : 3 (d) 2 : 5

Sol : [Board Term-1 SQP 2012]

Since, $M_o = 3M_d - 2M$

$$Z = 3M - 2X \quad ...(1)$$

Now $X:M = 9:8$

$$\frac{X}{M} = \frac{9}{8}$$

$$X = \frac{9M}{8}$$

Substituting the value of X in equation (1), we get

$$Z = 3M - 2 \times \frac{9M}{8}$$

$$= 3M - \frac{9M}{4}$$

$$Z = \frac{3M}{4}$$

$$\frac{M}{Z} = \frac{4}{3}$$

or $M:Z = 4:3$

Thus (c) is correct option.

23. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 min and summarised in the table give below.

Number of cars	Frequency
0-10	7
10-20	14
20-30	13
30-40	12
40-50	20
50-60	11
60-70	15
70-80	08

Then, the mode of the data is
(a) 34.7 (b) 44.7
(c) 54.7 (d) 64.7

Sol : [Board Term-1 Delhi 2014]

Here, modal class is 40-50. Since, it has maximum frequency which is 20.

So, $l = 40$, $f_1 = 20$, $f_0 = 12$, $f_2 = 11$ and $h = 10$

$$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{20 - 12}{40 - 12 - 11}\right) \times 10$$

$$= 40 + \frac{80}{17}$$

$$= 40 + 4.7 = 44.7 \text{ cars}$$

Thus (b) is correct option.

24. If the mean of a, b, c is M and $ab + bc + ca = 0$, the mean of a^2, b^2 and c^2 is KM^2, then K is equal to
(a) 3 (b) 9
(c) 6 (d) 4

Sol : [Board Term-1 OD 2016]

We have $\frac{a+b+c}{3} = M$

$$a + b + c = 3M$$

and $\frac{a^2+b^2+c^2}{3} = KM^2$

Now, $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$$(a+b+c)^2 = a^2 + b^2 + c^2$$

$$[ab + bc + ca = 0]$$

$$(3M)^2 = 3KM^2$$

$$9M^2 = 3KM^2 \Rightarrow K = 3$$

Thus (a) is correct option.

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(i) How much blue PVC coated fabric is required?

(ii) How much white fabric is required ?

(iii) If labour charge for the construction of tent is ₹ 15 per sq. meter what is the total cost of tent ?

(iv) If space requirement of a pilgrims is 6 sq. meter, how many pilgrims can be accommodate in a tent?

(v) If total 50000 pilgrims are expected to visit fair, how many tents are required ?

Sol :

(i) Common radius of conical and cylindrical part is $\frac{42}{2} = 21$ m. Height of conical part is equal to its radius 21 m.

Surface area of conical part of tent

$$\begin{aligned}\pi rl &= \pi r\sqrt{h^2+r^2}\\ &= \frac{22}{7}\times 21\times\sqrt{(21)^2+(20)^2}\\ &= 22\times 3\times 29 = 1914 \text{ sq. meter}\end{aligned}$$

Thus, 1914 sq. meter of blue PVC coated fabric is required.

(ii) Surface area of cylindrical part of tent,

$$\begin{aligned}&= 2\pi rh\\ &= 2\times\frac{22}{7}\times 21\times 5\\ &= 660 \text{ sq. meter}\end{aligned}$$

(iii) Total cost is sum of material cost and construction cost of both type of fabric.

Total cost = White fabric cost + Blue fabric cost

$$\begin{aligned}&= (60+15)\times 1914+(70+15)\times 660\\ &= 75\times 1914+85\times 660\\ &= 75(1914+17\times 44)\\ &= ₹199650\end{aligned}$$

(iv) Total floor area of tent,

$$\begin{aligned}\pi r^2 &= \frac{22}{7}\times 21\times 21\\ &= 22\times 3\times 21\\ &= 1386 \text{ sq. meter}\end{aligned}$$

$$\text{Pilgrims in a tent} = \frac{1386}{6} = 231 \text{ pilgrims}$$

(v) Total requirement of tent

$$= \frac{50000}{231} = 216.45$$

Thus, 217 tent are required.

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(i) Which of the following is correct expression for the surface area of cylindrical part of boiler?

(ii) Which of the following is correct expression for the total surface area of boiler?

(iii) Which of the following is correct expression for the volume of boiler?

(iv) What is the ratio of volume to the surface area?

(v) If $r = 3$ m, what is the volume of boiler?

Sol :

(i) Radius of cylindrical part is equal to the radius of hemispherical part and length of cylindrical part is three times of radius.

Surface area of cylindrical part of boiler $= 2\pi rl$

Since $l = 3r$,

$$2\pi rl = 2\pi r \times 3r = 6\pi r^2$$

(ii) Total surface area of boiler

= SA of cylindrical part + SA of two hemisphere

$$= 6\pi r^2 + 2\left(\frac{4\pi r^2}{2}\right)$$

$$= 6\pi r^2 + 4\pi r^2 = 10\pi r^2$$

(iii) Volume of boiler,

= Volume of cylinder + Volume of two hemisphere

$$= \pi r^2 l + 2\left(\frac{2\pi}{3} \times r^3\right)$$

$$= \pi r^2 \cdot 3r + \frac{4\pi}{3} \times r^3$$

$$= \left(3 + \frac{4}{3}\right)\pi r^3 = \frac{13}{3}\pi r^3$$

(iv) Ratio of volume to the surface

$$= \frac{\frac{13}{3}\pi r^3}{10\pi r^2} = \frac{13}{30}r$$

(v) At $r = 3$ m volume of boiler,

$$= \frac{13}{3}\pi r^3$$

$$= \frac{13}{3} \times \pi \times 3^3$$

$$= 13 \times \pi \times 9 = 117\pi \text{ m}^3$$

141. Kumbh Mela is a major pilgrimage and festival in Hinduism. It is celebrated in a cycle of approximately 12 years at four river-bank pilgrimage sites: the Prayagraj (Ganges-Yamuna Sarasvati rivers confluence), Haridwar (Ganges), Nashik (Godavari), and Ujjain (Shipra).

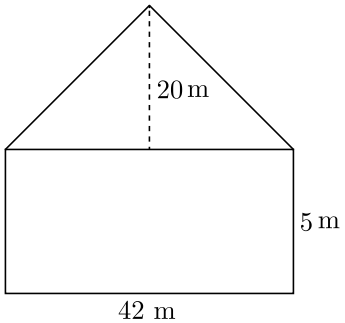
The festival is marked by a ritual dip in the waters. The seekers believe that bathing in these rivers is a means to prayascitta for past mistakes, and that it cleanses them of their sins.

Government of UP is planing to procure tent for the pilgrims during Kumbh Mela. The specification of tent is given below.

(1) Lower cylindrical part must have a white colored thick fabric whose cost is ₹ 60 per square meter.

(2) Top conical part must have PVC coated blue fabric whose cost is ₹ 70 per square meter.

The front view of tent is given below with dimension.



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We prepare the following cumulative table

Height x (in cm)	Number of Students (f)	cf
150-155	15	15
155-160	13	28
160-165	10	38
165-170	08	46
170-175	09	55
175-180	08	63
	$N = 63$	

We have, $N = 63;$

$$\frac{N}{2} = \frac{63}{2}$$

$$= 31.5$$

The cumulative frequency just greater than $\frac{N}{2}$ is 38 and the corresponding class is 160-165. Thus upper limit is 165.

Thus (a) is correct option.

7. For finding the popular size of readymade garments, which central tendency is used?

(a) Mean

(b) Median

(c) Mode

(d) Both Mean and Mode

Sol : [Board Term-1 Delhi 2014]

For finding the popular size of ready made garments, mode is the best measure of central tendency.

Thus (c) is correct option.

8. If the difference of mode and median of a data is 24, then the difference of median and mean is

(a) 12 (b) 24

(c) 08 (d) 36

Sol : [Board Term-1 OD 2011]

We have, $M_o - M_d = 24$

We know $M_o = 3M_d - 2M$

Now $M_o - M_d = 2M_d - 2M$

$$24 = 2(M_d - M)$$

$$M_d - M = 12$$

Thus (a) is correct option.

9. If the mean of the numbers $27 + x$, $31 + x$, $89 + x$ $107 + x, 156 + x$ is 82, then the mean of $130 + x$, $126 + x$, $68 + x$, $50 + x$, and $1 + x$ is

(a) 75 (b) 157

(c) 82 (d) 80

Sol : [Board Term-1 SQP 2012]

Given,

$$82 = \frac{(27+x)+(31+x)+(89+x)+(107+x)+(156+x)}{5}$$

$$82 \times 5 = 410 + 5x$$

$$410 - 410 = 5x \Rightarrow x = 0$$

Required mean is,

$$\overline{x} = \frac{130+x+126+x+68+x+50+x+1+x}{5}$$

$$= \frac{375+5x}{5} = \frac{375+0}{5}$$

$$= \frac{375}{5} = 75$$

Thus (a) is correct option.

10. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then the median of the new set

(a) Is increased by 2

(b) Is decreased by 2

(c) Is two times the original median

(d) Remains the same as that of the original set

Sol : [Board Term-1 Foreign 2017]

Since, $n = 9$

then, median term $= \left(\frac{9+1}{2}\right)^{\text{th}} = 5^{\text{th}}$ item.

Now, last four observations are increased by 2, but median is 5^{th} observation, which is remaining unchanged. Hence there will be no change in median.

Thus (d) is correct option.

11. If the coordinates of the point of intersection of less than ogive and more than ogive is (13.5,20), then the value of median is

(a) 13.5 (b) 20

(c) 33.5 (d) 7.5

Sol : [Board Term-1 Delhi 2013]

The abscissa of point of intersection gives the median of the data. So, median is 13.5.

Thus (a) is correct option.

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12. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is

(a) 6 (b) 7

(c) 8 (d) 10

Sol : [Board Term-1 SQP 2012]

Mode of the data is 8 as it is repeated maximum number of times. Thus (c) is correct option.

13. If the mean of the observation $x, x+3, x+5, x+7$ and $x+10$ is 9, the mean of the last three observation is

(a) $10\frac{1}{3}$ (b) $10\frac{2}{3}$

(c) $11\frac{1}{3}$ (d) $11\frac{2}{3}$

Sol : [Board Term-1 SQP 2015]

$$\text{Mean} = \frac{\text{Sum of all the observations}}{\text{Total no. of observation}}$$

$$9 = \frac{x+x+3+x+5+x+7+x+10}{5}$$

$$9 = \frac{5x+25}{5}$$

$$x = 4$$

So, mean of last three observation,

$$= \frac{x+5+x+7+x+10}{3} = \frac{5x+22}{3}$$

$$\frac{3x+22}{3} = \frac{3\times 4+22}{3}$$

$$= \frac{12+22}{3} = \frac{34}{3} = 11\frac{1}{3}$$

Thus (c) is correct option.

14. The mean and median of the data a, b and c are 50 and 35 respectively, where $a<b<c$. If $c-a=55$, then $(b-a)$ is

(a) 8 (b) 7

(c) 3 (d) 5

Sol : [Board Term-1 Delhi 2014]

Since, a, b and c and are in ascending order, therefore median is b i.e $b=35$.

$$\text{Mean} \quad \frac{a+b+c}{3} = 50$$

$$a+b+c = 150$$

$$a+c = 150-35 = 115 \qquad ...(1)$$

Also, it is given that $c-a = 55$ ...(2)

Subtracting equation (2) and (1), we get

$$a = 30$$

Hence, $b-a = 35-30 = 5$

Thus (d) is correct option.

15. Observations of some data are $\frac{x}{5}$, x, $\frac{x}{3}$, $\frac{2x}{3}$, $\frac{x}{4}$, $\frac{2x}{5}$ and $\frac{3x}{4}$ where $x>0$. If the median of the data is 4, then the value of x is

(a) 5 (b) 15

(c) 9 (d) 10

Sol : [Board Term-1 Foreign 2016]

Given observations are $\frac{x}{5}$, x, $\frac{x}{3}$, $\frac{2x}{3}$, $\frac{x}{4}$, $\frac{2x}{5}$ and $\frac{3x}{4}$ where $x>0$. On arranging the above observations in ascending order, we get

$$\frac{x}{5}, \frac{x}{4}, \frac{x}{3}, \frac{2x}{5}, \frac{2x}{3}, \frac{3x}{4}, x$$

Here, total number of observations are 7, which is odd.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$

$$= \left(\frac{7+1}{2}\right)^{\text{th}} \text{ observation}$$

$$= 4^{\text{th}} \text{ observation} = \frac{2x}{5}$$

$$\text{Median} = \frac{2x}{5} = 4$$

$$x = \frac{4\times 5}{2} = 10$$

Thus (d) is correct option.

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16. If the mean of the squares of first n natural numbers is 105, then the first n natural numbers is

(a) 8 (b) 9

(c) 10 (d) 11

Sol : [Board Term-1 OD 2010]

Sum of square, $\sum x^2 = \frac{n(n+1)(2n+1)}{6}$

Mean of squares of first n natural numbers,

$$105 = \frac{(n+1)(2n+1)}{6}$$

$$2n^2+3n+1 = 630$$

$$2n^2+3n-629 = 0$$

$$2n^2+37n-34n-629 = 0$$

$$n(2n+37)-17(2n+37) = 0$$

$$(2n+37)(n-17) = 0 \Rightarrow n = 17$$

Since, n is odd, therefore median is

$= \left(\frac{17+1}{2}\right)^{\text{th}} = 9^{\text{th}}$ observation.

Thus (b) is correct option.

17. Mode of the following grouped frequency distribution is

Class	Frequency
3-6	2
6-9	5
9-12	10
12-15	23
15-18	21
18-21	12
21-24	03

(a) 13.6 (b) 15.6

(c) 14.6 (d) 16.6

Sol : [Board 2010]

We observe that the class 12-15 has maximum frequency 23. Therefore, this is the modal class.

We have, $l = 12$, $h = 3$, $f_1 = 23$, $f_0 = 10$ and $f_2 = 21$

$$M_o = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 12 + \frac{23 - 10}{46 - 10 - 21} \times 3$$

$$= 12 + \frac{13}{15} \times 3$$

$$= 12 + \frac{13}{5} = 14.6$$

Thus (c) is correct option.

18. While computing the mean of grouped data, we assume that the frequencies are

(a) evenly distributed over all the classes

(b) centred at the class marks of the classes

(c) centred at the upper limits of the classes

(d) centred at the lower limits of the classes

Sol : [Board Term-1 Delhi 2013]

While computing mean of ground data, we assume that the frequencies distribution table.

Thus (b) is correct option.

19. If median is 137 and mean is 137.05, then the value of mode is

(a) 156.90 (b) 136.90

(c) 186.90 (d) 206.90

Sol : [Board 2009]

$$M_o = 3M_d - 2M$$

$$= 3(137) - 2(137.05)$$

$$= 411 - 274.10 = 136.90$$

Thus (b) is correct option.

20. The following data gives the distribution of total household expenditure (in Rs) of manual workers in a city.

Expenditure (in Rs)	Frequency
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	07

Then, find the average expenditure which is done by the maximum number of manual workers.

(a) 1747.26 (b) 1847.26

(c) 1947.26 (d) 2047.26

Sol : [Board Term-1 Delhi 2013]

We observe that the class 1500-2000 has maximum frequency 40. Therefore, this is the modal class.

We have $l = 1500$, $h = 500$, $f_1 = 40$, $f_0 = 24$ and $f_2 = 23$

$$M_o = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 1500 + \frac{40 - 24}{80 - 24 - 33} \times 500$$

$$= 1500 + \frac{16}{23} \times 500$$

$$= 1847.26$$

Thus (b) is correct option.

21. For the following distribution

Marks	Number of Students	Marks	Number of students
Below 10	3	Below 40	57
Below 20	12	Below 50	75
Below 30	28	Below 60	80

The modal class is

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CHAPTER 13

Statistics

OBJECTIVE QUESTIONS

1. The middle most observation of every data arranged in order is called

(a) mode (b) median

(c) mean (d) deviation

Sol : [Board 2024 OD Standard]

The middle most observation, after arranging all observations in ascending or descending order, is called the median.

Thus (b) is correct option.

2. For some data $x_1, x_2, \ldots\ldots\ldots\ldots x_n$ with respective frequencies $f_1, f_2, \ldots\ldots f_n$, the value of $\sum_1^n f_i(x_i - \overline{x})$ is equal to

(a) $n\overline{x}$ (b) 1

(c) $\sum f_i$ (d) 0

Sol : [Board 2024 OD Standard]

$$\sum_1^n f_i(x_i - \overline{x}) = 0$$

Thus (d) is correct option.

3. The median and mode respectively of a frequency distribution are 26 and 29, Then its mean is

(a) 27.5 (b) 24.5

(c) 28.4 (d) 25.8

Sol : [Board 2020 Delhi Basic]

We have $M_o = 3M_d - 2M$

$29 = 3 \times 26 - 2M$

$2M = 78 - 29$

$= 49$

$M = \frac{49}{2}$

$= 24.5$

Thus (b) is correct option.

4. The cumulative frequency table is useful in determining

(a) Mean (b) Median

(c) Mode (d) All of these

Sol : [Board 2020 OD Basic]

Cumulative frequency is defined as a running total of frequencies. It is helpful in finding the mean, median and mode.

Thus (d) is correct option.

5. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is

(a) 6 (b) 7

(c) 8 (d) 12

Sol :

Let x be the upper limit and y be the lower limit.

Since the mid value of the class is 10.

Hence, $\frac{x+y}{2} = 10$

$x + y = 20$ ...(1)

Since width of the class is 6,

$x - y = 6$ ...(2)

Solving (1) and (2), we get $y = 7$

Hence, lower limit of the class is 7.

Thus (b) is correct option.

6. Consider the following frequency distribution of the heights of 60 students of a class

Height (in cm)	150-155	155-160	160-165	165-170	170-175	175-180
Number of students	15	13	10	8	9	5

The upper limit of the median class in the given data is

(a) 165 (b) 155

(c) 160 (d) 170

Sol : [Board 2020 SQP Standard]

NODIA

25. In the formula $\overline{x} = a + \frac{\sum f_i d_i}{\sum f_i}$, for finding the mean of

grouped data d_i's are deviation from a of

(a) lower limits of the classes

(b) upper limits of the classes

(c) mid-points of the classes

(d) frequencies of the class marks

Sol : [Board Term-1 OD 2014]

Mid-point of the classes $= (x_i - a)$,

where, $x_i = \frac{\text{upper limit} + \text{lower limit}}{2}$

So, the option (c) is correct, which is the required answer.

Thus (c) is correct option.

26. While computing mean of grouped data, we assume that the frequencies are

(a) evenly distributed over all the classes

(b) centred at the class marks of the classes

(c) centred at the upper limits of the classes

(d) centred at the lower limits of the classes

Sol : (b) centred at the class marks of the classes

Frequencies are centred at the class-marks of the classes.

So, the option (b) is correct, which is the required answer.

Thus (b) is correct option.

27. If x_i's are the mid-points of the class intervals of grouped data, f_i's are the corresponding frequencies and $\overline{x}$ is the mean, then $\sum(f_i x_i - \overline{x})$ is equal to

(a) 0 (b) -1

(c) 1 (d) 2

Sol : [Board Term-1 Foreign 2017]

$$\sum(f_i x_i - \overline{x}) = \sum f_i x_i - \sum \overline{x}$$
$$= \sum f_i x_i - n\overline{x}$$
$$= \sum f_i x_i - \sum f_i x_i = 0 \quad \left(\overline{x} = \frac{\sum f_i x_i}{n}\right)$$

So, the option (a) is correct, which is the required answer.

28. In the formula $\overline{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$, for finding the mean of grouped frequency distribution, u_i is equal to

(a) $\frac{x_i + a}{h}$ (b) $h(x_i - a)$

(c) $\frac{x_i - a}{h}$ (d) $\frac{a - x_i}{h}$

Sol : [Board 2008]

We know that, $u_i = \frac{x_i - a}{h}$

So, the option (c) is correct, which is the required answer.

Thus (c) is correct option.

29. For the following distribution.

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

the sum of lower limits of the median class and modal class is

(a) 15 (b) 25

(c) 30 (d) 35

Sol : [Board Term-1 Comp 2018]

Here,

Class	Frequency	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66

Now, $\frac{N}{2} = \frac{33}{2} = 33$, which lies in the interval 10-15. Therefore, lower limit of the median class is 10.

The highest frequency is 20, which lies in the interval 15-20. Therefore, lower limit of modal class is 15. Hence, required sum is $10 + 15 = 25$.

Thus (b) is correct option.

30. Consider the following frequency distribution

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

The upper limit of the median class is

(a) 17 (b) 17.5

(c) 18 (d) 18.5

Sol : [Board 2007]

Given, classes are not continuous, so we make continuous by subtracting 0.5 from lower limit and adding 0.5 to upper limit of each class.

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Class	Frequency	Cumulative frequency
-0.5-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	8	46
23.5-29.5	11	57

Here, $\frac{N}{2} = \frac{57}{2} = 28.5$, which lies in the interval $11.5 - 17.5$. Hence, the upper limit is 17.5.

Thus (b) is correct option.

31. For the following distribution:

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

The modal class is

(a) 10-20 (b) 20-30

(c) 30-40 (d) 50-60

Sol : [Board Term-1 Comp 2017]

Marks	Number of students
0-10	$3 - 0 = 3$
10-20	$12 - 3 = 9$
20-30	$27 - 12 = 15$
30-40	$57 - 27 = 30$
40-50	$75 - 57 = 18$
50-60	$80 - 75 = 5$

Class 30-40 has the maximum frequency 30, therefore this is model class.

Thus (c) is correct option.

32. Consider the data:

Class	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is

(a) 0 (b) 19

(c) 20 (d) 38

Sol : [Board Term-1 Foreign 2013]

Class	Frequency	Cumulative frequency
65-85	4	7
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	7	63
185-205	4	67

Here, $\frac{N}{2} = \frac{67}{2} = 33.5$, which lies in the interval $125 - 145$. Hence, upper limit of median class is 145.

Here, we see that the highest frequency is 20 which lies in 125-145. Hence, the lower limit of modal class is 125.

Required difference

= Upper limit of median class

$-$ Lower limit of modal class

$= 145 - 125 = 20$

Thus (c) is correct option.

33. The times, in seconds, taken by 150 athletes to run a 110 m hurdle race are tabulated below

Class	Frequency
13.8-14	2
14-14.2	4
14.2-14.4	5
14.4-14.6	71
14.6-14.8	48
14.8-15	20

The number of athletes who completed the race in less than 14.6 second is :

(a) 11

(b) 71

(c) 82

(d) 130

Sol : [Board Term-1 Delhi 2011]

The number of athletes who completed the race in less than 14.6

$$= 2 + 4 + 5 + 71 = 82$$

Thus (c) is correct option.

NODIA

34. Consider the following distribution :

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30-40 is :

(a) 3 (b) 4

(c) 48 (d) 51

Sol : [Board Term-1 Delhi 2012]

Marks obtained	Number of students
0-10	$(63-58)=5$
10-20	$(58-55)=3$
20-30	$(55-51)=4$
30-40	$(51-48)=3$
40-50	$(48-42)=6$
50-60	$42=42$

Hence, frequency in the class interval 30-40 is 3.

Thus (a) is correct option.

35. The mean weight of 9 students is 25 kg. If one more student is joined in the group the mean is unaltered, then the weight of the 10^{th} student is

(a) 25 kg (b) 24 kg

(c) 26 kg (d) 23 kg

Sol : [Board Term-1 Foreign 2012]

The sum of the weights of the 9 students $= 25 \times 9 = 225$. If one more student is joined in the group, then total number of students is 10 and mean is 25.

Hence, the sum of the weights of the 10^{th} students $= 25 \times 10 = 250$.

Hence, the weight of the 10^{th} student is $250 - 225 = 25$ kg.

However we can answer this question without any calculation. If mean is not altered on adding more data, then added data must be of mean value.

Thus (a) is correct option.

36. **Assertion :** If the number of runs scored by 11 players of a cricket team of India are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27 then median is 30.

Reason : Median $= \left(\frac{n+1}{2}\right)^{th}$ value, if n is odd.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Arranging the terms in ascending order,

0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

$$\text{Median value} = \left(\frac{11+1}{2}\right)^{th}$$

$$= 6^{th} \text{ value} = 27$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

ONE MARK QUESTIONS

37. Find the class-marks of the classes 10-25 and 35-66.

Sol : [Board 2020 OD Standard]

Class mark of $10-25$, $= \frac{10+25}{2}$

$$= \frac{35}{2} = 17.5$$

and class mark of $35-55$,

$$= \frac{35+55}{2} = \frac{90}{2} = 45$$

PRACTICE

- Find the class marks of the classes 15-35 and 45-60.
  [Board 2020 OD Standard]
  Ans : 20 and 52.5

- Find the class marks of the classes 20-50 and 35-60.
  [Board 2020 OD Standard]
  Ans : 35 and 47.5

38. If the mean of the first n natural number is 15, then find n.

Sol : [Board 2020 Delhi Standard]

Given : 1, 2, 3, 4, ... to n terms.

NODIA

Sol : [Board Term-1 2015]

Class 30-35 has the maximum frequency 25, therefore this is model class.

Now $l = 30,\ f_1 = 25\ \ f_0 = 10\ ,\ f_2 = 7,\ h = 5$

$$\text{Mode} \quad M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \frac{25 - 10}{50 - 10 - 7} \times 5$$

$$= 30 + 2.27 \text{ or } 32.27 \text{ approx.}$$

81. Find the value of λ, if the mode of the following data is 20 :

15, 20, 25, 18, 13, 15, 25, 15, 18, 17, 20, 25, 20, λ, 18.

Sol : [Board Term-1 2015]

First we prepare the following table as discrete frequency distribution.

x_i	13	15	17	18	20	λ	20
f_i	1	3	1	3	3	1	3

Frequency of 20 must be highest to be mode of the frequency distribution, $\lambda = 20$.

82. The mean and median of 100 observation are 50 and 52 respectively. The value of the largest observation is 100. It was later found that it is 110. Find the true mean and median.

Sol : [Board Term-1 2016]

$$\text{Mean,} \quad M = \frac{\sum fx}{\sum f}$$

$$50 = \frac{\sum fx}{100}$$

$$\sum fx = 5000$$

$$\text{Correct,} \quad \sum fx' = 5000 - 100 + 110 = 5010$$

$$\text{Correct Mean} = \frac{5010}{100} = 50.1$$

Median will remain same i.e median is 52.

83. Find the arithmetic mean of the following frequency distribution :

x_i	3	4	5	7	10
f_i	3	4	8	5	10

Sol : [Board Term-1 2015]

We prepare the following table to fine mean.

x_i	f_i	$f_i x_i$
3	3	9
4	4	16
5	8	40
7	5	35
10	10	100
Total	$\sum f_i = 30$	$\sum f_i x_i = 200$

$$\text{Mean,} \quad M = \frac{\sum f_i x_i}{\sum f_i} = \frac{200}{30} = 6.67$$

84. Given below is the distribution of weekly pocket money received by students of a class. Calculate the pocket money that is received by most of the students.

Pocket Money (in Rs.)	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Number of students.	2	2	3	12	18	5	2

Sol : [Board Term-1 2015]

Class Interval	Frequency
0-20	2
20-40	2
40-60	3
60-80	12
80-100	18
100-120	5
120-140	2
Total	44

Class 80-100 has the maximum frequency 18, therefore this is model class.

We have $l = 80,\ f_1 = 18,\ f_2 = 5,\ f_0 = 12\ ,\ h = 20$

$$\text{Mode,} \quad M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 80 + \left(\frac{18 - 12}{36 - 12 - 5}\right) \times 20$$

$$= 80 + \frac{6}{19} \times 20$$

$$= 80 + 6.31$$

$$= 86.31$$

85. Find the mean of the following distribution :

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	5	4	1	6	4

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upper limit is 165.

45. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then what is the median of the new set?

Sol : [Board Term-1 OD 2013]

Since, $n = 9$

then, median term $= \left(\frac{9+1}{2}\right)^{\text{th}} = 5^{\text{th}}$ item.

Now, last four observations are increased by 2, but median is 5^{th} observation, which is remaining unchanged. Hence there will be no change in median.

46. If the mean of the observation $x, x+3, x+5, x+7$ and $x+10$ is 9, the find the mean of the last three observation

Sol : [Board Term-1 OD 2014]

$$\text{Mean} = \frac{\text{Sum of all the observations}}{\text{Total no. of observation}}$$

$$9 = \frac{x+x+3+x+5+x+7+x+10}{5}$$

$$9 = \frac{5x+25}{5}$$

$$x = 4$$

So, mean of last three observation,

$$= \frac{x+5+x+7+x+10}{3} = \frac{5x+22}{3}$$

$$\frac{3x+22}{3} = \frac{3\times 4+22}{3}$$

$$= \frac{12+22}{3} = \frac{34}{3} = 11\frac{1}{3}$$

PRACTICE

If the mean of the numbers $27+x$, $31+x$, $89+x$ $107+x, 156+x$ is 82, then what is the mean of $130+x$, $126+x$, $68+x$, $50+x$, and $1+x$?

[Board 2010]

Ans : 75

47. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. What is the mode of this set of numbers?

Sol : [Board Term-1 Delhi 2012]

Mode of the data is 8 as it is repeated maximum number of times.

48. The mean weight of 9 students is 25 kg. If one more student is joined in the group the mean is unaltered, then find the weight of the 10^{th} student.

Sol : [Board Term-1 Delhi 2013]

The sum of the weights of the 9 students $= 25 \times 9 = 225$. If one more student is joined in the group, then total number of students is 10 and mean is 25.

Hence, the sum of the weights of the 10^{th} students $= 25 \times 10 = 250$.

Hence, the weight of the 10^{th} student is $250 - 225 = 25$ kg.

However we can answer this question without any calculation. If mean is not altered on adding more data, then added data must be of mean value.

NODIA

49. The mean and median of the data a, b and c are 50 and 35 respectively, where $a < b < c$. If $c - a = 55$, then find the value of $(b-a)$.

Sol : [Board Term-1 OD 2014]

Since, a, b and c and are in ascending order, therefore median is b i.e $b = 35$.

$$\text{Mean} \quad \frac{a+b+c}{3} = 50$$

$$a+b+c = 150$$

$$a+c = 150-35 = 115 \qquad \ldots(1)$$

Also, it is given that $c - a = 55$ $\ldots(2)$

Subtracting equation (2) and (1), we get

$$a = 30$$

Hence, $b - a = 35 - 30 = 5$

50. If the mean of the squares of first n natural numbers is 105, then find the first n natural numbers.

Sol : [Board Term-1 Delhi 2011]

Sum of square, $\Sigma x^2 = \frac{n(n+1)(2n+1)}{6}$

Mean of squares of first n natural numbers,

$$\text{Mean} \quad \frac{\Sigma x^2}{n} = \frac{(n+1)(2n+1)}{6}$$

$$105 = \frac{(n+1)(2n+1)}{6}$$

$$2n^2+3n+1 = 630$$

$$2n^2+3n-629 = 0$$

$$2n^2+37n-34n-629 = 0$$

$$n(2n+37)-17(2n+37) = 0$$

$$(2n+37)(n-17) = 0 \Rightarrow n = 17$$

Since, n is odd, therefore median is $=\left(\frac{17+1}{2}\right)^{\text{th}} = 9^{\text{th}}$ observation.

51. Observations of some data are $\frac{x}{5}$, x, $\frac{x}{3}$, $\frac{2x}{3}$, $\frac{x}{4}$, $\frac{2x}{5}$ and $\frac{3x}{4}$ where $x > 0$. If the median of the data is 4, then what is the value of x?

Sol : [Board Term-1 Delhi 2013]

Given observations are $\frac{x}{5}$, x, $\frac{x}{3}$, $\frac{2x}{3}$, $\frac{x}{4}$, $\frac{2x}{5}$ and $\frac{3x}{4}$ where $x > 0$. On arranging the above observations in ascending order, we get

$$\frac{x}{5}, \frac{x}{4}, \frac{x}{3}, \frac{2x}{5}, \frac{2x}{3}, \frac{3x}{4}, x$$

Here, total number of observations are 7, which is odd.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$
$$= \left(\frac{7+1}{2}\right)^{\text{th}} \text{ observation}$$
$$= 4^{\text{th}} \text{ observation} = \frac{2x}{5}$$
$$\text{Median} = \frac{2x}{5} = 4$$
$$x = \frac{4 \times 5}{2}$$
$$= 10$$

52. If the coordinates of the point of intersection of less than ogive and more than ogive is (13.5, 20), then find the value of median.

Sol : [Board Term-1 Delhi 2012]

The abscissa of point of intersection gives the median of the data. So, median is 13.5.

53. Find the mode of the following grouped frequency distribution.

Class	Frequency
3-6	2
6-9	5
9-12	10
12-15	23
15-18	21
18-21	12
21-24	03

Sol :

We observe that the class 12-15 has maximum frequency 23. Therefore, this is the modal class.

We have, $l = 12$, $h = 3$, $f_1 = 23$, $f_0 = 10$ and $f_2 = 21$

$$M_o = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$
$$= 12 + \frac{23 - 10}{46 - 10 - 21} \times 3$$
$$= 12 + \frac{13}{15} \times 3$$
$$= 12 + \frac{13}{5} = 14.6$$

54. What is abscissa of the point of intersection of the "Less than type" and of the "More than type" cumulative frequency curve of a grouped data ?

Sol :

The abscissa of the point of intersection of the "Less than type" and "More than type" cumulative frequency curve of a grouped data is median.

55. If median is 137 and mean is 137.05, then what is the value of mode ?

Sol : [Board Term-1 OD 2016]

$$M_o = 3M_d - 2M$$
$$= 3(137) - 2(137.05)$$
$$= 411 - 274.10$$
$$= 136.90$$

PRACTICE

✍ Median of a data is 52.5 and its mean is 54, use empirical relationship between three measure of central tendency to find its mode. [Board Term-1 2012]

Ans : 49.5

56. The following data gives the distribution of total household expenditure (in <) of manual workers in a city.

Expenditure (in <)	Frequency
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	07

Then, find the average expenditure which is done

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by the maximum number of manual workers.

Sol : [Board Term-1 Delhi 2015]

We observe that the class 1500-2000 has maximum frequency 40. Therefore, this is the modal class.

We have $l = 1500$, $h = 500$, $f_1 = 40$, $f_0 = 24$ and $f_2 = 23$

$$\begin{aligned} M_o &= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \\ &= 1500 + \frac{40 - 24}{80 - 24 - 33} \times 500 \\ &= 1500 + \frac{16}{23} \times 500 \\ &= 1847.26 \end{aligned}$$

57. If x_i's are the mid-points of the class intervals of grouped data, f_i's are the corresponding frequencies and $\overline{x}$ is the mean, then find $\sum(f_i x_i - \overline{x})$.

Sol : [Board Term-1 Delhi 2017]

$$\begin{aligned} \sum(f_i x_i - \overline{x}) &= \sum f_i x_i - \sum \overline{x} \\ &= \sum f_i x_i - n\overline{x} \\ &= \sum f_i x_i - \sum f_i x_i = 0 \quad \left(\overline{x} = \frac{\sum f_i x_i}{n}\right) \end{aligned}$$

58. What is the model class for the following distribution?

Marks	**Number of Students**	**Marks**	**Number of students**
Below 10	3	Below 40	57
Below 20	12	Below 50	75
Below 30	28	Below 60	80

Sol :

Let us first construct the following frequency distribution table.

Marks	**Number of Students**
0-10	3
10-20	9
20-30	16
30-40	29
40-50	18
50-60	5

Since, the maximum frequency is 29 and the class corresponding to this frequency is 30-40. So, the modal class is 30-40.

59. For the following distribution find the sum of lower limits of the median class and modal class.

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

Sol : [Board Term-1 Foreign 2016]

Here,

Class	Frequency	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66

Now, $\frac{N}{2} = \frac{33}{2} = 33$, which lies in the interval 10-15. Therefore, lower limit of the median class is 10.

The highest frequency is 20, which lies in the interval 15-20. Therefore, lower limit of modal class is 15. Hence, required sum is $10 + 15 = 25$.

NODIA

60. Consider the following frequency distribution

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

What is the upper limit of the median class ?

Sol : [Board Term-1 OD 2015]

Given, classes are not continuous, so we make continuous by subtracting 0.5 from lower limit and adding 0.5 to upper limit of each class.

Class	Frequency	Cumulative frequency
-0.5-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	8	46
23.5-29.5	11	57

Here, $\frac{N}{2} = \frac{57}{2} = 28.5$, which lies in the interval $11.5 - 17.5$. Hence, the upper limit is 17.5.

61. For the following distribution what is the modal class:

Marks	Number of students
Below 10	3

(ii) Class mark of the class : $10-25 = \frac{10+25}{2}$

$= \frac{35}{2} = 17.5$

71. Find the mean of the data using an empirical formula when it is given that mode is 50.5 and median in 45.5.

Sol : [Board Term-1 2015]

Mode, $M = 50.5$

Median, $M_d = 45.5$

Now $3M_d = M_o + 2M$

$3 \times 45.5 = 50.5 + 2M$

Mean, $M = \frac{136.5 - 50.5}{2}$

$= 43$

Hence mean is 43.

72. Find the mean of first odd multiples of 5.

Sol : [Board Term-1 2012]

The first five odd multiples of 5, according to the problem are : 5, 15, 25, 35, 45

Mean $= \frac{5+15+25+35+45}{5}$

$= \frac{125}{5} = 25$

TWO MARKS QUESTIONS

73. Find the mean the following distribution :

Class	3-5	5-7	7-9	9-11	11-13
Frequency	5	10	10	7	8

Sol : [Board 2020 Delhi Standard]

Class	Frequency (f_i)	Mid-Value (x_i)	$f_i x_i$
3-5	5	4	20
5-7	10	6	60
7-9	10	8	80
9-11	7	10	70
11-13	8	12	96
	$\sum f_i = 40$		$\sum f_i x_i = 326$

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{326}{40} = 8.15$

74. Find the mode of the following data :

Class	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	6	8	10	12	6	5	3

Sol : [Board 2020 Delhi Standard]

Class 60-80 has the maximum frequency 12, therefore this is model class.

Hence, $l = 60$, $f_1 = 12$, $f_0 = 6$, $f_2 = 6$ and $h = 20$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$= 60 + \frac{12-10}{2 \times 12 - 10 - 6} \times 20$

$= 60 + \frac{2 \times 20}{24 - 16} = 60 + \frac{40}{8}$

$= 60 + 5 = 65$

75. The mode of the following frequency distribution is 36. Find the missing frequency f.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	f	16	12	6	7

Sol : [Board 2020 OD Basic]

Mode is 36 which lies in class 30-40, therefore this is model class.

Here, $f_0 = f$, $f_2 = 16$, $f_2 = 12$, $l = 30$ and $h = 10$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$36 = 30 + \frac{16-f}{2 \times 16 - f - 12} \times 10$

$6 = \frac{16-f}{20-f} \times 10$

$120 - 6f = 160 - 10f$

$4f = 40 \Rightarrow f = 10$

76. Find the median for the given frequency distribution :

Class	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Frequency	2	3	8	6	6	3	2

Sol : [Board 2020 OD Basic]

Class	Frequency	$c.f.$
40-45	2	2
45-50	3	5

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Marks obtained	Number of students
0-10	$(63-58)=5$
10-20	$(58-55)=3$
20-30	$(55-51)=4$
30-40	$(51-48)=3$
40-50	$(48-42)=6$
50-60	$42=42$

Hence, frequency in the class interval 30-40 is 3.

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67. From the following frequency distribution, find the median class :

Cost of living index	1400-1500	1550-1700	1700-1850	1850-2000
Number of weeks	8	15	21	8

Sol : [Board Term-1 2015]

We prepare following cumulative frequency table to find median class.

Cost of living index	Number of weeks f	c.f.
1400-1500	8	8
1550-1700	15	23
1700-1850	21	44
1850-2000	8	52

We have $N = 52$; $\frac{N}{2} = 26$

Cumulative frequency just greater than $\frac{N}{2}$ is 44 and the corresponding class is 1700-1850. Thus median class is 1700-1850.

68. Find median of the data, using an empirical relation when it is given that Mode = 12.4 and Mean = 10.5.

Sol : [Board Term-1 2015]

Mode, $M_o = 12.4$

Mean, $M = 10.5$

Median, $M_d = \frac{1}{3}M + \frac{2}{3}M_o$

$= \frac{1}{3}(12.4) + \frac{2}{3}(10.5)$

$= \frac{12.4}{3} + \frac{21}{3}$

$= \frac{12.4+21}{3} = \frac{33.4}{3} = 11.13$

69. In the following frequency distribution, find the median class.

Height (in cm)	104-145	145-150	150-155	155-160	160-165	165-170
Frequency	5	15	25	30	15	10

Sol : [Board Term-1 OD 2015]

We prepare following cumulative frequency table to find median class.

Height	Frequency	c.f.
140-145	5	5
145-150	15	20
150-155	25	45
155-160	30	75
160-165	15	90
165-170	10	100
	$N=100$	

We have $N = 100$; $\frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 75 and the corresponding class is 155-160. Thus median class is 155-160.

70. Consider the following distribution :

Marks Obtained	0 or more	10 or more	20 or more	30 or more	40 or more	50 or more
Number of students	63	58	55	51	48	42

(i) Calculate the frequency of the class 30 - 40.

(ii) Calculate the class mark of the class 10 - 25.

Sol : [Board Term-1 2014]

Class Interval	c.f.	f
0-10	63	5
10-20	58	3
20-30	55	4
30-40	51	3
40-50	48	6
50-60	42	42

(i) Frequency of the class 30 - 40 is 3.

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Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

Sol :

Marks	Number of students
0-10	$3-0=3$
10-20	$12-3=9$
20-30	$27-12=15$
30-40	$57-27=30$
40-50	$75-57=18$
50-60	$80-75=5$

Class 30-40 has the maximum frequency 30, therefore this is model class.

62. Which central tendency is obtained by the abscissa of point of intersection of less than type and more than type ogives ?

Sol :

Median.

63. Consider the data:

Class	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Frequency	4	5	13	20	14	7	4

What is the difference of the upper limit of the median class and the lower limit of the modal class?

Sol : [Board Term-1 OD 2017]

Class	Frequency	Cumulative frequency
65-85	4	7
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	7	63
185-205	4	67

Here, $\frac{N}{2}=\frac{67}{2}=33.5$, which lies in the interval $125-145$. Hence, upper limit of median class is 145.

Here, we see that the highest frequency is 20 which lies in 125-145. Hence, the lower limit of modal class is 125.

Required difference

= Upper limit of median class

$-$ Lower limit of modal class

$= 145-125=20$

64. The times, in seconds, taken by 150 athletes to run a 110 m hurdle race are tabulated below

Class	Frequency
13.8-14	2
14-14.2	4
14.2-14.4	5
14.4-14.6	71
14.6-14.8	48
14.8-15	20

What is the number of athletes who completed the race in less than 14.6 second?

Sol :

The number of athletes who completed the race in less than 14.6 second,

$$= 2+4+5+71=82$$

65. If the median of a series exceeds the mean by 3, find by what number the mode exceeds its mean?

Sol : [Board Term-1 Delhi 2015]

We have $M_d = M+3$

Now $M_o = 3M_d-2M$

$= 3(M+3)-2M$

$= 3M+9-2M=M+9$

Hence mode exceeds mean by 9.

66. In the following distribution what is the frequency of the class 30-40?

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

Sol :

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Class	Frequency	*c.f.*
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30
	$N = 30$	

We have $N = 30$; $\frac{N}{2} = 15$

Cumulative frequency just greater than $\frac{N}{2}$ is 19 and the corresponding class is 55-60. Thus median class is 55-60.

Now $l = 55,\ f = 6,\ F = 13,\ h = 5$

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$

$= 55 + \left(\frac{15 - 13}{6}\right) \times 5$

$= 55 + \frac{5}{3}$

$= 55 + 1.67 = 56.67$

77. Find the mean of the following distribution :

Class	10-25	25-40	40-55	55-70	70-85	85-100
Frequency	2	3	7	6	6	6

Sol : [Board 2020 Delhi Basic]

Let $a = 62.5$ be assumed mean.

Class Interval	Frequency (f_i)	*c.f.*	x_i	$U_i = \frac{x - a}{h}$	$f_i u_i$
10-25	2	2	17.5	-3	-6
25-40	3	5	32.5	-2	-6
40-55	7	12	47.5	-1	-7
55-70	6	18	$62.50 = a$	0	0
70-85	6	24	77.5	1	6
85-100	6	30	92.5	2	12
	$\sum f_i = 30$				$\sum f u_i = -1$

Mean, $\overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 62.5 + \frac{-1}{30} \times 15$

$= 62.5 - \frac{1}{2} = 62.5 - 0.5 = 62$

78. Find the mean of the following data :

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	20	35	52	44	38	31

Sol : [Board 2019 Comp]

Let $a = 70$ be assumed mean.

C.I.	Frequency f	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0-20	20	10	-3	-60
20-40	35	30	-2	-70
40-60	52	50	-1	-52
60-80	44	70	0	0
80-100	38	90	1	38
100-120	31	110	2	62
	$\sum f_i = 220$			$\sum f_i u_i = -82$

Mean, $\overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 70 + \frac{(-82)}{220} \times 20$

$= 70 - \frac{82}{11} = 70 - 7.45 = 62.55$

79. Find the mode of the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	10	16	12	6	7

Sol : [Board 2019 Delhi]

Class 30-40 has the maximum frequency 16, therefore this is model class.

We have $l = 30, f_0 = 10, f_1 = 16, f_2 = 12, h = 10$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$= 30 + \left(\frac{16 - 10}{2 \times 16 - 10 - 12}\right) \times 10$

$= 30 + \left(\frac{6}{32 - 22}\right) \times 10$

$= 30 + \left(\frac{6}{10}\right) \times 10$

$= 30 + 6 = 36$

80. The data regarding marks obtained by 48 students of a class in a class test is given below. Calculate the modal marks of students.

Marks obtained	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
Number of students	1	0	2	0	0	10	25	7	2	1

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The sum of first n natural numbers

$$S_n = \frac{n(n+1)}{2}$$

Mean, $$M = \frac{n(n+1)}{2 \times n}$$

$$15 = \frac{n(n+1)}{2 \times n}$$

$$15 = \frac{n+1}{2}$$

$$n+1 = 30 \Rightarrow n = 29$$

39. What is abscissa of the point of intersection of the "Less than type" and of the "More than type" cumulative frequency curve of a grouped data ?

Sol :

The abscissa of the point of intersection of the "Less than type" and "More than type" cumulative frequency curve of a grouped data is median.

40. The median and mode respectively of a frequency distribution are 26 and 29, find its mean.

Sol : [Board 2020 Delhi Basic]

We have $$M_o = 3M_d - 2M$$

$$29 = 3 \times 26 - 2M$$

$$2M = 78 - 29 = 49$$

$$M = \frac{49}{2}$$

$$= 24.5$$

41. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. What is the lower limit of the class?

Sol : [Board Term-1 Delhi 2014]

Let x be the upper limit and y be the lower limit.

Since the mid value of the class is 10.

Hence, $$\frac{x+y}{2} = 10$$

$$x + y = 20 \quad ...(1)$$

Since width of the class is 6,

$$x - y = 6 \quad ...(2)$$

Solving (1) and (2), we get $y = 7$

Hence, lower limit of the class is 7.

42. For finding the popular size of readymade garments, which central tendency is used?

Sol : [Board Term-1 OD 2012]

For finding the popular size of ready made garments, mode is the best measure of central tendency.

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43. If the difference of mode and median of a data is 24, then what is the difference of median and mean?

Sol : [Board Term-1 OD 2011]

We have, $$M_o - M_d = 24$$

We know $$M_o = 3M_d - 2M$$

Now $$M_o - M_d = 2M_d - 2M$$

$$24 = 2(M_d - M)$$

$$M_d - M = 12$$

44. Consider the following frequency distribution of the heights of 60 students of a class

Height (in cm)	150-155	155-160	160-165	165-170	170-175	175-180
Number of students	15	13	10	8	9	5

What is the upper limit of the median class in the given data?

Sol : [Board 2020 SQP Standard]

We prepare the following cumulative table

Height x (in cm)	Number of Students (f)	cf
150-155	15	15
155-160	13	28
160-165	10	38
165-170	08	46
170-175	09	55
175-180	08	63
	$N = 63$	

We have, $$N = 63; \frac{N}{2} = \frac{63}{2} = 31.5$$

The cumulative frequency just greater than $\frac{N}{2}$ is 38 and the corresponding class is 160-165. Thus

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Sol : [Board Term-1 2015]

x_i	f_i	$f_i x_i$
3	5	15
9	4	36
15	1	15
21	6	126
27	4	108
Total	$\sum f_i = 20$	$\sum f_i x_i = 300$

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{300}{20} = 15$

86. Find the sum of the lower limit of the median class and the upper limit of the modal class :

Classes	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	1	3	5	9	7	3

Sol : [Board Term-1 2012]

We prepare following cumulative frequency table to find median class.

Class	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	1	3	5	9	7	3
Cumulative Frequency	1	4	9	18	25	28

We have $N = 28$; $\frac{N}{2} = \frac{28}{2} = 14$

Cumulative frequency just greater than $\frac{N}{2}$ is 18 and the corresponding class is 40 - 50. Thus median class is 40-50.

Lower limit is 40 and upper limit is 5. Their sum is $= 40 + 50 = 90$

87. The following table gives the life time in days of 100 bulbs :

Life time in days	Less than 50	Less than 100	Less than 150	Less than 200	Less than 250	Less than 300
Number of Bulbs	8	23	55	81	93	100

Change the above distribution as frequency distribution.

Sol : [Board Term-1 2012]

Life time in days (Class Interval)	Number of Bulbs (Frequency)
0-50	8
50-100	15
100-150	32
150-200	26
150-200	12
150-200	7
Total	100

88. Find the unknown values in the following table :

Class Interval	Frequency	Cumulative Frequency
0-10	5	5
10-20	7	x_1
20-30	x_2	18
30-40	5	x_3
40-50	x_4	30

Sol : [Board Term-1 2016]

We have $x_1 = 5 + 7 = 12$

$x_2 = 18 - x_1 = 18 - 12 = 6$

$x_3 = 18 + 5 = 23$

and $x_4 = 30 - x_3 = 30 - 23 = 7$

89. Calculate the median from the following data :

Marks	0-10	10-20	20-30	30-40	40-50
Number of Students	5	15	30	8	2

Sol : [Board Term-1 2012]

We prepare following cumulative frequency table to find median class.

Marks	No. of students	c.f.
0-10	5	5
10-20	15	20
20-30	30	50
30-40	8	58
40-50	2	60
	$N = 60$	

We have $N = 60$; $\frac{N}{2} = 30$

Cumulative frequency just greater than $\frac{N}{2}$ is 50 and the corresponding class is 20-30. Thus median class is 20-30.

Now $l = 20,\ f = 30,\ F = 20,\ h = 10$

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$

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Also $\sum f_i = 25 + x + y = 50$

$x + y = 25$

$x = 25 - 13 = 12$

Thus $x = 12$ and $y = 13$

116. The table below shows the daily expenditure on food of 25 households in a locality. Find the mean daily expenditure on food.

Daily expenditure (in ₹)	100-150	150-200	200-250	250-300	300-350
Number of households	4	5	12	2	2

Sol : [Board 2019 Delhi]

Let $a = 225$ be assumed mean,

Daily Expenditure (in ₹)	No. of household (f_i)	(x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100-150	4	125	-2	-8
150-200	5	175	-1	-5
200-250	12	225	0	0
250-300	2	275	1	2
300-350	2	325	2	4
	$\sum f_i = 25$			$\sum f_i u_i = -7$

Mean, $M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 225 + \frac{(-7)}{25} \times 50$

$= 225 - 14 = 211$

Hence, mean of daily expenditure on food is ₹211.

117. The weekly expenditure of 500 families is tabulated below :

Weekly Expenditure(Rs.)	Number of families
0-1000	150
1000-2000	200
2000-3000	75
3000-4000	60
4000-5000	15

Find the median expenditure.

Sol : [Board Term-1 2015]

We prepare following cumulative frequency table to find median class.

Expenditure	f (families)	c.f.
0-1000	150	150
1000-2000	200	350
2000-3000	75	425
3000-4000	60	485
4000-5000	15	500
	$\sum f = 500$	

We have $N = 500$; $\frac{N}{2} = 250$

Cumulative frequency just greater than $\frac{N}{2}$ is 350 and the corresponding class is 1000-2000. Thus median class is 1000-2000.

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right) h$

$= 1000 + \frac{250 - 150}{200} \times 1000$

$= 1000 + 500 = 1,500$

Thus median expenditure is Rs. 1500 per week.

118. Following frequency distribution shows the expenditure on milk of 30 households in a locality :

Daily expenditure on milk (Rs.)	0-30	30-60	60-90	90-120	120-150
Number of households	5	6	9	6	4

Find the mode for the above data.

Sol : [Board Term-1 2016]

Class 60-90 has the maximum frequency 9, therefore this is model class.

Here, $l_1 = 60$, $f_1 = 9$, $f_0 = 6$, $f_2 = 6$ and $h = 30$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$

$= 60 + \left(\frac{9 - 6}{2 \times 9 - 6 - 6}\right) \times 30$

$= 60 + \frac{30 \times 3}{6} = 60 + 15 = 75$

119. Find the median of the following data :

Height (in cm)	Less than 120	Less than 140	Less than 160	Less than 180	Less than 200
Number of students	12	26	34	40	50

Sol : [Board Term-1 2015]

We prepare following cumulative frequency table

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95. The frequency distribution of agricultural holdings in a village below :

Area of land (in hectare)	1-3	3-5	5-7	7-9	9-11	11-13
Number of families	20	45	80	55	40	12

Find the modal agricultural holding of the village.

Sol : [Board Term-1 2012]

Class 5-7 has the maximum frequency 80, therefore this is model class.

Here $l = 5, f_1 = 80, f_0 = 45,\ h = 2,\ f_2 = 55$

$$\text{Mode,} \quad M_o = l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$$

$$= 5 + \frac{80 - 45}{160 - 45 - 55} \times 2$$

$$= 5 + \frac{35 \times 2}{60} = 6.17$$

96. Write the median class of the following distribution:

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

Sol : [Board Term-1 OD 2012]

We prepare following cumulative frequency table to find median class.

Classes	Frequency	Less than c.f.
0-10	4	4
10-20	4	8
20-30	8	16
30-40	10	26
40-50	12	38
50-60	8	46
60-70	4	50
	$N = 50$	

We have $N = 50\ ;\ \frac{N}{2} = 25$

Cumulative frequency just greater than $\frac{N}{2}$ is 26 and the corresponding class is 30-40. Thus median class is 20-20.

97. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day :

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	60	42	55	70	53	20

Form the "less than type" cumulative frequency distribution table.

Sol : [Board Term-1 2012]

Age	Number of Patients
Less then 20	60
Less then 30	102
Less then 40	157
Less then 50	227
Less then 60	280
Less then 70	300

98. Find the mean of the following data :

Class	Frequency
0.5-5.5	13
5.5-10.5	16
10.5-15.5	22
15.5-20.5	18
20.5-25.5	11

Sol : [Board Term-1 Delhi 2012]

We prepare following table to find mean.

Class	$x_i = \frac{l_1 + l_2}{2}$	f_i	$f_i x_i$
0.5-5.5	3	13	39
5.5-10.5	8	16	128
10.5-15.5	13	22	286
15.5-20.5	18	18	324
20.5-25.5	23	11	253
	Total	$\sum f_i = 80$	1,030

$$\text{Mean} \quad \bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{1,030}{80} = 12.9$$

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99. Find the mean number of plants per house from the following data :

Number of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of houses	1	2	1	5	6	2	3

Sol : [Board Term-1 2012]

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We prepare following table to find mean.

Class	$x_i = \frac{l_1 + l_2}{2}$	f_i	$f_i x_i$
0-2	1	1	1
2-4	3	2	6
4-6	5	1	5
6-8	7	5	35
8-10	9	6	54
10-12	11	2	22
12-14	13	3	39
	Total	20	162

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{162}{20} = 8.1$

Mean number of plants per house is 8.1.

100. Given below is a frequency distribution showing the marks by 50 students of a class :

Marks	Number of students
Below 20	17
Below 40	22
Below 60	29
Below 80	37
Below 100	50

Form the distribution table for the above data.

Sol : [Board Term-1 2012]

Class	Frequency
0-20	17
20-40	5
40-60	7
60-80	8
80-100	13
Total	50

101. Find the mode of the following frequency distribution :

Classes	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

Sol : [Board Term-1 2013]

Class 18-24 has the maximum frequency 12, therefore this is model class.

Now $l = 18,\ f_1 = 12,\ f_0 = 10,\ f_2 = 6,\ h = 6$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$= 18 + \frac{12 - 10}{24 - 10 - 6} \times 6$

$= 18 + 1.5 = 19.5$

102. Find the mean of the following frequency distribution :

Class	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

Sol : [Board Term-1 2011]

We prepare following table to find mean.

Classes	x_i	f_i	$f_i x_i$
0-6	3	7	21
6-12	9	5	45
12-18	15	10	150
18-24	21	12	252
24-30	27	6	162
		$\sum f_i = 40$	$\sum f_i x_i = 630$

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{630}{40} = 15.75$

103. The mean of the following frequency distribution is 25. Find the value of p.

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	10	6	p

Sol : [Board Term-1 2015]

We prepare following table to find mean.

Class-Interval	Mid-Point x_i	f_i	$f_i x_i$
0-10	5	4	20
10-20	15	6	90
20-30	25	10	250
30-40	35	6	210
40-50	45	p	$45\ p$
		$26 + p$	$570 + 45p$

We have $M = \frac{\sum f_i x_i}{\sum f_i}$

$25 = \frac{570 + 45p}{26 + p}$

$650 + 25p = 570 + 45p$

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$$650 - 570 = 45p - 25p$$

Thus $p = 4$

104. The data regarding the height of 50 girls of class X of a school is given below :

Height (in cm)	120-130	130-140	140-150	150-160	160-170	Total
Number of girls	2	8	12	20	8	50

Change the above distribution to 'more than type' distribution.

Sol : [Board Term-1 2012]

Heights	No. of Girls
more than 120	50
more than 130	48
more than 140	40
more than 150	28
more than 160	6

105. Convert the following distribution to more than type, cumulative frequency distribution :

Class	50-60	60-70	70-80	80-90	90-100
Frequency	12	18	10	15	5

Sol : [Board Term-1 2012]

We prepare following cumulative frequency table.

Class	Cumulative Frequency
More than 50	60
More than 60	48
More than 70	30
More than 80	20
More than 90	5

106. Convert the following cumulative distribution to a frequency distribution :

Height (in cm)	less than 140	less than 145	less than 150	less than 155	less than 160	less than 165
Number of students	4	11	29	40	46	51

Sol : [Board Term-1 2012]

We prepare following cumulative frequency table.

Class	Frequency	Cumulative Frequency
135-140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51

107. Prepare a cumulative frequency distribution of 'more than type' for the following data :

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	3	8	15	7	5

Sol : [Board Term-1 2012]

We prepare following cumulative frequency table.

Marks	Cumulative Frequency
More than 0	38
More than 10	35
More than 20	27
More than 30	12
More than 40	5

108. Change the following distribution to 'more than type' of distribution :

Daily income (in Rs.)	100-120	120-140	140-160	160-180	180-200
Number of students	12	14	8	6	10

Sol : [Board Term-1 2012]

We prepare following cumulative frequency table.

Daily income	No. of workers
More than 100	50
More than 120	38
More than 140	24
More than 160	16
More than 180	10

109. Given below is a frequency distribution table showing daily income of 100 workers of a factory :

Daily income of workers (in Rs.)	200-300	300-400	400-500	500-600	600-700
Number of workers	12	18	35	20	15

Convert this table to a cumulative frequency distribution table of 'more than type'.

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$$= 20 + \left(\frac{30-20}{30}\right) \times 10$$

$$= 20 + \frac{100}{30}$$

$$= 20 + 3.33$$

Thus $Md = 23.33$

90. Write the relationship connecting three measures of central tendencies. Hence find the median of the give data if mode is 24.5 and mean is 29.75.

Sol : [Board Term-1 2012]

Mode, $M_o = 24.5$

and mean, $M = 29.75$

The relationship connecting measures of central tendencies is,

$$3M_d = M_o + 2M$$

Thus $3M_d = 24.5 + 2 \times 59.50$

$$= 24.5 + 59.50$$

$$= 84.0$$

Median $M_d = \frac{84}{3} = 28$

91. The following distribution shows the marks scored by 140 students in an examination. Calculate the mode of the distribution :

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	20	24	40	36	20

Sol : [Board Term-1 2012]

Class 20-30 has the maximum frequency 40, therefore this is model class.

Here, $l = 20, f_1 = 40, f_0 = 24, f_2 = 36, h = 10$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 20 + \frac{(40-24)}{80-24-36} \times 10$$

$$= 20 + \frac{16 \times 10}{20} = 28$$

92. Find the unknown entries a, b, c, d in the following distribution of heights of students in a class :

Height (in cm)	Frequency	Cumulative Frequency
150-155	12	12
155-160	a	25
160-165	10	b
165-170	c	43
170-175	5	48
175-180	2	d

Sol : [Board Term-1 2012]

From the table,

$12 + a = 25 \Rightarrow a = 25 - 12 = 13$

$25 + 10 = b \Rightarrow b = 35,$

$b + c = 43 \Rightarrow$

$c = 43 - b = 13 - 35 = 8$

and $48 + 2 = d \Rightarrow d = 50$

93. Find the mode of the following distribution :

Classes	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

Sol :

Class 35-40 has the maximum frequency 50, therefore this is model class.

Now $l = 35$, $f_1 = 50$, $f_2 = 42$, $f_0 = 34$, $h = 5$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 35 + \frac{50-34}{100-34-42} \times 5$$

$$= 35 + \frac{16 \times 5}{24} = 38.33$$

94. Find x and y from the following cumulative frequency distribution :

Classes	Frequency	c.f.
0-8	15	15
8-16	x	28
16-24	15	43
24-32	18	y
32-40	09	70

Sol : [Board Term-1 2012]

From the cumulative frequency distribution,

$15 + x = 28$

$x = 28 - 15 = 13$

and $43 + 18 = y \Rightarrow y = 61$

Hence, $x = 13$ and $y = 61$

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Sol : [Board 2020 OD Standard]

Class	Class Mark	Frequency	$f_i x_i$
11-13	12	3	36
13-15	14	6	84
15-17	16	9	144
17-19	18	13	234
19-21	20	f	$20f$
21-23	22	5	110
23-25	24	4	96
	Total	$40+f$	$704+20f$

We have $\sum f_i = 40+f$

$\sum f_i x_i = 704+20f$

Mean, $M = \frac{\sum f_i x_i}{\sum f_i}$

$18 = \frac{704\times 20f}{40+f}$

$720+18f = 704+20f$

$f = 8$

113. Find the mode of the following frequency distribution:

Class	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	3	8	9	10	3	2

Sol : [Board 2020 OD Standard]

Class 30-35 has the maximum frequency 10, therefore this is model class.

Now $l = 30,\ f_0 = 9,\ f_1 = 10,\ f_2 = 3$ and $h = 5$

Mode, $M_o = l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)h$

$= 30+\left(\frac{10-9}{2\times 10-9-3}\right)\times 5$

$= 30+\frac{5}{8}$

$= 30+0.625$

$= 30.625$

114. The marks obtained by 110 students in an examination are given below

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Number of Students	14	16	28	23	18	8	3

Find the mean marks of the students.

Sol : [Board 2019 OD]

Marks	f_i	x_i	$u_i = \frac{x_i-a}{h}$	$f_i u_i$
30-35	14	32.5	-3	-42
35-40	16	37.5	-2	-32
40-45	28	42.5	-1	-28
45-50	23	47.5	0	0
50-55	18	52.5	1	18
55-60	8	57.5	2	16
60-65	3	62.5	3	9
	$\sum f_i = 110$			$\sum f_i u_i = -59$

Let a be assumed mean, where $a = 47.5$

Mean $M = a+\frac{\sum f_i u_i}{N}\times h$

$= 47.5+\frac{(-59)}{110}\times 5$

$= 47.5-2.682$

$= 44.818$

115. The mean of the following distribution is 48 and sum of all the frequency is 50. Find the missing frequencies x and y.

Class	20-30	30-40	40-50	50-60	60-70
Frequency	8	6	x	11	y

Sol : [Board Term-1 2015, 2016}

We prepare following table to find mean.

C.I.	f_i	x_i	$u_i = \frac{x_i-a}{h}$	$f_i u_i$
20-30	8	25	-2	-16
30-40	6	35	-1	-6
40-50	x	45=a	0	0
50-60	11	55	1	11
60-70	y	65	2	$2y$
Total	$\sum f_i = 25+x+y$			$\sum f_i u_i = 2y-11$

Mean, $M = a+\frac{\sum f_i u_i}{\sum f_i}\times h$

$48 = 45+\frac{2y-11}{50}\times 10$

$48-45 = \frac{2y-11}{5}$

$3\times 5 = 2y-11$

$15 = 2y-11 \Rightarrow y = 13$

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122. Calculate the mean of the following frequency distribution :

Class	10-30	30-50	50-70	70-90	90-110
Frequency	15	18	25	10	2

Sol :

We prepare following table to find mean.

C.I.	f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
10-30	15	20	-2	-30
30-50	18	40	-1	-18
50-70	25	$60 = a$	0	0
70-90	10	80	1	10
90-110	2	100	2	4
Total	$\sum f = 70$			$\sum f_i u_i = -34$

Let mid point of class 50-60 be assumed mean a.

$$a = 60$$

Mean, $$M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 60 + \frac{-34}{70} \times 20$$

$$= 60 - 9.71 = 50.29$$

123. Heights of students of class X are given in the following distribution :

Heights (in cm)	150-155	155-160	160-165	165-170	170-175
Number of students	15	8	20	12	5

Find the modal height.

Sol : [Board Term-1 2015]

Class 160-165 has the maximum frequency 20, therefore this is model class.

Now $l = 160,\ f_1 = 20,\ f_0 = 8,\ f_2 = 12,\ h = 5$

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 160 + \left(\frac{20 - 8}{40 - 8 - 12}\right) \times 5$$

$$= 160 + \left(\frac{12}{20}\right) \times 5$$

$$= 163$$

Thus modal height is 163 cm.

124. A school conducted a test (of 100 marks) in English for students of Class X. The marks obtained by students are shown in the following table :

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of students	1	2	4	15	15	25	15	10	2	1

Find the modal marks.

Sol : [Board Term-1 2015]

Class 50-60 has the maximum frequency 25, therefore this is model class.

Here $l = 50,\ f_1 = 25,\ f_0 = 15,\ f_2 = 15,\ h = 10$

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 50 + \frac{25 - 15}{50 - 15 - 15} \times 10$$

$$= 50 + \frac{10}{20} \times 10$$

$$= 50 + 10 = 55$$

125. The following frequency distribution shows the number of runs scored by some batsman of India in one-day cricket matches :

Run scored	2000-4000	4000-6000	6000-8000	8000-10000	10000-12000
Number of batsmen	9	8	10	2	1

Find the mode for the above data.

Sol : [Board Term-1 2015]

Class 6000-8000 has the maximum frequency 10, therefore this is model class.

Here $f_0 = 8,\ f_1 = 10,\ f_2 = 2,\ h = 2000,$ and $l = 6000$

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 6000 + \left(\frac{10 - 8}{20 - 8 - 2}\right) \times 2000$$

$$= 6000 + \frac{2}{10} \times 2000$$

$$= 6000 + 400 = 6400$$

126. A group of students conducted a survey of their locality to collect the data regarding number of plants and recorded it in the following table :

Number of plants	0-3	3-6	6-9	9-12	12-15
Number of houses	2	4	5	1	2

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to find median class.

Height	Frequency	$c.f.$
100-120	12	12
120-140	14	26
140-160	8	34
160-180	6	40
180-200	10	50
Total	$N = 50$	

We have $N = 50$; $\frac{N}{2} = 25$

Cumulative frequency just greater than $\frac{N}{2}$ is 26 and the corresponding class is 120-140. Thus median class is 120-140.

$$\text{Median, } M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$= 120 + \left(\frac{25 - 12}{14}\right) \times 20$$

$$= 120 + \frac{260}{14}$$

$$= 120 + 18.57 = 138.57$$

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120. Find the mean of the following distribution :

Height (in cm)	Less than 75	Less than 100	Less than 125	Less than 150	Less than 175	Less than 200
No. of students	5	11	14	18	21	28
Height (in cm)	Less than 225	Less than 250	Less than 275	Less than 300		
No. of students	33	37	45	50		

Sol : [Board Term-1 2016]

We prepare following table to find mean.

Class Interval Height (in cm)	Frequency f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
50-75	5	62.5	-5	-25
75-100	6	87.5	-4	-24
100-125	3	112.5	-3	-9
125-150	4	137.5	-2	-8
150-175	3	162.5	-1	-3
175-200	7		0	0
200-225	5	212.5	1	5
225-250	4	237.5	2	8
250-275	8	262.5	3	24
275-300	5	287.5	4	20
	$\sum f_i = 50$			$\sum f_i y_i = -12$

Here, $\sum f_i u_i = -12$; $\sum f_i = 50$, $h = 25$

$$\text{Mean } M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 187.5 + \frac{-12}{50} \times 25$$

$$= 187.5 - 6 = 181.5$$

121. The mean of the following distribution is 314. Determine the missing frequency x.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	x	10	12	7	8

Sol : [Board Term-1 2016]

We prepare following table to find mean.

C.I.	x	f	$u_i = \frac{x - f}{h}$	$f_i u_i$
1-10	5	5	-3	-15
10-20	15	x	-2	$-2x$
20-30	25	10	-1	-10
30-40	35	12	0	0
40-50	45	7	1	7
50-60	55	8	2	16
Total		$42 + 2$		$-2x - 2$

Let mid point of class 30-40 be assumed mean a.

$$a = 35$$

$$\text{Mean } M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$31.4 = 35 + \frac{-2x - 2}{42 + x} \times 10$$

$$(2x + 2)10 = (42 + x)(3.6)$$

$$20x + 20 = 151.2 + 3.6x$$

$$16.4x = 131.2 \Rightarrow x = 8$$

Sol : [Board Term-1 2016]

Cumulative frequency distribution table (more than type)

Daily income of workers (in Rs.)	Number of workers
More than 200	100
More than 300	88
More than 400	70
More than 500	35
More than 600	15
More than 700	0

Convert the following data into 'more than type' distribution :

Class	50-55	55-60	60-65	65-70	70-75	75-80
Frequency	2	8	12	24	38	16

Sol : [Board Term-1 2012]

We prepare following cumulative frequency table.

Class	Frequency
More than 50	100
More than 55	98
More than 60	90
More than 65	78
More than 70	54
More than 75	16

THREE MARKS QUESTIONS

110. 250 apples of a box were weighted and the distribution of masses of the apples is given in the following table :

Mass (in grams)	80-100	100-120	120-140	140-160	160-180
Number of apples	20	60	70	x	60

(i) Find the value of x and the mean mass of the apples.

(ii) Find the modal mass of the apples.

Sol : [Board 2023 OD Standard]

(i) $20 + 60 + 70 + x + 60 = 250$

$210 + x = 250$

$x = 250 - 210 = 40$

We prepare following cumulative frequency distribution table.

Weight	f_i	$c.f.$	x_i	$f_i x_i$
80-100	20	20	90	1800
100-120	60	80	110	6600
120-140	70	150	130	9100
140-160	40	190	150	6000
160-180	60	250	170	10200
	$\sum f_i = 250$			$\sum f_i x_i = 33700$

$$M = \frac{\sum f_i x_i}{x_i} = \frac{33700}{250} = 134.8 \text{ gm}$$

(ii) Class 120-140 has the maximum frequency 70, therefore this is model class.

Here, $l = 120$, $f_1 = 70$, $f_0 = 60$, $f_2 = 60$ and $h = 20$

$$\text{Mode,} \quad M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$

$$= 120 + \frac{70 - 60}{2 \times 70 - 60 - 40} \times 20$$

$$= 120 + \frac{20}{40} \times 10 = 125$$

111. Compute the mode for the following frequency distribution:

Size of items (in cm)	0- 4	4- 8	8-12	12-16	16-20	20-24	24-28
Frequency	5	7	9	17	12	10	6

Sol : [Board 2020 OD Standard]

Class 12-16 has the maximum frequency 17, therefore this is model class.

We have $l = 12$, $f_1 = 17$, $f_0 = 9$, $f_2 = 12$ and $h = 4$

$$\text{Mode} \quad M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 12 + \left(\frac{17 - 9}{2 \times 17 - 9 - 12}\right) \times 4$$

$$= 12 + \frac{8 \times 4}{13}$$

$$= 12 + 2.46$$

$$= 14.46$$

112. The mean of the following frequency distribution is 18. The frequency f in the class interval 19-21 is missing. Determine f.

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

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Find the mode for the above data.

Sol : [Board Term-1 2015]

Class 6-9 has the maximum frequency 5, therefore this is model class.

Now $l_1 = 6,\ f_1 = 5,\ f_0 = 4,\ f_2 = 1,\ h = 3$

$$\text{Mode,} \quad M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 6 + \frac{5-4}{10-4-1} \times 3$$

$$= 6 + \frac{1}{5} \times 3 = 6 + 0.6 = 6.6$$

127. If the median for the following frequency distribution is 28.5, find the value of x and y :

Class	Frequency
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Sol : [Board Term-1 2013]

We prepare following cumulative frequency table to find median class.

Class	f	$c.f.$
0-10	5	5
10-20	x	$x+5$
20-30	20	$x+25$
30-40	15	$x+40$
40-50	y	$x+y+40$
50-60	5	$x+y+45$
	$\sum f = 60$	

Since, median is 28.5 which lies between 20-30. Thus model class is 20-30.

From table $N = x+y+45$

$$60 = x+y+45$$

$$x+y = 60-45 = 15 \quad \ldots(1)$$

$$\text{Median,} \quad M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$28.5 = 20 + \frac{[30-(x+5)]}{20} \times 10$$

$$8.5 = \frac{25-x}{2}$$

$$25 - x = 17 \Rightarrow x = 25 - 17 = 8$$

From (1), $y = 15 - 8 = 7$

Hence, $x = 8$ and $y = 7$

128. Find the mean and mode of the following frequency distribution :

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	3	8	10	15	7	4	3

Sol : [Board Term-1 2013]

We prepare following table to find mean.

Classes	x_i	f_i	$f_i x_i$
0-10	5	3	15
10-20	15	8	120
20-30	25	10	250
30-40	35	15	525
40-50	45	7	315
50-60	55	4	220
60-70	65	3	195
		$\sum f_i = 50$	$\sum f_i x_i = 1640$

$$\text{Mean} \quad M = \frac{\sum f_i x_i}{\sum f_i} = \frac{1640}{50} = 32.8$$

Class 30-40 has the maximum frequency 35, therefore this is model class.

Here $l = 30,\ f_1 = 15,\ f_2 = 7,\ f_0 = 10,\ h = 10$

$$\text{Mode,} \quad M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 30 + \frac{15-10}{30-10-7} \times 10$$

$$= 30 + \frac{5}{13} \times 10$$

$$= 30 + \frac{50}{13}$$

$$= 30 + 3.85$$

$$= 33.85$$

129. Find the mean of the following distribution by step deviation method :

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	13	20	15	7	5

Sol : [Board Term-1 2011]

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The mean of the new numbers is

$$\overline{x}+\frac{\frac{n(n+1)}{2}}{n}=\overline{x}=\frac{(n+1)}{2}$$

144. Find the mode of the following frequency distribution :

Class-Interval	f
25-35	7
35-45	31
45-55	33
55-65	17
65-75	11
75-85	1

Sol : [Board Term-1 2015]

Class 44-45 has the maximum frequency 33, therefore this is model class.

Now $l_1=45,\ f_0=31,\ f_1=33\ f_2=17,\ h=10$

$$\text{Mode,}\quad M_o=l+h\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)$$
$$=45+\frac{33-31}{66-31-17}\times 10$$
$$=45+\frac{2}{18}\times 10$$
$$=46.1$$

145. The mode of a distribution is 55 and the modal class is 45-60 and the frequency preceding the modal class is 5 and the frequency after the modal class is 10. Find the frequency of the modal class.

Sol : [Board Term-1 Foreign 2015]

Mode, $M_o=55$

Modal class $=45-60$

Frequency of the class preceding,

$f_1=5$

Frequency of the class succeeding the modal class,

$f_2=10$

Let the frequency of modal class be f.

$$\text{Mode}\quad M_o=l+\frac{f-f_1}{2f-f_1-f_2}\times h$$
$$55=45+\frac{f-5}{2f-5-10}\times 15$$
$$10=\left(\frac{f-5}{2f-15}\right)\times 15$$
$$\frac{10}{15}=\frac{f-5}{2f-15}$$
$$\frac{2}{3}=\frac{f-15}{2f-15}$$
$$4f-30=3f-15$$
$$4f-3f=-15+30$$
$$f=15$$

146. The sum of deviations of a set of values $x_1,\ x_2,\ x_3$, ..., x_n, measured from 50 is -10 and the sum of deviations of the values from 46 is 70. Find the value of n and the mean.

Sol : [Board 2018 Comp]

We have,

$$\sum_{i=1}^{n}(x_i-50)=-10 \text{ and } \sum_{i=1}^{n}(x_i-46)=70$$
$$\sum_{i=1}^{n}x_i-50n=-10 \qquad ...(1)$$
and $$\sum_{i=1}^{n}x_i-46n=70 \qquad ...(2)$$

Subtracting (2) from (1) we get,

$$-4n=-80\Rightarrow n=20$$

Substituting this value of n in equation (1) we have

$$\sum_{i=1}^{n}x_i-50\times 20=-10$$
$$\sum_{i=1}^{n}x_i=990$$

Mean $$M=\frac{1}{n}\left(\sum_{i=1}^{n}x_i\right)=\frac{990}{20}=49.5$$

Hence, $n=20$ and mean $=49.5$

147. Prove that $\sum(x_i-\overline{x})=0$

Sol :

We have $$\overline{x}=\frac{1}{n}\left(\sum_{i=1}^{n}x_i\right)$$
$$n\overline{x}=\sum_{i=1}^{n}x_i$$

Now, $$\sum_{i=1}^{n}(x_i-\overline{x})=(x_1-\overline{x})+(x_2-\overline{x})+...+(x_n-\overline{x})$$
$$\sum_{i=1}^{n}(x_i-\overline{x})=(x_1+x_2+...+x_n)-n\overline{x}$$
$$\sum_{i=1}^{n}(x_i-\overline{x})=\sum_{i=1}^{n}x_i-n\overline{x}$$
$$=\sum_{i=1}^{n}(x_i-\overline{x})$$

Hence, $$\sum_{i=1}^{n}(x_i-\overline{x})=0$$

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Classes	f_i	$c.f.$
0-100	15	15
100-200	17	32
200-300	f	$32+f$
300-400	12	$44+f$
400-500	9	$53+f$
500-600	5	$58+f$
600-700	2	$60+f$

From table, $N = 60+f \Rightarrow \frac{N}{2} = \frac{60+f}{2}$

Since median is 240 which lies between class 200-300. Thus median class is 200-300.

$$\text{Median, } M_d = l+\left(\frac{\frac{N}{2}-F}{f}\right)h$$

$$240 = 200+\left[\frac{\frac{60+f}{2}-32}{f}\right]\times 100$$

$$40 = \left[\frac{60+f-64}{2f}\right]\times 100$$

$$8f = 10f-40$$

$$2f = 40$$

$$f = 20$$

133. The following table shows the weights (in gms) of a sample of 100 apples, taken from a large consignment:

Weight (in gms)	50-60	60-70	70-80	80-90	90-100	100-110	110-120	120-130
No. of Apples	8	10	12	16	18	14	12	10

Find the median weight of apples.

Sol : [Board Term-1 2011]

C.I.	50-60	60-70	70-80	80-90	90-100	100-110	110-120	120-130
f	8	10	12	16	18	14	12	10
$c.f.$	8	18	30	46	64	78	90	100

We have $N = 100$; $\frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 64 and the corresponding class is 90-100. Thus median class is 90-100.

$$\text{Median, } M_d = l+\left(\frac{\frac{N}{2}-F}{f}\right)h$$

$$= 90+\left(\frac{50-46}{18}\right)\times 10$$

$$= 90+\frac{40}{18} = 92.2$$

$$= 92.2 \text{ gm.}$$

Thus median weight is 92.2.

134. Find the mean of the following data :

Classes	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	6	8	10	12	8	6

Sol : [Board Term-1 2011]

We prepare following table to find mean.

Classes	Frequency	Mid points	$f_i x_i$
0-20	6	10	60
20-40	8	30	240
40-60	10	50	500
60-80	12	70	840
80-100	8	90	720
100-120	6	110	660
	$\sum f_i = 50$		$\sum f_i x_i = 3020$

Mean, $M = \frac{\sum x_i f_i}{\sum f_i} = \frac{3020}{50} = 60.4$

135. The mean of the following distribution is 53. Find the missing frequency p :

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	p	13

Sol : [Board Term-1 2011]

Class	x_i (Class marks)	f_i	$f_i x_i$
0-20	10	12	120
20-40	30	15	450
40-60	50	32	1600
60-80	70	p	$70p$
80-100	90	13	1170
	Total	$\sum f_i = 72+p$	$\sum f_i x_i = 3340+70p$

Mean, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$$53 = \frac{3340+70p}{72+p}$$

$$3340+70p = 53(72+p)$$

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$$3340 + 70p = 3816 + 53p$$
$$70p - 53p = 3816 - 3340$$
$$17p = 476$$
$$p = \frac{476}{17} = 28$$

136. Weekly income of 600 families is given below :

Income (in Rs.)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No. of Families	250	190	100	40	15	5

Find the median.

Sol : [Board 2010]

We prepare following cumulative frequency table to find median class.

Income	No. of Families	*c.f.*
0-1000	250	250
1000-2000	190	440
2000-3000	100	540
3000-4000	40	580
4000-5000	15	595
5000-6000	5	600
	$N = 600$	

We have $N = 600$; $\frac{N}{2} = 300$

Cumulative frequency just greater than $\frac{N}{2}$ is 440 and the corresponding class is 1000-2000. Thus median class is 1000-2000.

$$\text{Median, } M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$
$$= 1000 + \left(\frac{300 - 250}{190}\right) \times 1000$$
$$= 1000 + \frac{50}{190} \times 1000$$
$$= 1000 + \frac{5000}{19}$$
$$= 1000 + 263.16$$
$$= 1263.16 \text{ = Rs. } 1263.16$$

137. Find the mean for the following data :

Class	24.5-29.5	29.5-34.5	34.5-39.5	39.5-44.5	44.5-49.5	49.5-54.5	54.5-59.5
Frequency	4	14	22	16	6	5	3

Sol : [Board Term-1 2011]

We prepare following table to find mean.

Class	Class marks (x_i)	f_i	$f_i x_i$
24.5-29.5	27	4	108
29.5-34.5	32	14	448
34.5-39.5	37	22	814
39.5-44.5	42	16	672
44.5-49.5	47	6	282
49.5-54.5	52	5	260
54.5-59.5	57	3	171
		$\sum f_i = 70$	$\sum f_i x_i = 2,755$

$$\text{Mean } M = \frac{\sum f_i x_i}{\sum f_i} = \frac{2755}{70} = 39.36$$

138. Find the mode of following data :

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
Number of students	8	20	45	58	70

Sol : [Board Term-1 OD 2013]

Class-Interval	Frequency
0-10	8
10-20	12
20-30	25
30-40	13
40-50	12
Total	70

Class 20-30 has the maximum frequency, therefore this is model class.

Now $l = 20$, $f_1 = 25$, $f_2 = 13$, $f_0 = 12$, $h = 10$

$$\text{Mode, } M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$
$$= 20 + \left(\frac{25 - 12}{50 - 12 - 13}\right) \times 10$$
$$= 20 + \frac{13}{25} \times 10$$
$$= 20 + 5.2 = 25.2$$

139. Find the mean of the following data :

Class	Less than 20	Less than 40	Less than 60	Less than 80	Less than 100
Frequency	15	37	74	99	120

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Sol : [Board Term-1 2011]

We prepare following table to find mean.

C.I.	f_i	x_i	$x_i f_i$
0-20	15	10	150
20-40	22	30	660
40-60	37	50	1850
60-80	25	70	1750
80-100	21	90	1890
	$\sum f_i = 120$		$\sum x_i f_i = 6{,}300$

Mean $M = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{6300}{120} = 52.5$

140. The sum of deviations of a set of values x_1, x_2, x_3 , ..., x_n, measured from 50 is -10 and the sum of deviations of the values from 46 is 70. Find the value of n and the mean.

Sol : [Board Term-1 Delhi 2013]

We have,

$$\sum_{i=1}^{n}(x_i - 50) = -10 \text{ and } \sum_{i=1}^{n}(x_i - 46) = 70$$

$$\sum_{i=1}^{n} x_i - 50n = -10 \quad ...(1)$$

and $$\sum_{i=1}^{n} x_i - 46n = 70 \quad ...(2)$$

Subtracting (2) from (1) we get,

$$-4n = -80 \Rightarrow n = 20$$

Substituting this value of n in equation (1) we have

$$\sum_{i=1}^{n} x_i - 50 \times 20 = -10$$

$$\sum_{i=1}^{n} x_i = 990$$

Mean $$M = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{990}{20} = 49.5$$

Hence, $n = 20$ and mean $= 49.5$

141. Prove that $\sum(x_i - \overline{x}) = 0$

Sol : [Board Term-1 Delhi 2014]

We have $$\overline{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)$$

$$n\overline{x} = \sum_{i=1}^{n} x_i$$

Now, $$\sum_{i=1}^{n}(x_i - \overline{x}) = (x_1 - \overline{x}) + (x_2 - \overline{x}) + ... + (x_n - \overline{x})$$

$$\sum_{i=1}^{n}(x_i - \overline{x}) = (x_1 + x_2 + ... + x_n) - n\overline{x}$$

$$\sum_{i=1}^{n}(x_i - \overline{x}) = \sum_{i=1}^{n} x_i - n\overline{x} = \sum_{i=1}^{n}(x_i - \overline{x})$$

Hence, $$\sum_{i=1}^{n}(x_i - \overline{x}) = 0$$

142. Compute the median from the following data :

Mid-values	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

Sol : [Board Term-1 Delhi 2015]

Here, the mid-values are given So, we should first find the upper and lower limits of the various classes. The difference between two consecutive values is $h = 125 - 115 = 10$

Lower limit of a class = Mid-value $-\dfrac{h}{2}$

Upper limit = Mid- value $+\dfrac{h}{2}$

Mid-value	Class Groups	Frequency	Cumulative Frequency
115	110-120	6	6
125	120-130	25	31
135	130-140	48	79
145	140-150	72	151
155	150-160	116	267
165	106-170	60	327
175	170-180	38	365
185	180-190	22	387
195	190-200	3	390

Now $N = 390$; $\frac{N}{2} = 195$

Cumulative frequency just greater than $\frac{N}{2}$ is 36 and the corresponding class is 150-160. Thus median class is 150-160.

Here, $l = 150$, $f = 116$, $h = 10$, $F = 151$

Median, $M_d = l + \left(\dfrac{\frac{N}{2} - F}{f}\right)h$

$= 150 + \dfrac{195 - 151}{116} \times 10$

$= 153.8$

143. The mean of n observations is $\overline{x}$, if the first term is increased by 1, second by 2 and so on. What will be the new mean ?

Sol : [Board Term-1 Delhi 2015]

I term $+1$

II term $+2$

III term $+3$

n terms $+n$

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Hence, modal daily wages of the workers is ₹ 166.67.

150. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

Sol : [Board 2019 Delhi]

Class	Frequency (f)	Cumulative Frequency (cf)
0-10	f_1	f_1
10-20	5	f_1+5
20-30	9	f_1+14
30-40	12	f_1+26
40-50	f_2	f_1+f_2+26
50-60	3	f_1+f_2+29
60-70	2	f_1+f_2+31
	$N=\sum f=40$	

Now, $f_1+f_2+31 = 40$

$f_1+f_2 = 9$

$f_2 = 9-f_1$ ...(1)

Since median is 32.5, which lies in 30-40, median class is 30-40.

Here $l = 30$, $\frac{N}{2} = \frac{40}{2} = 20$, $f = 12$ and $F = 14+f_1$

Now, Median $= 3.25$

$$l+\left(\frac{\frac{N}{2}-cf}{f}\right)\times h = 32.5$$

$$30+\left(\frac{20-(14+f_1)}{12}\right)\times 10 = 32.5$$

$$\left(\frac{6-f_1}{12}\right)\times 10 = 2.5$$

$$\frac{60-10f_1}{12} = 2.5$$

$$60-10f_1 = 30$$

$$10f_1 = 30 \Rightarrow f_1 = 3$$

From equation (1), we get $f_2 = 9-3 = 6$

Hence, $f_1 = 3$ and $f_2 = 6$

151. The arithmetic mean of the following frequency distribution is 53. Find the value of k.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	k	13

Sol : [Board 2019 Delhi]

Class Interval	Class Marks (x_i)	Frequency (f_i)	f_ix_i
0-20	10	12	120
20-40	30	15	450
40-60	50	32	1600
60-80	70	k	$70k$
80-100	90	13	1170
	Total	$72+k$	$3340+70k$

Mean, $M = \frac{\sum f_ix_i}{\sum f_i}$

$$53 = \frac{3340+70k}{72+k}$$

$$53(72+k) = 3340+70k$$

$$3816+53k = 3340+70k$$

$$70k-53k = 3816-3340$$

$$17k = 476 \Rightarrow k = 28$$

Hence, value of k is 28.

152. Find the mode of the following frequency distribution

Class Interval	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

Sol : [Board 2019 OD Standard]

Class Interval	Frequency
25-30	25
30-35	34
35-40	50
40-45	42
45-50	38
50-55	14

Class 35-40 has the maximum frequency 50, therefore this is model class.

Now, $l = 35$ $f_1 = 50$, $f_0 = 34$, $f_2 = 42$, $h = 5$

Mode, $M_o = l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)h$

$$= 35+\left(\frac{50-34}{2\times 50-34-42}\right)\times 5$$

$$= 35+\frac{16\times 5}{24}$$

$$= 35+\frac{10}{3} = \frac{105+10}{3} = \frac{115}{3}$$

$$= 38.33$$

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FIVE MARKS QUESTIONS

148. The median of the following data is 525. Find the values of x and y, if total frequency is 100 :

Class	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Sol : [Board 2020 Delhi STD]

We prepare cumulative frequency table as given below.

Class Interval	Frequency (f)	Cum. freq. *c.f.*
0-100	2	2
100-200	5	7
200-300	x	$7+x$
300-400	12	$19+x$
400-500	17	$36+x$
500-600	20	$56+x$
600-700	y	$56+x+y$
700-800	9	$65+x+y$
800-900	7	$72+x+y$
900-1000	4	$76+x+y$
	$N=100$	

From table we have

$$76+x+y = 100$$

$$x+y = 100-76 = 24 \quad ...(1)$$

Here median is 525 which lies between class $500-600$. Thus median class is 500-600.

$$\text{Median, } M_d = l+\left(\frac{\frac{N}{2}-F}{f}\right)h$$

$$525 = 500+\left[\frac{\frac{100}{2}-(36+x)}{20}\right]\times 100$$

$$25 = (50-36-x)5$$

$$14-x = \frac{25}{5} = 5$$

$$x = 14-5 = 9$$

Substituting the value of x is equation (1), we get

$$y = 24-9 = 15$$

Hence, $x = 9$ and $y = 15$

149. Daily wages of 110 workers, obtained in a survey, are tabulated below :

Daily Wages (in ₹)	100-120	120-140	140-160	160-180	180-200	200-220	220-240
Number of Workers	10	15	20	22	18	12	13

Compute the mean daily wages an modal daily wages of these workers.

Sol : [Board 2020 SQP STD]

Let $a = 170$ be assumed mean.

CI	x_i	(f)	x_i-170	$\frac{x_i-170}{20}$	f_iu_i
100-120	110	10	-60	-3	-30
120-140	130	15	-40	-2	-30
140-160	150	20	-20	-1	-20
160-180	170	22	0	0	0
180-200	190	18	20	1	18
200-220	210	12	40	2	24
220-240	230	13	60	3	39
		$\sum f_i = 110$			$\sum f_iu_i = 1$

$$\text{Mean, } \quad \overline{x} = a+\frac{\sum f_iu_i}{\sum f_i}\times h$$

$$= 170+\frac{1}{110}\times 20$$

$$= 170+0.1818 \quad = 170.1818$$

Hence, mean daily wages of the workers is ₹170.1818Here the maximum frequency is 22 and the corresponding class is 160-180. So, 160-180 is modal class.

Now $l = 160$, $h = 20$, $f = 22$, $f_1 = 20$ and $f_2 = 18$

$$\text{Mode} \quad M_o = l+\frac{f-f_1}{2f-f_1-f_2}\times h$$

$$= 160+\frac{22-20}{2\times 22-20-18}\times 20$$

$$= 160+\frac{40}{6} = 160+6.666 = 166.67$$

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Class	x_i (Class Marks)	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
0-10	5	-3	5	-15
10-20	15	-2	13	-26
20-30	25	-1	20	-20
30-40	35	0	15	0
40-50	45	1	7	7
50-60	55	2	5	10
	Total		$\sum f_i = 65$	$\sum f u_i = -44$

Let assumed mean, $a = 35$ and given $h = 10$.

$$\text{Mean,} \quad M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 35 + \frac{-44}{65} \times 10$$

$$= 35 - 6.76 = 28.24$$

130. If the mean of the following data is 14.7, find the values of p and q.

Class	0-6	6-12	12-18	18-24	24-30	30-36	36-42	Total
Frequency	10	p	4	7	q	4	1	40

Sol : [Board Term-1 2013]

Class	x_i	f_i	$f_i x_i$
0-6	3	10	30
6-12	9	p	$9p$
12-18	15	4	60
18-24	21	7	147
24-30	27	q	$27q$
30-36	33	4	132
36-42	39	1	39
	Total	$\sum f_i = 26 + p + q = 40$	$\sum f_i x_i = 408 + 9p + 27q$

We have $\sum f_i = 40,$

$$26 + p + q = 40$$

$$p + q = 14 \qquad ...(1)$$

Mean $M = \frac{\sum x_i f_i}{\sum f_i}$

$$14.7 = \frac{408 + 9p + 27q}{40}$$

$$588 = 408 + 9p + 27q$$

$$180 = 9p + 27q$$

$$p + 3q = 20 \qquad ...(2)$$

Subtracting equation (1) from (2) we have,

$$2q = 6 \Rightarrow q = 3$$

Substituting this value of q in equation (2) we get

$$p = 14 - q = 14 - 3 = 11$$

Hence, $p = 11,\ q = 3$

131. Find the mean and median for the following data :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	8	16	36	34	6

Sol : [Board Term-1 2011]

We prepare following cumulative frequency table to find median class.

Class	x_i(class marks)	f_i	$f_i x_i$	*c.f.*
0-10	5	8	40	8
10-20	15	16	240	24
20-30	25	36	900	60
30-40	35	34	1190	94
40-50	45	6	270	100
		$\sum f_i = 100$	$\sum f_i x_i = 2640$	

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{2640}{100} = 26.4$

We have $N = 100\ ;\ \frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 60 and the corresponding class is 20-30. Thus median class is 20-30.

$$\text{Median,} \quad M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right) h$$

$$= 20 + \frac{50 - 24}{36} \times 10$$

$$= 20 + 7.22 = 27.22$$

132. If the median of the following data is 240, then find the value of f :

Classes	0-100	100-200	200-300	300-400	400-500	500-600	600-700
Frequency	15	17	f	12	9	5	2

Sol : [Board Term-1 2011]

We prepare following cumulative frequency table to find median class.

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153. The table below show the salaries of 280 persons:

Salary (In thousand ₹)	No. of Persons
5-10	49
10-15	133
15-20	63
20-25	15
25-30	6
30-35	7
35-40	4
40-45	2
45-50	1

Calculate the median salary of the data.

Sol : [Board 2018]

Salary (In thousand ₹)	No. of Persons (f)	c.f.
5-10	49	49
10-15	133	182
15-20	63	245
20-25	15	260
25-30	6	266
30-35	7	273
35-40	4	277
40-45	2	279
45-50	1	280

We have $\frac{N}{2} = \frac{280}{2} = 140$

Commutative frequency greater than just greater than $\frac{N}{2} = 140$ is 182 and the corresponding class is 10-15. Thus median class is 10-15.

$$\text{Median } M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$
$$= 10 + \frac{(140 - 49)}{133} \times 5$$
$$= 10 + \frac{91 \times 5}{133}$$
$$= 13.42$$

Median salary is ₹ 13.42 thousand or ₹ 13420 (approx)

154. On annual day of a school, 400 students participated in the function. Frequency distribution showing their ages is as shown in the following table :

Ages (in years)	05-07	07-09	09-11	11-13	13-15	15-17	17-19
Number of students	70	120	32	100	45	28	5

Find mean and median of the above data.

Sol : [Board Term-1 OD 2015]

We prepare following cumulative frequency table to find mean and median.

C.I.	f_i	$c.f.$	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
05-07	70	70	6	-3	-210
07-09	120	190	8	-2	-240
09-11	32	222	10	-1	-32
11-13	100	322	12	0	0
13-15	45	367	14	1	45
15-17	28	395	16	2	56
17-19	5	400	18	3	15
	$\sum f_i = 400$				$\sum f_i u_i = -366$

Assumed mean, $a = 12$

$$\text{Mean, } M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$
$$= 12 + \frac{-366}{400} \times 2$$
$$= 12 - \frac{183}{100} = 12 - 1.83$$
$$= 10.17$$

We have $N = 400$; $\frac{N}{2} = 200$

Cumulative frequency just greater than $\frac{N}{2}$ is 222 and the corresponding class is 9-11. Thus median class is 9-11.

$$\text{Median, } M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$
$$= 9 + \frac{200 - 190}{32} \times 2$$
$$= 9 + \frac{10}{32} \times 2 = 9 + 0.625$$
$$= 9.625$$

155. The mean of the following distribution is 18. Find

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the frequency of the class 19-21.

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

Sol : [Board 2018]

Class	Class mark	Frequency (f)	$f_i x_i$
11-13	12	3	36
13-15	14	6	84
15-17	16	9	144
17-19	18	13	234
19-21	20	f	$20f$
21-23	22	5	110
23-25	24	4	96
		$40+f$	$704+20f$

Mean, $$M = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$18 = \frac{704+20f}{40+f}$$

$$720+18f = 704+20f \Rightarrow f = 8$$

156. Literacy rates of 40 cities are given in the following table. It is given that mean literacy rate is 63.5, then find the missing frequencies x and y.

Literacy rate (in %)	35-40	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
Number of cities	1	2	3	x	y	6	8	4	2	3	2

Sol : [Board Term-1 2016]

We prepare following table to find mean.

C.I.	x_i	u_i	f_i	$f_i u_i$
35-40	37.5	-5	1	-5
40-45	42.5	-4	2	-8
45-50	75.5	-3	3	-9
50-55	52.5	-2	x	$-2x$
55-60	57.5	-1	y	$-y$
60-65	$62.5 = a$	0	6	0
65-70	67.5	1	8	8
70-75	72.5	2	4	8
75-80	77.5	3	2	6
80-85	82.5	4	3	12
85-90	87.5	5	2	10
Total			$\sum f_i = 31+x+y$	$\sum f_i u_i = 22-2x-y$

Here, $$\sum f_i = 31+x+y = 40$$

$$x+y = 9 \quad ...(1)$$

$$\sum f_i u_i = 22-2x-y$$

Mean $$M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$63.5 = 62.5 + \frac{(22-2x-y)}{40} \times 5$$

$$2x+y = 14 \quad ...(2)$$

Solving equation (1) and (2) we have $x = 5$ and $y = 4$

157. The following distribution gives the weights of 60 students of a class. Find the mean and mode weights of the students.

Weight (in kg)	40-44	44-48	48-52	52-56	56-60	60-64	64-68	68-72
Number of students	4	6	10	14	10	8	6	2

Sol : [Board Term-1 Delhi 2015]

We prepare following table to find mean.

C.I.	x_i	f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
40-44	42	4	-3	-12
44-48	46	6	-2	-12
48-52	50	10	-1	-10
52-56	$54 = a$	14	0	0
56-60	58	10	1	10
60-64	62	8	2	16
64-68	66	6	3	18
68-72	70	2	4	8
		$\sum f_i = 60$		$\sum f_i u_i = 18$

Let a be assumed mean where $a = 54$

Mean, $$M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 54 + \frac{18}{60} \times 4 = 55.2$$

Class 52-56 has the maximum frequency 14, therefore this is model class.

Here $l = 52$, $f_1 = 14$, $f_0 = 10$, $f_2 = 10$, $h = 4$

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

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$$= 52 + \frac{14-10}{28-10-10} \times 4 = 54$$

Hence mean is 55.2 and mode is 5.

158. On the sports day of a school, 300 students participated. Their ages are given in the following distribution :

Age (in years)	5-7	7-9	9-11	11-13	13-15	15-17	17-19
Number of students	67	33	41	95	36	13	15

Find the mean and mode of the data.

Sol : [Board Term-1 Foreign 2015]

We prepare following table to find mean.

Age	x_i	f_i	$f_i x_i$
5-7	6	67	402
7-9	8	33	264
9-11	10	41	410
11-13	12	95	1140
13-15	14	36	504
15-17	16	13	208
17-19	18	15	270
		$\sum f_i = 300$	$\sum f_i x_i = 3,198$

Mean, $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{3,198}{300} = 10.66$

Class 11-13 has the maximum frequency 95, therefore this is model class.

Now $l = 11,\ f_1 = 95,\ f_0 = 41,\ f_2 = 36,\ h = 2$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 11 + \frac{95-41}{190-41-36} \times 2$$

$$= 11 + \frac{54}{113} \times 2$$

$$= 11 + 0.95 = 11.95$$

159. The median of the following data is 525. Find the values of x and y if the total frequency is 100.

Class Interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Sol : [Board Term-1 2013]

We prepare following cumulative frequency table to find median class.

Class Interval	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	x	$7+x$
300-400	12	$19+x$
400-500	17	$36+x$
500-600	20	$56+x$
600-700	y	$56+x+y$
700-800	9	$65+x+y$
800-900	7	$72+x+y$
900-1000	4	$76+x+y$
	$N = 100$	

Here median is 525, which lies between class 500-600. Thus median class is 500-600.

Now, $76 + x + y = 100$

$$x + y = 100 - 76 = 24 \quad \ldots(1)$$

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$$525 = 500 + \left[\frac{\frac{100}{2} - (36+x)}{20}\right] \times 100$$

$$25 = (50 - 36 - x)5$$

$$(14 - x) = \frac{25}{5} = 5$$

$$x = 14 - 5 = 9$$

Substituting the value of x in equation (1),

$$y = 24 - 9 = 15$$

Hence, $x = 9$ and $y = 15$

160. Monthly expenditures on milk in 100 families of a housing society are given in the following frequency distribution :

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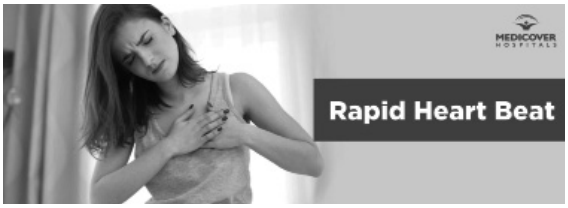
$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{8650}{52} = 166.4$$

(ii) Since maximum frequency is 20 and corresponding class is 160-170. Thus lower limit of modal class is 160.

(iii) Cumulative frequency just greater than $\frac{N}{2} = \frac{52}{2} = 26$ is 35 and the corresponding class is 160-170. Thus median class is 160-170 and upper limit is 170.

177. Heart Rate : The heart rate is one of the 'vital signs,' or the important indicators of health in the human body. It measures the number of times per minute that the heart contracts or beats. The speed of the heartbeat varies as a result of physical activity, threats to safety, and emotional responses. The resting heart rate refers to the heart rate when a person is relaxed. While a normal heart rate does not guarantee that a person is free of health problems, it is a useful benchmark for identifying a range of health issues. After the age of 10 years, the heart rate of a person should be between 60 and 100 beats per minute while they are resting.



Thirty women were examined by doctors of AIIMS and the number of heart beats per minute were recorded and summarised as follows.

Number of heart beats per minute	Number of women (f_i)
65-68	2
68-71	4
71-74	3
74-77	8
77-80	7
80-83	4
83-86	2

Based on the above information, answer the following questions.

(i) What is the mean heart beats per minute for these women ?

(ii) What is the upper limit of median value of heart beats per minute for these women ?

(iii) What is the lower limit of mode value of heart beats per minute for these women ?

(iv) How many women are having heart beat in range 68-77?

Sol :

(i) We prepare the following frequency distribution table.

Heart beats per minute	Class mark (x_i)	Women (f_i)	cf	$u_i = \frac{x_i - a}{3}$	$f_i u_i$
65-68	66.5	2	2	-3	-6
68-71	69.5	4	6	-2	-8
71-74	72.5	3	9	-1	-3
74-77	75.5	8	17	0	0
77-80	78.5	7	24	1	7
80-83	81.5	4	28	2	8
83-86	84.5	2	30	3	6
Total		$\sum f_i = 30$			$\sum f_i u_i = 4$

We have, $$\text{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 75.5 + \frac{4 \times 3}{30}$$

$$= 75.5 + 0.4 = 75.9$$

(i) The mean heart beat per minute is 75.9.

(ii) Cumulative frequency just greater than $\frac{N}{2} = \frac{30}{2} = 15$ is 17 and the corresponding class is 74-77. Thus median class is 74-77 and upper limit is 77.

(iii) Class 74-77 has the maximum frequency 8, therefore this is model class and lower limit is 74.

(iv) $4 + 3 + 8 = 15$ women are having heart beat in range 68-77.

178. Sulfur dioxide (SO_2) is a colorless gas at ambient temperature and pressure. It is soluble in water and forms sulfurous acid which is slowly oxidized to sulfuric acid by dissolved oxygen.

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163. If the mean of the following frequency distribution is 91, and sum of frequency is 150, find the missing frequency x and y :

Class	0-30	30-60	60-90	90-120	120-150	150-180
Frequency	12	21	x	52	y	11

Sol : [Board Term-1 2011]

We prepare following table to find mean.

Class	x_i	f_i	$f_i x_i$
0-30	15	12	180
30-60	45	21	945
60-90	75	x	$75x$
90-120	105	52	5460
120-150	135	y	$135y$
150-180	165	11	1815
	Total	$\sum f_i = x+y+96=150$	$\sum f_i x_i = 8400+75x+135y$

$$96+x+y = 150$$
$$x+y = 54 \quad ...(1)$$
$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$$
$$91 = \frac{8400+75x+135y}{150}$$
$$13650 = 8,400+75x+135y$$
$$75x+135y = 5250$$
$$5x+9y = 350 \quad ...(2)$$

Solving equation (1) and (2) we get $x = 34$ and $y = 20$

164. Find the median of the following data :

Profit (in lakh of rupee)	Number of shops
More than of equal to 5	30
More than of equal to 10	28
More than of equal to 15	16
More than of equal to 20	14
More than of equal to 25	10
More than of equal to 30	7
More than of equal to 35	3

Sol : [Board Term-1 2013]

We prepare following cumulative frequency table to find median class.

Class	f	$c.f.$
5-10	2	2
10-15	12	14
15-20	2	16
20-25	4	20
25-30	3	23
30-35	4	27
35-40	3	30
Total	$\sum f = 30 = N$	

We have $N = 30$; $\frac{N}{2} = 15$

Cumulative frequency just greater than $\frac{N}{2}$ is 16 and the corresponding class is 15-20. Thus median class is 15-20.

Median, $M_d = l + \left(\frac{\frac{N}{2}-F}{f}\right)h$

Now, $l = 15$, $N = 30$, $F = 14$, $f = 2$ $h = 5$

Median, $M_d = l + \left(\frac{\frac{N}{2}-F}{f}\right)h$

$$= 15 + \left(\frac{15-14}{2}\right) \times 5$$
$$= 15 + 2.5 = 17.5$$

165. Find the value of x and y, if the median for the following data is 31.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	6	y	6	5	40

Sol : [Board Term-1 2011]

We prepare following cumulative frequency table to find median class.

C.I.	f	$c.f.$
0-10	5	5
10-20	x	$5+x$
20-30	6	$11+x$
30-40	y	$11+x+y$
40-50	6	$17+x+y$
50-60	5	$22+x+y$
	Total 40	

Since median is 31, which lies between 30-40.

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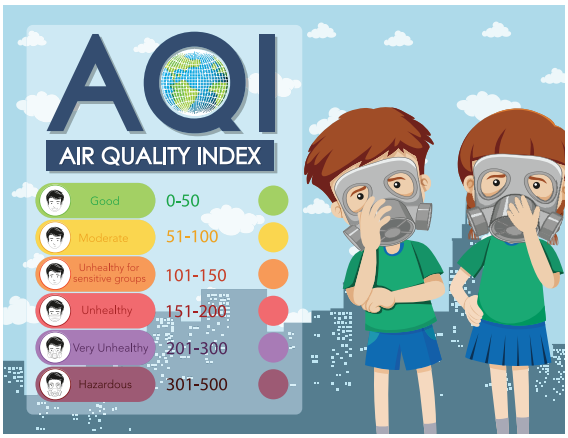
five major air pollutants :

1. Ground-level ozone
2. Particle pollution/particulate matter (PM2.5/pm 10)
3. Carbon Monoxide
4. Sulfur dioxide
5. Nitrogen dioxide

The higher the AQI value, the greater the level of air pollution and the greater the health concerns.



Following frequency distribution shows the Air Quality Index of different localities of Delhi on 27th December 2020 reported by Times of India Newspaper on 28th December 2020.

AIQ	Number of weeks f
270-280	4
280-290	10
290-300	14
300-310	20
310-320	24
320-330	8
Total	80

Based on the above information, answer the following questions.

(i) Estimate the mean AQI.

(ii) In which class does the median of distribution lie?

(iii) In which class does the mode of distribution lie ?

(iv) What is the median AQI?

Sol :

(i) We prepare the following commutative frequency distribution table.

AIQ	Number of weeks f	cf	Mid-point x_i	fx_i
270-280	4	4	275	1100
280-290	10	14	285	2850
290-300	14	28	295	4130
300-310	20	48	305	6100
310-320	24	72	315	7560
320-330	8	80	325	2600
Total	80			24340

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{24340}{80} = 304.25$$

(ii) Cumulative frequency just greater than $\frac{N}{2} = \frac{80}{2} = 40$ is 48 and the corresponding class is 300-310. Thus median class is 300-310.

(iii) Since 310-320 has highest frequency i.e. 24, thus 310-320 is model class.

(iv) Here $l = 300,,\ N = 80,\ F = 28,\ f = 20$ and $h = 10$

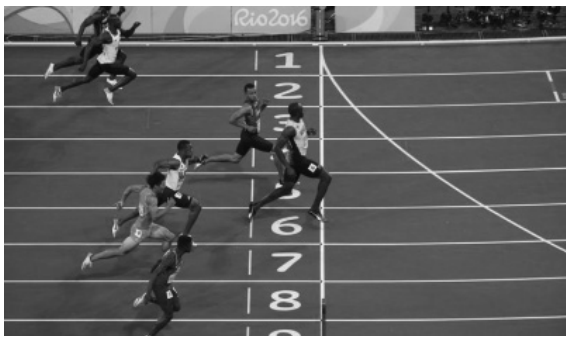
$$\text{Median,} \quad M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$= 300 + \left(\frac{40 - 28}{20}\right) \times 10$$

$$= 300 + 6$$

$$= 306$$

175. 100 Metres Race : The 100 metres is a sprint race in track and field competitions. The shortest common outdoor running distance, it is one of the most popular and prestigious events in the sport of athletics. It has been contested at the summer Olympics since 1896 for men and since 1928 for women. The World Championships 100 metres has been contested since 1983. The reigning 100 m Olympic or world champion is often named "the fastest man or woman in the world".



Downloaded from www.pdf.tube

We prepare following cumulative frequency table to find mean and median.

C.I.	f_i	$c.f.$	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
05-07	70	70	6	-3	-210
07-09	120	190	8	-2	-240
09-11	32	222	10	-1	-32
11-13	100	322	12	0	0
13-15	45	367	14	1	45
15-17	28	395	16	2	56
17-19	5	400	18	3	15
	$\sum f_i = 400$				$\sum f_i u_i = -366$

Assumed mean, $a = 12$

Mean, $M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 12 + \frac{-366}{400} \times 2$

$= 12 - \frac{183}{100} = 12 - 1.83$

$= 10.17$

We have $N = 400$; $\frac{N}{2} = 200$

Cumulative frequency just greater than $\frac{N}{2}$ is 222 and the corresponding class is 9-11. Thus median class is 9-11.

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right) h$

$= 9 + \frac{200 - 190}{32} \times 2$

$= 9 + \frac{10}{32} \times 2 = 9 + 0.625$

$= 9.625$

169. The following distribution gives the weights of 60 students of a class. Find the mean and mode weights of the students.

Weight (in kg)	40-44	44-48	48-52	52-56	56-60	60-64	64-68	68-72
Number of students	4	6	10	14	10	8	6	2

Sol : [Board Term-1 2015]

We prepare following table to find mean.

C.I.	x_i	f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
40-44	42	4	-3	-12
44-48	46	6	-2	-12
48-52	50	10	-1	-10
52-56	$54 = a$	14	0	0
56-60	58	10	1	10
60-64	62	8	2	16
64-68	66	6	3	18
68-72	70	2	4	8
		$\sum f_i = 60$		$\sum f_i u_i = 18$

Let a be assumed mean where $a = 54$

Mean, $M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 54 + \frac{18}{60} \times 4 = 55.2$

Class 52-56 has the maximum frequency 14, therefore this is model class.

Here $l = 52$, $f_1 = 14$, $f_0 = 10$, $f_2 = 10$, $h = 4$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$

$= 52 + \frac{14 - 10}{28 - 10 - 10} \times 4 = 54$

Hence mean is 55.2 and mode is 5.

170. The following table gives the life time of 200 bulbs. Calculate the mean life time of a bulb by step deviation method :

Life time (in hours)	400-499	500-599	600-699	700-799	800-899	900-999
Number of bulbs	24	47	39	42	34	14

Sol : [Board Term-1 2012]

We prepare following table to find mean.

Life time (in hrs)	x_i	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
399.5-499.5	449.5	-2	24	-48
499.5-599.5	549.5	-1	47	-47
599.5-699.5	649.5	0	39	0
699.5-799.5	749.5	1	42	42

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Thus median class is 30-40.

Here from table, $N = 22 + x + y$

$$40 = 22 + x + y$$

$$x + y = 18 \quad ...(1)$$

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$$31 = 30 + \left[\frac{20 - (11 + x)}{y}\right] \times 10$$

$$1 = \frac{(9 - x) \times 10}{y}$$

$$y = 90 - 10x$$

$$10x + y = 90 \quad ...(2)$$

Solving equation (1) and (2) we get $x = 8$ and $y = 10$

166. Find the mode of the following distribution of marks obtained by the students in an examination:

Marks obtained	0-20	20-40	40-60	60-80	80-100
Number of students	15	18	21	29	17

Given the mean of the above distribution is 53, using empirical relationship estimate the value of its median.

Sol : [Board Term-1 SQP 2017]

Class 60-80 has the maximum frequency 29, therefore this is model class.

Here, $l = 60$, $f_1 = 29$, $f_0 = 21$, $f_2 = 17$ and $h = 20$

Mode, $M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$$= 60 + \frac{8}{58 - 38} \times 20 = 60 + 8 = 68$$

Now $3M_d = M_o + 2M = 68 + 2 \times 53$

$$M_d = \frac{174}{3} = 58$$

Hence median is 58.

167. Daily wages of 110 workers, obtained in a survey, are tabulated below :

Daily Wages (in ₹)	100-120	120-140	140-160	160-180	180-200	200-220	220-240
Number of Workers	10	15	20	22	18	12	13

Compute the mean daily wages an modal daily wages of these workers.

Sol : [Board 2020 SQP Standard]

Let $a = 170$ be assumed mean.

CI	x_i	(f)	$x_i - 170$	$\frac{x_i - 170}{20}$	$f_i u_i$
100-120	110	10	-60	-3	-30
120-140	130	15	-40	-2	-30
140-160	150	20	-20	-1	-20
160-180	170	22	0	0	0
180-200	190	18	20	1	18
200-220	210	12	40	2	24
220-240	230	13	60	3	39
		$\sum f_i = 110$			$\sum f_i u_i = 1$

Mean, $\overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$$= 170 + \frac{1}{110} \times 20$$

$$= 170 + 0.1818$$

$$= 170.1818$$

Hence, mean daily wages of the workers is ₹ 170.1818Here the maximum frequency is 22 and the corresponding class is 160-180. So, 160-180 is modal class.

Now $l = 160$, $h = 20$, $f = 22$, $f_1 = 20$ and $f_2 = 18$

Mode $M_o = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$

$$= 160 + \frac{22 - 20}{2 \times 22 - 20 - 18} \times 20$$

$$= 160 + \frac{40}{6}$$

$$= 160 + 6.666$$

$$= 166.67$$

Hence, modal daily wages of the workers is ₹ 166.67.

168. On annual day of a school, 400 students participated in the function. Frequency distribution showing their ages is as shown in the following table :

Ages (in years)	05-07	07-09	09-11	11-13	13-15	15-17	17-19
Number of students	70	120	32	100	45	28	5

Find mean and median of the above data.

Sol : [Board Term-1 2015]

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COMPETENCEY BASED QUESTIONS

173. BINGO is game of chance. The host has 75 balls numbered 1 through 75. Each player has a BINGO card with some numbers written on it.

The participant cancels the number on the card when called out a number written on the ball selected at random. Whosoever cancels all the numbers on his/her card, says BINGO and wins the game.

The table given below, shows the data of one such game where 48 balls were used before Tara said 'BINGO'.

Numbers announced	Number of times
0 – 15	8
15 – 30	9
30 – 45	10
45 – 60	12
60 – 75	9

Based on the above information, answer the following:

(i) Write the median class.

(ii) When first ball was picked up, what was the probability of calling out an even number?

(iii) Find median of the given data.

(iv) Find mode of the given data.

Sol :[Board 2024 OD Standard]

We draw cumulative frequency table as shown below.

Numbers announced	Number of times	Cumulative frequency
0 – 15	8	8
15 – 30	9	17
30 – 45	10	27
45 – 60	12	39
60 – 75	9	48

Here sum of all frequencies,

$$N = 48$$

$$\frac{N}{2} = \frac{48}{2} = 24$$

(i) Median Class

Class $30-45$ has frequency 27 which is just greater than 24. Thus $30-45$ is median class.

(ii) Probability (Even number)

Here we have 75 ball out of which even numbers are 2, 4, ..., 74 . Thus we have 37 ball with even number.

Total number of ball,

$$n(S) = 75$$

Number of favorable cases,

$$n(E) = 37$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{37}{75}$$

(iii) Median

Now, $l = 30$, $\frac{N}{2} = 24$, $F = 17$, $f = 10$ and $h = 15$

Median, $$M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$= 30 + \left(\frac{24-17}{10}\right) \times 15$$

$$= 30 + \frac{7}{10} \times 15 = 40.5$$

Thus, the median is 40.5.

(iv) Mode

As class 45 – 60 has highest frequency 12, modal class is 45 – 60.

Here, $l = 45$, $f_1 = 12$, $f_0 = 10$, $f_2 = 9$ and $h = 15$

Mode, $$M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$

$$= 45 + \left(\frac{12-10}{2 \times 12 - 10 - 9}\right) \times 15$$

$$= 45 + \frac{2}{5} \times 15 = 51$$

Thus, the mode is 51.

174. Air Quality Iindex : AQI is an index for reporting air quality on a daily basis. The purpose of the AQI is to help people know how the local air quality impacts their health. The Environmental Protection Agency (EPA) calculates the AQI for

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799.5-899.5	849.5	2	34	68
899.5-999.5	949.5	3	14	42
Total			$\sum f_i = 200$	$\sum f_i u_i = 57$

Let a be assumed mean, then $a = 649.5$

$$\text{Mean,} \quad M = a + \left(\frac{\sum f_i u_i}{\sum f_i} \times h\right)$$

$$= 649.5 + \frac{57}{200} \times 100$$

$$= 649.5 + 28.5 = 678$$

Hence, mean life time of a bulb is 678 hours.

171. If the mean of the following frequency distribution is 91, and sum of frequency is 150, find the missing frequency x and y :

Class	0-30	30-60	60-90	90-120	120-150	150-180
Frequency	12	21	x	52	y	11

Sol : [Board Term-1 2012]

We prepare following table to find mean.

Class	x_i (Class marks)	f_i	$f_i x_i$
0-30	15	12	180
30-60	45	21	945
60-90	75	x	$75x$
90-120	105	52	5460
120-150	135	y	$135y$
150-180	165	11	1815
	Total	$\sum f = x + y + 96 = 150$	$\sum f x_i = 8400 + 75x + 135y$

$$96 + x + y = 150$$

$$x + y = 54 \quad \ldots(1)$$

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$91 = \frac{8400 + 75x + 135y}{150}$$

$$13650 = 8,400 + 75x + 135y$$

$$75x + 135y = 5250$$

$$5x + 9y = 350 \quad \ldots(2)$$

Solving equation (1) and (2) we get $x = 34$ and $y = 20$

172. The following table gives the daily income of 50 workers of a factory.

Daily income (in Rs.)	100-120	120-140	140-160	160-180	180-200
Number of Workers	12	14	8	6	10

Find the mean, mode and median of the above data.

Sol : [Board Term-1 2009]

We prepare following table to find mean.

C.I.	f_i	$c.f.$	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100-120	12	12	110	-2	-24
120-140	14	26	130	-1	-14
140-160	8	34	150	0	0
160-180	6	40	170	1	6
180-200	10	50	190	2	20
	$\sum f = 50$				$\sum f_i u_i = -12$

Let a be assumed mean be $a = 150$

$$\text{Mean} \quad M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 150 + \frac{-12}{50} \times 20$$

$$= 150 - 4.8 = 145.2$$

Now $N = 50$; $\frac{N}{2} = 25$

Cumulative frequency just greater than $\frac{N}{2}$ is 26 and the corresponding class is 120-140. Thus median class is 120-140.

Now $l = 120$, $f = 14$, $F = 12$ and $h = 20$

$$\text{Median,} \quad M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$= 120 + \left(\frac{25 - 12}{14}\right) \times 20$$

$$= 120 + 18.57 \times 138.57$$

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

$$= 3 \times 138.57 - 2 \times 145.2$$

$$= 415.71 - 290.4 = 125.31$$

Hence, mean is 145.2, median is 138.57 and mode is 125.31.

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A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (in sec)	0-20	20-40	40-60	60-80	80-100
No. of students	8	10	13	6	3

Based on the above information, answer the following questions.

(i) Estimate the mean time taken by a student to finish the race.

(ii) What will be the upper limit of the modal class ?

(iii) What is the sum of lower limits of median class and modal class ?

(iv) How many students finished the race within 1 minute?

Sol :

(i) We prepare the following commutative frequency distribution table.

Time (in sec)	Number of students (f_i)	Mid-value x_i	$f_i x_i$	cf
0-20	8	10	80	8
20-40	10	30	300	18
40-60	13	50	650	31
60-80	6	70	420	37
80-100	3	90	270	40
Total	$\sum f_i = 40$		$\sum f_i x_i = 1720$	

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1720}{40} = 43$$

Mean time taken by a student to finish the race is 43 sec.

(ii) Since 40-60 has highest frequency i.e. 13 upper limit of modal class is 60.

(iii) Cumulative frequency just greater than $\frac{N}{2} = \frac{40}{2} = 20$ is 31 and the corresponding class is 40-60. Thus median class is 40-60 and lower limit is 40.

Sum of lower limits of median class and modal class $= 40 + 40 = 80$.

(iv) The number of students finished the race within 1 min (i.e. 60 sec)

= cumulative frequency of class 40-60

= 31

176. Living Index : Cost of living indexes are meant to compare the expenses an average person can expect to incur to acquire food, shelter, transportation, energy, clothing, education, healthcare, childcare, and entertainment in different regions. A cost of living index is also used to track how much the costs of basic expenses rise over a period.

The weekly observation on cost of living index in a certain city for a particular year are given below. Observe the following table:

Cost of Living Index	Number of weeks
140-150	5
150-160	10
160-170	20
170-180	9
180-190	6
190-200	2
Total	52

Based on the above information, answer the following questions.

(i) What is the approximate mean weekly cost-of living index ?

(ii) What will be the lower limit of the modal class ?

(iii) What is the upper limit of median class ?

Sol :

(i) We prepare the following commutative frequency distribution table.

Cost of Living Index	Number of weeks f	cf	Mid-point x_i	$f_i x_i$
140-150	5	5	145	725
150-160	10	15	155	1550
160-170	20	35	165	3300
170-180	9	44	175	1575
180-190	6	50	185	1110
190-200	2	52	195	390
Total	52			860

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Monthly expenditure (in Rs.)	0-175	175-350	350-525	525-700	700-875	875-1050	1050-1125
Number of families	10	14	15	21	28	7	5

Find the mode and median for the distribution.

Sol : [Board Term-1 2016]

We prepare following cumulative frequency table to find median class.

C.I.	f	$c.f.$
0-175	10	10
157-350	14	24
350-525	15	39
525-700	21	60
700-875	28	88
875-1050	7	95
1050-1225	5	100
	$N = 100$	

We have $N = 100 \; ; \; \frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 60 and the corresponding class is 525-700. Thus median class is 525-700.

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$= 525 + \frac{50 - 39}{21} \times 175$

$= 525 + \frac{11}{21} \times 175$

$= 525 + 91.6 = 616.6$

Class 700-875 has the maximum frequency 28, therefore this is model class.

Here $l = 700, \; f_0 = 21, \; f_1 = 28 \; f_2 = 7, \; h = 175$

Mode, $M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$= 700 + \left(\frac{28 - 21}{2 \times 28 - 21 - 7}\right) \times 175$

$= 700 + \frac{7}{28} \times 175$

$= 700 + 43.75 = 743.75$

161. Calculate the average daily income (in Rs.) of the following data about men working in a company :

Daily income (Rs.)	100	200	300	400	500
Number of men	12	28	34	41	50

Sol : [Board Term-1 2012]

We prepare following table to find mean.

Class	x_i (class mark)	f_i	$f_i x_i$
0-100	50	12	600
100-200	150	16	2400
20-300	250	6	1500
300-400	350	7	2450
400-500	450	9	4050
		$\sum f_i = 50$	$\sum f_i x_i = 11{,}000$

Mean $M = \frac{\sum x_i f_i}{\sum f_i} = \frac{11000}{50} = 200$

Average daily income is Rs. 220.

162. The following table gives the life time of 200 bulbs. Calculate the mean life time of a bulb by step deviation method :

Life time (in hours)	400-499	500-599	600-699	700-799	800-899	900-999
Number of bulbs	24	47	39	42	34	14

Sol : [Board Term-1 2014]

We prepare following table to find mean.

Life time (in hrs)	x_i	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
399.5-499.5	449.5	-2	24	-48
499.5-599.5	549.5	-1	47	-47
599.5-699.5	649.5	0	39	0
699.5-799.5	749.5	1	42	42
799.5-899.5	849.5	2	34	68
899.5-999.5	949.5	3	14	42
Total			$\sum f_i = 200$	$\sum f_i u_i = 57$

Let a be assumed mean,

$a = 649.5$

Mean, $M = a + \left(\frac{\sum f_i u_i}{\sum f_i} \times h\right)$

$= 649.5 + \frac{57}{200} \times 100$

$= 649.5 + 28.5$

$= 678$

Hence, mean life time of a bulb is 678 hours.

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Sulfur dioxide is a major air pollutant and has significant impacts upon human health. In addition, the concentration of sulfur dioxide in the atmosphere can influence the habitat suitability for plant communities, as well as animal life. Sulfur dioxide emissions are a precursor to acid rain and atmospheric particulates.

To find out the concentration of SO_2 in the air (in parts per million, i.e. ppm), the data was collected for 30 localities in a delhi and is presented below:

Concentration of SO_2 (in ppm)	Frequency
0.00-0.04	4
0.04-0.08	9
0.08-0.12	9
0.12-0.16	2
0.16-0.20	4
0.20-0.24	2

Based on the above information, answer the following questions.

(i) What is the mean concentration of SO_2 in the air?

(ii) What is the lower limit of median value of concentration of SO_2 in the air?

(iii) Find the median value concentration of SO_2 in the air.

(iv) How many localities are having SO_2 in range 0.04-0.16 ppm ?

Sol :

(i) Let assumed mean, $a = 0.10$

Here, class size $h = 0.04$

(i) We prepare the following frequency distribution table.

SO_2 (in ppm)	Class mark (x_i)	Frequency (f_i)	cf	$u_i = \frac{x_i - 0.10}{0.04}$	$f_i u_i$
0.00-0.04	0.02	4	4	-2	-8
0.04-0.08	0.06	9	13	-1	-9
0.08-0.12	0.10	9	22	0	0
0.12-0.16	0.14	2	24	1	2
0.16-0.20	0.18	4	28	2	8
0.20-0.24	0.22	2	30	3	6
Total		$\sum f_i = 30$			$\sum f_i u_i = -1$

We have, $\text{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$$= 0.10 + \frac{(-1) \times 0.04}{30}$$

$$= 0.10 - 0.001 = 0.099 \text{ ppm}$$

(ii) Cumulative frequency just greater than $\frac{N}{2} = \frac{30}{2} = 15$ is 22 and the corresponding class is 0.08-0.12. Thus median class is 0.08-0.12 and lower limit is 0.08 ppm.

(iii) Here $l = 0.08$, $N = 30$, $F = 13$, $f = 9$ and $h = 0.04$

$$\text{Median, } M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

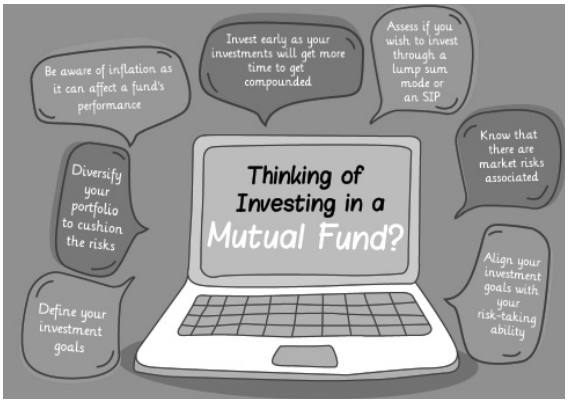
$$= 0.08 + \left(\frac{\frac{30}{2} - 13}{9}\right) \times 0.04$$

$$= 0.08 + \frac{0.08}{9}$$

$$= 0.08 + 0.00888 = 0.0889 \text{ ppm}$$

(iv) $9 + 9 + 2 = 20$ localities are having SO_2 in range 0.04-0.16 ppm ?

179. Mutual Fund : A mutual fund is a type of financial vehicle made up of a pool of money collected from many investors to invest in securities like stocks, bonds, money market instruments, and other assets. Mutual funds are operated by professional money managers, who allocate the fund's assets and attempt to produce capital gains or income for the fund's investors.


Be aware of inflation as it can affect a fund's performance
Invest early as your investments will get more time to get compounded
Assess if you wish to invest through a lump sum mode or an SIP
Know that there are market risks associated
Diversify your portfolio to cushion the risks
Thinking of Investing in a Mutual Fund?
Align your investment goals with your risk-taking ability
Define your investment goals

Net asset value (NAV) represents a fund's per share market value. It is the price at which investors buy ("bid price") fund shares from a fund company and sell them ("redemption price") to a fund company.

The following table below shows the net asset value (NAV) per unit of mutual funds of ICICI mutual funds.

NAV (Rs)	No. of mutual funds
0-5	13

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6-10	16
11-15	22
16-20	18
21-25	11

Based on the above information, answer the following questions.

(i) What is the lower limit of model class of NAV?

(ii) What is the mode NAV of mutual funds ?

(iii) What is the mean NAV of mutual funds ?

(iv) What is the median NAV of mutual funds ?

Sol :

The given classes are not continuous. So, let us make them continuous and rewrite the table.

Class	Frequency f	cf
0.5-5.5	13	13
5.5-10.5	16	29
10.5-15.5	22	51
15.5-20.5	18	69
20.5-25.5	11	80

(i) Class 10.5-15.5 has the maximum frequency 22, therefore this is model class.

(ii) Now $l = 10.5$, $f_1 = 22$, $f_2 = 18$, $f_0 = 16$, $h = 5$

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$
$$= 10.5 + \frac{22 - 16}{44 - 18 - 16} \times 5$$
$$= 10.5 + 3 = 13.5$$

(iii) We prepare following table to find mean.

Class	$x_i = \frac{l_1 + l_2}{2}$	f_i	$f_i x_i$
0.5-5.5	3	13	39
5.5-10.5	8	16	128
10.5-15.5	13	22	286
15.5-20.5	18	18	324
20.5-25.5	23	11	253
	Total	$\sum f_i = 80$	1030

Mean $$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1030}{80} = 12.9$$

(iv) $$3M_d = M_o + 2M$$
$$3M_d = 13.5 + 2 \times 12.9 = 39.3$$
$$M_d = \frac{39.3}{3} = 13.1$$

180. Salary Cut : Our personal and professional lives have taken quite a turn in the light of the coronavirus pandemic. Worklife has been revamped across the globe, in that working from home has become the norm for those employees and organizations that continue to carry on with their operations and functioning in the lockdown. Increments, incentives, businesses have all been impacted.

The Covid-19 pandemic has stalled economic activity at an unprecedented scale globally, raising the spectre of job losses and salary cuts. Most of companies decided to bring down the salary by upto 50 %.

The following table shows the salaries (in percent) received by 80 employee during lockdown.

Salary Received (in %)	40-50	50-60	60-70	70-80	80-90	90-100
Number of employee	4	10	14	20	24	8

Based on the above information, answer the following questions.

(i) What is the mean salary (in %) received ?

(ii) How many employee received more than 70 % salary ?

(iii) Find the total number of employee whose salary is reduced by more than 40 % ?

(iv) What is the lower limit of mode class of salary (in %) received?

(v) Which is the median class of salary (in %) received?

Sol :

(i) We prepare following cumulative frequency table.

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C.I.	f_i	$c.f.$	x_i	$f_i x_i$
40-50	4	4	45	180
50-60	10	14	55	550
60-70	14	28	65	910
70-80	20	48	75	1500
80-90	24	72	85	2040
90-100	8	80	95	760
	$\sum f_i = 80$			$\sum f_i x_i = 5940$

$$M = \frac{\sum f_i x_i}{\sum f_i} = \frac{5940}{80} = 74.25$$

(ii) From cumulative frequency we get that 28 employee received up to 70 % salary. Rest $80 - 28 = 52$ employee received more than 70 % salary.

(iii) From cumulative frequency we get that 10 employee's salary is cut by more than 40 %.

(iv) Class 80-90 has the maximum frequency 24, therefore this is model class. Lower limit is 80.

(v) Cumulative frequency just greater than $\frac{N}{2} = \frac{80}{2} = 40$ is 48 and the corresponding class is 70-80. Thus median class is 70-80.

181. Traffic Management : A traffic enforcement camera is a camera which may be mounted beside or over a road or installed in an enforcement vehicle to detect motoring offenses, including speeding, vehicles going through a red traffic light. A worldwide review of studies found that speed cameras led to a reduction of 11% to 44% for fatal and serious injury crashes. The British Medical Journal recently reported that speed cameras were effective at reducing accidents and injuries in their vicinity and recommended wider deployment.

In order to monitor reckless driving on Mumbai road, special cameras have been installed at many traffic light. The following table shows a frequency distribution table for the speed of 100 vehicles passing through a particular spot on a day.

Speed (in km/h)	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of Vehicles	1	3	7	16	35	29	7	2

Based on the above information, answer the following questions.

(i) Find the number of vehicles whose speed is more than 70 km/h ?

(ii) Find the number of vehicles whose speed is less than 50 km/h ?

(iii) What is the mode value of speed ?

(iv) What is the median value of speed ?

(v) Find the mean value of speed using empirical relation.

Sol :

We prepare following cumulative frequency table

Speed (in km/h)	f	$c.f.$
20-30	1	1
30-40	3	4
40-50	7	11
50-60	16	27
60-70	35	62
70-80	29	91
80-90	7	98
90-100	2	100
Total	$\sum f = 100 = N$	

(i) From cummulative frequency we get that 62 vehicles have speed upto 70 km/h. Rest $100 - 62 = 38$ vehicles have speed more than 70 km/h.

(ii) From cummulative frequency we get that 11 vehicles have speed upto 50 km/h.

(iii) Class 60-70 has the maximum frequency 35, therefore this is model class.

Now $l = 60,\ f_1 = 35,\ f_2 = 29,\ f_0 = 16,\ h = 10$

$$\text{Mode,} \quad M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 60 + \frac{35 - 16}{120 - 29 - 16} \times 10$$

$$= 60 + 2.53 = 62.53 \text{ km/h}$$

(iv) Cumulative frequency just greater than $\frac{N}{2} = \frac{100}{2} = 50$ is 62 and the corresponding class is

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$$= 35 + \left[\frac{50-45}{33}\right] \times 5$$

$$= 35 + \frac{25}{33}$$

$$= 35 + 0.76$$

$$= 35.76 \text{ years}$$

Thus, the median age 35.76 years.

(ii) Class 35-40 has the maximum frequency 78, therefore this is model class.

(iii) Now $l = 35,\ f_1 = 33,\ f_2 = 11,\ f_0 = 21,\ h = 5$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 35 + \frac{33-21}{66-21-11} \times 5$$

$$= 35 + \frac{12}{33} \times 5$$

$$= 35 + \frac{20}{11}$$

$$= 35 + 1.82 = 36.82 \text{ years}$$

(iv) Empirical relation,

$$3M_d = M_o + 2M$$

$$3 \times 35.76 = 36.82 + 2M$$

$$107.28 = 36.82 + 2M$$

$$107.28 = 36.82 + 2M$$

$$2M = 107.28 - 36.82 = 70.46$$

$$M = \frac{70.46}{2} = 35.23 \text{ years}$$

187. The Kendriya Vidyalaya Sangathan is a system of premier central government schools in India that are instituted under the aegis of the Ministry of Education (MHRD), Government of India. As of October 2020, it has a total of 1239 schools. It is one of the world's largest chains of schools. The system came into being in 1963 under the name 'Central Schools'. Later, the name was changed to Kendriya Vidyalaya. Its schools are all affiliated to the Central Board of Secondary Education (CBSE). The objective of KVS is to cater to the educational needs of the children of transferable Central Government employees including Defence and Para-Military personnel by providing a common programme of education.

NODIA

Commissioner of Regional office Jaipur prepare a table of the marks obtained of 100 students which is given below

Marks obtained	0-20	20-40	40-60	60-80	80-100
Number of students	15	18	21	29	p

He was told that mean marks of a student is 53.

(i) How many students got marks between 80-100?

(ii) What is the lower limit of model class ?

(iii) What is the value of model marks ?

(iv) What is the value of median marks ?

(v) What is the upper limit of median class ?

Sol :

(i) Since numbers of students are 100,

$$15 + 18 + 21 + 29 + p = 100$$

$$83 + p = 100$$

$$p = 100 - 83 = 17$$

(ii) Class 60-80 has the maximum frequency 29, therefore this is model class. Lower limit of this class is 60.

(iii) Here, $l = 60,\ f_1 = 29,\ f_0 = 21,\ f_2 = 17$ and $h = 20$

Mode, $M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$$= 60 + \frac{29-21}{2 \times 29 - 21 - 17} \times 20$$

$$= 60 + \frac{8}{58-38} \times 20$$

$$= 60 + 8 = 68$$

(iv) Now $3M_d = M_o + 2M = 68 + 2 \times 53$

$$M_d = \frac{174}{3} = 58$$

electric scooter, Hero Scooter Pvt Ltd wants to declare the mileage of their electric scooters. For this, they recorded the mileage (km/charge) of 50 scooters of the same model. Details of which are given in the following table.

Mileage (km/charge)	100-120	120-140	140-160	160-180
Number of scooters	7	12	18	13

Based on the above information, answer the following questions.

(i) What is the average mileage.

(ii) What is the modal value of mileage ?

(iii) What is the median value of mileage ?

(iv) What about the mileage can be claimed by the manufacturer for his scooter ?

Sol :

We prepare following cumulative frequency table

Class interval	Class mark (x_i)	Frequency (f_i)	$x_i f_i$	*c.f.*
100-120	110	7	770	7
120-140	130	12	1560	19
140-160	150	18	2700	37
160-180	170	13	2210	50
Total		50	7240	

(i) $M = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{7240}{50} = 144.8$ km/charge

Thus average mileage is 144.8 km/charge.

(ii) Class 140-160 has the maximum frequency 18, therefore this is model class.

Now $l = 140,\ f_1 = 18,\ f_2 = 13,\ f_0 = 12,\ h = 20$

Mode, $M_o = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$= 140 + \dfrac{18 - 12}{36 - 12 - 13} \times 20$

$= 140 + \dfrac{6}{11} \times 20$

$= 140 + \dfrac{120}{11} = 140 + 10.91$

$= 150.91$ km/charge

(iii) Cumulative frequency just greater than $\frac{N}{2} = \frac{50}{2} = 25$ is 37 and the corresponding class is 140-160. Thus median class is 140-160.

Now, $l = 140,\ \frac{N}{2} = 25,\ F = 19,\ f = 18$ and $h = 20$

Median, $M_d = l + \left(\dfrac{\frac{N}{2} - F}{f}\right)h$

$= 140 + \dfrac{25 - 19}{18} \times 20$

$= 140 + \dfrac{60}{9} = 146.67$ km/charge

Median of the distance travelled is 224.12 km

(iv) Since, Mean is 144.8, Mode is 150.91 and Median is 146.67 and minimum of which is 144.8, therefore manufacturer can claim the mileage for his scooter 144 km/charge.

184. Transport department of a Jaipur wants to buy some Electric buses for the city. For which they wants to analyse the distance travelled by existing public transport buses in a day.

The following data shows the distance travelled by 60 existing public transport buses in a day.

Daily distance travelled (in km)	200-209	210-219	220-229	230-239	240-249
Number of buses	4	14	26	10	6

Base on the above information, answer the following questions.

(i) Find the median class of daily distance travelled ?

(ii) What is the cumulative frequency of the class preceding the median class ?

(iii) Find the median of the distance travelled.

(iv) If the mode of the distance travelled is 223.78 km, find the mean of the distance travelled by

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the bus .

Sol :

Here, class intervals are in inclusive form. So, we first convert them in exclusive form. The frequency distribution table in exclusive form is as follows :

Class interval	Frequency (f_i)	($c.f.$)
199.5-209.5	4	4
209.5-219.5	14	18
219.5-229.5	26	44
229.5-239.5	10	54
239.5-249.5	6	60

(i) Here, $\sum f_i$ i.e., $N = 60$

$$\frac{N}{2} = 30$$

Cumulative frequency just greater than $\frac{N}{2} = \frac{60}{2} = 30$ is 44 and the corresponding class is 219.5-229.5. Thus median class is 219.5-229.5.

(ii) Clearly, the cumulative frequency of the class preceding the median class 219.5-229.5 is 18.

(iii) Now, $l = 219.5,\ N = 60,\ F = 18,\ f = 26$

$h = 10$

$$\text{Median, } M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$= 219.5 + \left(\frac{30 - 18}{26}\right) \times 10$$

$$= 219.5 + \frac{12 \times 10}{26}$$

$$= 219.5 + 4.62 = 224.12$$

Median of the distance travelled is 224.12 km.

(iv) $$3M_d = M_o + 2M$$

$$3 \times 224.12 = 223.78 + 2M$$

$$672.36 - 223.78 = 2M$$

$$448.58 = 2M$$

$$M = \frac{448.58}{2} = 224.29 \text{ km}$$

185. An inspector in an enforcement squad of electricity department visit to a locality of 100 families and record their monthly consumption of electricity, on the basis of family members, electronic items in the house and wastage of electricity, which is summarise in the following table.

Monthly Consumption (in kwh)	Number of families
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Inspector calculated that median of the above data is 525 and after that he lost two data which is given as x and y in table.

Based on the above information, answer the following questions.

(i) What is the value of lost data x ?

(ii) What is the value of lost data y ?

(iii) What will be the upper limit of the modal class?

Sol :

We prepare following cumulative frequency table

Monthly Consumption (in kwh)	Number of families	Cumulative Frequency
0-100	2	2
100-200	5	7
200-300	x	$7 + x$
300-400	12	$19 + x$
400-500	17	$36 + x$
500-600	20	$56 + x$

600-700	y	$56+x+y$
700-800	9	$65+x+y$
800-900	7	$72+x+y$
900-1000	4	$76+x+y$
Total	$76+x+y$	

Since total frequency is 100,

$$76+x+y = 100$$
$$x+y = 100-76=24$$

Here median is 525, thus median class is 500-600. Also $\frac{N}{2}=\frac{100}{2}=50$.

Now, $l=500$, $\frac{N}{2}=50$, $F=36+x$, $f=20$ and $h=100$.

$$\text{Median, } M_d = l+\left(\frac{\frac{N}{2}-F}{f}\right)h$$
$$525 = 500+\left(\frac{50-36-x}{20}\right)\times 100$$
$$25 = (14-x)\times 5$$
$$25 = 70-5x$$
$$x = \frac{70-25}{5}=9$$
$$\text{Now } y = 24-9=15$$

(i) Thus $x=9$

(ii) $y=15$

(iii) Class 500-600 has the maximum frequency 20, therefore this is model class.

186. Life insurance is a contract between an insurance policy holder and an insurer or assurer, where the insurer promises to pay a designated beneficiary a sum of money upon the death of an insured person (often the policy holder). Depending on the contract, other events such as terminal illness or critical illness can also trigger payment. The policy holder typically pays a premium, either regularly or as one lump sum.

SBI life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

(i) What is the median value of age ?

(ii) What will be the upper limit of the modal class?

(iii) What is the mode value of age ?

(iv) Find the mean value of age using empirical relation.

Sol :

The given table is cumulative frequency distribution. We write the frequency distribution as given below:

Class interval	Cumulative Frequency	Frequency
15-20	2	$2-0=2$
20-25	6	$6-2=4$
25-30	24	$24-6=18$
30-35	45	$45-24=21$
35-40	78	$78-45=33$
40-45	89	$89-78=11$
45-50	92	$92-89=3$
50-55	98	$98-92=6$
55-60	100	$100-98=2$

We have, $\sum f_i = N = 100$

(ii) Cumulative frequency just greater than $\frac{N}{2}=\frac{100}{2}=50$ is 78 and the corresponding class is 35-40. Thus median class is 35-40.

Now, $l=35$, $\frac{N}{2}=50$, $F=45$, $f=33$ and $h=5$

$$\text{Median, } M_d = l+\left(\frac{\frac{N}{2}-F}{f}\right)h$$

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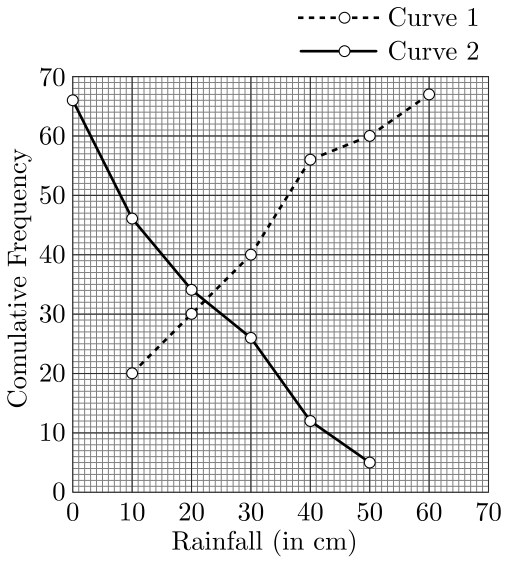

Based on the above graph, answer the following questions :

(i) Identify less than type ogive and more than type ogive from the given graph.

(ii) Find the median rainfall of Dispur.

(iii) Obtain the Mode of the data if mean rainfall is 23.4 cm

Sol : [Board 2020 SQP Standard]

(i) Curve-1 shows less than ogive and curve-2 shows more than ogive.

(ii) The abscissa of intersecting point of less than and more than ogive give the median. Thus median is 21 cm.

(iii) Mode of data,

$$M_o = 3M_d - 2M$$
$$= 3 \times 21 - 2 \times 23.4$$
$$= 63 - 46.8$$
$$= 16.2 \text{ cm}$$

190. Amul, is an Indian dairy cooperative society, based at Anand in the Gujarat. Formed in 1946, it is a cooperative brand managed by a cooperative body, the Gujarat Co-operative Milk Marketing Federation Ltd. (GCMMF), which today is jointly owned by 36 lakh (3.6 million) milk producers in Gujarat. Amul spurred India's White Revolution, which made the country the world's largest producer of milk and milk products.

Survey manager of Amul dairy has recorded monthly expenditures on milk in 100 families of a housing society. This is given in the following frequency distribution :

Monthly expenditure (in Rs.)	Number of families
0-175	10
175-350	14
350-525	15
525-700	x
700-875	28
875-1050	7
1050-1225	5

(i) How many families spend between Rs 350-700 on milk ?

(ii) What is the upper limit of median class ?

(iii) What is the median expenditure on milk?

(iv) What is the lower limit of model class ?

(v) What is the model expenditure on milk?

Sol :

(i) Since number of families is 100,

$$10 + 14 + 15 + x + 28 + 7 + 5 = 100$$
$$79 + x = 100$$
$$x = 100 - 79 = 21$$

Thus $15 + 21 = 36$ families spend between Rs 350- 700 on milk.

(ii) We prepare following cumulative frequency table to find median class.

C.I.	f	$c.f.$
0-175	10	10
157-350	14	24
350-525	15	39

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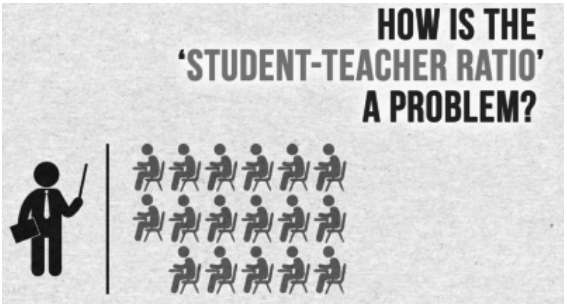
Hence median is 58.

(v) Since median is 58 and corresponding class is 40-60. Upper limit of this class is 60.

188. Student-teacher ratio expresses the relationship between the number of students enrolled in a school and the number teachers employed by the school. Student-teacher ratio is important for a number of reasons. It can be used as a tool to measure teacher workload as well as the allocation of resources. A low student-teacher ratio indicates the burden on a single teacher of teaching multiple students as well as the lack of time that each student gets.



A survey was conducted in the 100 secondary school of Rajasthan and following frequency distribution table was prepared :

Students per teacher	Number of School
20-25	5
25-30	15
30-35	25
35-40	30
40-45	15
45-50	10

(i) What is the upper limit of median class ?

(ii) What is the median value of students per teacher?

(iii) What is the lower limit of model class ?

(iv) What is the model value of students per teacher ?

(v) What is the mean value of students per teacher ?

Sol :

(i) We prepare following cumulative frequency table to find median class.

Students per teacher	Number of School	c.f
20-25	5	5
25-30	15	20
30-35	25	45
35-40	30	75
40-45	15	90
45-50	10	100
	$N = 100$	

We have $N = 100$; $\frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 75 and the corresponding class is 35-40. Thus median class is 35-40 and upper limit is 40.

(ii) Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$= 35 + \frac{50 - 45}{30} \times 5$

$= 35 + \frac{5}{6} = \frac{215}{6} = 35.83$

(iii) Class 35-40 has the maximum frequency 30, therefore this is model class. Lower limit of this class is 35.

(iv) Here, $l = 35$, $f_1 = 30$, $f_0 = 25$, $f_2 = 15$ and $h = 5$

Mode, $M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$= 35 + \frac{30 - 25}{60 - 25 - 15} \times 5$

$= 35 + \frac{5}{20} \times 5$

$= 35 + 1.25 = 36.25$

(v) Now $3M_d = M_o + 2M$

$\frac{3 \times 215}{6} = 36.25 + 2M$

$2 \times 215 = 4 \times 36.25 + 8M$

$430 = 145 + 8M$

$8M = 430 - 145 = 285$

$M = \frac{285}{8} = 35.625$

189. A TV reporter was given a task to prepare a report on the rainfall of the city Dispur of Indian in a particular year. After collecting the data, he analysed the data and prepared a report on the rainfall of the city, Using this report, he drew the following graph of a particular time period of 66 days

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60-70. Thus median class is 60-70.

Now, $l = 60,\ N = 100,\ F = 27,\ f = 35\ h = 10$

$$\text{Median, } M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$
$$= 60 + \left(\frac{50 - 27}{35}\right) \times 10$$
$$= 60 + 6.57 = 66.57 \text{ km/h}$$

(v) Empirical relation,

$$3M_d = M_o + 2M$$
$$3 \times 66.57 = 62.53 + 2M$$
$$2M = 3 \times 66.57 - 62.53$$
$$= 137.18 \text{ km/h}$$
$$M = \frac{137.18}{2} = 68.59 \text{ km/h}$$

182. Toll Tax : In India, for every state or national highway/expressway, a fee is charged for raising the cost incurred in constructing as well as for maintaining the roads. This fee is called toll and is a kind of tax. Once the cost of the highway is recovered, the fee is collected at a lessened rate of 40 percent, for the purpose of maintenance of the road.

On a particular day, National Highway Authority of India (NHAI) checked the toll tax collection of a particular toll plaza in Rajasthan. The following table shows the toll tax paid by drivers and the number of vehicles on that particular day.

Toll tax (in ₹)	30-40	40-50	50-60	60-70	70-80
Number of vehicles	80	110	120	70	40

Based on the above information, answer the following questions.

(i) What is the mean of toll tax received by NHAI?

(ii) What is the toll tax received by NHAI in a day, from that particular toll plaza ?

(iii) What is the mode of toll tax received by NHAI?

(iv) Find the median value of speed using empirical relation.

Sol :

We prepare following cumulative frequency table

Class	Class marks (x_i)	$d_i = x_i - A$	Frequency (f_i)	$f_i d_i$
30-40	35	-20	80	-1600
40-50	45	-10	110	-1100
50-60	$55 = A$	0	120	0
60-70	65	10	70	700
70-80	75	20	40	800
Total			$\sum f_i = 420$	$\sum f_i d_i = -1200$

(i) $$\text{Required Mean} = A + \frac{\sum f_i d_i}{\sum f_i}$$
$$= 55 - \frac{1200}{420}$$
$$= ₹\ 52.14$$

(ii) Total toll tax received by a vehicle $= ₹\ 52.14$

Total number of vehicles $= 420$

Average toll tax received in a day

$$= ₹\ (52.14 \times 420)$$
$$= ₹\ 21898.80$$

(iii) Class 50-60 has the maximum frequency 120, therefore this is model class.

Now $l = 50,\ f_1 = 120,\ f_2 = 70,\ f_0 = 110,\ h = 10$

$$\text{Mode, } M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$
$$= 50 + \frac{120 - 110}{240 - 110 - 70} \times 10$$
$$= 50 + \frac{10}{6}$$
$$= 50 + 1.67 = 51.67$$

(iv) Empirical relation,

$$3M_d = M_o + 2M$$
$$3M_d = 51.67 + 2 \times 52.14$$
$$3M_d = 155.95$$
$$M_d = \frac{155.95}{3} = 51.98$$

183. Electric scooters are plug-in electric vehicles with two or three wheels. The electricity is stored on board in a rechargeable battery, which drives one or more electric motors. Leading manufacturer of

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C.I.	f	$c.f.$
525-700	21	60
700-875	28	88
875-1050	7	95
1050-1225	5	100
	$N = 100$	

We have $N = 100$; $\frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 60 and the corresponding class is 525-700. Thus median class is 525-700 and upper limit is 700.

(iii) Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$= 525 + \frac{50 - 39}{21} \times 175$

$= 525 + \frac{11}{21} \times 175$

$= 525 + 91.6 = 616.6$

(iv) Class 700-875 has the maximum frequency 28, therefore this is model class and lower limit is 700.

(v) Here $l = 700$, $f_0 = 21$, $f_1 = 28$ $f_2 = 7$, $h = 175$

Mode, $M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$= 700 + \left(\frac{28 - 21}{2 \times 28 - 21 - 7}\right) \times 175$

$= 700 + \frac{7}{28} \times 175$

$= 700 + 43.75$

$= 743.75$

191. Formula one Portugese Grand Prix technical team at the Algarve International Circuit are analysing last year data of drivers' performance to provide valuable inferences to commentators on how the drivers can improve this year.

The length of time taken by 80 drivers to complete a journey is given in the table below:

Times (in minutes)	70-80	80-90	90-100	100-110	110-120	120-130
Number of drivers	4	10	14	20	24	8

(i) What is the estimate of the mean time taken to complete the journey ?

(ii) In which interval does the median of the distribution lie?

(iii) In which interval does the mode of the distribution lie?

(iv) What is the model time taken to complete journey ?

(v) What is the median time taken to complete journey ?

Sol :

(i) We prepare the following table :

C.I.	f_i	$c.f.$	x_i	$f_i x_i$
70-80	4	4	75	300
80-90	10	14	85	850
90-100	14	28	95	1330
100-110	20	48	105	2100
110-120	24	72	115	2760
120-130	8	80	125	1000
	$\sum f_i = 80$			$\sum f_i x_i = 8340$

$$M = \frac{\sum f_i x_i}{\sum f_i} = \frac{8390}{80} = 104.25$$

(ii) Cumulative frequency just greater than $\frac{N}{2} = \frac{80}{2} = 40$ is 48 and the corresponding class is 100-110. Thus median class is 100-110.

(iii) Class 110-120 has the maximum frequency 24, therefore this is model class.

(iv) Here, $l = 110$, $f_1 = 24$, $f_0 = 20$, $f_2 = 8$ and $h = 10$

Mode, $M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$= 110 + \frac{24 - 20}{2 \times 24 - 20 - 8} \times 10$

$= 110 + \frac{4}{20} \times 10 = 112$

(v) Now $3M_d = M_o + 2M$

$= 112 + 2 \times 104.25$

$= 320.5$

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$P(\text{number less game}),\ P(\overline{E}) = 1 - P(E)$
$= 1 - 0.07$
$= 0.93$

PRACTICE

✍ If the probability of winning a game is $\frac{5}{11}$, find the probability of losing the game.
[Board Term-2 2014]
Ans : $\frac{6}{11}$

✍ A girl calculates the probability of her winning the game in a match and find it 0.08. What is the probability of her losing the game?
[Board Term-2 2012]
Ans : 0.92

43. If a card is selected from a deck of 52 cards, then find the probability of its being a red face card?
Sol : [Board Term-2 Delhi 2011]
In a deck of 52 cards, there are 12 face cards i.e., 6 red and 6 black cards.

$n(S) = 52$
$n(E) = 6$

So, probability of getting a red face card,

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

44. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. Find the number of outcomes favourable to E.
Sol : [Board Term-2 OD 2017]
In a deck of 52 cards, there are 13 cards of heart and 1 is ace of heart.
Hence, the number of outcomes favourable

$n(E) = 52 - 1 = 51$

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45. When a die is thrown, what is the probability of getting an odd number less than 3?
Sol : [Board Term-2 Delhi 2013]
Odd number less than 3 is 1 only.

$n(S) = 6$
$n(E) = 1$

So, probability of getting an odd number less than 3,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

PRACTICE

✍ What is the probability of getting a number greater then 3 in throwing a die?
[Board 2009]
Ans :

46. The probability of getting a bad egg in a lot of 400 eggs is 0.035. Find the number of bad eggs in the lot.
Sol : [Board Term-2 2012]
Number x be number of bad eggs.

$n(E) = x$
Total eggs, $n(S) = 400$
$P(\text{bad eggs})$ $P(E) = 0.035$
$P(\text{bad eggs}),\ P(E) = \frac{n(E)}{n(S)}$
$0.035 = \frac{x}{400}$
$x = 400 \times 0.035 = 14$

Thus there are 14 bad eggs in lot.

PRACTICE

✍ The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap ?
[Board 2010]
Ans : 162

47. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, then how many tickets has she bought?
Sol :
Total number of sold tickets are 6000. Let she bought x tickets.
Now $n(S) = 6000$
$n(E) = x$
$P(E) = \frac{n(E)}{n(S)}$
$0.08 = \frac{x}{6000}$
$x = 0.08 \times 6000 = 480$

Hence, she bought 480 tickets.

48. Find the probability of an impossible event.
Sol : [Board Term-2 2012]
Probability of impossible event is 0.

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CHAPTER 14

Probability

OBJECTIVE QUESTIONS

1. Two dice are rolled together. The probability of getting sum of numbers on the two dice as 2, 3 or 5, is:

(a) $\frac{7}{36}$ (b) $\frac{11}{36}$

(c) $\frac{5}{36}$ (d) $\frac{4}{9}$

Sol : [Board 2024 OD Standard]

Total number of outcomes is 36.

Here, all favorable possible outcome is (1, 1), (1, 2), (2, 1), (2, 3), (3, 1), (1, 4) and (4, 1).

$$n(S) = 36$$

No. of favourable outcome,

$$n(E) = 7$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{7}{36}$$

Thus (a) is correct option.

2. From the data 1, 4, 7, 9, 16, 21, 25, if all the even numbers are removed, then the probability of getting at random a prime number from the remaining is

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$

Sol : [Board 2024 OD Standard]

If we remove even number from data 1, 4, 7, 9, 16, 21, 25 we have 1, 7, 9, 21, 25 only. Here prime number is only 7.

Thus $n(S) = 5$

Favorable outcomes,

$$n(E) = 1$$

Probability of getting prime number,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{5}$$

Thus (b) is correct option.

3. One card is drawn at random from a well shuffled deck of 52 playing cards. The probability that it is a red ace card, is

(a) $\frac{1}{13}$ (b) $\frac{1}{26}$

(c) $\frac{1}{52}$ (d) $\frac{1}{2}$

Sol : [Board 2024 OD Standard]

In a deck of 52 cards, there are 2 cards of red ace.

Thus $n(S) = 52$

Favorable outcomes,

$$n(E) = 2$$

Probability of getting red ace,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

Thus (b) is correct option.

4. A card is drawn at random from a well shuffled deck of 52 playing cards. The probability of getting a face card is

(a) $\frac{1}{2}$ (b) $\frac{3}{13}$

(c) $\frac{4}{13}$ (d) $\frac{1}{13}$

Sol : [Board 2023 OD Standard]

Total cards, $n(S) = 52$

There are 12 face cards.

$$n(E) = 13$$

P(face card) $P(E) = \frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

Thus (b) is correct option.

5. A bag contains 5 pink, 8 blue and 7 yellow balls. One ball is drawn at random from the bag. What is the probability of getting neither a blue nor a pink ball ?

(a) $\frac{1}{4}$ (b) $\frac{2}{5}$

(c) $\frac{7}{20}$ (d) $\frac{13}{20}$

Sol : [Board 2023 OD Standard]

There are total $5 + 7 + 8 = 20$ balls in bag. Thus total possible outcomes,

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P(sum of two numbers will be multiple of 4)

$$P(E) = \frac{n(E)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

Thus (d) is correct option.

32. A letter is chosen at random from the letters of the word ASSASSINATION, then the probability that the letter chosen is a vowel is in the form of $\frac{6}{2x+1}$, then x is equal to

(a) 5 (b) 6

(c) 7 (d) 8

Sol : [Board Term-2 Delhi 2012]

There are 13 letters in the word ASSASSINATION out of which one letter can be chosen in 13 ways. Hence, total number of outcomes are 13. There are 6 vowels in the word ASSISSINATION. So, there are 6 ways of selecting a vowel.

$$n(S) = 13$$

$$n(E) = 6$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{13}$$

But given that, $\frac{6}{2x+1} = \frac{6}{13}$

$$2x+1 = 13 \Rightarrow x = 6$$

Thus (b) is correct option.

33. Ramesh buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random a tank containing 5 male fish and 9 female fish. Then, the probability that the fish taken out is a male fish, is

(a) $\frac{5}{13}$ (b) $\frac{5}{14}$

(c) $\frac{6}{13}$ (d) $\frac{7}{13}$

Sol : [Board Term-2 SQP 2011]

There are $14 = (5+9)$ fish out of which one can be chosen in 14 ways.

There are 5 male fish out of which one male fish can be chosen in 5 ways.

$$n(S) = 14$$

$$n(E) = 5$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{5}{14}$$

Thus (b) is correct option.

34. A number x is selected from the numbers 1, 2, 3 and then a second number y is randomly selected from the numbers 1, 4, 9 then the probability that the product xy of the two numbers will be less than 9 is

(a) $\frac{3}{7}$ (b) $\frac{4}{9}$

(c) $\frac{5}{9}$ (d) $\frac{7}{9}$

Sol : [Board Term-2 Foreign 2013]

Number x can be selected in three ways and corresponding to each such way there are three ways of selecting number y.

Therefore two numbers can be selected in 9 ways as (1, 1), (1, 4), (1, 9), (2, 1), (2, 4), (2, 9), (3, 1), (3, 4), (3, 9) So, total numbers of possible outcomes are 9.

The product xy will be less than 9, if x and y are chosen in one of the following ways: (1, 1), (1, 4), (2, 1), (2, 4), (3, 1)

$$n(S) = 9$$

$$n(E) = 5$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{9}$$

Thus (c) is correct option.

35. There are 1000 sealed envelopes in a box. 10 of them contain a cash prize of ₹ 100 each, 100 of them contain a cash prize of ₹ 50 each and 200 of them contain a cash prize of ₹ 10 each and rest do not contain any cash prize. If they are well-shuffled and an envelope is picked up out, then the probability that is contains no cash prize is

(a) 0.65 (b) 0.69

(c) 0.54 (d) 0.57

Sol : [Board Term-2 Delhi 2011]

Total number of envelopes in the box = 1000

Number of envelopes containing cash prize

$$= 10 + 100 + 200 = 310$$

Number of envelopes containing no cash

$$= 1000 - 310 = 690$$

Now $n(S) = 1000$

$$n(E) = 690$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{690}{1000} = 0.69$$

Thus (b) is correct option.

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Total number of sold tickets are 6000. Let she bought x tickets.

Now $n(S) = 6000$

$n(E) = x$

$P(E) = \frac{n(E)}{n(S)}$

$0.08 = \frac{x}{6000}$

$x = 0.08 \times 6000 = 480$

Hence, she bought 480 tickets.

Thus (c) is correct option.

13. The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4 is

(a) $\frac{4}{15}$ (b) $\frac{2}{15}$

(c) $\frac{1}{15}$ (d) $\frac{1}{5}$

Sol : [Board 2020 Delhi Basic]

Total possible outcome, $n(S) = 15$

Number of multiples of 4 between 1 to 15 are 4, 8, 12 i.e. 3 favourable outcome.

$n(E) = 3$

Required Probability, $P(E) = \frac{n(E)}{n(S)} = \frac{3}{15} = \frac{1}{5}$

Thus (d) is correct option.

14. Two coins are tossed simultaneously. The probability of getting at most one head is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Sol : [Board 2020 OD Basic]

All possible outcomes are {HH, HT, TH, TT}.

Thus $n(S) = 4$

Favourable outcomes are {HT, TH, TT}.

$n(E) = 3$

Probability of getting at most one head,

$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$

Thus (d) is correct option.

15. If an event cannot occur, then its probability is

(a) 1 (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) 0

Sol : [Board Term-2 OD 2012]

The event which cannot occur is said to be impossible event and probability of impossible event is zero.

Thus (d) is correct option.

16. Which of the following cannot be the probability of an event?

(a) $\frac{1}{3}$ (b) 0.1

(c) 3% (d) $\frac{17}{16}$

Sol : [Board Term-2 Foreign 2013]

Probability of an event always lies between 0 and 1.

Thus (d) is correct option.

17. If the probability of an event is p, then the probability of its complementary event will be

(a) $p - 1$ (b) p

(c) $1 - p$ (d) $1 - \frac{1}{p}$

Sol : [Board Term-2 OD 2017]

Since,

$P(E) + P(\overline{E}) = 1$

$P(E) = 1 - P(\overline{E})$

$= 1 - p$

Thus (c) is correct option.

18. The probability expressed as a percentage of a particular occurrence can never be

(a) less than 100

(b) less than 0

(c) greater than 1

(d) anything but a whole number

Sol : [Board Term-2 Foreign 2011]

Probability expressed as a percentage always lie between 0 and 100. So, it cannot be less than 0.

Thus (b) is correct option.

19. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$

(c) $\frac{4}{5}$ (d) $\frac{1}{3}$

Sol : [Board Term-2 OD 2016]

Multiples of 5 are 5, 10, 15, 20, 25, 30, 35 and 40 thus 8 outcome.

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$$n(S) = 20$$

7 yellow ball are neither a blue or nor a pink black balls are not red. Thus favourable outcome

$$n(E) = 7$$

P(Neither a blue or now a pink),

$$P(E) = \frac{n(E)}{n(S)} = \frac{7}{20}$$

Thus (c) is correct option.

6. An event is very unlikely to happen. Its probability is closest to

(a) 0.0001 (b) 0.001

(c) 0.01 (d) 0.1

Sol : [Board 2022 Term-1 SQP STD]

The probability of an event which is very unlikely to happen is closest to zero and from the given options 0.0001 is closest to zero.

Thus (a) is correct option.

7. The $P(A)$ denotes the probability of an event A , then

(a) $P(A) < 0$ (b) $P(A) > 1$

(c) $0 \le P(A) \le 1$ (d) $-1 \le P(A) \le 1$

Sol : [Board 2022 Term-1 SQP Basic]

Probability of an event always lies between 0 and 1.

Thus (c) is correct option.

8. If a card is selected from a deck of 52 cards, then the probability of its being a red face card is

(a) $\frac{3}{26}$ (b) $\frac{3}{13}$

(c) $\frac{2}{13}$ (d) $\frac{1}{2}$

Sol : [Board 2022 Term-1 Basic]

In a deck of 52 cards, there are 12 face cards i.e., 6 red and 6 black cards.

$$n(S) = 52$$
$$n(E) = 6$$

So, probability of getting a red face card,

$$P(E) = \frac{n(E)}{n(S)}$$
$$= \frac{6}{52} = \frac{3}{26}$$

Thus (a) is correct option.

9. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is

(a) 4 (b) 13

(c) 48 (d) 51

Sol : [Board 2022 Term-1 STD]

In a deck of 52 cards, there are 13 cards of heart and 1 is ace of heart.

Hence, the number of outcomes favourable

$$n(E) = 52 - 1 = 51$$

Thus (d) is correct option.

10. When a die is thrown, the probability of getting an odd number less than 3 is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) 0

Sol : [Board 2022 Term-1 STD]

Odd number less than 3 is 1 only.

$$n(S) = 6$$
$$n(E) = 1$$

So, probability of getting an odd number less than 3,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

Thus (a) is correct option.

11. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is

(a) 7 (b) 14

(c) 21 (d) 28

Sol : [Board 2022 Term-1 Basic]

We have

$$n(S) = 400$$
$$n(E) = x$$
$$P(E) = \frac{n(E)}{n(S)}$$
$$0.035 = \frac{x}{400}$$
$$x = 0.035 \times 400 = 14$$

Thus (b) is correct option.

12. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, then how many tickets has she bought?

(a) 40 (b) 240

(c) 480 (d) 750

Sol : [Board Term-2 Foreign 2014]

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$$n(E) = 1$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

Thus (b) is correct option.

26. If a letter is chosen at random from the letter of English alphabet, then the probability that it is a letter of the word DELHI is

(a) $\frac{1}{5}$ (b) $\frac{1}{26}$

(c) $\frac{5}{26}$ (d) $\frac{21}{26}$

Sol : [Board Term-2 Delhi 2017]

The English alphabet has 26 letters in all. The word DELHI has 5 letter, so the number of favourable outcomes is 5.

$$n(S) = 26$$

$$n(E) = 5$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{26}$$

Thus (c) is correct option.

27. The probability that a two digit number selected at random will be a multiple of 3 and not a multiple of 5 is

(a) $\frac{2}{15}$ (b) $\frac{4}{15}$

(c) $\frac{1}{15}$ (d) $\frac{4}{90}$

Sol : [Board Term-2 SQP 2011]

24 out of the 90 two digit numbers are divisible by 3 and not by 5.

$$n(S) = 90$$

$$n(E) = 24$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{24}{90} = \frac{4}{15}$$

Thus (b) is correct option.

28. If in a lottery, there are 5 prizes and 20 blanks, then the probability of getting a prize is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{5}$ (d) 1

Sol : [Board 2010]

We have $n(S) = 20 + 5 = 25$

$$n(E) = 5$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{25} = \frac{1}{5}$$

Thus (c) is correct option.

29. If a number x is chosen at random from the numbers $-2, -1, 0, 1, 2$. Then, the probability that $x^2 < 2$ is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{5}$ (d) $\frac{3}{5}$

Sol : [Board Term-2 OD 2015]

Total number of possible outcomes are 5.

We observe that $x^2 < 2$ when x takes anyone of the following three values -1, 0 and 1.

We have $n(S) = 5$

$$n(E) = 3$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{5}$$

Thus (d) is correct option.

30. Which of the following relationship is the correct?

(a) $P(E) + P(\overline{E}) = 1$

(b) $P(\overline{E}) - P(E) = 1$

(c) $P(E) = 1 + P(\overline{E})$

(d) None of these

Sol : [Board Term-2 Foreign 2016]

$$P(E) + P(\overline{E}) = 1$$

Thus (a) is correct option.

31. Two dice are thrown together. The probability that sum of the two numbers will be a multiple of 4, is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

Sol : [Board Term-2 Comp. 2014]

Total number of outcomes is 36.

Here, all possible outcome is (1, 3), (2, 2), (2, 6), (3, 1), (3, 5), (4, 4), (5, 3), (6, 2) and (6, 6),

$$n(S) = 36$$

$$n(E) = 9$$

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$$n(S) = 40$$
$$n(E) = 8$$
$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{40} = \frac{1}{5}$$

Thus (a) is correct option.

20. Someone is asked to take a number from 1 to 100. The probability that it is a prime, is

(a) $\frac{8}{25}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{13}{50}$

Sol : [Board Term-2 Delhi 2017]

Prime numbers between 1 to 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97, i.e 25 outcome.

$$n(S) = 100$$
$$n(E) = 25$$
$$P(E) = \frac{n(E)}{n(S)} = \frac{25}{100} = \frac{1}{4}$$

Thus (c) is correct option.

21. The probability of getting a number greater then 3 in throwing a die is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

Sol : [Board Term-2 OD 2017]

$$n(S) = 6$$
$$n(E) = 2$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

Thus (d) is correct option.

22. Out of one digit prime numbers, one number is selected at random. The probability of selecting an even number is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

Sol : [Board Term-2 SQP 2016]

One digit prime numbers are 2, 3, 5, 7. Out of these numbers, only the number 2 is even.

$$n(S) = 4$$
$$n(E) = 1$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$

Thus (b) is correct option.

23. A bag contains 3 red and 2 blue marbles. If a marble is drawn at random, then the probability of drawing a blue marble is :

(a) $\frac{2}{5}$ (b) $\frac{1}{4}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

Sol : [Board Term-2 Delhi 2017]

There are 5 marbles in the bag. Out of these 5 marbles one can be choose in 5 ways. Since, the bag contains 2 blue marbles. Therefore, one blue marble can be drawn in 2 ways.

$$n(S) = 5$$
$$n(E) = 2$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{5}$$

Thus (b) is correct option.

24. A single letter is selected at random from the word PROBABILITY. The probability that the selected letter is a vowel is

(a) $\frac{2}{11}$ (b) $\frac{3}{11}$

(c) $\frac{4}{11}$ (d) 0

Sol : [Board Term-2 OD 2016]

There are 11 letter in word PROBABILITY. Out of these 11 letter, 4 letter are vowels.

$$n(S) = 11$$
$$n(E) = 4$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{11}$$

Thus (c) is correct option.

25. A fair die is thrown once. The probability of getting a composite number less than 5 is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{2}{3}$ (d) 0

Sol : [Board Term-2 SQP 2011]

The outcomes are 1, 2, 3, 4, 5, 6. Out of these, 4 is the only composite number which is less than 5.

$$n(S) = 6$$

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36. **Assertion :**A card is drawn at random from a well shuffled deck of 52 playing cards. The probability of getting a face card is 1.1.

Reason : Probability of an event always lies between 0 and 1.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Sol :

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

ONE MARK QUESTIONS

37. What is the probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4?

Sol : [Board 2020 Delhi Basic]

Total possible outcome, $n(S) = 15$

Number of multiples of 4 between 1 to 15 are 4, 8, 12 i.e. 3 favourable outcome.

$$n(E) = 3$$

Required Probability, $P(E) = \frac{n(E)}{n(S)} = \frac{3}{15} = \frac{1}{5}$

PRACTICE

✎ One ticket is drawn at random from a bag containing tickets numbered 1 to 40. Find the probability that the selected ticket has a number which is a multiple of 5.

[Board 2010]

Ans : $\frac{1}{5}$

38. Two coins are tossed simultaneously. What is the probability of getting at most one head ?

Sol : [Board 2020 OD Basic]

All possible outcomes are {HH, HT, TH, TT}.

Thus $n(S) = 4$

Favourable outcomes are {HT, TH, TT}.

$$n(E) = 3$$

Probability of getting at most one head,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

39. A die is thrown once. What is the probability of getting a number less than 3?

Sol : [Board 2020 Delhi Standard]

There are 6 possible outcome for a die.

$$n(S) = 6$$

Favourable outcome are 1 and 2 i.e. two outcomes.

$$n(E) = 2$$

P(number less than 3)

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

40. A number is chosen at random from the numbers $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$. Then the probability that square of this number is less than or equal to 1 is ..... .

Sol : [Board 2020 SQP Standard]

Given numbers are $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ and their squares are 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, 25.

Total number of outcomes $n(S) = 11$.

Favourable outcome are $-1, 0, 1$, thus number of favourable outcomes is $n(E) = 3$.

Required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{3}{11}$

41. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting a red king.

Sol : [Board 2020 OD Basic]

Total no. of cards, $n(S) = 52$

Number of red kings, $n(E) = 2$

P(a red king), $P(E) = \frac{n(E)}{n(S)}$

$$= \frac{2}{52} = \frac{1}{26}$$

PRACTICE

✎ A card drawn at random from a well shuffled deck of 52 playing cards. What is the probability of getting a black king?

[Board 2020 OD Basic]

Ans :

42. If the probability of wining a game is 0.07, what is the probability of losing it?

Sol : [Board 2020 Delhi Standard]

P(winning the game), $P(E) = 0.07$

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$$M_d = \frac{320.5}{3} = 106.83$$

192. Apples are most widely planted and are commercially the most important fruit crop in Jammu and Kashmir. The cultivation of apple crop in Jammu and Kashmir shows particular interest for a number of reasons. In terms of both area and production, apple is very beneficial fruit crop. This provides a major source of income and employment in Jammu and Kashmir.

Horticultural department has tasked their statistical officer to create a model for farmers to be able to predict their produce output based on various factors.

A box containing 250 apples was opened and each apple was weighed. The distribution of the weight of the apples is given in the following table :

Weight (in grams)	No. of f_i Apples
80-100	20
100-120	60
120-140	70
140-160	40
160-180	60

(i) How many apples are in the range 140-160 grams?

(ii) What is the mean weight of the apples?

(iii) What is the upper limit of the median class?

(iv) What is the modal mass of the apples?

(v) What is the median mass of the apples?

Sol :

(i) $20 + 60 + 70 + x + 60 = 250$

$210 + x = 250$

$x = 250 - 210$

$= 40$

(ii) We prepare following cumulative frequency distribution table.

Weight	f_i	$c.f.$	x_i	$f_i x_i$
80-100	20	20	90	1800
100-120	60	80	110	6600
120-140	70	150	130	9100
140-160	40	190	150	6000
160-180	60	250	170	10200
	$\sum f_i = 250$			$\sum f_i x_i = 33700$

$$M = \frac{\sum f_i x_i}{x_i} = \frac{33700}{250} = 134.8 \text{ gm}$$

(iii) Cumulative frequency just greater than $\frac{N}{2} = \frac{250}{2} = 125$ is 150 and the corresponding class is 120-140. Thus median class is 120-140 and upper limit is 140.

(iv) Class 120-140 has the maximum frequency 70, therefore this is model class.

Here, $l = 120$, $f_1 = 70$, $f_0 = 60$, $f_2 = 60$ and $h = 20$

Mode, $M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$= 120 + \frac{70 - 60}{2 \times 70 - 60 - 40} \times 20$

$= 120 + \frac{20}{40} \times 10 = 125$

(v) Now $3M_d = M_o + 2M$

$3M_d = 125 + 2 \times 135 = 395$

$M_d = \frac{395}{3} = 131.67$ grams

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NODIA

49. Someone is asked to take a number from 1 to 100. What is the probability that it is a prime ?

Sol : [Board 2006]

Prime numbers between 1 to 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97, i.e 25 outcome.

$$n(S) = 100$$
$$n(E) = 25$$
$$P(E) = \frac{n(E)}{n(S)} = \frac{25}{100} = \frac{1}{4}$$

50. Out of one digit prime numbers, one number is selected at random. What is the probability of selecting an even number?

Sol : [Board Term-2 OD 2014]

One digit prime numbers are 2, 3, 5, 7. Out of these numbers, only the number 2 is even.

$$n(S) = 4$$
$$n(E) = 1$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$

51. A bag contains 3 red and 2 blue marbles. If a marble is drawn at random, then find the probability of drawing a blue marble.

Sol : [Board Term-2 Delhi 2013]

There are 5 marbles in the bag. Out of these 5 marbles one can be choose in 5 ways. Since, the bag contains 2 blue marbles. Therefore, one blue marble can be drawn in 2 ways.

$$n(S) = 5$$
$$n(E) = 2$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{5}$$

52. A single letter is selected at random from the word PROBABILITY. What is the probability that the selected letter is a vowel?

Sol : [Board 2007]

There are 11 letter in word PROBABILITY. Out of these 11 letter, 4 letter are vowels.

$$n(S) = 11$$
$$n(E) = 4$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{11}$$

53. A die is thrown once. Find the probability of getting "at most 2."

Sol : [Board Term-2 OD Compt 2017]

All possible outcome i.e. sample space,

$$S = \{1, 2, 3, 4, 5, 6\}$$

Number of all possible outcome,

$$n(S) = 6$$

Favourable outcomes,

$$E = \{1, 2\}$$

Number of favourable outcome,

$$n(E) = 2$$

Thus $$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

54. If E be an event such that $P(E) = \frac{3}{7}$, what is P (not E) equal to?

Sol : [Board Term-2 Delhi 2014]

We have $$P(E) = \frac{3}{7}$$
$$P(\text{not } E) = 1 - P(E)$$
$$= 1 - \frac{3}{7} = \frac{4}{7}$$

PRACTICE

If $P(E) = 0.20$, then what is the probability of 'not E'?

[Board Term-2 Delhi 2013]

Ans : 0.80

55. A bag contains lemon flavoured candies only. Shalini takes out one candy without looking into the bag. What is the probability that she takes out an orange flavoured candy?

Sol : [Board Term-2 2012]

Bag contains only lemon flavoured candies. So, getting an orange flavoured candy is an impossible.

$$P(E) = 0$$

56. Out of 200 bulbs in a box, 12 bulbs are defective. One bulb is taken out at random from the box. What is the probability that the drawn bulb is not defective?

Sol : [Board Term-2 SQP 2016]

Total number of bulbs,

$$n(S) = 200$$

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Number of favourable cases,

$$n(E) = 200 - 12 = 188$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{188}{200} = \frac{47}{50}$$

57. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability of getting neither a red card nor a queen.

Sol : [Board Term-2 OD 2016]

There are 26 red cards out of total 52 cards and 2 black queen also.

Total number of cards, $n(S) = 52$

Cards which are neither red nor queen,

$$n(E) = 52 - (26 + 2) = 24$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{24}{52} = \frac{6}{13}$$

58. A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant.

Sol : [Board Term-2 Delhi 2015, 2020 Delhi STD]

In the English language there are 26 alphabets. Consonant are 21. The probability of chosen a consonant

$$n(S) = 26$$
$$n(E) = 21$$
$$P(E) = \frac{n(E)}{n(S)} = \frac{21}{26}$$

59. Cards marked with number 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number.

Sol : [Board Term-2 2016]

Total number of outcomes,

$$n(S) = 48$$

Favourable outcomes are 4, 9, 16, 25, 36 and 49.

No. of favourable outcomes,

$$n(E) = 6$$

P(perfect square number),

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{48} \text{ or } = \frac{1}{8}$$

60. 20 tickets, on which numbers 1 to 20 are written, are mixed thoroughly and then a ticket is drawn at random out of them. Find the probability that the number on the drawn ticket is a multiple of 3 or 7.

Sol : [Board Term-2 Foreign 2016]

Total number of cases,

$$n(S) = 20$$

Favourable outcome,

$$E = \{3, 6, 7, 9, 12, 14, 15, 18\}$$

Number of favourable cases,

$$n(E) = 8$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

61. What is the probability that a non-leap year has 53 Mondays ?

Sol : [Board Term-2 2015]

There are 365 days in a non-leap year.

365 days = 52 weeks + 1 day

One day can be M, T, W, Th, F, S, S i.e. total 7 possible outcomes and only one favourable outcome.

Thus $n(S) = 7$ and $n(E) = 1$

P(53 Mondays in non-leap year)

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{7}$$

62. Two different dice are tossed together. Find the probability that the product of the number on the top of the dice is 6.

Sol : [Board Term-2 OD 2015]

Total number of possible outcomes,

$$n(S) = 6 \times 6 = 36$$

Product of 6 are (1, 6), (2, 3), (6, 1) and (3, 2).

Number of favourable outcomes,

$$n(E) = 4$$

Total number of chances

$$n(S) = 6 \times 6 = 36$$

P(Product of 6)

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

63. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes. Find the probability that the arrow will point at any factor of 8 ?

Sol : [Board Term-2 Foreign 2015]

NODIA

Total number of points are 8. Thus, total number of possible outcomes

$$n(S) = 8$$

Favourable outcomes are 1, 2, 4, and 8

No. of favourable outcomes,

$$n(E) = 3$$

P(factor of 8)

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{8} = \frac{1}{2}$$

64. A bag contains cards numbered from 1 to 25. A card is drawn at random from the bag. Find the probability that number is divisible by both 2 and 3.

Sol : [Board Term-2 Foreign 2014]

Since bag contains 25 cards,

$$n(S) = 25$$

The numbers divisible by 2 and 3 both are 6, 12, 18, 24 which are 4 numbers.

Thus $n(E) = 4$

P(number divisible by 2 and 3)

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{25}$$

65. A number is selected at random from 1 to 30. Find the probability that it is a prime number.

Sol : [Board Term-2 2014]

Number of possible outcomes,

$$n(S) = 30$$

Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Number of favourable outcomes, $n(E) = 10$

P(prime), $P(E) = \frac{n(E)}{n(S)} = \frac{10}{30} = \frac{1}{3}$

66. A box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from box, find the probability that it bears a prime number less than 23.

Sol : [Board Term-2 2012]

Number of possible outcomes,

$$n(S) = 90$$

Prime numbers less than 23 are 2, 3, 5, 7, 11, 13, 17, 19.

Number of favourable outcomes

$$n(E) = 8$$

P(prime no. less than 23)

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{90} = \frac{4}{45}$$

67. From the number 3, 5, 5, 7, 7, 7, 9, 9, 9, 9, one number is selected at random, what is the probability that the selected number is mean?

Sol : [Board Term-2 2012]

Total outcomes, $n(S) = 10$

Mean,

$$M = \frac{3+5+5+7+7+7+9+9+9+9}{10} = \frac{70}{10} = 7$$

Thus 7 is the mean of given numbers and frequency of 7 is 3 in given data.

Number of favourable outcomes,

$$n(E) = 3$$

P(mean), $P(E) = \frac{n(E)}{n(S)} = \frac{3}{10}$

68. A die is thrown once. Find the probability of getting a prime number.

Sol : [Board Term-2 2012]

Total outcomes, $n(S) = 6$

Prime numbers are 2, 3, 5.

$$n(E) = 3$$

P(prime no.), $P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

69. In tossing a die, what is the probability of getting an odd number or number less than 4 ?

Sol : [Board Term-2 2012]

Total outcome, $n(S) = 6$

Odd numbers are 1, 3, 5 and number less than 4 are 1, 2, 3. Thus there are 4 favourable outcome.

$$n(E) = 4$$

P(an odd no. or a no. <4),

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

70. Find the probability of getting a sum of 9, when two dice are thrown simultaneously.

Sol : [Board Term-2 OD 2011]

If two dice are thrown there are $6 \times 6 = 36$ possible outcomes. Thus there are 4 favourable outcome (3, 6), (6, 3), (4, 5) and (5, 4). In these case sum of both faces are 9.

Number of total outcomes,

$$n(S) = 36$$

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Possible outcome are $-3, -2, -1. 0, 1, 2, 3$ i.e 7 outcomes.

Thus $n(S) = 7$

Favourable outcomes are $x^2 < 4$ i.e. $= -1, 0, 1$.

$$n(E) = 3$$

$$P(x^2 < 4), \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{7}$$

PRACTICE

- If a number x is chosen a random from the number $-3, -2, -1, 0, 1, 2, 3$. What is probability that $x^2 \leq 4$?
  [Board 2020 Delhi Standard]
  Ans : $\frac{5}{7}$

- A number is chosen at random from the numbers $-3, -2, -1, 0, 1, 2, 3$. What will be the probability that square of this number is less than or equal to 1.
  [Board Term-2 Delhi 2017]
  Ans : $\frac{3}{7}$

84. A bag contains cards with numbers written on it from 1–80. A card is pulled out at random. Find the probability that the card shows a perfect square.

Sol : [Board Term-2 2016]

We have $S = \{1, 2 \ldots\ldots\ldots 80\}$

Number of possible outcomes,

$$n(S) = 80$$

Favourable outcome are $\{1, 4, 9, 16, 25, 36, 49, 64\}$

Number of favourable outcomes,

$$n(E) = 8$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{8}{80} = \frac{1}{10}$$

85. A bag contains 6 red and 5 blue balls. Find the probability that the ball drawn is not red.

Sol : [Board Term-2 2014, 2015]

No. of possible outcomes,

$$n(S) = 6 + 5 = 11$$

Since 5 blue balls are favourable outcome,

$$n(E) = 5$$

$$P(\text{not red}), \quad P(E) = \frac{n(E)}{n(S)} = \frac{5}{11}$$

86. There are 30 cards of the same size in a bag in which the numbers 1 to 30 are written. One card is taken out of the bag at random. Find the probability that the number on the selected card is not divisible by 3.

Sol : [Board Term-2 Foreign 2014]

Total cards $n(S) = 30$

Number divisible by 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27 and 30 i.e. 10 numbers.

Number of favourable outcomes,

$$n(E) = 30 - 10 = 20$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{20}{30} = \frac{2}{3}$$

87. A bag contains cards bearing numbers from 11 to 30. A card is taken out from the bag at random. Find the probability that the selected card has multiple of 5 on it.

Sol : [Board Term-2 Delhi 2014, 2012]

No. of cards $n(S) = 20$

Multiples of 5 from 11 to 30 are 15, 20, 25 and 30 i.e 4 numbers .

Thus number of favourable outcomes,

$$n(E) = 4$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{20} = \frac{1}{5}$$

88. Two different dice are tossed together. Find the probability :

(i) that the number on each die is even.

(ii) that the sum of numbers appearing on the two dice is 5.

Sol : [Board Term-2 OD 2014]

In both case, $n(S) = 36$

(i) Even numbers events are (2, 2) (2, 4) (2, 6) (4, 2) (4, 4) (4, 6) (6, 2), (6,4) and (6, 6) which are 9 event.

$$n(E_1) = 9$$

P(number of each die is even),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) Sum of numbers is 5 in (1, 4) (2, 3) (3, 2) (4, 1)

$$n(E_2) = 4$$

P(sum of numbers appearing on two dice is 5)

NODIA

balls. If the probability of drawing a blue ball is double that of red ball, determine the number of blue balls in the bag.

[Board Term-2 2013]

Ans : 10

✍ A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of blackballs in the bag.

[Board Term-2 2016]

Ans : 45

✍ A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of a red ball, find the number of blue balls in the bag.

[Board Term-2 2012]

Ans : 12

78. If a pair of dice is thrown once, then what is the probability of getting a sum of 8?

Sol : [Board 2020 Delhi Basic]

Number of possible outcomes,

$$n(S) = 6^2 = 36$$

The favourable outcomes are (sum of getting 8) $\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$ i.e. 5 outcomes.

Number of favourable outcome,

$$n(E) = 5$$

Probability (getting sum of 8),

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$

PRACTICE

✍ Two different dice are thrown together, find the probability that the sum of the numbers appeared is less than 5.

[Board 2020 OD Basic]

Ans : $\frac{1}{6}$

✍ Two dice are rolled simultaneously. Find the probability that the sum of numbers appearing is 10.

[Board Term-2 Foreign 2014]

Ans : $\frac{1}{12}$

79. In a lottery, there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Sol : [Board 2020 OD Basic]

Total number of possible outcomes,

$$n(S) = 10 + 25 = 35$$

Total number of prizes,

$$n(E) = 10$$

Probability of getting a prize,

$$P(E) = \frac{n(E)}{n(S)} = \frac{10}{35} = \frac{2}{7}$$

80. Two different coins are tossed simultaneously, What is the probability of getting at least one head?

Sol : [Board 2020 Delhi OD Basic]

All possible outcomes are {HH, HT, TH, TT}.

Thus $n(S) = 4$

Favourable outcomes are {HT, TH, HH}.

$$n(E) = 3$$

Probability of getting at least one head,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

81. A pair of dice is thrown once. What is the probability of getting a doublet?

Sol : [Board 2020 Delhi Standard]

There are total $6^2 = 36$ possible outcomes. Thus

$$n(S) = 36$$

Favourable outcomes are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6).

Number of favourable outcomes,

$$n(E) = 6$$

P(getting doublet),

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

82. A die is thrown once. What is the probability of getting a prime number.

Sol : [Board 2020 OD Standard]

There are 6 possible outcome for a die.

$$n(S) = 6$$

Favourable outcome are 1 and 2 i.e. two outcome.

$$n(E) = 2$$

P (number less than 3),

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

83. If a number x is chosen at random from the numbers $-3, -2, -1.$ 0, 1, 2, 3, then find the probability of $x^2 < 4$.

Sol : [Board 2020 OD Standard]

$$P(E_2) = \frac{n(E_2)}{n(S)}$$
$$= \frac{6}{36} = \frac{1}{6}$$

104. In a family of three children, find the probability of having at least two boys.

Sol : [Board 2020 OD Basic]

If there are three children in family all possible outcome are {BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}.

So, the total number of outcomes,

$$n(S) = 2^3 = 8$$

At-least two of them are boys means all those cases in which we have either 2 or 3 boys. Thus favourable outcome are {BBB, BBG, BGB, GBB}

Number of favourable outcome,

$$n(E) = 4$$

The probability of having at least two boys

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

105. A child has a die whose six faces show the letters as shown below:

$\boxed{A}$ $\boxed{A}$ $\boxed{B}$ $\boxed{C}$ $\boxed{C}$ $\boxed{C}$

The die is thrown once. What is the probability of getting (i) A, (ii) C?

Sol : [Board 2020 OD Standard]

Total possible outcomes, $n(S) = 6$

(i) Probability of getting letter A,

$$n(E_1) = 2.$$
$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

(ii) Probability of getting letter C,

$$n(E_2) = 3$$
$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{6}$$

PRACTICE

A child has a die whose six faces show the letters as shown below:

$\boxed{A}$ $\boxed{B}$ $\boxed{C}$ $\boxed{D}$ $\boxed{E}$ $\boxed{A}$

The die is thrown once. What is the probability of getting (i) A, (ii) D?

[Board 2020 OD Standard]

Ans : $\frac{1}{3}, \frac{1}{6}$

106. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

Sol : [Board 2019 Delhi]

Possible outcomes are {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}.

Total possible outcomes,

$$n(S) = 2^3 = 8$$

Number of outcomes where the game lost,

$$n(E) = 8 - 2 = 6$$

Probability of losing the game,

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

PRACTICE

A game consists of tossing a one-rupee coin 3 times and noting the outcome each time. Ramesh will win the game if all the tosses show the same result, (i.e either all three heads or all three tails) and loses the game otherwise. Find the probability that Ramesh will lose the game.

[Board Term-2 Foreign 2016, Delhi 2017]

Ans : $\frac{3}{4}$

107. A die is thrown once. Find the probability of getting a number which (i) is a prime number (ii) lies between 2 and 6.

Sol : [Board 2019 Delhi]

Total outcomes $n(S) = 6$

(i) is a prime number

Prime numbers are 2, 3 and 5.

$$n(E_1) = 3$$

P(prime no.),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) lies between 2 and 6

$$n(E_2) = 3$$

P(lies between 2 and 6),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

108. A die is thrown twice. Find the probability that
(i) 5 will come up at least once.
(ii) 5 will not come up either time.

Sol : [Board 2019 OD]

There are $6 \times 6 = 36$ possible outcome. Thus sample space for two die is

$$n(S) = 36$$

(i) 5 will come up at least once

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$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

89. A letter of English alphabet is chosen at random, find the probability that the letter so chosen is :

(i) a vowel,

(ii) a consonant.

Sol : [Board Term-2 Delhi 2014]

Since total number in English alphabet is 26, in which 5 vowels and 21 consonants.

In both case total possible outcome

$$n(S) = 26$$

(i) a vowel,

$$n(E_1) = 5$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{5}{26}$$

(ii) a consonant,

$$n(E_2) = 21$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{21}{26}$$

90. Harpreet tosses two different coins simultaneously. What is the probability that she gets :

(i) at least one head ?

(ii) one head and one tail ?

Sol : [Board Term-2 Foreign 2014]

All possible outcomes are {HH, TT, TH, HT}

$$n(S) = 4$$

(i) At least one head,

All favourable outcome are {HH, TH, HT}

$$n(E_1) = 3$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{4}$$

(ii) One head and one tail

All favourable outcome are {TH, HT}

$$n(E_2) = 2$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

91. A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag, find the probability of getting :

(i) not a white ball,

(ii) neither a green nor a red ball.

Sol : [Board Term-2 2012, 2014]

Bag contains 5 red, 8 green and 7 white balls i.e. total 20 ball.

Total number of possible outcomes,

$$n(S) = 20$$

(i) not a white ball,

There are 5 red and 8 green balls which are not white.

Thus number of favourable outcome,

$$n(E_1) = 13$$

P (not a white ball),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{13}{20}$$

(ii) neither a green nor a red ball.

There are 7 white balls which are neither a green nor a red ball.

$$n(E_2) = 7$$

P (neither a green nor a red ball)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{7}{20}$$

NODIA

92. A bag contains 3 red, 4 green and 5 white candles, one candle is drawn at random from the bag, find the probability that candle is not red.

Sol : [Board Term-2 2014]

Total number of possible outcomes are $3 + 4 + 5 = 12$.

$$n(S) = 12$$

When candles not red, there are 9 possibilities,

$$n(E) = 9$$

P(candle is not red),

$$P(E) = \frac{n(E)}{n(S)} = \frac{9}{12} = \frac{3}{4}$$

93. In a family of two children find the probability of having at least one girl.

Sol : [Board Term-2 2017]

All possible outcomes,

$$S = \{GG, GB, BG, BB\}$$

Total number of possible outcomes,

$$n(S) = 4$$

Favourable outcomes are GG, GB and BG.

Thus $n(E) = 3$

P(at least one girl),

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

94. Find the probability that a leap year has 53 Sundays

Sol : [Board Term-2 2014]

366 days = 52 weeks + 2 days

2 days can be MT, TW, WTh, ThF, FS, SS, SM out of which SS and SM are favourable outcome.

Total number of possible outcomes,

$$n(S) = 7$$

Thus number of favourable outcome,

$$n(E) = 2$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$$

95. Two coins are tossed together. Find the probability of getting both heads or both tails.

Sol : [Board Term-2 2012]

Possibilities are HH, HT TH, TT out of which HH and TT are favourable.

$$n(S) = 4$$

$$n(E) = 2$$

$$P(\text{HH or TT}),\ P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

96. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting :

(i) a non face card,

(ii) a black king.

Sol : [Board Term-2 OD 2012]

Total cards, $n(S) = 52$

(i) There are 12 face cards and thus 40 non-face cards.

$$n(E_1) = 40$$

$$P(\text{non-faces}),\quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{40}{52} = \frac{10}{13}$$

(ii) There are 2 black king

$$n(E_2) = 2$$

$$P(\text{black king}),\quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

97. A lot consists of 144 ball pens of which 20 are defective and others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that :

(i) she will buy it ?

(ii) she will not buy it ?

Sol : [Board Term-2 2012]

Total no. of pens, $n(S) = 144$

No. of good pen, $n(E) = 144 - 20 = 124$

Probability of purchasing pen,

$$P(E) = \frac{n(E)}{n(S)} = \frac{124}{144} = \frac{31}{36}$$

Probability of not purchasing pen,

$$P(\overline{E}) = 1 - P(E)$$

$$= 1 - \frac{5}{36} = \frac{31}{36}$$

98. Two different dice are thrown together. Find the probability that the product of the number appeared is less than 18.

Sol : [Board Term-2 Foreign 2017]

There are $6 \times 6 = 36$ possible outcomes.

$$n(S) = 36$$

Favourable outcomes are (4, 2), (4, 3), (4, 5), (5, 1), (5, 2), (5, 3), (5, 3), (6, 1), (6, 2), (1, 1), (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (2, 5) and (4, 1).

No. of favourable outcomes,

$$n(E) = 26$$

P(Product appears is less than 18)

$$P(E) = \frac{n(E)}{n(S)} = \frac{26}{36} = \frac{13}{18}$$

99. A box contains cards numbered 11 to 123. A card is drawn at random from the box. Find the probability that the number of the drawn card is

(i) A perfect square number

(ii) A multiple of 7.

Sol : [Board Term-2 SQP 2017]

Total number of all possible outcomes,

$$n(S) = 113$$

(i) Perfect square numbers between 11 to 123 are 16, 25, 36, 49, 64, 81, 100 and 121.

No. of all favourable outcomes

$$n(E_1) = 8$$

P(Number drawn is perfect square),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{8}{113}$$

(ii) No. of multiples of 7 from 11 to 123 is 16 i.e 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98,

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105, 112 and 119.

No. of all favourable outcomes.

$$n(E_2) = 16$$

P(number drawn card is multiple of 7)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{16}{113}$$

100. A box contains 12 balls of which some are red in colour. If 6 more red balls are put in the box and a ball is drawn at random the probability of drawing a red ball doubles than what it was before. Find the number of red balls in the bag.

Sol : [Board Term-2 SQP 2017]

Let x be red balls in the box out of 12 balls.

$$P(R) = \frac{x}{12}$$

After putting 6 red balls in the bag, total numbers of balls in box is $12+6=18$ and red ball are $x+6$.

$$P'(R) = \frac{x+6}{18}$$

According to the problem

$$2 \times \frac{x}{12} = \frac{x+6}{18}$$

$$18x = 6x+36 \Rightarrow x = 3$$

Hence there were 8 red ball.

THREE MARKS QUESTIONS

101. An integer is chosen between 70 and 100. Find the probability that it is
(i) a prime number
(ii) divisible by 7

Sol : [Board 2020 SQP Standard]

There are 29 integer from 70 to 100. Total number of outcomes are 29 in both case.

$$n(S) = 29$$

(i) There are 6 prime numbers between 70 and 100 as 71, 73, 79, 83, 89 and 97 i.e. 6 favourable outcome.

$$n(E_1) = 6$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{29}$$

(ii) There are 4 numbers between 70 and 100 which are divisible by 7 as 77, 84, 91 and 98 i.e. 4 favourable outcome.

$$n(B) = 4$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{29}$$

102. Find the probability that 5 Sundays occur in the month of November of a randomly selected year.

Sol : [Board 2020 Delhi Basic]

Total no. of days in November = 30

So, it has 4 weeks and 2 days. 4 weeks have 4 Sundays. The two remaining days should be
Sunday, Monday
Monday, Tuesday
Tuesday, Wednesday
Wednesday, Thursday
Thursday, Friday
Friday, Saturday
Saturday, Sunday

Thus number of possible outcomes,

$$n(S) = 7$$

Number of favourable outcome,

$$n(E) = 2$$

So, the probability of getting 5 Sunday in the month of November,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$$

103. Two dice are tossed simultaneously. Find the probability of getting
(i) an even number on both dice.
(ii) the sum of two numbers more than 9.

Sol : [Board 2020 OD Basic]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) an even number on both dice.

Favourable outcome are (2, 2), (2, 4), (2, 6), (4, 2) (4, 4), (4, 6), (6, 2), (6, 4) and (6, 6).

Number of favourable outcomes

$$n(E_1) = 9$$

P(an even number on both dice),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) sum of two numbers more than 9

Favourable outcome are (4, 6), (5, 5), (5, 6), (6, 4), (6, 5) and (6, 6).

Number of favourable outcomes

$$n(E_2) = 6$$

P (sum of two numbers more than 9),

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Number of favourable outcomes

$$n(E) = 4$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

71. Can 1.1 be probability of an event ?

Sol : [Board Term-2 2014]

No. Since the probability of an event cannot be more than 1.

72. A bag contains 3 red and 5 black balls. A ball is drawn at random from the bag. What is the probability that the drawn ball is not red ?

Sol : [Board Term-2 Delhi 2017]

There are total $3 + 5 = 8$ balls in bag. Thus total possible outcomes,

$$n(S) = 8$$

5 black balls are not red. Thus favourable outcome

$$n(E) = 5$$

P(drawn ball is not red),

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{8}$$

73. If three different coins are tossed together, then find the probability of getting two heads.

Sol : [Board Term-2 OD Compt. 2017]

All possible outcomes are {HHH, THH, HTH, HHT, TTT, TTH, THT, HTT}.

Number of possible outcomes,

$$n(S) = 8$$

Number of favourable outcomes,

$$n(E) = 3$$

P(getting two heads),

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

74. A number x is chosen from 25, 24, 23, $-2, -1$, 0, 1, 2, 3. Find the probability that $|x| < 3$.

Sol : [Board Term-2 2015]

Total possible outcomes,

$$n(S) = 9$$

Favourable outcome are $-2, -1$, 0, 1, and 2.

Favourable outcomes $n(E) = 5$

$$P(|x| < 3) = \frac{n(E)}{n(S)} = \frac{5}{9}$$

75. A die thrown once. What is the probability of getting an even prime number?

Sol : [Board 2020 Delhi Standard]

Total possible outcomes of die is 6.

$$n(S) = 6$$

Favourable outcomes is only 2 i.e. there is one possible outcome.

$$n(E) = 1$$

P (getting an even prime number),

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

TWO MARKS QUESTIONS

76. In a pack of 52 playing cards one card is lost. From the remaining cards, a card is drawn at random. Find the probability that the drawn card is queen of heart, if the lost card is a black card.

Sol : [Board 2024 OD Standard]

One card lost is a black card, thu we hav 51 card now. There is only one queen of heart.

Total number of outcomes,

$$n(S) = 51$$

Number of favorable cases,

$$n(E) = 1$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{51}$$

77. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball at random from the bag is three times that of a red ball, find the number of blue balls in the bag.

Sol : [Board 2020 Delhi Basic]

Let x be blue balls.

Total balls, $n(S) = 5 + x$

$$n(R) = 5 \text{ and } n(B) = x$$

P (red ball), $P(R) = \frac{n(R)}{n(S)} = \frac{5}{5+x}$

P (blue ball), $P(R) = \frac{n(B)}{n(S)} = \frac{x}{5+x}$

As per question we have

$$\frac{x}{5+x} = \frac{3 \times 5}{5+x}$$

Thus $x = 15$

Hence, bag contains 15 blue balls.

PRACTICE

A bag contains 5 red balls and some blue

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Favourable case are $(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)(1,5),(2,5),(3,5),(4,5)$ and $(6,\ 5)$thus 11 case.

Number of favourable outcome,

$$n(E_1) = 11$$

Probability that 5 will come up at least once,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{11}{36}$$

(ii) 5 will not come up either time

Probability that 5 will come up either time

$$P(\overline{E}) = 1 - P(E)$$
$$= 1 - \frac{11}{36} = \frac{36-11}{36} = \frac{25}{36}$$

109. Two different dice are tossed together. Find the probability:

(i) of getting a doublet

(ii) of getting a sum 10, of the numbers on the two dice.

Sol : [Board 2018]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) of getting a doublet

Doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6) which are 6 doublets.

Number of favourable outcomes,

$$n(E_1) = 6$$

P(doublet), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii) of getting a sum 10, of the numbers on the two dice.

Favourable outcomes are (4, 6), (5, 5), (6, 4) i.e., 3.

Number of favourable outcomes,

$$n(E_2) = 3$$

P(sum 10), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

110. An integer is chosen at random between 1 and 100. Find the probability that it is:

(i) divisible by 8.

(ii) not divisible by 8.

Sol : [Board 2018]

Total number of outcomes,

$$n(S) = 100 - 2 = 98$$

(i) divisible by 8.

Favourable outcomes are 8, 16, 24, ..., 96, i.e., 12.

Number of favourable outcomes,

$$n(E_1) = 12$$

P(Divisible by 8),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{12}{98} = \frac{6}{49}$$

(ii) not divisible by 8.

P(not divisible by 8),

$$P(\overline{E}) = 1 - P(E)$$
$$= 1 - \frac{6}{49} = \frac{43}{49}$$

111. From a pack of 52 playing cards, Jacks, Queens and Kings of red colour are removed. From the remaining, a card is drawn at random. Find the probability that drawn card is

(i) a black king,

(ii) a card of red colour,

(iii) a card of black colour.

Sol : [Board Term-2 OD 2015]

There are total 52 cards out of which 6 cards are removed.

Total number of all possible outcomes,

$$n(S) = 52 - 6 = 46$$

Number of black king,

$$n(E_1) = 2$$

(i) a black king,

Probability of drawing black king

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{46} = \frac{1}{23}$$

(ii) a card of red colour,

Total red card,

$$n(E_2) = 26 - 6 = 20$$

Probability of drawing red colour card

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{20}{46} = \frac{10}{23}$$

(iii) a card of black colour.

Total card of black colour,

$$n(E_3) = 26$$

Probability of drawing black colour card

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{26}{46} = \frac{13}{23}$$

112. A bag contains cards numbered 1 to 49. Find the probability that the number on the drawn card is :

(i) an odd number

(ii) a multiple of 5

(iii) Even prime

Sol : [Board Term-2 2014]

NODIA

Prime numbers greater than 80 and less than 100 are 83, 89 and 97 i.e. 3 numbers.

$$n(E_2) = 3$$

Required probability,

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{100}$$

119. Cards numbered 2 to 101 are placed in a box. A card is selected at random from the box, find the probability that the card selected :

(i) has a number which is a perfect square.

(ii) has an odd number which is not less than 70.

Sol : [Board Term-2 2012]

Since box consists of 100 cards, there are 100 possible outcomes.

$$n(S) = 100$$

(i) has a number which is a perfect square.

Perfect squares are 4, 9, 16, 25, 36, 49, 64, 81 and 100.

Number of favourable outcomes,

$$n(E_1) = 9$$

P(Perfect square),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{100}$$

(ii) has an odd number which is not less than 70.

Favourable outcomes are 71, 73, 75, ...101.

Number of favourable outcomes,

$$n(E_2) = 16$$

P(odd number not less than 70),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{16}{100} = \frac{4}{25}$$

120. All red face cards are removed from a pack of playing cards. The remaining cards are well shuffled and then a card is drawn at random from them. Find the probability that the drawn card is :

(i) a red card

(ii) a face card

(iii) a card of clubs

Sol : [Board Term-2 Delhi 2015]

Since red face cards are removed, number of all possible outcomes are $52 - 6 = 46$

$$n(S) = 46$$

(i) a red card

No. of remaining red cards,

$$n(E_1) = 26 - 6 = 20$$

P(red card), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{20}{46} = \frac{10}{26}$

(ii) a face card

Number of remaining face cards,

$$n(E_2) = 12 - 6 = 6$$

P(a face card),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{46} = \frac{3}{23}$$

(iii) a card of clubs

Number of cards clubs

$$n(E_3) = 13$$

P(a card of clubs),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{13}{46}$$

121. The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange balls, find the total number of ball in the jar.

Sol : [Board Term-2 OD 2015]

Probability of red ball, $P(R) = \frac{1}{4}$

Probability of blue ball, $n(B) = \frac{1}{3}$

Probability of orange,

$$P(O) = 1 - [P(R) + P(B)] = 1 - \left(\frac{1}{4} + \frac{1}{3}\right) = \frac{5}{12}$$

Now $P(O) = \frac{n(O)}{n(S)}$ $\frac{5}{12} = \frac{10}{n(S)}$

Total numbers of balls,

$$n(S) = \frac{10 \times 12}{5} = 24$$

PRACTICE

A bag contains, white, black and red balls only. A ball is drawn at random from the bag. If the probability of getting a white ball is $\frac{3}{10}$ and that of a black ball is $\frac{2}{5}$, then find the probability of getting a red ball. If the bag contains 20 black balls, then find the total number of balls in the bag.

[Board Term-2 OD 2015]

Ans : 50

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(5, 6) and (6, 5) i.e. 6 outcomes.

$$n(E_1) = 6$$

P(a total of 9 or 11)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

116. A box consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Ramesh, a shopkeeper will buy only those shirts which are good but 'Kewal' another shopkeeper will not buy shirts with major defects. A shirt is taken out of the box at random. What is the probability that :

(i) Ramesh will buy the selected shirt ?

(ii) Kewal will buy the selected shirt ?

Sol : [Board Term-2 Delhi 2016]

Since box consists of 100 shirts, there are 100 possible outcomes.

$$n(S) = 100$$

(i) Ramesh will buy the selected shirt ?

Number of good shirts

$$n(E_1) = 88$$

P(Ramesh buys the shirt).

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{88}{100} = \frac{22}{25}$$

(ii) Kewal will buy the selected shirt ?

Number of shirts without major defect,

$$n(E_2) = 88 + 8 = 96$$

P(Kewal buys a shirt)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{96}{100} = \frac{24}{25}$$

PRACTICE

A box contains 125 shirts of which 110 are good 12 have minor defects and 3 have major defects. Ram Lal will buy only those shirts which are good while Naveen will reject only those which have major defects. A shirt is taken out at random from the box. Find the probability that :

(i) Ram Lal will buy it

(ii) Naveen will buy it

[Board Term-2 OD 2017]

Ans : $\frac{22}{25}$, $\frac{122}{125}$

117. A box contains 100 cards marked from 1 to 100. If one card is drawn at random from the box, find the probability that it bears :

(i) a single digit number

(ii) a number which is a perfect square

(iii) a number which is divisible by 7

Sol : [Board Term-2 2016]

Since box consists of 100 cards, there are 100 possible outcomes.

$$n(S) = 100$$

(i) a single digit number

Number of favourable outcomes,

$$n(E_1) = 9$$

P(single digit number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{100}$$

(ii) a number which is a perfect square

Perfect square number are 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.

No. of favourable outcomes,

$$n(E_2) = 10$$

P(perfect square),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{100} = \frac{1}{10}$$

(iii) a number which is divisible by 7

Number divisible by 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91 and 98 i.e. 14 numbers.

No. of favourable outcomes,

$$n(E_3) = 14$$

P(a number divisible by 7),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{14}{100}$$

118. There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card.

(i) is divisible by 9 and is a perfect square.

(ii) is a prime number greater than 80.

Sol : [Board Term-2 OD 2016]

Since bag consists of 100 cards, there are 100 possible outcomes.

$$n(S) = 100$$

(i) is divisible by 9 and is a perfect square.

Number divisible by 9 and perfect square are 9, 36 and 81 i.e. 3 numbers.

$$n(E_1) = 3$$

Required probability,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{100}$$

(ii) is a prime number greater than 80.

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(iii) A multiple of 6.

Sol : [Board Term-2 SQP 2016]

We have 48 cards and thus there are 48 possible outcomes.

$$n(S) = 48$$

(i) Divisible by 7.

Number of cards divisible by 7 are 7, 14, 21, 28, 35, 35, 42 and 49.

No. of favourable outcomes,

$$n(E_1) = 7$$

P(cards divisible by 7),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{7}{48}$$

(ii) A perfect square.

Number of cards having a perfect square are 4, 9, 16, 25, 36 and 49.

No. of favourable outcomes,

$$n(E_2) = 6$$

P(cards having a perfect square),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{48} = \frac{1}{8}$$

(iii) A multiple of 6.

Number of multiples of 6 from 3 to 50 are 6, 12, 24, 30, 36, 42, and 48.

No. of favourable outcomes,

$$n(E_3) = 6$$

P(multiple of 6 from 3 to 50),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{8}{48} = \frac{1}{6}$$

141. All the red face cards are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the drawn card is

(i) of red colour

(ii) a queen

(iii) an ace

(iv) a face card.

Sol : [Board Term-2 OD 2015]

There are $52 - 6 = 46$ cards after removing black face cards. We have 46 cards and thus there are 48 possible outcomes.

$$n(S) = 46$$

(i) red colour

Number of red cards, $n(E_1) = 26 - 6 = 20$

P(red colour), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{20}{46} = \frac{10}{23}$

(ii) a queen

No. of queen, $n(E_2) = 4 - 2 = 2$

P(a queen), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{2}{46} = \frac{1}{23}$

(iii) an ace

No. of ace, $n(E_3) = 4$

P(an ace), $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{4}{46} = \frac{2}{23}$

(iv) a face card

Number of face cards, $n(E_4) = 12 - 6 = 6$

P(a face card) $P(E_4) = \frac{n(E_4)}{n(S)} = \frac{6}{46} = \frac{3}{23}$

NODIA

PRACTICE

All the black face cards are removed from a pack of 52 cards. Find the probability of getting a

(i) face card

(ii) red card

(iii) black card

(iv) king

[Board Term-2 2014]

Ans : $\frac{3}{23}, \frac{13}{23}, \frac{10}{23}, \frac{1}{23}$

142. A box contains 20 cards from 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is

(i) divisible by 2 or 3

(ii) a prime number

Sol : [Board Term-2 2015]

We have 20 cards and thus there are 20 possible outcomes.

$$n(S) = 20$$

(i) divisible by 2 or 3

Number divisible by 2 or 3 are 6, 12, 18.

Number of favourable outcomes,

$$n(E_1) = 3$$

P(divisible by 2 or 3),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{20}$$

(ii) a prime number

Prime numbers are 2, 3, 5, 7, 11, 13, 17 and 19 i.e. 8 numbers.

Number of favourable outcomes,

$$n(E_2) = 8$$

P(a prime no.),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

122. Two different dice are thrown together. Find the probability of :

(i) getting a number greater than 3 on each die.

(ii) getting a total of 6 or 7 of the numbers on two dice.

Sol : [Board Term-2 Delhi 2016]

When two dice are thrown there are $6 \times 6 = 36$ possible outcomes.

$$n(S) = 36$$

(i) getting a number greater than 3 on each die.

Favourable outcomes are (4, 5), (4, 4), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5) and (6, 6).

No. of favourable outcomes,

$$n(E_1) = 9$$

P(a number > 3 on each die)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) getting a total of 6 or 7 of the numbers on two dice.

Favourable outcomes are (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1).

No. of favourable outcomes $n(B) = 11$

P(a total of 6 to 7),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{11}{36}$$

123. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting

(i) Non face card,

(ii) Black king or a Red queen,

(iii) Spade card.

Sol : [Board Term-2 SQP 2016]

Total cards $n(S) = 52$

(i) Non face card

Total number of non-face card,

$$n(E_1) = 52 - 12 = 40$$

P(non-face cards),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{40}{52} = \frac{10}{13}$$

(ii) Black king or a red queen,

Number of black kings $= 2$

Number of red queens $= 2$

Thus there are 4 favourable outcome.

$$n(E_2) = 4$$

P(a black Kind or a red queen),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(iii) Spade Card

Number of spade cards,

$$n(E_3) = 13$$

P(Spade cards),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

124. Three coins are tossed simultaneously once. Find the probability of getting :

(i) at least one tail,

(ii) no tail.

Sol : [Board Term-2 2012]

Sample space for three coins tossed is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$$n(S) = 2^3 = 8$$

(i) at least one tail,

Number of favourable outcomes,

$$n(E_1) = 7$$

P(at least one tail),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{7}{8}$$

(ii) no tail,

Number of favourable outcomes,

$$n(E_2) = 1$$

P(no tail), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{8}$

125. A game consists of tossing a one-rupee coin three times and noting its outcome each time. Find the probability of getting :

(i) three heads,

(ii) at least two tails.

Sol : [Board Term-2 Foreign 2015]

Sample space for three coins tossed is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

$$n(S) = 2^3 = 8$$

(i) three heads,

Favourable outcome is {HHH} i.e. only one outcome.

Thus $n(E_1) = 1$

P(three heads), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{8}$

(ii) at least two tails.

Favourable outcome are {TTT, TTH, THT, HTT}.

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$$n(E) = 13$$

P(getting a number multiple of 7),

$$P(E) = \frac{n(E)}{n(S)} = \frac{13}{90}$$

134. A box contains cards, number 1 to 90. A card is drawn at random from the box. Find the probability that the selected card bears a :

(i) Two digit number.

(ii) Perfect square number

Sol : [Board Term-2 Delhi Compt. 2017]

We have 90 cards and thus there are 90 possible outcomes.

$$n(S) = 90$$

(i) No. of cards having 2 digit number $90 - 9 = 81$

Number of favourable outcomes,

$$n(E_1) = 81$$

P(selected card bears two digit number)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{81}{90} = \frac{9}{10}$$

(ii) Perfect square number between 1 to 90 are 1, 4, 9, 16, 25, 36, 49, 64 and 81 i.e. 9 numbers.

No. of favourable outcomes,

$$n(E_2) = 9$$

P(perfect square numbers)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{9}{90} = \frac{1}{10}$$

135. Two different dice are thrown together. Find the probability that the number obtained :

(i) have a sum less than 7.

(ii) have a product less than 16.

(iii) is a doublet of odd numbers.

Sol : [Board Term-2 Delhi 2017]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) have a sum less than 7.

Favourable outcome are (1, 1), (1, 2), (1, 3), (1, 4), (1,5) (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2) and (5, 1).

Number of favourable outcomes

$$n(E_1) = 15$$

P(have sum less than 7),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

(ii) have a product less than 16.

Favourable outcome are (1, 2), (1, 3), (1, 4), (1, 5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1) and (6, 2).

No. of favourable outcomes,

$$n(E_2) = 24$$

P(have a product less than 16),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{24}{36} = \frac{2}{3}$$

(iii) is a doublet of odd numbers.

Favourable outcome are (1, 1), (3, 3) and (5, 5).

No. of favourable outcomes,

$$n(E_3) = 3$$

P(a doublet of odd number),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

FIVE MARKS QUESTIONS

136. What is the probability that a randomly taken leap year has 52 Sundays?

Sol : [Board 2020 OD Standard]

Number of days in a leap year = 366

$$\text{Number of weeks} = \frac{366}{7} = 52.28$$

So, there will be 52 weeks and 2 days

So, every leap year has 52 Sundays

Now, the probability depends on remaining 2 days

The Possible pairing of days are

Sunday – Monday
Monday – Tuesday
Tuesday – Wednesday
Wednesday – Thursday
Thursday – Friday
Friday – Saturday
Saturday – Sunday

There are total 7 pairs and out of 7 pairs, only 2 pairs have Sunday. The remaining 5 pairs does not include Sunday.

$$n(S) = 7$$

$$n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{7}$$

Therefore, the probability of only 52 Sunday in a Leap year is $\frac{5}{7}$.

NODIA

P(red balls), $P'(R) = \frac{n'(R)}{n'(S)} = \frac{x+2}{20}$

Now, according to the question,

$$\frac{x+2}{20} = \frac{9}{8} \times \frac{x}{18}$$
$$180x = 144x + 288$$
$$36x = 288$$
$$x = \frac{288}{36} = 8$$

Now substituting $x = 8$ we have

$$P(\overline{R}) = \frac{18-8}{18} = \frac{10}{18} = \frac{5}{9}$$

130. Cards numbered 1 to 30 are put in a bag. A card is drawn at random. Find the probability that the drawn card is

(i) prime number > 7

(ii) not a perfect square

Sol : [Board Term-2 2014]

We have 30 cards and thus there are 30 possible outcomes.

$$n(S) = 30$$

(i) prime number > 7

Favourable outcomes are 11, 13, 17, 19, 23, 29. Thus number of favourable outcomes,

$$n(E_1) = 6$$

P(prime no. > 7) $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{30} = \frac{1}{5}$

(ii) not a perfect square

Favourable outcomes are 1, 4, 9, 16, 25. Thus number of favourable outcomes,

P(not a perfect square),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{25}{30} = \frac{5}{6}$$

131. Two dice are thrown at the same time. Find the probability of getting :

(i) same number on both dice

(ii) sum of two numbers appearing on both the dice is 8.

Sol : [Board Term-2 2012]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) same number on both dice

Favourable outcome are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6).

Thus number of favourable outcome

$$n(E_1) = 6$$

P(Same number on both dice)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) sum of two numbers appearing on both the dice is 8.

Favourable outcome are (2, 6), (3, 5), (4, 4), (6, 2) and (5, 3). Thus number of favourable outcomes,

$$n(E_2) = 5$$

P(Sum is 8), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{5}{36}$

132. Five cards, ten, Jack, Queen, King and Ace of diamonds are well shuffled. One card is picked up from them.

(i) Find the probability that the drawn card is Queen.

(ii) If Queen is put aside, then find the probability that the second card drawn is an ace.

Sol : [Board Term-2 2014]

We have 5 cards and thus there are 5 possible outcomes.

$$n(S) = 5$$

(i) Drawn card is queen

No. of favourable outcomes,

$$n(E_1) = 1$$

P(queen), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{5}$

(ii) second card drawn is an ace

Since, queen was kept, number of all possible outcomes

$$n(S) = 5 - 1 = 4$$

Number of favourable outcomes

$$n(E_2) = 1$$

P(second card drawn is an ace),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{4}$$

133. From all the two digit numbers a number is chosen at random. Find the probability that the chosen number is a multiple of 7.

Sol : [Board Term-2 OD Compt. 2017]

All possible outcomes are 10, 11, 12 ....., 98 and 99.

No. of all possible outcomes

$$n(S) = 90$$

All favourable outcomes are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91 and 98 i.e 13 outcome.

No. of favourable outcomes

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Total cards, $n(S) = 49$

(i) an odd number

Odd number are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47 and 49.

Total odd number, $n(E_1) = 25$

P(odd number), $P(E_1) = \dfrac{n(E_1)}{n(S)}$

$= \dfrac{n(O)}{n(S)} = \dfrac{25}{49}$

(ii) a multiple of 5

Multiple of 5 are 5, 10, 15, 20, 25, 30, 35, 40 and 45.

Total multiple of 5 number,

$n(E_2) = 5$

P(multiple of 5), $P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{9}{49}$

(iii) Even prime

Only 2 is even prime number. Therefore

$n(E_3) = 1$

P(even prime), $P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{1}{49}$

113. Two unbiased coins are tossed simultaneously. Find the probability of getting :

(i) at least one head,

(ii) almost one head,

(iii) no head.

Sol : [Board Term-2 2012, 2014]

There are 4 possible outcome when two unbiased coins are tossed simultaneously.

Sample space $S = \{HH, HT, TH, TT\}$

$n(S) = 4$

(i) at least one head,

Favourable outcomes are {HT, TH, HH}.

$n(E_1) = 3$

$P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{3}{4}$

(ii) almost one head,

Favourable outcomes are {HT, TH, HH}.

$n(E_2) = 3$

$P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{3}{4}$

(iii) no head.

Favourable outcomes is {TT} only.

$n(E_3) = 1$

$P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{1}{4}$

114. Three different coins are tossed together. Find the probability of getting

(i) exactly two heads.

(ii) at least two heads

(iii) at least two tails.

Sol : [Board Term-2 OD 2016]

Sample space for three coins tossed is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$n(S) = 8$

(i) Exactly two heads

Sample space $E_1 = \{HHT, HTH, THH\}$

$n(E_1) = 3$

$P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{3}{8}$

(ii) At least two heads.

Sample space $E_2 = \{HHT, HTH, THH, HHH\}$

$n(E_2) = 4$

$P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$

(iii) At least two tails,

Sample space $E_3 = \{TTH, THT, HTT, TTT\}$

$n(E_3) = 4$

$P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$

115. In a single throw of a pair of different dice, what is the probability of getting

(i) a prime number on each dice ?

(ii) a total of 9 or 11 ?

Sol : [Board Term-2 Delhi 2016]

When two dice are thrown there are $6 \times 6 = 36$ possible outcomes.

$n(S) = 36$

(i) a prime number on each dice ?

Favourable outcomes are (2, 2) (2, 3) (2, 5) (3, 2) (3, 3) (3, 5) (5, 2) (5, 3) and (5, 5) i.e. 9 outcomes.

$n(E_1) = 9$

P(a prime number on each die)

$P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{9}{36} = \dfrac{1}{4}$

(ii) a total of 9 or 11 ?

Favourable outcomes are (3, 6) (4, 5) (5, 4) (6, 3)

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137. Jayanti throws a pair of dice and records the product of the numbers appearing on the dice. Pihu throws 1 dice and records the squares the number that appears on it. Who has the better chance of getting the number 36? Justify?

Sol : [Board 2020 SQP Standard]

Jayanti throws two dice together. There are $6^2 = 36$ total number of possible outcomes.

$$n(S) = 36$$

She get 36 only when she gets (6, 6),

No. of favourable outcomes,

$$n(E_1) = 1$$

P(getting the numbers of product 25)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{36}$$

Pihu throws one dice. There are 6 total number of all possible outcomes.

$$n(S) = 6$$

The number where square is 36 is 6.

No. of favourable outcomes,

$$n(E_2) = 1$$

P(getting a number whose square is 36)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{6}$$

$$P(E_2) > P(E_1)$$

Hence, Pihu has better chances to the number square 36.

138. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the numbers 25.

Sol : [Board Term-2 Delhi 2017]

Peter throws two dice together. There are $6^2 = 36$ total number of possible outcomes.

$$n(S) = 36$$

He get 25 only when he gets (5, 5),

No. of favourable outcomes,

$$n(E_1) = 1$$

P(getting the numbers of product 25),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{n(A)}{n(S)} = \frac{1}{36}$$

Rina throws one dice. There are 6 total number of all possible outcomes.

$$n(S) = 6$$

The number where square is 25 is 5.

No. of favourable outcomes,

$$n(E_2) = 1$$

P(getting a number whose square is 25)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{6}$$

$$P(E_2) > P(E_1)$$

Hence, Rina has better chances to the number square 25.

139. The probability of selecting a blue marble at random from a jar that contains only blue, black and green marbles is $\frac{1}{5}$. The probability of selecting a black marble at random from the same jar is $\frac{1}{4}$. If the jar contains 11 green marbles, find the total number of marbles in the jar.

Sol : [Board 2019 OD]

Let x and y be the number of blue and black marbles. No of green marbles $= 11$

Total number of marbles $= x + y + 11$

According to the problem,

$$P(\text{black marbles}) = \frac{1}{4}$$

$$\frac{y}{x+y+11} = \frac{1}{4}$$

$$x = 3y - 11 \quad \ldots(1)$$

Again, $P(\text{blue marble}) = \frac{1}{5}$

$$\frac{x}{x+y+11} = \frac{1}{5}$$

$$5x = x + y + 11$$

$$x = \frac{y+11}{4} \quad \ldots(2)$$

From equation (1) and (2), we have

$$3y - 11 = \frac{y+11}{4}$$

$$12y - 44 = y + 11$$

$$12y - y = 11 + 44$$

$$11y = 55 \Rightarrow y = 5$$

From equation (1) we have

$$x = 3 \times 5 - 11 = 4$$

Hence, total number of marbles in the jar,

$$x + y + 11 = 4 + 5 + 11 = 20$$

140. Cards marked with numbers 3, 4, 5, ...50 are placed in a bag and mixed thoroughly. One card is drawn at random from the bag. Find the probability that number on the card drawn is :

(i) Divisible by 7.

(ii) A perfect square.

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Number of favourable outcomes,

$$n(E_2) = 4$$

P (at least two tails), $P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$

126. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting :

(i) a red face card,

(ii) a spade,

(iii) either a king or a black cards.

Sol : [Board Term-2 2012, 2015]

Total cards, $n(S) = 52$

(i) Red face card

Total number of red-face card,

$$n(E_1) = 6$$

P(red face cards)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

(ii) Spade card

Number of spade cards

$$n(E_2) = 13$$

P(Spade cards),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(iii) Black king or a red queen,

Number of kings $= 4$

Number of black cards $= 26 - 2 = 24$

Thus there are 4 favourable outcome.

$$n(E_3) = 24 + 4 = 28$$

P(a black Kind or a red queen)

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{28}{52} = \frac{7}{13}$$

127. Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3 respectively. They are thrown and the sum of the numbers on them is noted. What is the probability of getting even sum :

Sol :

Total number of outcomes $= 6 \times 6 = 36$

Possible sum of two numbers on the two dice are 2, 3, 4, 5, 6, 7, 8, 9. i.e. outcomes favourable to event are (1, 1), (1, 1), (2, 2), (3, 1), (3, 1), (1, 3), (1, 3), (3, 3), (4, 2), (4, 2), (5, 1), (5, 1), (5, 3), (5, 3), (6, 2), (6, 2)

Hence, number of outcomes favourable to E is 18.

$$n(S) = 36$$

$$n(E) = 18$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

128. Three unbiased coins are tossed together. Find the probability of getting :

(i) at least two heads,

(ii) almost two heads.

Sol : [Board Term-2 2015]

Sample space for three coins tossed is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

$$n(S) = 8$$

(i) Sample space for at least 2 heads is {HHH, HHT, HTH, THH}

Number of favourable outcomes,

$$n(E_1) = 4$$

P(at least two heads).

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(ii) Sample space for almost two heads is {HHT, HTH, TTT, THH, THT, TTH, HTT}

Number of favourable outcomes,

$$n(E_2) = 7$$

P(almost 2 heads), $P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{7}{8}$

129. A bag contains 18 balls out of which x balls are red.

(i) If one ball is drawn at random from the bag, what is the probability that it is not red ?

(ii) If 2 more red balls are put in the bag, the probability of drawing a red ball will be $\frac{9}{8}$ times the probability of drawing a red ball in the first case. Find the value of x.

Sol : [Board Term-2 Foreign 2015]

Total ball, $n(S) = 18$

Red ball $n(R) = x$

(i) not red

P(red ball), $P(R) = \dfrac{n(R)}{n(S)} = \dfrac{x}{18}$

P(no red ball),

$$P(\overline{R}) = 1 - \frac{x}{18} = \frac{18 - x}{18}$$

(ii) Now two more red balls are added.

Now total ball $n'(S) = 18 + 2 = 20$

There are total $x + 2$ red ball.

$$n'(R) = x + 2$$

143. A box contains cards bearing numbers from 6 to 70. If one card is drawn at random from the box, find the probability that it bears,

(i) a one digit number.

(ii) a number divisible by 5.

(iii) an odd number less than 30.

(iv) a composite number between 50 and 70.

Sol : [Board Term-2 Foreign 2015]

We have $70 - 5 = 65$ cards and thus there are 65 possible outcomes.

$$n(S) = 65$$

(i) a one digit number.

One digit numbers are 6, 7, 8 and 9.

Number of favourable outcomes

$$n(E_1) = 4$$

P(one digit number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{65}$$

(ii) a number divisible by 5.

Number divisible by 5 are 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65 and 70 i.e. 13 numbers.

Number of favourable outcomes,

$$n(E_2) = 13$$

P(a number divisible by 5),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{13}{65} = \frac{1}{5}$$

(iii) an odd number less than 30.

Odd number less than 30 are 7, 9, 11, 13, 15, 17, 19 23, 25, 27 and 29.

Number of favourable outcomes,

$$n(E_3) = 12$$

P(a odd number less than 30),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{12}{65}$$

(iv) a composite number between 50 and 70

Composite number between 50 and 70 are 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68 and 69.

Number of favourable outcomes,

$$n(E_4) = 15$$

P(a composite number between 50 and 70)

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{15}{65} = \frac{3}{13}$$

144. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is :

(i) a card of spade or an ace.

(ii) a black king.

(iii) neither a jack nor a king.

(iv) either a king or a queen.

Sol : [Board Term-2 OD 2015]

We have 52 cards and thus there are 52 possible outcomes.

$$n(S) = 52$$

(i) a card of spade or an ace

Cards of spade or an ace,

$$n(E_1) = 13 + 3 = 16$$

P(spade or an ace),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{16}{52} = \frac{4}{13}$$

(ii) a black king

Number of black kings,

$$n(E_2) = 2$$

P(a black king),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(iii) neither a jack nor a king

There are $4 + 4 = 8$ Jack or king.

Number of neither jack nor a king,

$$n(E_3) = 52 - 8 = 44$$

P(neither jack nor a king),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{44}{52} = \frac{11}{13}$$

(iv) either a king or a queen

There are $4 + 4 = 8$ king or queen.

$$n(E_4) = 8$$

P(either a king or a queen),

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{8}{52} = \frac{2}{13}$$

145. A bag contains 15 balls of which x are blue and the remaining are red. If the number of red balls are increased by 5, the probability of drawing the red balls doubles. Find :

(i) P(red ball)

(ii) P(blue ball)

(iii) P(blue ball if of 5 extra red balls are actually added)

Sol : [Board Term-2 2015]

Total ball, $n(S) = 15$

Blue ball $n(B) = x$

Red ball $n(R) = 15 - x$

Now probability of drawing red ball,

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If a family is selected at random, find the following probability.

(i) Find the probability that the family has two or three children.

(ii) Find the probability that the family has more than one child.

(iii) Find the probability that the family has less than three children.

(iv) Based on the answers to parts (i), (ii) and (iii) which is most likely to occur? Explain why.

Sol :

We make the following table for ease of calculation.

Children	Frequency
No children	51
One child	20
Two child	19
Three child	7
Four or More child	3
Total	100

Data are given in percentage however we have taken simple number out of 100. It doesn't make any difference.

Total possible outcomes in all case is 100 because frequency is 100 and out of which we have to select one.

Thus $n(S) = 100$

(i) Family has two or three children,

Let E_1 be the event that the family has two or three children.

Favourable outcome, $n(E_1) = 19 + 7 = 26$

Probability, $P(E_1) = \dfrac{n(E_1)}{n(S)}$

$= \dfrac{26}{100} = 0.26$

(ii) Family has more than one child,

Let E_2 be the event that the family has more than one child.

Favourable outcome, $n(E_2) = 19 + 7 + 3 = 29$

Probability, $P(E_2) = \dfrac{n(E_2)}{n(S)}$

$= \dfrac{29}{100} = 0.29$

(iii) Family has less than three children,

Let E_3 be the event that the family has less than three children.

Favourable outcome,

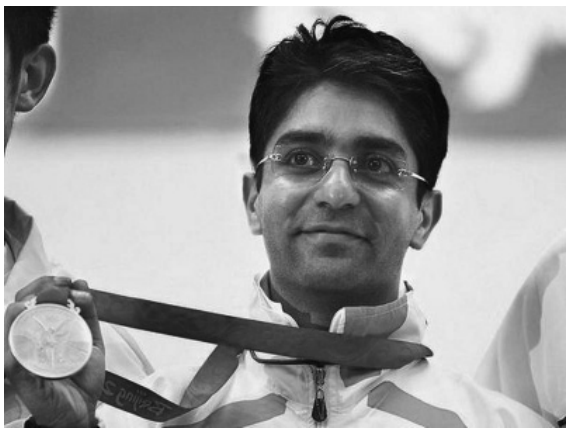
$n(E_3) = 51 + 20 + 19 = 90$

Probability, $P(E_3) = \dfrac{n(E_3)}{n(S)}$

$= \dfrac{90}{100} = 0.90$

(iv) Here (iii) is most likely to occur because probability is maximum.

165. Abhinav Bindra is retired sport shooter and currently India's only individual Olympic gold medalist. His gold in the 10-meter air rifle event at the 2008 Summer Olympics was also India's first Olympic gold medal since 1980. He is the first Indian to have held concurrently the world and Olympic titles for the men's 10-meter air rifle event, having earned those honors at the 2008 Summer Olympics and the 2006 ISSF World Shooting Championships. Bindra has also won nine medals at the Commonwealth Games and three gold medals at the Asian Games.



A circular dartboard has a total radius of 8 inch, with circular bands that are 2 inch wide, as shown in figure. Abhinav is still skilled enough to hit this board 100% of the time so he always score at least two points each time he throw a dart. Assume the probabilities are related to area, on the next dart that he throw.

(i) What is the probability that he score at least

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148. Two different dice are rolled together once. Find the probability of numbers coming on the tops whose product is a perfect square.

Sol : [Board Term-2 OD Compt. 2017]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

Favourable outcome are (2, 2), (3, 3), (4, 4) (5, 5), (6, 6), (1, 1), (4, 1) and (1, 4).

Number of favourable outcomes

$$n(E) = 8$$

P(product is a prefect square),

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{36} = \frac{2}{9}$$

149. A box contains 90 discs which are numbered 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two digit number,

(ii) number divisible by 5.

Sol : [Board Term-2 Foreign 2017]

Total number of discs in the box are 90.

Thus we have 90 possible outcomes.

$$n(S) = 90$$

(i) a two digit number,

Discs with two digit number are 10, 11, ...89 and 90 which are 81 numbers.

No. of favourable outcomes,

$$n(E_1) = 81$$

P(a disc with two digit number)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{81}{90} = \frac{9}{10}$$

(ii) number divisible by 5

The numbers divisible by 5 between 1 to 90 are 5, 10, 15 ..... 85 and 90 which are 18 numbers.

No. of favourable outcomes,

$$n(E_2) = 18$$

P (a disc with a number divisible by 5)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{18}{90} = \frac{1}{5}$$

150. Cards on which numbers 1, 2, 3 ..... 100 are written (one number on one card and no number is repeated), put in a bag and are mixed thoroughly. A card is drawn at random from the bag. Find the probability that card taken out has

(i) an even number

(ii) a number which is a multiple of 13.

(iii) a perfect square number.

(iv) a prime number less than 20.

Sol : [Board Term-2 Delhi Compt 2017]

There are 100 cards in bags. Thus we have 100 possible outcomes.

$$n(S) = 100$$

(i) an even number

Even numbers 1 to 100 are 50.

Number of favourable outcomes,

$$n(E_1) = 50$$

P(an even number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{50}{100} = \frac{1}{2}$$

(ii) A number which is a multiple of 13

Numbers multiples of 13, 26, 39, 52, 65, 78 and 91.

No. of favourable outcomes,

$$n(E_2) = 7$$

P(card taken out has multiple of 13),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{7}{100}$$

(iii) a perfect square number

Perfect square number in 1 to 100 are 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.

No. of all favourable outcomes,

$$n(E_3) = 10$$

P(perfect square number),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{10}{100} = \frac{1}{10}$$

(iv) a prime number less than 20

Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

No. of all favourable outcomes,

$$n(E_4) = 8$$

P(prime number less than 20),

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{8}{100} = \frac{2}{25}$$

151. Two different dice are thrown together. Find the probability that the numbers obtained have

(i) even sum, and

(ii) even product.

Sol :

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

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(i) even sum

Favourable outcome are (1, 3), (1, 5), (1, 1), (2, 2), (2, 4), (2, 6), (3, 1) (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3) (5, 5) (6, 2), (6, 4) and (6, 6).

Number of favourable outcomes,

$$n(E_1) = 18$$

P(even sum), $P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{18}{36} = \dfrac{1}{2}$ or 0.5

(ii) Even product

Favourable outcome are (1, 2), (1, 4), (1, 6), (2, 1), (2,2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2) (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5) and (6, 6).

Number of favourable outcomes

$$n(E_2) = 27$$

P(have a product less than 16),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{27}{36} = \frac{3}{4} = 0.75$$

Probability of getting even product is $\dfrac{3}{4}$ or 0.75.

152. From a deck of 52 playing cards, Jacks and kings of red colour and Queen and Aces of black colour are removed. The remaining cards are mixed and a card is drown at random. Find the probability that the drawn card is

(i) a black queen

(ii) a card of red colour

(iii) a Jack of black colour

(iv) a face card

Sol : [Board Term-2 OD Compt 2017]

There are $52 - (2 + 2 + 2 + 2) = 44$ cards in deck.

Thus we have 44 possible outcomes.

$$n(S) = 44$$

(i) a black queen

Number of black Queens in the deck,

$$n(E_1) = 0$$

P(getting a black queen),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{0}{44} = 0$$

Hence it is an impossible event

(ii) a card of red colour

Number of red cards,

$$n(E_2) = 26 - 4 = 22$$

P(getting a red card),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{22}{44} = \frac{1}{2}$$

(iii) a Jack of black colour

Number of Jacks (black),

$$n(E_3) = 2$$

P(getting a black coloured Jack),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{2}{44} = \frac{1}{22}$$

(iv) a face card

Number of face cards in the deck,

$$n(E_4) = 12 - 6 = 6$$

P(getting a face card),

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{6}{44} = \frac{3}{22}$$

153. A bag contains 20 balls out of which x balls are red.

(i) If one ball is drawn at random from the bag, find the probability that it is not red.

(ii) If 4 more red balls are out into the bag, the probability of drawing a red ball will be $\frac{5}{4}$ times the probability of drawing a red ball in the first case. Find the value of x.

Sol : [Board Term-2 Foreign 2015]

Total ball, $n(S) = 20$

Red ball $n(R) = x$

(i) not red

P(red ball), $P(R) = \dfrac{n(R)}{n(S)} = \dfrac{x}{20}$

P(no red ball),

$$P(\overline{R}) = 1 - \frac{x}{20} = \frac{20 - x}{20} \quad ...(1)$$

(ii) Now two more red balls are added.

Total ball $n'(S) = 20 + 4 = 24$

There are total $x + 4$ red ball.

$$n'(R) = x + 4$$

P(red balls), $P'(R) = \dfrac{n'(R)}{n'(S)} = \dfrac{x+4}{24}$

Now, according to the question,

$$\frac{x+4}{24} = \frac{5}{4} \times \frac{x}{20}$$

$$\frac{x+4}{24} = \frac{x}{16}$$

$$16x + 64 = 24x$$

$$64 = 8x \Rightarrow x = 8$$

For first case, substituting $x = 8$ in equation (1) we have

$$P(\overline{R}) = \frac{20 - 8}{20} = \frac{12}{20} = \frac{3}{5}$$

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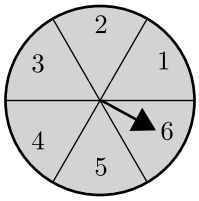
154. In Figure a disc on which a player spins an arrow twice. The fraction $\frac{a}{b}$ is formed, where a is the number of sector on which arrow stops on the first spin and 'b' is the number of the sector in which the arrow stops on second spin, On each spin, each sector has equal chance of selection by the arrow.

Find the probability that the fraction $\frac{a}{b} > 1$



Sol : [Board Term-2 Foreign 2016]

For $\frac{a}{b} > 1$, when $a = 1$, b can not take any value.

For $a = 2$, b can take 1 value i.e. 1.

For $a = 3$, b can take 2 values, i.e. 1 and 2.

For $a = 4, b$ can take 3 values i.e. 1, 2, and 3.

For $a = 5, b$ can take 4 values i.e. 1, 2, 3 and 4.

For $a = 6, b$ can take 5 values i.e. 1, 2, 3, 4 and 5

Total possible outcomes,

$$n(S) = 36$$

Favourable outcomes,

$$n(E) = 0 + 1 + 2 + 3 + 4 + 5 = 15$$

$$p\left(\frac{a}{b} > 1\right), \qquad P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

155. A bag contains 25 cards numbered from 1 to 25. A card is drawn at random from the bag. Find the probability that the number on the drawn card is :

(i) divisible by 3 or 5

(ii) a perfect square number.

Sol : [Board Term-2 2015]

Total cards $n(S) = 25$

(i) divisible by 3 or 5

Number divisible by 3 are 3, 6, 9, 12, 15, 16, 21, 24, and number divisible by 5 are 5, 10, 15, 20 and 25.

Thus number divisible by 3 or 5,

$$n(E_1) = 12$$

P(divisible by 3 or 5),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{12}{25}$$

(ii) a perfect square number.

Perfect square number are 1, 4, 9, 16 and 25.

$$n(E_2) = 5$$

P(a perfect square no.),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{5}{25} = \frac{1}{5}$$

156. A dice is rolled twice. Find the probability that :

(i) 5 will not come up either time.

(ii) 5 will come up exactly one time.

Sol : [Board Term-2 Delhi 2014]

When a dice is rolled twice, total number of outcomes,

$$n(S) = 6^2 = 36$$

There are 25 outcomes when 5 not come up either time.

Thus $n(E_1) = 25$

P(5 will not come up either time),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{25}{36}$$

(ii) 5 will come up exactly one time.

Possible outcomes are (1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6) and (6, 5).

$$n(E_2) = 10$$

P(5 will come up exactly one time)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

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COMPETENCEY BASED QUESTIONS

157. "Eight Ball" is a game played on a pool table with 15 balls numbered 1 to 151 and a "cue ball" that is solid and white. Of the 15 numbered balls, eight are solid (non-white) coloured and numbered 1 to 8 and seven are striped balls numbered 9 to 15.

The 15 numbered pool balls (no cue ball) are placed in a large bowl and mixed, then one ball is drawn out at random.

$$P(R) = \frac{n(R)}{n(S)} = \frac{15-x}{15} \quad ...(1)$$

If the number of red balls are increased by 5, i.e. total the probability of drawing the red balls doubles.

In this case, number of total ball,

$$n(S') = 15+5 = 20$$

and number of red ball,

$$n(R') = 15-x+5 = 20-x\,.$$

Now in this case probability of drawing red ball,

$$P(R') = \frac{n'(R)}{n'(S)} = \frac{20-x}{20}$$

According to the question, we have

$$P(R') = 2P(R)$$

$$\frac{20-x}{20} = 2\left(\frac{15-x}{15}\right)$$

$$1-\frac{x}{20} = 2-\frac{2x}{15}$$

$$\frac{2x}{15}-\frac{x}{20} = 2-1$$

$$\frac{8x-3x}{60} = 1$$

$$5x = 60 \Rightarrow x = 12$$

(i) P(red ball)

$$P(R) = \frac{n(R)}{n(S)} = \frac{15-12}{15} = \frac{3}{15} = \frac{1}{5}$$

(ii) P(blue ball)

$$P(R) = \frac{n(B)}{n(S)} = \frac{12}{15} = \frac{4}{5}$$

(iii) P(blue ball if of 5 extra red balls are actually added)

$$P'(R) = \frac{n'(R)}{n'(S)} = \frac{3+5}{15+5} = \frac{8}{20} = \frac{2}{5}$$

146. Three digit number are made using the digits 4, 5, 9 (without repetition). If a number among them is selected at random, what is the probability that the number will :

(i) be a multiple of 5 ?

(ii) be a multiple of 9 ?

(iii) will end with 9 ?

Sol : [Board Term-2 2014]

Total number of three digit numbers are 459, 495, 549, 594, 945 and 954. Thus we have 6 possible outcomes.

$$n(S) = 6$$

(i) be a multiple of 5

Multiple of 5 are 495 and 945.

$$n(E_1) = 2$$

P(multiple of 5),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

(ii) be a multiple of 9

All are multiple of 9.

$$n(E_2) = 6$$

P(multiple of 9),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{6} = 1$$

(iii) will end with 9

Numbers 459 and 549 ends with 9.

$$n(E_3) = 2$$

P(ending with 9),

$$P(E_3) = \frac{n(E_2)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

147. A number x is selected at random from the numbers 1, 2, 3 and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that product of x and y is less than 16.

Sol : [Board Term-2 OD 2016]

We have,

Total possible outcome are 1, 2, 3, 4, 4, 8, 9, 12, 16, 16, 18, 27, 32, 36 48 and 64 which are shown in following table.

×	1	2	3	4
1	1	2	3	4
4	4	8	12	16
9	9	18	27	36
16	16	32	48	64

There are 16 possible outcomes,

$$n(S) = 16$$

Total favourable number having product less than 16 are 1, 2, 3, 4, 4, 8, 9 and 12.

Number of favourable outcomes

$$n(E) = 8$$

P(product of x and y is less than 16),

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{16} = \frac{1}{2}$$

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(iv) Person is a Independent men or green party men,

Let E_4 be the event that the selected person is Independent men or green party men.

Favourable outcome, $n(E_4) = 7 + 3 = 10$

Probability, $P(E_4) = \frac{n(E_4)}{n(S)}$

$= \frac{10}{100} = 0.1$

(v) Person is not a Independent Women or Republican men,

Let E_5 be the event that the selected person is not a Independent Women or Republican men.

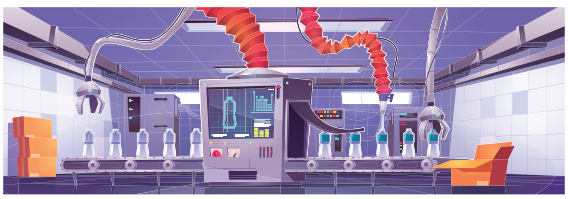
Favourable outcome,

$n(E_5) = 100 - (8 + 20) = 72$

Probability, $P(E_5) = \frac{n(E_5)}{n(S)}$

$= \frac{72}{100} = 0.72$

159. A large water bottling plant can remain at full production as long as one of its two generators is functioning. Due to past experience and the age difference between the systems, the plant manager estimates the probability of the main generator failing is 0.05, the probability of the secondary generator failing is 0.01, and the probability of both failing is 0.007.



What is the probability the plant remains in full production today?

Sol :

Although the answer may seem complicated, note that probability the plant remains in full production and the probability of both failing are complements.

$P(\text{Full Production}) = 1 - P(\text{Both Generator Fail})$

$= 1 - 0.007 = 0.993$

There is 99.3 % probability plant remains in full production today.

160. A hydrofoil is a lifting surface, or foil, that operates in water. They are similar in appearance and purpose to aerofoils used by aeroplanes. Boats that use hydrofoil technology are also simply termed hydrofoils. As a hydrofoil craft gains speed, the hydrofoils lift the boat's hull out of the water, decreasing drag and allowing greater speeds.

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Inter-Island Waterways has just opened hydrofoil service between several islands. The hydrofoil is powered by two engines, one forward and one aft, and will operate if either of its two engines is functioning. Due to testing and past experience, the company knows the probability of the aft engine failing is P (aft engine fails)= 0.05 the probability of the forward engine failing is P (forward engine fails)= 0.03, and the probability that both fail is P (both engines simultaneously fail)= 0.013. What is the probability the hydrofoil completes its next trip?

Sol :

Although the answer may seem complicated, note that P (trip is completed) and P (both engines simultaneously fail) are complements.

$P(\text{trip is completed})$

$= 1 - P(\text{both engines simultaneously fail})$

$= 1 - 0.013 = 0.987$

There is 99.7% probability the trip will be completed.

161. A fire station gets an emergency call from a shopping mall in the mid-afternoon. From a study of traffic patterns, Chief fire officer DK Jain knows the probability the most direct route is clogged with traffic is 0.11, while the probability of the secondary route being clogged is 0.13. The probability both are clogged is 0.03.

What is the probability they can respond to the call unimpeded using one of these routes?

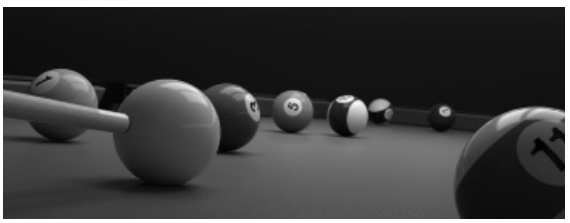
Sol :

Although the answer may seem complicated, note that probability both route are clogged and the probability that they can respond to the call unimpeded using one of these routes are complements.

$$P(\text{Resnose}) = 1 - P(\text{Both Route are Clogged})$$
$$= 1 - 0.03 = 0.97$$

There is 98 % probability hey can respond to the call unimpeded using one of these routes.

162. Eight Ball : This is a game played on a pool table with 15 balls numbered 1 through 15 and a cue ball that is solid white. Of the 15 numbered balls, 8 are a solid (nonwhite) color and numbered 1 through 8, and seven are striped balls numbered 9 through 15.



The fifteen numbered pool balls (no cueball) are placed in a large bowl and mixed, then one is drawn out.

(i) What is the probability of drawing the eight ball ?

(ii) What is the probability of drawing a number greater than fifteen ?

(iii) What is the probability of drawing an even number ?

(iv) What is the probability of drawing a multiple of three ?

(v) What is the probability of drawing a solid color and an even number ?

Sol :

Total possible outcomes in all case is 15 because there are total 15 ball and out of which we have to select one ball.

Thus $n(S) = 15$

(i) The probability of drawing the eighth ball,

Let E_1 be the event that the selected ball is the eight ball.

Favourable outcome, $n(E_1) = 1$

Probability, $P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{1}{15}$

(ii) The probability of drawing a number greater than fifteen ,

Let E_2 be the event that the selected ball is having a number greater than fifteen.

Favourable outcome, $n(E_2) = 0$

Probability, $P(E_2) = \dfrac{n(E_2)}{n(S)}$

$= \dfrac{0}{15} = 0$

(iii) The probability of drawing an even number,

Let E_3 be the event that the selected ball is having an even number i.e. 2, 4, 6, 8, 10, 12, 14

Favourable outcome, $n(E_3) = 7$

Probability, $P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{7}{15}$

(iv) The probability of drawing a multiple of three,

Let E_4 be the event that the selected ball is having multiple of three i.e 3, 6, 9, 12, 15

Favourable outcome, $n(E_4) = 5$

Probability, $P(E_4) = \dfrac{n(E_4)}{n(S)}$

$= \dfrac{5}{15} = \dfrac{1}{3}$

(v) The probability of drawing a solid colour and an even number

Let E_5 be the event that the that the selected ball is having a solid colour and an even number i.e. 2, 4, 6, 8.

Favourable outcome, $n(E_5) = 4$

Probability, $P(E_5) = \dfrac{n(E_5)}{n(S)}$

$= \dfrac{4}{15}$

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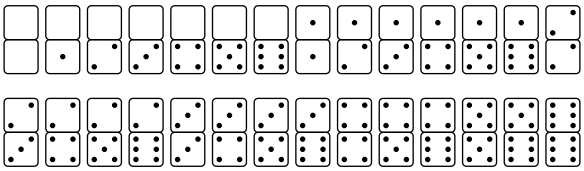
163. Double-six Dominos : It is a game played with the 28 numbered tiles shown in the diagram.



The 28 dominos are placed in a bag, shuffled, and then one domino is randomly drawn. Give the following answer.

(i) What is the probability the total number of dots on the domino is three or less ?

(ii) What is the probability the total number of dots on the domino is greater than three ?

(iii) What is the probability the total number of dots on the domino does not have a blank half ?

(iv) What is the probability the total number of dots on the domino is not a "double" (both sides the same) ?

Sol :

Total possible outcomes in all case is 28 because there are total 28 dominos.

Therefore $n(S) = 28$

(i) Total number of dots on the domino is three or less,

Let E_1 be the event that the total number of dots on the domino is three or less.

Favourable outcome, $n(E_1) = 6$

Probability, $P(E_1) = \frac{n(E_1)}{n(S)}$

$= \frac{6}{28} = \frac{3}{14}$

(ii) Total number of dots on the domino is greater than three,

Let E_2 be the event that the total number of dots on the domino is greater than three.

Favourable outcome, $n(E_2) = 22$

Probability, $P(E_2) = \frac{n(E_2)}{n(S)}$

$= \frac{22}{28} = \frac{11}{14}$

Alternative :

$P(E_2) = 1 - P(E_1) = 1 - \frac{3}{14} = \frac{11}{14}$

(iii) Total number of dots on the domino does not have a blank half,

Let E_3 be the event that the total number of dots on the domino does not have a blank half . Let $\overline{E_3}$ be the event that the total number of dots on the domino have a blank half .

Favourable outcome, $n(\overline{E_3}) = 7$

Probability, $P(\overline{E_3}) = \frac{n(\overline{E_3})}{n(S)}$

$= \frac{7}{28} = \frac{1}{4}$

Probability, $P(E_3) = 1 - P(\overline{E_3})$

$= 1 - \frac{1}{3} = \frac{3}{4}$

(iv) Total number of dots on the domino is not a "double" (both sides the same) ?

Let E_4 be the event that the total number of dots on the domino is not a double. Let $\overline{E_4}$ be the event that the total number of dots on the domino is a double.

Favourable outcome, $n(\overline{E_4}) = 7$

Probability, $P(E_4) = \frac{n(\overline{E_4})}{n(S)}$

$= \frac{7}{28} = \frac{1}{4}$

Probability, $P(E_4) = 1 - P(\overline{E_4})$

$= 1 - \frac{1}{3} = \frac{3}{4}$

164. Family Structures : For a recent year, 51% of the families in the United States had no children under the age of 18; 20% had one child; 19% had two children; 7% had three children; and 3% had four or more children.

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Based on the above information, answer the following question :

(i) What is the probability that the drawn ball bears number 8?

(ii) What is probability that the drawn ball bears an even number?

(iii) What is the probability that the drawn ball bears a number, which is a multiple of 3?

(iv) What is the probability that the drawn ball is a solid coloured and bears an .even number?

Sol : [Board 2023 OD Standard]

(i) Total number of balls $= 15$

Number of Ball bears number $8 = 1$

$P(\text{Getting ball bears number } 8) = \frac{1}{15}$

(ii) Number of balls having even numbers $= 7$

$P(\text{Getting even number balls}) = \frac{7}{15}$

(iii) Number of balls bearing a number, which is multiple of $3 = 5$

$P(\text{Getting balls having multiple of } 3) = \frac{5}{15}$

$= \frac{1}{3}$

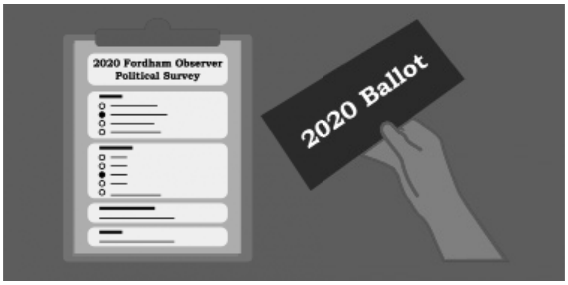
(iv) Solid coloured balls $= 8$

Number of solid coloured balls having an even number $= 4$.

P (Getting Solid Coloured even number Ball)

$= \frac{4}{15}$

158. Political survey questions are questions asked to gather the opinions and attitudes of potential voters. Political survey questions help you identify supporters and understand what the public needs. Using such questions, a political candidate or an organization can formulate policies to gain support from these people.



A survey of 100 voters was taken to gather information on critical issues and the demographic information collected is shown in the table. One out of the 100 voters is to be drawn at random to be interviewed on the India Today News on prime time.

	Women	Men	Totals
Republican	17	20	37
Democrat	22	17	39
Independent	8	7	15
Green Party	6	3	5
Totals	53	47	100

(i) What is the probability the person is a woman or a Republican ?

(ii) What is the probability the person is a Democrat ?

(iii) What is the probability the person is a Independent men ?

(iv) What is the probability the person is a Independent men or green party men ?

(v) What is the probability the person is not a Independent Women or Republican men ?

Sol :

Total possible outcomes in all case is 100 because there are total 100 person and out of which we have to select one for interview.

$n(S) = 100$

(i) Person is a woman (W) or a Republican (R),

Let E_1 be the event that the selected person is a woman or a Republican.

Favourable outcome, $n(E_1) = 53 + 20 = 73$

Probability, $P(E_1) = \frac{n(E_1)}{n(S)}$

$= \frac{73}{100} = 0.73$

(ii) Person is a democrat,

Let E_2 be the event that the selected person is democrat.

Favourable outcome, $n(E_2) = 39$

Probability, $P(E_2) = \frac{n(E_2)}{n(S)}$

$= \frac{39}{100} = 0.39$

(iii) Person is a Independent men,

Let E_3 be the event that the selected person is Independent men.

Favourable outcome, $n(E_3) = 7$

Probability, $P(E_3) = \frac{n(E_3)}{n(S)}$

$= \frac{7}{100} = 0.07$

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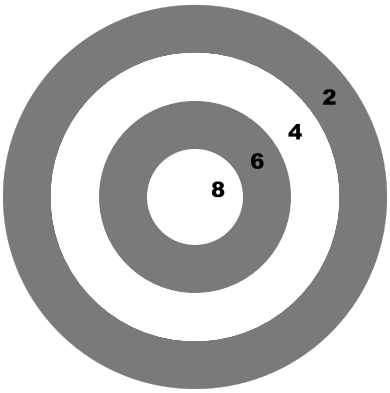
4 ?

(ii) What is the probability that he score at least 6 ?

(iii) What is the probability that he hit bull's eye ?

(iv) What is the probability that he score exactly 4 points ?



Sol :

Here sample space is area of circle of 8 inch.

Thus $n(S) = \pi 8^2$ in^2.

(i) The probability that you score at least 4,

Let E_1 be the event that she hit the circle in the radius of 6 inch to score 4. Favourable outcome will be area of this 6 inch circle.

Favourable outcome, $n(E_1) = \pi 6^2$ in^2.

Probability, $P(E_1) = \dfrac{n(E_1)}{n(S)}$

$= \dfrac{\pi 6^2}{\pi 8^2} = \dfrac{9}{16}$

(ii) The probability that you score at least 6,

Let E_2 be the event that she hit the circle in the radius of 4 inch to score 6. Favourable outcome will be area of this 4 inch circle.

Favourable outcome, $n(E_2) = \pi 4^2$ in^2.

Probability, $P(E_2) = \dfrac{n(E_2)}{n(S)}$

$= \dfrac{\pi 4^2}{\pi 8^2} = \dfrac{1}{4}$

(iii) The probability that you hit the bull's eye,

Let E_3 be the event that she hit the bulls eye. Favourable outcome will be area of 2 inch circle.

Favourable outcome, $n(E_3) = \pi 2^2$ in^2.

Probability, $P(E_3) = \dfrac{n(E_4)}{n(S)}$

$= \dfrac{\pi 2^2}{\pi 8^2} = \dfrac{1}{16}$

(iv) The probability that you score exactly 4 points,

Let E_4 be the event that she hit the circular band in the radius of 4 inch to 6 inch. Favourable outcome will be area of this circular band.

Favourable outcome, $n(E_4) = \pi(6^2 - 4^2)$ in^2.

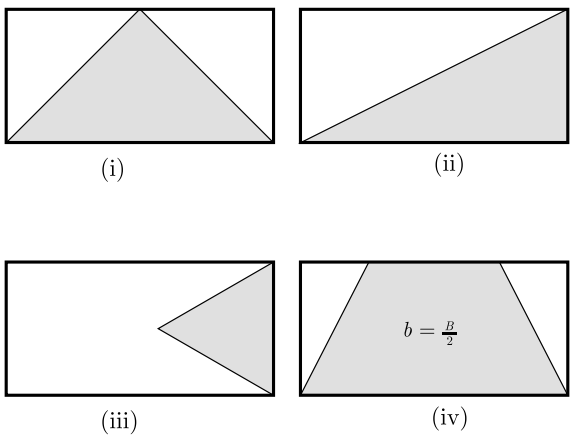
Probability, $P(E_4) = \dfrac{n(E_4)}{n(S)}$

$= \dfrac{\pi(6^2 - 4^2)}{\pi 8^2}$

$= \dfrac{20}{64} = \dfrac{5}{16}$

166. Apurvi Singh Chandela is an Indian Shooting player who competes in the 10 metre air rifle event. She won the gold medal in the 2019 ISSF World Cup in New Delhi. She is the world champion and currently ranked number one in 10 metre air rifle shooting. She is a recipient of Arjuna award.

Apurvi is able to hit a 10 in. by 20 in. target 100% of the time at a range of 100 metre. Assuming the probability she hits a target is related to its area, what is the probability she hits the shaded portions shown?



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probability of his rolling the die ?

(iii) If the player is allowed to roll the dice and getting a number greater than 4 entitles him to get prize, what is the probability of his winning the prize?

(iv) If getting a square number on the wheel allows a player to roll the dice, then what is the probability of his rolling the dice ?

(v) If the player is allowed to roll the die and getting a prime number on die entitles him to get prize, then what is the probability of his winning the prize?

Sol :

(i) Total outcomes $n(S) = 8$

Favourable outcome are $\{2, 4, 6, 8\}$, therefore

$$n(E) = 4$$

Probability of getting an even number on the wheel,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(ii) Total outcomes $n(S) = 8$

Favourable outcome are $\{3, 5, 7, 9\}$, therefore

$$n(E) = 4$$

Probability of getting an odd number on the wheel,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iii) Dice has total six outcome, thus total outcome

$$n(S) = 6$$

Number greater than 4 on dice are $\{5, 6\}$, therefore

$$n(E) = 2$$

Probability of getting a number greater than 4,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

(iv) Total outcomes $n(S) = 8$

Favourable outcome are $\{4, 9\}$, therefore

$$n(E) = 2$$

Probability of getting a square number on the wheel,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{8} = \frac{1}{4}$$

(v) Dice has total six outcome, thus total outcome

$$n(S) = 6$$

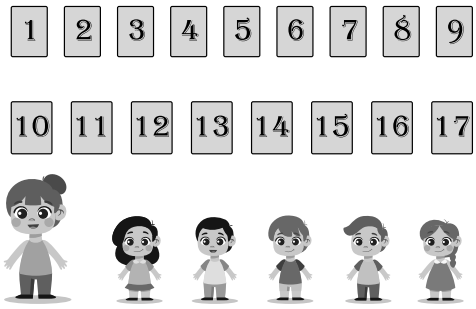
Prime number on dice are $\{2, 3, 5\}$, therefore

$$n(E) = 3$$

Probability of getting a prime number on dice,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

179. Five friends and one of their mother are having a picnic. The mother deicide to play card game. 17 cards numbered 1, 2, 3 ... 17 are put in a box and mixed thoroughly. The mother asks each boy to draw a card and after each draw she shows some magic tricks based on card number.



(i) What is the probability of drawing an odd number card in the first draw by the first boy?

(ii) Now in second draw, card drawn in first draw is replaced. What is the probability of drawing a prime number card by the second boy?

(iii) If in second draw, boy got number 2 and the card is not replaced, what is the probability of drawing a card bearing a multiple of 3 greater than 5 by the third boy?

(iv) If the card is replaced after the third draw, what is the probability of drawing a card bearing a number greater than 17 by the fourth boy ?

(v) If the card is replaced after the fourth draw, what is the probability of drawing a card bearing a multiple of 3 or 7 by the fifth boy?:

Sol :

(i) Total outcomes $n(S) = 17$

Favourable outcomes are $\{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, therefore

$$n(E) = 9$$

Probability of drawing an odd number card in the first draw by the first boy,

$$P(E) = \frac{n(E)}{n(S)} = \frac{9}{17}$$

(ii) Total outcomes $n(S) = 17$

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Favourable outcome, $n(E_1) = 21$

Probability, $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{21}{50} = 0.42$

(ii) Person has type A or type B blood,

Let E_2 be the event that a person has type A or type B blood.

Favourable outcome, $n(E_2) = 22 + 5 = 27$

Probability, $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{27}{50} = 0.54$

(iii) Person has neither type A nor type O blood, i.e person has type B or type AB

Let E_3 be the event that the selected person has neither type A nor type O blood.

Favourable outcome, $n(E_3) = 5 + 2 = 7$

Probability, $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{7}{50} = 0.14$

(iv) Person does not have type AB blood,

Let E_4 be the event that the selected person does not have type AB blood. Let $\overline{E_4}$ be the event that the selected person have type AB blood.

Favourable outcome, $n(\overline{E_4}) = 2$

Probability, $P(\overline{E_4}) = \frac{n(\overline{E_4})}{n(S)} = \frac{2}{50} = 0.02$

$P(E_4) = 1 - P(\overline{E_4})$

$= 1 - 0.04 = 0.96$

168. Computers in Elementary Schools : Computer-based learning (CBL) refers to any teaching methodology that makes use of computers for information transmission or input. At an elementary school level, computer applications can be used to display multimedia lesson plans or students can input examination answers into a computer to facilitate grading and evaluation. Multimedia formats and applications that allow students to pause content are especially valuable in early childhood education especially since students are coming to school with different competencies, learning styles and speeds for absorbing information.

Total 1000 elementary and secondary schools of Rajasthan were classified by the number of computers they had.

Computers	1–10	11–20	21–50	51–100	100 more
Schools	250	210	290	170	80

Choose one school at random and find the following probability.

(i) Find the probability that it has 50 or fewer computers.

(ii) Find the probability that it has more than 100 computers.

(iii) Find the probability that it has no more than 20 computers.

(iv) Find the probability that it has more than 50 computers.

Sol :

From table we see that total 1000 elementary and secondary schools were classified. Thus total possible outcomes in all case is 1000 schools and out of which we have to select one.

$n(S) = 1000$

(i) E_1 : Probability that it has 50 or fewer computers.

Favourable outcome, $n(E_1) = 250 + 210 + 290$

$= 750$

Probability, $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{750}{1000} = 0.75$

(ii) E_2 : Probability that it has more than 100 computers

Favourable outcome, $n(E_2) = 80$

Probability, $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{80}{1000} = 0.08$

(iii) E_3 : Probability that it has no more than 20

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Sol :

Here sample space is area 10 in. by 20 in. i.e 200 in^2.

Thus $n(S) = 200$

(i) Isosceles triangle

Let E_1 be the event that she hit the isosceles triangle shown in question having base 20 inch and height 10 inch. Favourable outcome will be area of this isosceles triangle.

Favourable outcome,

$$n(E_1) = \frac{1}{2} \times 20 \times 10 = 100 \text{ in}^2.$$

Probability, $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{100}{200} = \frac{1}{2}$

(ii) Right Triangle

Let E_2 be the event that she hit the right triangle shown in question having base 20 inch and height 10 inch. Favourable outcome will be area of this right triangle.

Favourable outcome,

$$n(E_2) = \frac{1}{2} \times 20 \times 10 = 100 \text{ in}^2.$$

Probability, $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{100}{200} = \frac{1}{2}$

(iii) Equilateral Triangle

Let E_3 be the event that she hit the equilateral triangle shown in question having each side 10 inch. Favourable outcome will be area of this equilateral triangle.

Favourable outcome,

$$n(E_3) = \frac{\sqrt{3}}{4} \times 10^2 = 25\sqrt{3} \text{ in}^2.$$

Probability, $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{25\sqrt{3}}{200} = \frac{\sqrt{3}}{8}$

(iv) Isosceles Trapezoid ($b = \frac{B}{2}$)

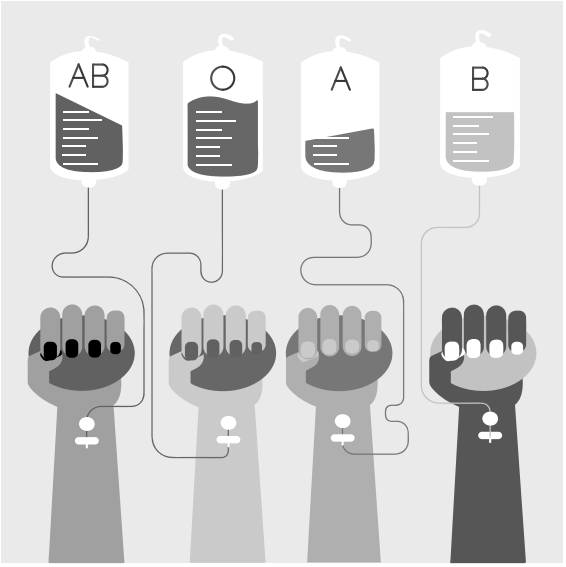
Let $n(E_4)$ be the event that she hit the isosceles trapezoid shown in question having each side 10 inch. Favourable outcome will be area of this isosceles trapezoid.

Favourable outcome,

$$n(E_4) = \frac{1}{2} \times 10 \times \left(20 + \frac{20}{2}\right) = 150 \text{ in}^2$$

Probability, $P(E_4) = \frac{n(E_4)}{n(S)} = \frac{150}{200} = \frac{3}{4}$

167. Blood Group : Blood type or blood group is a medical term. It describes the type of blood a person has. It is a classification of blood based on the presence or absence of inherited antigenic substances on the surface of red blood cells (RBCs). Blood types predict whether a serious reaction will occur in a blood transfusion.



In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

(i) What is the probability that a person has type O blood ?

(ii) What is the probability that a person has type A or type B blood ?

(iii) What is the probability that a person has neither type A nor type O blood ?

(iv) What is the probability that a person does not have type AB blood ?

Sol :

Frequency distribution table is as follows.

Type	Frequency
A	22
B	5
AB	2
O	21
Total	50

Total possible outcomes in all case is 50 because there are total 50 sample and out of which we have to select one.

Thus $n(S) = 50$

(i) Person has type O blood,

Let E_1 be the event that the selected person has type O blood.

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stayed at most 4 days i.e. 3 or 4 days.

Favourable outcome, $n(E_3) = 13 + 22 = 35$

Probability, $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{35}{100} = 0.35$

(iv) Patient stayed at least 5 days,

Let E_4 be the event that the selected stayed at least 5 days i.e. 5, 6, or 7 days

Favourable outcome, $n(E_4) = 45 + 14 + 6 = 65$

Probability, $P(E_4) = \frac{n(E_4)}{n(S)} = \frac{65}{100} = 0.65$

171. Cards on which numbers 1, 2, 3 ..... 100 are written (one number on one card and no number is repeated), put in a bag and are mixed thoroughly. A card is drawn at random from the bag. Find the following probability.

Sol :

(i) What is the probability that card taken out has a odd number ?

(ii) What is the probability that card taken out has a two digit odd number ?

(iii) What is the probability that card taken out has a odd number which is multiple of 11?

(iv) What is the probability that card taken out has an odd number which is not less than 70?

(v) What is the probability that card taken out has an odd number which is not multiple of 11 ?

Sol :

(i) There are 100 cards in bags. Thus we have 100 possible outcomes for all cases.

$$n(S) = 100$$

Odd numbers 1 to 100 are 50.

Number of favourable outcomes,

$$n(E_1) = 50$$

P(an odd number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{50}{100} = \frac{1}{2} = 0.5$$

(ii) Total odd number are 50 and 5 numbers are one digit odd number. Hence two digit odd number are 45. Thus favourable outcomes,

$$n(E_2) = 50 - 5 = 45$$

P(Two digit odd number),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{45}{100} = 0.45$$

(iii) Favourable outcomes are {11, 33, 55, 77, 99}. Number of favourable outcomes is 5. Therefore

$$n(E_3) = 5$$

P(odd number multiple of 11),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{5}{100} = 0.05$$

(iv) Favourable outcomes are 71, 73, 75, ...99. Number of favourable outcomes,

$$n(E_4) = 15$$

P(odd number not less than 70),

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{15}{100} = 0.15$$

(v) Total odd number are 50 and out of which {11, 33, 55, 77, 99} are multiple of 11. Thus $50 - 5 = 45$ numbers are not multiple of 11.

Therefore $n(E_5) = 45$

P(odd number not multiple of 11),

$$P(E_5) = \frac{n(E_5)}{n(S)} = \frac{45}{100} = 0.45$$

172. Insurance : An insurance company selected 1000 drivers at random in a particular city to determine a relationship between age and accidents. The data obtained are listed in following table.

Age	0	1	2	3	Over 3
Under 20	50	62	53	35	20
20–29	64	93	67	40	36
30–39	82	68	32	14	4
40–49	38	32	20	7	3
Over 49	43	50	35	28	24

Compute the probabilities of the following events for a driver chosen at random in the city:

(i) E_1 : being under 20 years old and having exactly three accidents in 1 year

(ii) E_2 : being 30–39 years old and having one or more accidents in 1 year

(iii) E_3 : having no accidents in 1 year

(iv) E_4 : being under 20 years old or having exactly three accidents in 1 year

Sol :

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From the given table we make the following table.

Age	0	1	2	3	Over 3
Under 20	50	62	53	35	20
20–29	64	93	67	40	36
30–39	82	68	32	14	4
40–49	38	32	20	7	3
Over 49	43	50	35	28	24
Total	277	305	207	124	87

Total possible outcomes in all case is 1000 because there are total 1000 driver and out of which we have to select one .

$$n(S) = 1000$$

(i) E_1 : being under 20 years old and having exactly three accidents in 1 year

Favourable outcome, $n(E_1) = 35$

Probability, $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{35}{1000} = 0.35$

(ii) E_2 : being 30–39 years old and having one or more accidents in 1 year

Favourable outcome,

$$n(E_2) = 68 + 32 + 14 + 4 = 118$$

Probability, $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{118}{1000} = 0.118$

(iii) E_3 : having no accidents in 1 year

Favourable outcome,

$$n(E_3) = 277$$

Probability, $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{277}{1000} = 0.277$

(iv) E_4 : being under 20 years old or having exactly three accidents in 1 year

Favourable outcome,

$n(E_4) = 50 + 62 + 53 + 35 + 20 + 40 + 14 + 7 + 28$

$= 309$

Probability, $P(E_4) = \frac{n(E_4)}{n(S)} = \frac{309}{1090} = 0.309$

173. Deepika Kumari is an Indian athlete who competes in the event of archery, is currently ranked World No. 9, and is a former world number one. She won a gold medal in the 2010 Commonwealth games in the women's individual recurve event. She also won a gold medal in the same competition in the women's team recurve event along with Dola Banerjee and Bombayala Devi. Kumari qualified for the 2012 Summer Olympics in London, where she competed in the Women's Individual and Women's team events, finishing in eighth place in the latter.

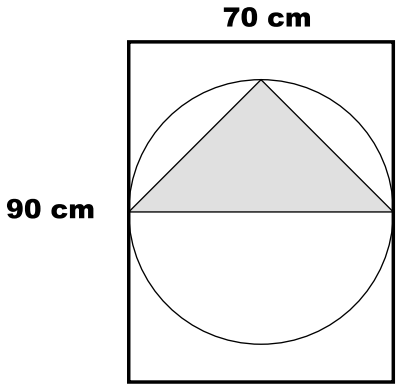
Deepika can hit the rectangular target shown 100% of the time at a range of 80 m. Assuming the probability the target is hit is related to its area, answer the following.

(i) What is the probability the Deepika hits within the triangle ?

(ii) What is the probability the Deepika hits within the circle ?

(iii) What is the probability the Deepika hits within the circle but outside the triangle ?

(iv) What is the probability the Deepika hits within the rectangle but outside the circle ?



Sol :

Deepika can hit the rectangular target shown 100% of the time, thus area of rectangle is sample space.

$$n(S) = 90 \times 70 = 6300 \text{ cm}^2$$

(i) E_1 : Probability the Deepika hits within the triangle,

Favourable outcome is the area of triangle. Base of triangle is 70 cm and height of triangle is 35 cm.

Favourable outcome, $n(E_1) = \frac{1}{2} \times 70 \times 35$

$= 1225 \text{ cm}^2$

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Probability, $P(E_1) = \frac{n(E_1)}{n(S)}$

$= \frac{1225}{6300} = \frac{7}{36}$

(ii) E_2 : Probability the Deepika hits within the circle,

Favourable outcome is the area of circle of radius 35 cm.

Favourable outcome, $n(E_2) = \pi 35^2$

$= \frac{22}{7} \times 35 \times 35$

$= 3850 \text{ cm}^2$

Probability, $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3850}{6300} = \frac{11}{18}$

(iii) E_3 : Probability the Deepika hits within the circle but outside the triangle,

Favourable outcome is the difference of area of circle and triangle.

Favourable outcome, $n(E_3) = 3850 - 1225$

$= 2626 \text{ cm}^2$

Probability, $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{2625}{6300} = \frac{5}{12}$

(iv) E_4 : Probability the Deepika hits within the rectangle but outside the circle,

Favourable outcome is the difference of area of rectangle and circle.

Favourable outcome, $n(E_4) = 6300 - 3850$

$= 2450 \text{ cm}^2$

Probability, $P(E_4) = \frac{n(E_4)}{n(S)}$

$= \frac{2450}{6300} = \frac{7}{18}$

Alternative :

Event E_4 is complement of E_2.

Thus $P(E_4) = 1 - P(E_2)$

$= 1 - \frac{11}{18} = \frac{7}{18}$

174. Remdesivir was the first drug approved by the FDA for treating the SARS-CoV-2 virus. It is indicated for treatment of COVID-19 disease in hospitalized adults and children aged 12 years and older who weigh at least 40 kg.

In a medical experiment, a Remdesivir is found to help 2,400 out of 4,000 people. If a doctor prescribes the Remdesivir for a particular patient, what is the approximate empirical probability that the patient will be helped?

Sol :

Here total sample space is 4000 people.

$n(S) = 4000$

Let E be the event that Remdesivir helps the patient.

Favourable outcome

$n(E) = 2400$

Probability that the patient will be helped,

$P(E) = \frac{n(E)}{n(S)}$

$= \frac{2400}{4000} = \frac{6}{10} = 0.6$

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175. Out of 460 times at bat, a baseball player gets 207 hits. What is the approximate empirical probability that the player will get a hit next time at bat?

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Favourable outcome, $n(E_3) = 250 + 210$

$= 460$

Probability, $P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{460}{1000} = 0.46$

(iv) E_4: Probability that it has more than 50 computers

Favourable outcome, $n(E_4) = 170 + 80 = 250$

Probability, $P(E_4) = \dfrac{n(E_4)}{n(S)} = \dfrac{250}{1000} = 0.25$

169. Consider the 210 discrete points found in the first and second quadrants where $-10 \leqslant x \leq 10$, $1 \leqslant y \leq 10$ and x and y are integers. The coordinates of each point is written on a slip of paper and placed in a box. One of the slips is then randomly drawn.

(i) What is the probability the point (x, y) is on the graph of $y = |x|$

(ii) What is the probability the point (x, y) is on the graph of $y = 2|x|$

(iii) What is the probability the point (x, y) has coordinates $(x \leq 5, y > -2)$

Sol :

Total possible outcomes in all case is 210 out of which we have to select one.

Thus $n(S) = 210$

(i) The probability the point (x, y) is on the graph of $y = |x|$,

Let E_1 be the event that the point (x, y) is on the graph of $y = |x|$. Here total 20 such point are possible.

Favourable outcome, $n(E_1) = 20$

Probability, $P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{20}{210} = \dfrac{2}{21}$

(ii) The probability the point (x, y) is on the graph of $y = 2|x|$

Let E_2 be the event the point (x, y) is on the graph of $y = 2|x|$. Here total 10 such point are possible.

Favourable outcome, $n(E_2) = 10$

Probability, $P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{10}{210} = \dfrac{1}{21}$

(iii) The probability the point (x, y) has coordinates $(x \leq 5, y > -2)$,

Let E_3 be the event that probability the point (x, y) has coordinates $(x \leq 5, y > -2)$. Here total 160 such point are possible.

Favourable outcome, $n(E_3) = 160$

Probability, $P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{160}{210} = \dfrac{16}{21}$

170. Hospital Stays : Hospital records indicated that maternity patients stayed in the hospital for the number of days shown in the distribution.

Number of days stayed	Frequency
3	13
4	22
5	45
6	14
7	6
	100

Find these probabilities.

(i) A patient stayed exactly 5 days.

(ii) A patient stayed less than 6 days.

(iii) A patient stayed at most 4 days.

(iv) A patient stayed at least 5 days.

Sol :

Total possible outcomes in all case is 1000 because frequency is 100 and out of which we have to select one.

Thus $n(S) = 100$

(i) Patient stayed exactly 5 days,

Let E_1 be the event that the selected patient stayed exactly 5 days,.

Favourable outcome, $n(E_1) = 45$

Probability, $P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{45}{100} = 0.45$

(ii) Patient stayed less than 6 days,

Let E_2 be the event that a patient stayed less than 6 days i.e 3, 4, or 5 days.

Favourable outcome,

$n(E_2) = 13 + 22 + 45 = 80$

Probability, $P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{80}{100} = 0.80$

(iii) Patient stayed at most 4 days,

Let E_3 be the event that the selected patient

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P(doublet of even number),

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(v) Favourable outcomes are {(3, 6), (6, 3), (4, 5), (5, 4), (4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6),}

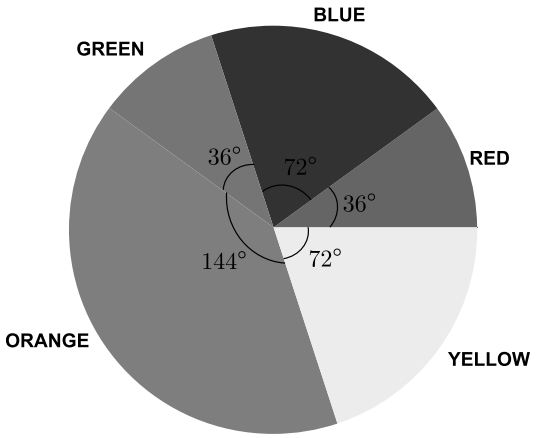
Thus number of favourable outcomes,

$$n(E_5) = 10$$

P(product of numbers greater than 16),

$$P(E_5) = \frac{n(E_5)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

177. A survey was taken at a high school, and the results were put in a circle graph. The students were asked to list their favourite colours. The measurement of each central angle is shown. If a person is chosen at random from the school, find the probability of each response.



(i) What is the probability of favourite colour being red ?

(ii) What is the probability of favourite colour being blue or green ?

(iii) What is the probability of favourite colour not being red or blue?

(iv) What is the probability of favourite colour not being orange or green ?

(v) What is the probability of favourite colour being red or blue?

Sol :

(i) $$\text{Probability} = \frac{\text{Area of region}}{\text{Area of circle}}$$

$$= \frac{\text{Total Angle in region}}{360^\circ}$$

$$P(\text{red}) = \frac{36^\circ}{360^\circ} = \frac{1}{10} = 0.1$$

(ii) P(blue of green)

$$= \frac{72^\circ + 36^\circ}{360^\circ} = \frac{108^\circ}{360^\circ} = \frac{3}{10} = 0$$

(iii)

$$P(\text{not red or blue}) = 1 - P(\text{red or blue})$$

$$= 1 - \frac{36^\circ + 72^\circ}{360^\circ} = 1 - \frac{108^\circ}{360^\circ}$$

$$= 1 - \frac{3}{10} = 1 - 0.3 = 0.7$$

(iv)

$$P(\text{not orange or green}) = 1 - P(\text{orange or green})$$

$$= 1 - \frac{144^\circ + 36^\circ}{360^\circ} = 1 - \frac{180^\circ}{360^\circ}$$

$$= 1 - \frac{1}{2} = 0.5$$

Thus (d) is correct option.

(v) $$P(\text{red or blue}) = 1 - P(\text{not red or blue})$$

$$= 1 - 0.7 = 0.3$$

178. A game at a stall in new year carnival involves spinning a wheel first as a first step to complete the game with certain rules. If the wheel stops at a particular number, then the player is allowed to roll a 6 faced unbiased dice.

Rules of Game:

(i) If the wheel stops at a particular number, then the player is allowed to roll a unbiased dice.

(ii) If the wheel stops at any other number, player get to try again and only one extra try allowed.

(ii) If player reach the next stage and roll a dice, he may get a prize depending on the number on dice.

(i) What is the probability of getting an even number on the wheel?

(ii) If getting an odd number on the wheel allows a player to roll the die, then what is the

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Sol :

Here total sample space is 460 batting.

$$n(S) = 460$$

Let E be the event that baseball player gets hits.

Favourable outcome

$$n(E) = 207$$

Probability that the player will get a hit next time,

$$P(E) = \frac{n(E)}{n(S)} = \frac{207}{460} = 0.45$$

176. In two dice game, the player take turns to roll both dice, they can roll as many times as they want in one turn. A player scores the sum of the two dice thrown and gradually reaches a higher score as they continue to roll. If a single number 1 is thrown on either die, the score for that whole turn is lost. Two dice are thrown simultaneously.

(i) What is the probability of getting the sum as an even number ?

(ii) What is the probability of getting the sum as a prime number ?

(iii) What is the probability of getting the sum of atleast 10?

(iv) What is the probability of getting a doublet of even number ?

(v) What is the probability of getting a product of numbers greater than 16?

Sol :

(i) All possible outcome are given as below:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Number of all possible outcomes in all case,

$$n(S) = 6 \times 6 = 36$$

Favourable outcome are $\{2, 4, 6, 8, 10, 12\}$. We may get as follows

{(1, 1), (1, 3), (3, 1), (2, 2), (1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (2, 6), (6, 2), (3, 5), (5, 3), (4, 4), (6, 4), (4, 6), (5, 5), (6, 6)}

Thus number of favourable outcomes,

$$n(E_1) = 18$$

P(sum as an even number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

(ii) Favourable outcome are $\{2, 3, 5, 7, 11\}$, which may be as follows

{(1, 1), (1, 2), (2, 1), (1, 4), (4, 1), (2, 3), (3, 2), (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3), (6, 5), (5, 6)}

Thus number of favourable outcomes,

$$n(E_1) = 15$$

P(sum as a prime number),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

(iii) Favourable outcomes are {(5, 5), (6, 4), (4, 6), (6, 5), (5, 6), (6, 6)}

Thus number of favourable outcomes,

$$n(E_3) = 6$$

P(sum of atleast 10),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(iv) Favourable outcomes are {(2, 2), (4, 4), (6, 6)}

Thus number of favourable outcomes,

$$n(E_3) = 3$$

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Favourable outcome are {2, 3, 5, 7, 11, 13, 17}, therefore

$$n(E) = 7$$

Probability of drawing prime number card by the second boy,

$$P(E) = \frac{n(E)}{n(S)} = \frac{7}{17}$$

(iii) If the card drawn is not replaced, then total number of cards remaining are 16.

Total outcomes $n(S) = 16$

Favourable outcome are {6, 9, 12, 15}, therefore

$$n(E) = 4$$

Probability of drawing a number multiple of 3 and greater than 5 by the third boy is,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{16} = \frac{1}{4}$$

(iv) Probability of getting number greater than 17 is zero because there is no card having number 17.

(v) Here we must note that we have 16 card because we have not replaced card in third draw.

Total outcomes $n(S) = 16$

Favourable outcome are {3, 6, 7, 9, 12, 14, 15}, therefore

$$n(E) = 7$$

Probability of drawing a number multiple of 3 or 7 by the fifth boy,

$$P(E) = \frac{n(E)}{n(S)} = \frac{7}{16}$$

****

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